A Practical Guide to the Art of N=4 Amplitudeology

A motive-ational introduction to recent progress in unlocking the structure of scattering amplitudes in maximally supersymmetric Yang-Mills theory.

M. Spradlin, Brown University

This Talk Has 2 Goals

For the in-crowd, a "pat on the back" review and a stirring motivational speech to keep working hard...

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For the in-crowd, a "pat on the back" review and a stirring motivational speech to keep working hard...

For newcomers, an attempt to convey the philosophy of our "cult" and a gentle introduction to its jargon.

The Appeal of Long-Lasting Vitality

The Zooh through and beyond? The From

Impossible → hard → trivial

phase transition often occurred on the timescale of months not years.

The Appeal of Long-Lasting Vitality

From Zoom through Zoll (and beyond?) the phase transition often occurred on the timescale of months not years.

Impossible → Hard → Trivial

Communal pride trumps (occasional) individual shame!

Many have contributed significantly to these developments including

L. Alday
N. Arkani-Hamed
T. Bargheer
N. Beisert
Z. Bern
R. Boels
J. Bourjaily
N.E. Bjerrum-Bohr
A. Brandhuber
M. Bullimore
R. Britto
F. Cachazo
S. Caron-Huot
J.J. Carrasco
V. Del Duca
L. Dixon
L. Dolan
C. Duhr
J. Drummond
B. Eden
H. Elvang
B. Feng
D. Freedman
D. Gaiotto
P. Goddard
A. Goncharov
J. Henn
P. Heslop
A. Hodges
Y. Huang
H. Ita
H. Johansson
V. Khoze
M. Kiermaier
G. Korchemsky
D. Kosower
L. Lipatov
F. Loebbert
D. Maitre
J. Maldacena
L. Mason
J. Plefka
R. Roiban
A. Sabio Vera
A. Sever
D. Skinner
A. Smirnov
E. Sokatchev
W. Spence
M. Staudacher
G. Travaglini
J. Trnka
C. Vergu
P. Vieira
A. Volovich
C. Wen
E. Witten

The goals of this research program are:

- to explore the rich mathematical structure of scattering amplitudes
- to exploit that structure to make previously impossible calculations trivial

Why $N=4$ Super-Yang Mills Theory?

- A nontrivial but apparently solvable four-dimensional QFT.

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- A nontrivial but apparently solvable four-dimensional QFT.
- But not just some random QFT, it is a gauge theory, and hence a cousin of QCD.
- Moreover it = quantum gravity in AdS space
- Most importantly, because it is there and

YES WE CAN!

S-Matrix Theory Reloaded?

The spirit behind much recent progress is similar to that of "S-Matrix Theory."

If we have seen further than the giants of the past, it is because

S-Matrix Theory Reloaded?

The spirit behind much recent progress is similar to that of "S-Matrix Theory."

If we have seen further than the giants of the past, it is because we're focused squarely on the well-manicured lawn beneath our feet ($\mathcal{N}=4$) rather than the dark jungle looming ahead (QCD).

A Completely Different Philosophy

Then, the idea was

(1) enumerate the principles
(locality, analyticity, etc.)

(2) solve!

A Completely Different Philosophy

Then, the idea was

(1) enumerate the principles
(locality, analyticity, etc.)

(2) solve!

Now, our aim is to

(1) write down the solution BY ANY MEANS NECESSARY

(2) deduce the principles from understanding its structure

BUCKLE
YOUR
SAFETY BELTS

Amplitude

I'll focus almost exclusively on planar, color-stripped partial amplitudes in $\mathcal{N}=4$ Yang-Mills theory.

$$\mathcal{A}(1,\ldots,n) = \sum_{\sigma \in S_n} A(\sigma(1),\ldots,\sigma(n))\, \mathrm{Tr}\left[T^{a_{\sigma(1)}} \ldots T^{a_{\sigma(n)}}\right]$$

color-ordered partial amplitude

Amplitude Representation

In massless ϕ^3 theory, two representations of the tree-level four-particle amplitude are

$$A = -\frac{1}{s} - \frac{1}{t} - \frac{1}{u}$$ and

$$A = \frac{10s^2}{tu} + \frac{20s}{t} - \frac{9s}{tu} + \frac{32s}{u} + \frac{10u}{t} - \frac{15}{t} + \frac{34t}{u} - \frac{15}{u} + 42 + \frac{22t}{s} - \frac{5u}{ts} + \frac{10u}{s} + \frac{12t^2}{us} - \frac{5t}{us} - \frac{11}{s}$$

They are equal, if you use

$$s + t + u = 0$$

A linear constraint $\Rightarrow$ switch to

free variables

For particles with spin we need spinor helicity variables.

Solve $k^2 = 0$ via $k^\mu = \sigma^\mu_{a\dot{a}} \underbrace{\lambda^a \bar{\lambda}^{\dot{a}}}$

2 2-component objects

Notation $\langle ij \rangle = \epsilon_{ab} \lambda_i^a \lambda_j^b \quad [ij] = \epsilon_{\dot{a}\dot{b}} \bar{\lambda}_i^{\dot{a}} \bar{\lambda}_j^{\dot{b}}$

$\langle i | j + k | \ell] = \langle ij \rangle [j\ell] + \langle ik \rangle [k\ell]$

Amplitude Representation
Spinor Helicity
MHV etc.

A spin-1 particle has 2 physical states
helicity $+1$ or -1

The n-particle MHV amplitude involves $n-2$ $+$ helicity & 2 $-$ helicity particles

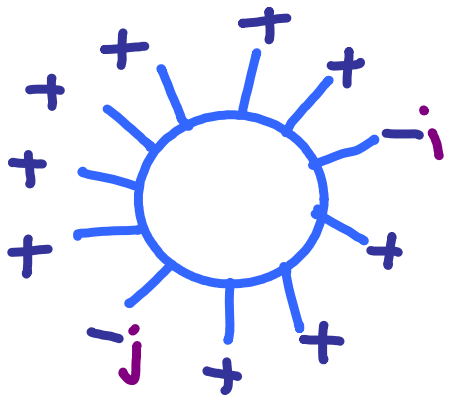
NMHV: $(n-3, 3)$

N^2MHV: $(n-4, 4)$

etc.

Amplitude
Representation
Spinor Helicity
MHV etc.
Parke-Taylor

The tree-level MHV amplitude is



$$= \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n1 \rangle}$$

(Parke Taylor '86, Berends Giele '88)

This is, psychologically, the most important formula in our field.

It's message is ...

The Philosophy of Amplitudeology

1. Simplifications do not happen by accident.

Amplitude Representation
Spinor Helicity
MHV etc.
Parke-Taylor Superamplitude

Working in $\mathcal{N}=4$ superspace we write

$$A_{MHV} = \frac{\delta^8\left(\sum_{i=1}^{n} \lambda_i^a \eta_i^A\right)}{\langle 12\rangle\langle 23\rangle \cdots \langle n1\rangle}$$

supermomentum conservation

$$A = A_{MHV}\left[1 + P_{NMHV} + P_{NNMHV} + \cdots\right]$$

$O(\eta^8)$ $O(\eta^4)$ $O(\eta^8)$ etc.

Amplitude
Representation
Spinor Helicity
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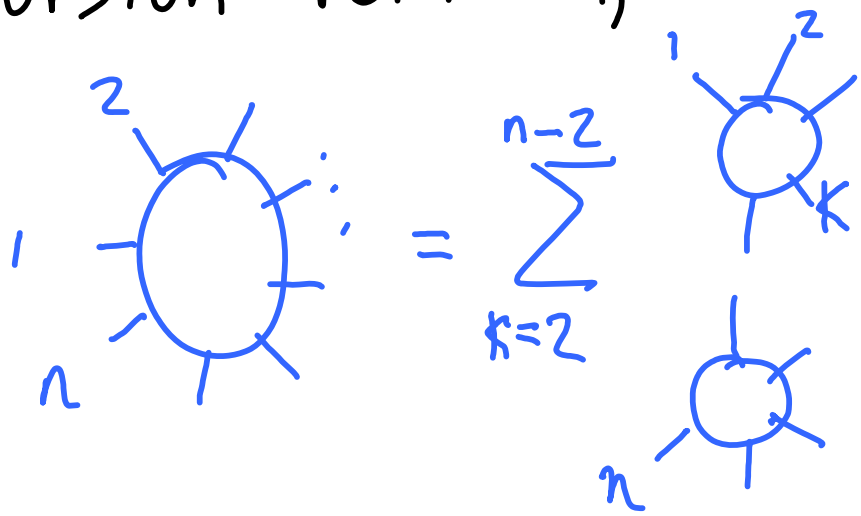
Parke-Taylor
Superamplitude
BCFW

BCFW is a quadratic on-shell recursion relation; schematically



It is important both conceptually and practically — for rapidly generating data.

There is no single "BCFW representation", different discrete choices give apparently different (but secretly =) results.

Example: The amplitude $A(1^-,2^-,3^-,4^+,5^+,6^+)$ may be expressed as either

$$\frac{[4|5+6|1\rangle^3}{[23][34]\langle 56\rangle\langle 61\rangle[2|3+4|5\rangle\, s_{234}} + \frac{[6|1+2|3\rangle^3}{[61][12]\langle 34\rangle\langle 45\rangle[2|3+4|5\rangle\, s_{612}}$$

or

$$\frac{s_{123}^3}{[12][23]\langle 45\rangle\langle 56\rangle[1|2+3|4\rangle[3|4+5|6\rangle} + \frac{\langle 12\rangle^3[45]^3}{[34]\langle 61\rangle[3|4+5|6\rangle[5|6+1|2\rangle\, s_{612}}$$

$$+ \frac{\langle 23\rangle^3[56]^3}{\langle 34\rangle[61][1|2+3|4\rangle[5|6+1|2\rangle\, s_{234}}$$

Equality of these two expressions can (essentially) only be checked by numerical experimentation.

[Richardson's theorem: there is no algorithm for determining if two mathematical expressions are equal.]

A. Hodges followed his nose and found that there is a reason why in this case (2 terms) = (3 terms).

Each expression computes the volume of a polytope in complex projective space, triangulated in different ways!

This is one of many examples showing that...

The Philosophy of Amplitudeology

1. Simplifications do not happen by accident.

2. This is an experimental science.
   (Get the answer first, by any means necessary, then analyze it.)

Amplitude
Representation
Spinor Helicity
MHV etc.
Parke-Taylor
Superamplitude
BCFW
A=Volumes
Twistor Space

"We can go to twistor space by carrying out a ½-fourier transform

$$A(\lambda_a, \mu^{\dot{a}}) = \int d^2\bar{\lambda}_{\dot{a}}\, e^{i\bar{\lambda}_{\dot{a}}\mu^{\dot{a}}}\, A(\lambda_a, \bar{\lambda}_{\dot{a}})$$

"

Since an overall rescaling of the 4 variables (λ, μ) leaves k^μ unchanged, they parametenize $\mathbb{CP}^3$.

points in $\mathbb{CP}^3$ $\longmapsto$ null rays in spacetime

lines in $\mathbb{CP}^3$ $\longmapsto$ points in spacetime

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Twistor String

Noting that momentum conserving delta function

$$A_{MHV} = \overbrace{\int d^4x \; \exp\left(i x_{a\dot{a}} \sum_{i=1}^{n} \lambda_i^a \tilde{\lambda}_i^{\dot{a}}\right)}^{} \frac{1}{\langle 12\rangle \cdots \langle n1\rangle}$$

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measure on the moduli space of lines in $\mathbb{P}^3$

Witten invented a topological string theory on $\mathbb{P}^3$ which computes the tree-level N^kMHV amplitude as an integral over curves of degree $k+1$!

MOVING ON TO LOOP LEVEL

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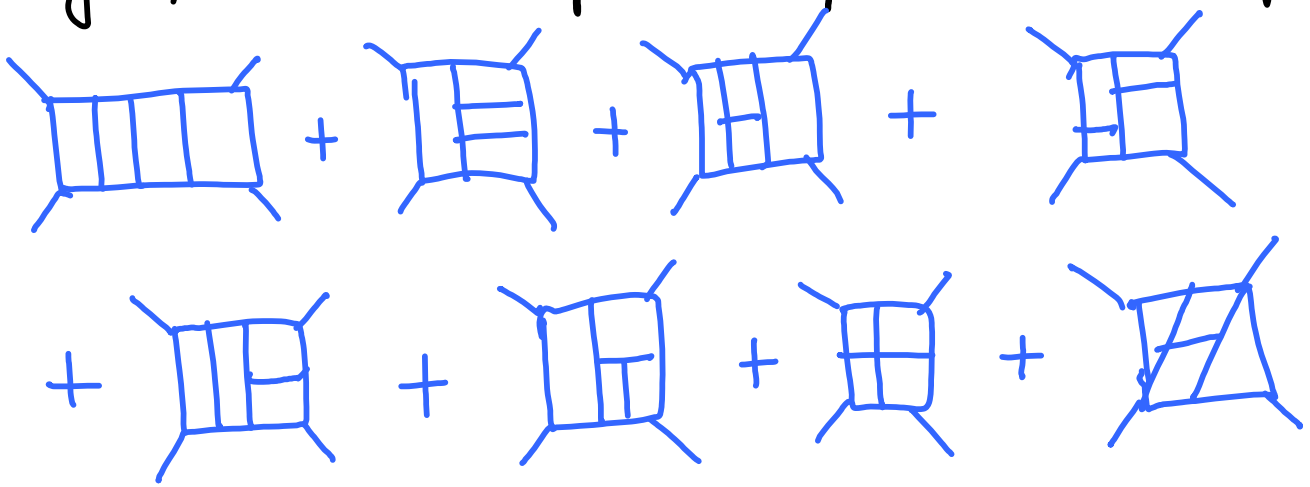
Gen. Unitarity

Union of the unitarity-based method (Bern, Dixon, Dunbar, Kosower) with more recent advances (Britto, Cachazo, Feng) allowed much more data to be collected

eg the four-loop four-particle amplitude



(Bern, Czakon, Dixon, Kosower, Smirnov)

A pattern in this data reveals a surprise...

What is Dual Conformal Symmetry?

A strange non-local symmetry under which scattering amplitudes are co-variant

- tree level gluon amplitudes in QCD
- all orders in SYM

} $N_{color} = \infty$

No understanding at the level of the Lagrangian.

What is Dual Conformal Symmetry?

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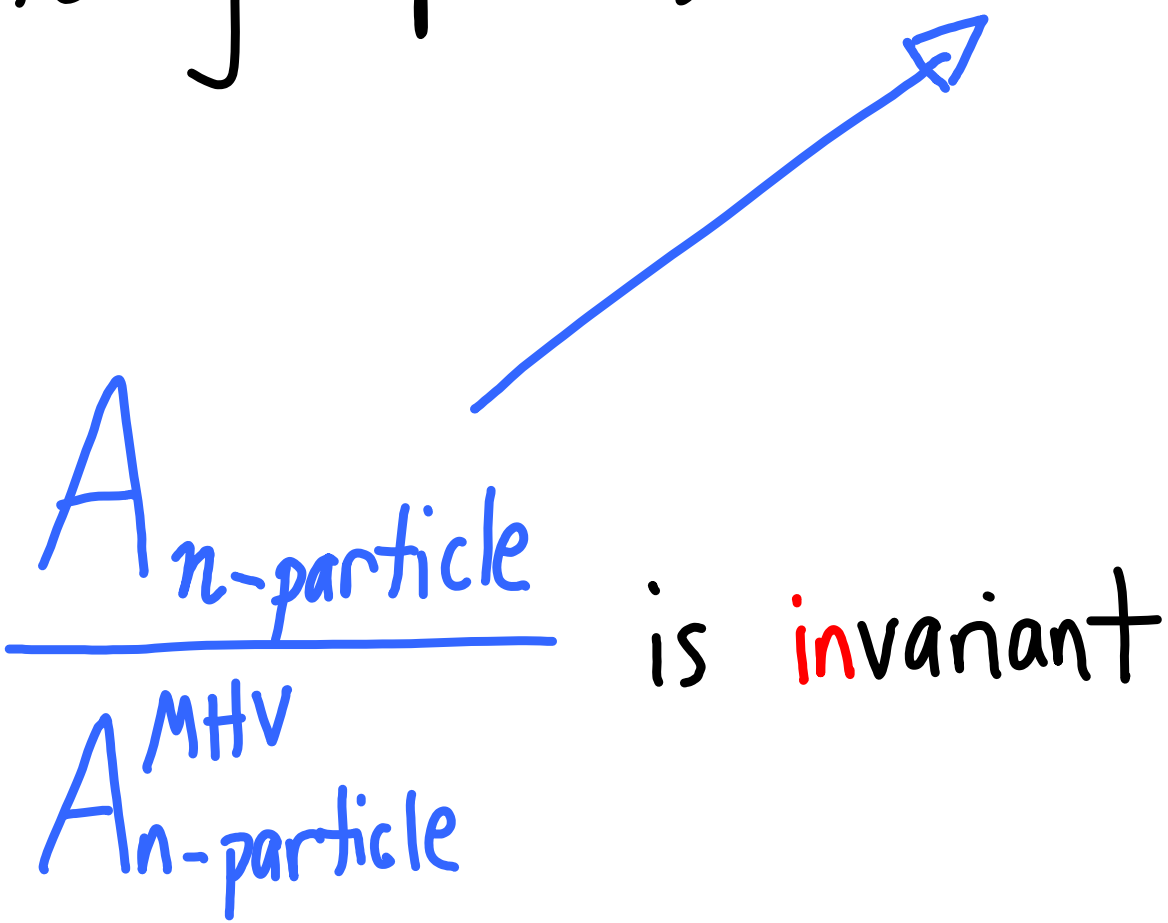


Dual conformal symmetry

(Drummond, Henn, Korchemsky, Sokatchev)

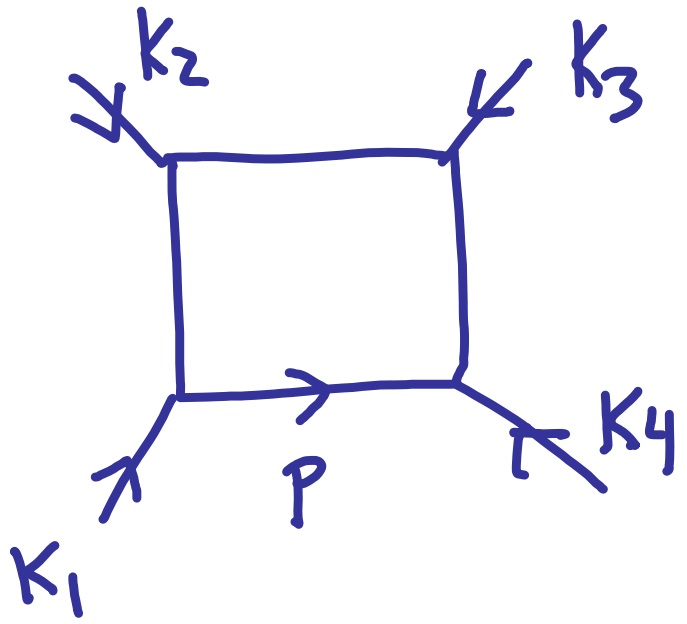
Parameterize

$$\left.\begin{aligned} K_1 &= X_1 - X_2 \\ K_2 &= X_2 - X_3 \\ &\vdots \\ K_n &= X_n - X_1 \end{aligned}\right\} \sum K_i = 0$$

$\Rightarrow$ Then amplitudes are invariant under conformal transformations on the X_i.

Example: 1-loop supersymmetric YM

$$\frac{A_4^{\text{1-loop}}}{A_4^{\text{tree}}} = -\frac{1}{2} st \int d^4P$$



[Green, Schwarz, Brink (1982)]

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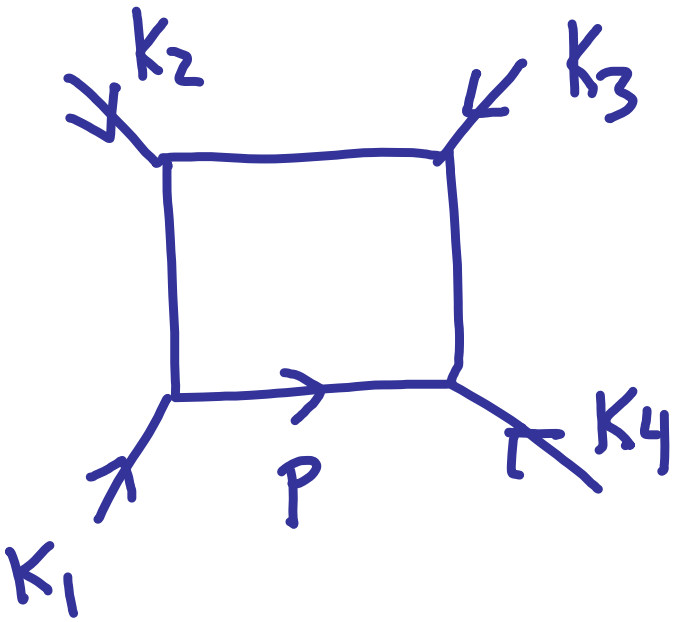
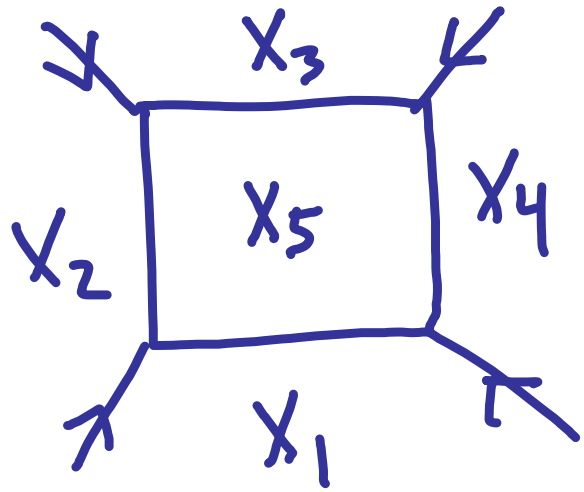


$$= -\frac{1}{2} st \int \frac{d^4P}{P^2(P+K_1)^2(P+K_1+K_2)^2(P-K_4)^2}$$

Now change to the dual "x" variables

Example: 1-loop supersymmetric YM

$$\frac{A_4^{\text{1-loop}}}{A_4^{\text{tree}}} = -\frac{1}{2} x_{13}^2 x_{24}^2 \int d^4 x_5$$



$$= -\frac{1}{2} \int d^4 x_5 \; \frac{x_{13}^2 \, x_{24}^2}{x_{15}^2 \, x_{25}^2 \, x_{35}^2 \, x_{45}^2}$$

where $\quad x_{ij}^2 = (x_i - x_j)^2$

Example: 1-loop supersymmetric YM

$$\frac{A_4^{\text{1-loop}}}{A_4^{\text{tree}}} = -\frac{1}{2}\int d^4x_5 \; \frac{x_{13}^2\, x_{24}^2}{x_{15}^2\, x_{25}^2\, x_{35}^2\, x_{45}^2}$$

which has manifest conformal symmetry, in particular under inversion $x_i^\mu \to x_i^\mu / x_i^2$.

$$x_{ij}^2 \to \frac{x_{ij}^2}{x_i^2\, x_j^2} \qquad d^4x_5 \to \frac{d^4x_5}{(x_5^2)^4}$$

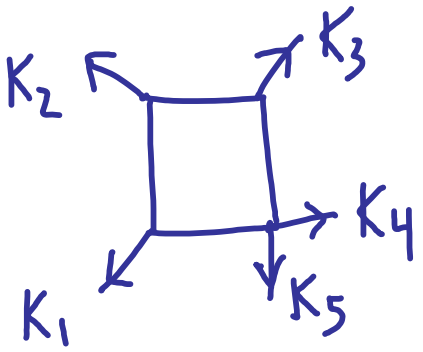
Important Note

Specifying all x's uniquely fixes all k's, but the converse is not quite true due to overall translation invariance.

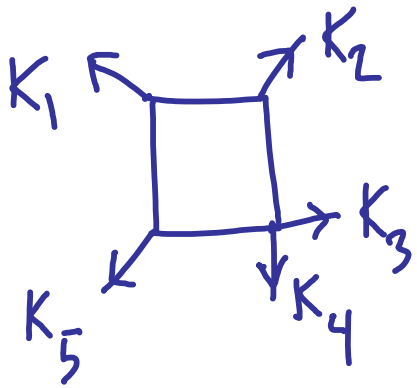
$$\left.\begin{aligned} k_1 &= x_1 - x_2 \\ k_2 &= x_2 - x_3 \\ &\vdots \\ k_n &= x_n - x_1 \end{aligned}\right\} \quad \sum k_i = 0$$

This has important consequences at loop level...

Normally the addition of distinct feynman diagrams underneath the $\int$ sign is ambiguous due to the lack of a natural origin in p-space



$$= \int d^D p \; F_1(k_1, k_2, k_3, k_4, p)$$

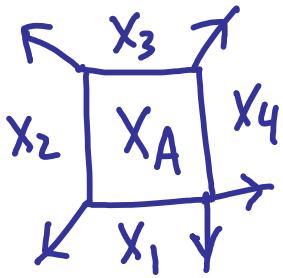


$$= \int d^D p \; F_2(k_1, k_2, k_3, k_4, p)$$

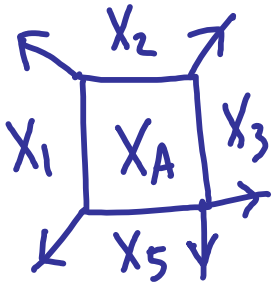
Their sum would be $\int d^D p \left[F_1 + F_2 \right]$

but you can make independent choices about which propagator to call "p."

This ambiguity does not exist when you use dual variables.



$$= \int d^D X_A \, \frac{1}{X_{A1}^2 \, X_{A2}^2 \, X_{A3}^2 \, X_{A4}^2}$$



$$= \int d^D X_A \, \frac{1}{X_{A1}^2 \, X_{A2}^2 \, X_{A3}^2 \, X_{A5}^2}$$

Their sum is unambiguous:

$$\int d^D X_A \, \frac{1}{X_{A1}^2 \, X_{A2}^2 \, X_{A3}^2} \left[\frac{1}{X_{A4}^2} + \frac{1}{X_{A5}^2} \right]$$

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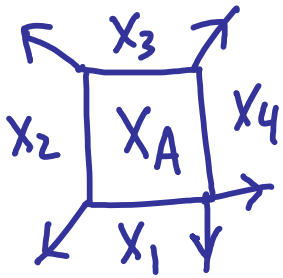

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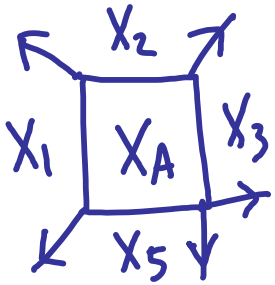

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Often there is enormous cancellation at the level of the integrand.

Amplitude
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Spinor Helicity
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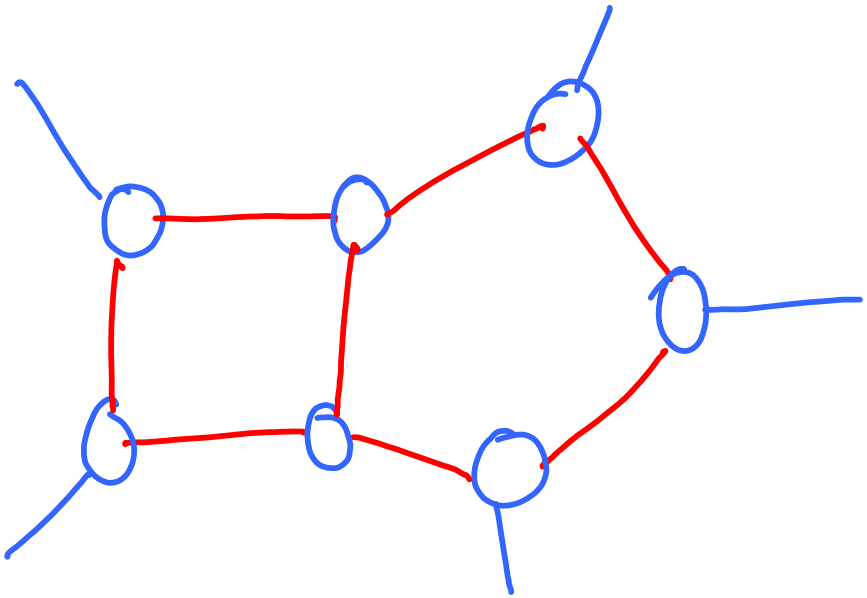
BCFW
A=Volumes
Twistor Space
Twistor String
Dual Conformal Inv.
The Integrand
Leading Sings.

The L-loop integrand is a rational function with (by definition) only simple poles in $\mathbb{C}^{4L}$ (the loop momentum variables).

A leading singularity is a residue of the integrand @ one of these poles.



has a pole when all 8 propagators go on-shell

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Representation
Spinor Helicity
MHV etc.
Parke-Taylor
Superamplitude
BCFW
A=Volumes
Twistor Space
Twistor String
Dual Conformal Inv.
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Leading Sings.
Yangian Symmetry

Yangian Symmetry

= the closure under commutation of
{ordinary conformal transformations,
dual conformal transformations}

= diffeomorphism invariance in
twistor space

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Representation
Spinor Helicity
MHV etc.
Parke-Taylor
Superamplitude
BCFW
A=Volumes
Twistor Space
Twistor String
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The Integrand
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Yangian Symmetry
R-Invariants

R-invariants = a natural collection of Yangian invariant objects which constitute the building blocks for e.g. tree-level BCFW representations & leading singularities of loop integrands

(= δ-functions constraining five points in $\mathbb{CP}^{3|4}$ to lie inside a common $\mathbb{CP}^3$ subspace.)

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Representation
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BCFW
A=Volumes
Twistor Space
Twistor String
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Yangian Symmetry
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Grassmannian Int.

Grassmannian Integral = A contour integral over the space of $k \times n$ matrices

$$\int [dC]_{k\times n} \frac{\delta^{4|4}(C_{ai} Z_i)}{(1\cdots k)(2\cdots k+1)\cdots(n+k-1)} \qquad Z_i = (\lambda_i, \mu_i)$$

minors of the matrix C

(Arkani-Hamed, Cachazo, Cheung, Kaplan)

which is a generating function for all n-particle N^kMHV Yangian invariants.

(It's residues are R-invariants.)

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Representation
Spinor Helicity
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BCFW
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Strong Coupling

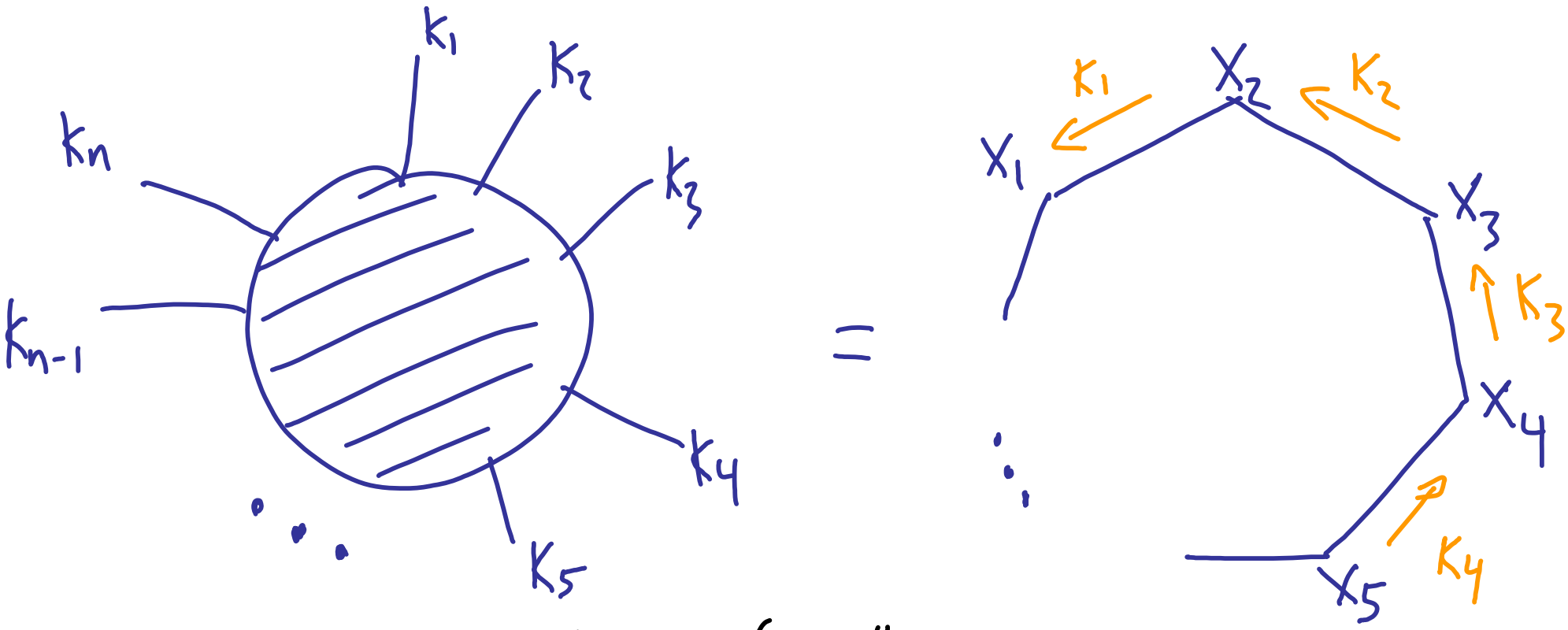
Using AdS/CFT, Alday & Maldacena showed that at strong coupling, amplitudes are computed as certain Wilson loops = minimal area surfaces in AdS_5.

Hence integrability methods can be brought to bear on the problem.

(Alday, Gaiotto, Maldacena)

Amplitudes and Wilson Loops

Another remarkable feature of $\mathcal{N}=4$ supersymmetric Yang-Mills is



The MHV scattering amplitude, for all n and to all orders in perturbation theory, = the expectation value of a Wilson loop on an n-sided polygon with lightlike edges.

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Strong Coupling
A = Wilson Loop

This was observed to be true in several cases

Alday & Maldacena
Drummond, Henn, Korchemsky, Sokatchev
Brandhuber, Heslop, Travaglini

and then proven in general @ level of the integrand

Caron-Huot
Mason & Skinner

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Yangian Symmetry
R-Invariants
Grassmannian Int.
Strong Coupling
A = Wilson Loop
A = Correlation Fn.

The integrand may in fact be computed yet another way, by correlation functions of the form

$$\langle \underbrace{\mathcal{O}(x_1)\cdots\mathcal{O}(x_n)}_{\text{lightlike separated}} \underbrace{\mathcal{L}(y_1)\cdots\mathcal{L}(y_L)}_{\text{Lagrangian insertions}} \rangle$$

½ BPS operators

Eden, Heslop, Korchemsky, Sokatchev

Adamo, Bullimore, Mason, Skinner

All such "proofs @ the level of the integrand" are made possible by...

Completely Free Variables For n-particle Kinematics

It is impossible to overstate the importance of good variables

Spinor helicity $\lambda_i^a, \tilde{\lambda}_i^{\dot{a}}$ makes the on-shell condition automatic

$$K_i^2 = 0$$

dual variables x_i^m make momentum conservation automatic

$$K_1 + K_2 + \cdots + K_n = 0$$

It is possible to explicitly solve both conditions simultaneously in terms of momentum twistor variables. (A. Hodges)

$$Z_i = (\lambda_i^1, \lambda_i^2, \mu_i^1, \mu_i^2) \Rightarrow x_i^{a\dot{a}} = \frac{\lambda_i^a \mu_{i+1}^{\dot{a}} - \lambda_{i+1}^a \mu_i^{\dot{a}}}{\langle i, i+1 \rangle}$$

You Say You Want a Revolution

One revolution has come to fruition

$$A = \int d^4p_1 \cdots d^4p_L \underbrace{\sum \text{feynman diagrams}}$$

$\Downarrow$ exciting new technology

$$= \int d^4p_1 \cdots d^4p_L \text{ (relatively simple integrand)}$$

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still very hard to evaluate!

But another is desperately needed,

Amplitude
Representation
Spinor Helicity
MHV etc.
Parke-Taylor
Superamplitude
BCFW
A=Volumes
Twistor Space
Twistor String
Dual Conformal Inv.
The Integrand
Leading Sings.
Yangian Symmetry
R-Invariants
Grassmannian Int.
Strong Coupling
A = Wilson Loop
A = Correlation Fn.
BDS Remainder

The natural "observable" for loop-level MHV amplitudes is the BDS remainder function which is what is left after appropriate subtraction of IR divergences, leaving a finite & dual conformal invariant quantity.

The first nontrivial loop amplitude in $\mathcal{N}=4$ SYM is the

2-loop (1-loop is absorbed into BDS subtraction)

6-particle ($\nexists$ any cross-ratios for $n<6$)

MHV remainder function.

Computed analytically in late 2009 by Del Duca, Duhr & Smirnov

$$R^{(2)}_{6,WL}(u_1,u_2,u_3) = \tag{H.1}$$
$$\frac{1}{24}\pi^2 G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1};1\right)+\frac{1}{24}\pi^2 G\left(\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)+\frac{1}{24}\pi^2 G\left(\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)+$$
$$\frac{1}{24}\pi^2 G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1};1\right)+\frac{1}{24}\pi^2 G\left(\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)+\frac{1}{24}\pi^2 G\left(\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)+$$
$$\frac{1}{24}\pi^2 G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1};1\right)+\frac{1}{24}\pi^2 G\left(\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)+\frac{1}{24}\pi^2 G\left(\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)+$$
$$\frac{3}{2}G\left(0,0,\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)+\frac{3}{2}G\left(0,0,\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)+\frac{3}{2}G\left(0,0,\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)+$$
$$\frac{3}{2}G\left(0,0,\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)+\frac{3}{2}G\left(0,0,\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)+\frac{3}{2}G\left(0,0,\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)-$$
$$\frac{1}{2}G\left(0,\frac{1}{u_1},0,\frac{1}{u_2};1\right)+G\left(0,\frac{1}{u_1},0,\frac{1}{u_1+u_2};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_1},0,\frac{1}{u_3};1\right)+$$
$$G\left(0,\frac{1}{u_1},0,\frac{1}{u_1+u_3};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_1},\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_1},\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)-$$
$$\frac{1}{2}G\left(0,\frac{1}{u_1},\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_1},\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_2},0,\frac{1}{u_1};1\right)+$$
$$G\left(0,\frac{1}{u_2},0,\frac{1}{u_1+u_2};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_2},0,\frac{1}{u_3};1\right)+G\left(0,\frac{1}{u_2},0,\frac{1}{u_2+u_3};1\right)-$$
$$\frac{1}{2}G\left(0,\frac{1}{u_2},\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_2},\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_2},\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)-$$
$$\frac{1}{2}G\left(0,\frac{1}{u_2},\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)+\frac{1}{4}G\left(0,\frac{u_2-1}{u_1+u_2-1},0,\frac{1}{1-u_1};1\right)+$$
$$\frac{1}{4}G\left(0,\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1},0;1\right)-\frac{1}{4}G\left(0,\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1},1;1\right)+$$
$$\frac{1}{4}G\left(0,\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)-\frac{1}{4}G\left(0,\frac{u_2-1}{u_1+u_2-1},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)-$$
$$\frac{1}{2}G\left(0,\frac{1}{u_3},0,\frac{1}{u_1};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_3},0,\frac{1}{u_2};1\right)+G\left(0,\frac{1}{u_3},0,\frac{1}{u_1+u_3};1\right)+$$
$$G\left(0,\frac{1}{u_3},0,\frac{1}{u_2+u_3};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_3},\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_3},\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)-$$
$$\frac{1}{2}G\left(0,\frac{1}{u_3},\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_3},\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)+$$
$$\frac{1}{4}G\left(0,\frac{u_1-1}{u_1+u_3-1},0,\frac{1}{1-u_3};1\right)+\frac{1}{4}G\left(0,\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3},0;1\right)-$$
$$\frac{1}{4}G\left(0,\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3},1;1\right)+\frac{1}{4}G\left(0,\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)-$$
$$\frac{1}{4}G\left(0,\frac{u_1-1}{u_1+u_3-1},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)+\frac{1}{4}G\left(0,\frac{u_3-1}{u_2+u_3-1},0,\frac{1}{1-u_2};1\right)+$$
$$\frac{1}{4}G\left(0,\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2},0;1\right)-\frac{1}{4}G\left(0,\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2},1;1\right)+$$
$$\frac{1}{4}G\left(0,\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)-\frac{1}{4}G\left(0,\frac{u_3-1}{u_2+u_3-1},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_1},1,\frac{1}{u_3},0;1\right)+\frac{1}{2}G\left(\frac{1}{1-u_1},\frac{1}{1-u_1},1,\frac{1}{1-u_1};1\right)+$$

$$\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},0,1;1\right)-\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},0,\frac{1}{1-u_1};1\right)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},1,0;1\right)-\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1},0;1\right)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1},1;1\right)-\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{u_2-1}{u_1+u_2-1},1;1\right)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)-G\left(\frac{1}{u_1},0,0,\frac{1}{u_2};1\right)+$$
$$\frac{1}{2}G\left(\frac{1}{u_1},0,0,\frac{1}{u_1+u_2};1\right)-G\left(\frac{1}{u_1},0,0,\frac{1}{u_3};1\right)+\frac{1}{2}G\left(\frac{1}{u_1},0,0,\frac{1}{u_1+u_3};1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)-\frac{1}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)-\frac{1}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)-\frac{1}{4}G\left(\frac{1}{1-u_2},1,\frac{1}{u_1},0;1\right)+$$
$$\frac{1}{2}G\left(\frac{1}{1-u_2},\frac{1}{1-u_2},1,\frac{1}{1-u_2};1\right)+\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},0,1;1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},0,\frac{1}{1-u_2};1\right)+\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},1,0;1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2},0;1\right)+\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2},1;1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{u_3-1}{u_2+u_3-1},1;1\right)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)-G\left(\frac{1}{u_2},0,0,\frac{1}{u_1};1\right)+$$
$$\frac{1}{2}G\left(\frac{1}{u_2},0,0,\frac{1}{u_1+u_2};1\right)-G\left(\frac{1}{u_2},0,0,\frac{1}{u_3};1\right)+\frac{1}{2}G\left(\frac{1}{u_2},0,0,\frac{1}{u_2+u_3};1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)-\frac{1}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)-\frac{1}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)-\frac{1}{4}G\left(\frac{1}{1-u_3},1,\frac{1}{u_2},0;1\right)+$$
$$\frac{1}{2}G\left(\frac{1}{1-u_3},\frac{1}{1-u_3},1,\frac{1}{1-u_3};1\right)+\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},0,1;1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},0,\frac{1}{1-u_3};1\right)+\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},1,0;1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3},0;1\right)+\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3},1;1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},\frac{u_1-1}{u_1+u_3-1},1;1\right)-\frac{79\pi^4}{360}+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)-G\left(\frac{1}{u_3},0,0,\frac{1}{u_1};1\right)-$$

$$G\left(\frac{1}{u_3},0,0,\frac{1}{u_2};1\right)+\frac{1}{2}G\left(\frac{1}{u_3},0,0,\frac{1}{u_1+u_3};1\right)+\frac{1}{2}G\left(\frac{1}{u_3},0,0,\frac{1}{u_2+u_3};1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)-\frac{1}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)-\frac{1}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)-\frac{1}{24}\pi^2\mathcal{G}\left(\frac{1}{1-u_1},u_{123};1\right)+\frac{1}{8}\pi^2\mathcal{G}\left(\frac{1}{1-u_1},v_{123};1\right)+$$
$$\frac{1}{8}\pi^2\mathcal{G}\left(\frac{1}{1-u_1},v_{132};1\right)-\frac{1}{24}\pi^2\mathcal{G}\left(\frac{1}{1-u_2},u_{231};1\right)+\frac{1}{8}\pi^2\mathcal{G}\left(\frac{1}{1-u_2},v_{213};1\right)+$$
$$\frac{1}{8}\pi^2\mathcal{G}\left(\frac{1}{1-u_2},v_{231};1\right)-\frac{1}{24}\pi^2\mathcal{G}\left(\frac{1}{1-u_3},u_{312};1\right)+\frac{1}{8}\pi^2\mathcal{G}\left(\frac{1}{1-u_3},v_{312};1\right)+$$
$$\frac{1}{8}\pi^2\mathcal{G}\left(\frac{1}{1-u_3},v_{321};1\right)-\frac{1}{4}\mathcal{G}\left(0,0,\frac{1}{1-u_1},v_{123};1\right)-\frac{1}{4}\mathcal{G}\left(0,0,\frac{1}{1-u_1},v_{132};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,0,\frac{1}{1-u_2},v_{213};1\right)-\frac{1}{4}\mathcal{G}\left(0,0,\frac{1}{1-u_2},v_{231};1\right)-\frac{1}{4}\mathcal{G}\left(0,0,\frac{1}{1-u_3},v_{312};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,0,\frac{1}{1-u_3},v_{321};1\right)-\frac{1}{4}\mathcal{G}\left(0,0,v_{123},\frac{1}{1-u_1};1\right)+\mathcal{G}\left(0,0,v_{132},0;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,0,v_{132},\frac{1}{1-u_1};1\right)+\mathcal{G}\left(0,0,v_{213},0;1\right)-\frac{1}{4}\mathcal{G}\left(0,0,v_{213},\frac{1}{1-u_2};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,0,v_{231},\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(0,0,v_{312},\frac{1}{1-u_3};1\right)+\mathcal{G}\left(0,0,v_{321},0;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,0,v_{321},\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},0,v_{123};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},0,v_{132};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(0,\frac{1}{1-u_1},\frac{1}{1-u_1},v_{123};1\right)-\frac{1}{2}\mathcal{G}\left(0,\frac{1}{1-u_1},\frac{1}{1-u_1},v_{132};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{123},1;1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{132},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},0,v_{213};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},0,v_{231};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(0,\frac{1}{1-u_2},\frac{1}{1-u_2},v_{213};1\right)-\frac{1}{2}\mathcal{G}\left(0,\frac{1}{1-u_2},\frac{1}{1-u_2},v_{231};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{213},1;1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{231},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},0,v_{312};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},0,v_{321};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(0,\frac{1}{1-u_3},\frac{1}{1-u_3},v_{312};1\right)-\frac{1}{2}\mathcal{G}\left(0,\frac{1}{1-u_3},\frac{1}{1-u_3},v_{321};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{312},1;1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{321},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{123},0,\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{1}{1-u_1},0;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{1}{1-u_1},1;1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{u_2-1}{u_1+u_2-1},1;1\right)+\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{1}{u_3},0;1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{231},0,\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{1}{u_1},0;1\right)-$$

$$\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{1}{1-u_2},0;1\right)+\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{1}{1-u_2},1;1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{u_3-1}{u_2+u_3-1},1;1\right)+\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{312},0,\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{1}{u_2},0;1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{1}{1-u_3},0;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{1}{1-u_3},1;1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{u_1-1}{u_1+u_3-1},1;1\right)+\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,v_{123},0,\frac{1}{1-u_1};1\right)-\frac{1}{2}\mathcal{G}\left(0,v_{123},1,\frac{1}{1-u_1};1\right)+\frac{1}{4}\mathcal{G}\left(0,v_{123},\frac{1}{1-u_1},0;1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(0,v_{123},\frac{1}{1-u_1},1;1\right)+\frac{1}{4}\mathcal{G}\left(0,v_{123},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(0,v_{132},0,\frac{1}{1-u_1};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,v_{132},\frac{1}{1-u_1},0;1\right)-\frac{1}{4}\mathcal{G}\left(0,v_{132},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(0,v_{213},0,\frac{1}{1-u_2};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,v_{213},\frac{1}{1-u_2},0;1\right)-\frac{1}{4}\mathcal{G}\left(0,v_{213},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)+\frac{1}{4}\mathcal{G}\left(0,v_{231},0,\frac{1}{1-u_2};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(0,v_{231},1,\frac{1}{1-u_2};1\right)+\frac{1}{4}\mathcal{G}\left(0,v_{231},\frac{1}{1-u_2},0;1\right)-\frac{1}{2}\mathcal{G}\left(0,v_{231},\frac{1}{1-u_2},1;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,v_{231},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)+\frac{1}{4}\mathcal{G}\left(0,v_{312},0,\frac{1}{1-u_3};1\right)-\frac{1}{2}\mathcal{G}\left(0,v_{312},1,\frac{1}{1-u_3};1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,v_{312},\frac{1}{1-u_3},0;1\right)-\frac{1}{2}\mathcal{G}\left(0,v_{312},\frac{1}{1-u_3},1;1\right)+\frac{1}{4}\mathcal{G}\left(0,v_{312},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,v_{321},0,\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(0,v_{321},\frac{1}{1-u_3},0;1\right)-\frac{1}{4}\mathcal{G}\left(0,v_{321},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,0,v_{123};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,0,v_{132};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},0,\frac{1}{1-u_1},v_{123};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},0,\frac{1}{1-u_1},v_{132};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{123},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{123},\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{132},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{132},\frac{1}{1-u_1};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},0,v_{123};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},0,v_{132};1\right)-\frac{3}{4}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},\frac{1}{1-u_1},v_{123};1\right)-$$
$$\frac{3}{4}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},\frac{1}{1-u_1},v_{132};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{123},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{132},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},0,1;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},0,\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},1,0;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{1}{1-u_1},0;1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{1}{1-u_1},1;1\right)+$$

$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{u_2-1}{u_1+u_2-1},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{1}{u_3},0;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},0,0;1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},0,1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},0,\frac{1}{1-u_1};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},1,0;1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},1,\frac{1}{1-u_1};1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1},0;1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1},1;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},0,0;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},0,1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},0,\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},1,0;1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},1,\frac{1}{1-u_1};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1},0;1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1},1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,0,v_{213};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,0,v_{231};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},0,\frac{1}{1-u_2},v_{213};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},0,\frac{1}{1-u_2},v_{231};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{213},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{213},\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{231},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{231},\frac{1}{1-u_2};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},0,v_{213};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},0,v_{231};1\right)-\frac{3}{4}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},\frac{1}{1-u_2},v_{213};1\right)-$$
$$\frac{3}{4}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},\frac{1}{1-u_2},v_{231};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{213},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{231},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},0,1;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},0,\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},1,0;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{1}{u_1},0;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{1}{1-u_2},0;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{1}{1-u_2},1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{u_3-1}{u_2+u_3-1},1;1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},0,0;1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},0,1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},0,\frac{1}{1-u_2};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},1,0;1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},1,\frac{1}{1-u_2};1\right)+$$

$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2},0;1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2},1;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},0,0;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},0,1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},0,\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},1,0;1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},1,\frac{1}{1-u_2};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2},0;1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2},1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,0,v_{312};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,0,v_{321};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},0,\frac{1}{1-u_3},v_{312};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},0,\frac{1}{1-u_3},v_{321};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{312},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{312},\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{321},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{321},\frac{1}{1-u_3};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},0,v_{312};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},0,v_{321};1\right)-\frac{3}{4}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},\frac{1}{1-u_3},v_{312};1\right)-$$
$$\frac{3}{4}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},\frac{1}{1-u_3},v_{321};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{312},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{321},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},0,1;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},0,\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},1,0;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{1}{u_2},0;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{1}{1-u_3},0;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{1}{1-u_3},1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{u_1-1}{u_1+u_3-1},1;1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},0,0;1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},0,1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},0,\frac{1}{1-u_3};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},1,0;1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},1,\frac{1}{1-u_3};1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3},0;1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3},1;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},0,0;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},0,1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},0,\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},1,0;1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},1,\frac{1}{1-u_3};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3},0;1\right)-$$

$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{123},0,1,\frac{1}{1-u_1};1\right)+\frac{1}{2}\mathcal{G}\left(v_{123},0,\frac{1}{1-u_1},1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{123},1,0,\frac{1}{1-u_1};1\right)-$$
$$\frac{5}{4}\mathcal{G}\left(v_{123},1,1,\frac{1}{1-u_1};1\right)+\frac{1}{2}\mathcal{G}\left(v_{123},1,\frac{1}{1-u_1},0;1\right)-\frac{5}{4}\mathcal{G}\left(v_{123},1,\frac{1}{1-u_1},1;1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{123},1,\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)+\frac{1}{2}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},0,1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},1,0;1\right)-$$
$$\frac{5}{4}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},1,1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},1,\frac{1}{1-u_1};1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},\frac{1}{1-u_1},1;1\right)-\frac{1}{4}\mathcal{G}\left(v_{132},1,1,\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(v_{132},1,\frac{1}{1-u_1},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(v_{132},\frac{1}{1-u_1},1,1;1\right)-\frac{1}{4}\mathcal{G}\left(v_{213},1,1,\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(v_{213},1,\frac{1}{1-u_2},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(v_{213},\frac{1}{1-u_2},1,1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{231},0,1,\frac{1}{1-u_2};1\right)+\frac{1}{2}\mathcal{G}\left(v_{231},0,\frac{1}{1-u_2},1;1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{231},1,0,\frac{1}{1-u_2};1\right)-\frac{5}{4}\mathcal{G}\left(v_{231},1,1,\frac{1}{1-u_2};1\right)+\frac{1}{2}\mathcal{G}\left(v_{231},1,\frac{1}{1-u_2},0;1\right)-$$
$$\frac{5}{4}\mathcal{G}\left(v_{231},1,\frac{1}{1-u_2},1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{231},1,\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)+\frac{1}{2}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},0,1;1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},1,0;1\right)-\frac{5}{4}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},1,1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},1,\frac{1}{1-u_2};1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},\frac{1}{1-u_2},1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{312},0,1,\frac{1}{1-u_3};1\right)+\frac{1}{2}\mathcal{G}\left(v_{312},0,\frac{1}{1-u_3},1;1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{312},1,0,\frac{1}{1-u_3};1\right)-\frac{5}{4}\mathcal{G}\left(v_{312},1,1,\frac{1}{1-u_3};1\right)+\frac{1}{2}\mathcal{G}\left(v_{312},1,\frac{1}{1-u_3},0;1\right)-$$
$$\frac{5}{4}\mathcal{G}\left(v_{312},1,\frac{1}{1-u_3},1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{312},1,\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)+\frac{1}{2}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},0,1;1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},1,0;1\right)-\frac{5}{4}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},1,1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},1,\frac{1}{1-u_3};1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},\frac{1}{1-u_3},1;1\right)-\frac{1}{4}\mathcal{G}\left(v_{321},1,1,\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(v_{321},1,\frac{1}{1-u_3},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(v_{321},\frac{1}{1-u_3},1,1;1\right)-\frac{3}{4}G\left(0,\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)-$$
$$\frac{3}{4}G\left(0,\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)-\frac{1}{4}G\left(0,\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}G\left(0,\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)-\frac{1}{4}G\left(0,\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}G\left(0,\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)H\left(0;u_1\right)-\frac{3}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)-$$
$$\frac{3}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)+\frac{1}{2}G\left(\frac{1}{u_1},\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{2}G\left(\frac{1}{u_1},\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)+\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)-\frac{1}{4}G\left(\frac{1}{1-u_2},1,\frac{1}{u_1};1\right)H\left(0;u_1\right)+$$

$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},1;1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)H\left(0;u_1\right)+\frac{1}{2}G\left(\frac{1}{u_2},0,\frac{1}{u_1};1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)+\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},0;1\right)H\left(0;u_1\right)+\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},\frac{u_1-1}{u_1+u_3-1};1\right)H\left(0;u_1\right)+\frac{1}{2}G\left(\frac{1}{u_3},0,\frac{1}{u_1};1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)+\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{123};1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{132};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{213};1\right)H\left(0;u_1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{231};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{312};1\right)H\left(0;u_1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{321};1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{1}{u_1};1\right)H\left(0;u_1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{1}{1-u_2};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{1}{1-u_3};1\right)H\left(0;u_1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{u_1-1}{u_1+u_3-1};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,v_{123},\frac{1}{1-u_1};1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(0,v_{132},\frac{1}{1-u_1};1\right)H\left(0;u_1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(0,v_{231},\frac{1}{1-u_2};1\right)H\left(0;u_1\right)+\frac{1}{2}\mathcal{G}\left(0,v_{312},\frac{1}{1-u_3};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{123};1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{132};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{123};1\right)H\left(0;u_1\right)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{132};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},1;1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},1;1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{213};1\right)H\left(0;u_1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{231};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{213};1\right)H\left(0;u_1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{231};1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},1;1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{1}{u_1};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{1}{1-u_2};1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},0;1\right)H\left(0;u_1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2};1\right)H\left(0;u_1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},0;1\right)H\left(0;u_1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2};1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{312};1\right)H\left(0;u_1\right)-$$

$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{321};1\right)H\left(0;u_1\right)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{312};1\right)H\left(0;u_1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{321};1\right)H\left(0;u_1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},0;1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{1}{1-u_3};1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{u_1-1}{u_1+u_3-1};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},0;1\right)H\left(0;u_1\right)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3};1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},0;1\right)H\left(0;u_1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{123},1,\frac{1}{1-u_1};1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},1;1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{132},1,\frac{1}{1-u_1};1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(v_{132},\frac{1}{1-u_1},1;1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{213},1,\frac{1}{1-u_2};1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(v_{213},\frac{1}{1-u_2},1;1\right)H\left(0;u_1\right)-$$
$$\frac{3}{4}\mathcal{G}\left(v_{231},1,\frac{1}{1-u_2};1\right)H\left(0;u_1\right)-\frac{3}{4}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},1;1\right)H\left(0;u_1\right)+$$
$$\frac{3}{4}\mathcal{G}\left(v_{312},1,\frac{1}{1-u_3};1\right)H\left(0;u_1\right)+\frac{3}{4}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},1;1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(v_{321},1,\frac{1}{1-u_3};1\right)H\left(0;u_1\right)-\frac{1}{4}\mathcal{G}\left(v_{321},\frac{1}{1-u_3},1;1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}G\left(0,\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H\left(0;u_2\right)-\frac{3}{4}G\left(0,\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H\left(0;u_2\right)-$$
$$\frac{3}{4}G\left(0,\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H\left(0;u_2\right)-\frac{1}{4}G\left(0,\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}G\left(0,\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H\left(0;u_2\right)+\frac{1}{4}G\left(0,\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},0;1\right)H\left(0;u_2\right)+\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{u_2-1}{u_1+u_2-1};1\right)H\left(0;u_2\right)+\frac{1}{2}G\left(\frac{1}{u_1},0,\frac{1}{u_2};1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_1+u_2};1\right)H\left(0;u_2\right)+\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H\left(0;u_2\right)-$$
$$\frac{3}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_1+u_2};1\right)H\left(0;u_2\right)-\frac{3}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_2+u_3};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H\left(0;u_2\right)+\frac{1}{2}G\left(\frac{1}{u_2},\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{2}G\left(\frac{1}{u_2},\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H\left(0;u_2\right)+\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},1,\frac{1}{u_2};1\right)H\left(0;u_2\right)+\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},1;1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)H\left(0;u_2\right)+\frac{1}{2}G\left(\frac{1}{u_3},0,\frac{1}{u_2};1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_2+u_3};1\right)H\left(0;u_2\right)+\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H\left(0;u_2\right)+$$

$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{123};1\right)H\left(0;u_2\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{132};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{213};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{231};1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{312};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{321};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{1}{1-u_1};1\right)H\left(0;u_2\right)-\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{u_2-1}{u_1+u_2-1};1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{1}{u_2};1\right)H\left(0;u_2\right)-\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{1}{1-u_3};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{2}\mathcal{G}\left(0,v_{123},\frac{1}{1-u_1};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(0,v_{213},\frac{1}{1-u_2};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,v_{231},\frac{1}{1-u_2};1\right)H\left(0;u_2\right)-\frac{1}{2}\mathcal{G}\left(0,v_{312},\frac{1}{1-u_3};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{123};1\right)H\left(0;u_2\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{132};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{123};1\right)H\left(0;u_2\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{132};1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},0;1\right)H\left(0;u_2\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{1}{1-u_1};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{u_2-1}{u_1+u_2-1};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},0;1\right)H\left(0;u_2\right)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1};1\right)H\left(0;u_2\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},0;1\right)H\left(0;u_2\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{213};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{231};1\right)H\left(0;u_2\right)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{213};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{231};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},1;1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},1;1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2};1\right)H\left(0;u_2\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{312};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{321};1\right)H\left(0;u_2\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{312};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{321};1\right)H\left(0;u_2\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},1;1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{1}{u_2};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{1}{1-u_3};1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},0;1\right)H\left(0;u_2\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},0;1\right)H\left(0;u_2\right)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3};1\right)H\left(0;u_2\right)+$$
$$\frac{3}{4}\mathcal{G}\left(v_{123},1,\frac{1}{1-u_1};1\right)H\left(0;u_2\right)+\frac{3}{4}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},1;1\right)H\left(0;u_2\right)-$$

$$\frac{1}{4}\mathcal{G}\left(v_{132},1,\frac{1}{1-u_1};1\right)H\left(0;u_2\right)-\frac{1}{4}\mathcal{G}\left(v_{132},\frac{1}{1-u_1},1;1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{213},1,\frac{1}{1-u_2};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(v_{213},\frac{1}{1-u_2},1;1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{231},1,\frac{1}{1-u_2};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},1;1\right)H\left(0;u_2\right)-$$
$$\frac{3}{4}\mathcal{G}\left(v_{312},1,\frac{1}{1-u_3};1\right)H\left(0;u_2\right)-\frac{3}{4}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},1;1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{321},1,\frac{1}{1-u_3};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(v_{321},\frac{1}{1-u_3},1;1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)H\left(0;u_2\right)+\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1};1\right)H\left(0;u_1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312};1\right)H\left(0;u_1\right)H\left(0;u_2\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312};1\right)H\left(0;u_1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321};1\right)H\left(0;u_1\right)H\left(0;u_2\right)+\frac{5}{24}\pi^2H\left(0;u_1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}G\left(0,\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H\left(0;u_3\right)-\frac{1}{4}G\left(0,\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}G\left(0,\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)H\left(0;u_3\right)-\frac{3}{4}G\left(0,\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H\left(0;u_3\right)-$$
$$\frac{3}{4}G\left(0,\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H\left(0;u_3\right)-\frac{1}{4}G\left(0,\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_1},1,\frac{1}{u_3};1\right)H\left(0;u_3\right)+\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},1;1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)H\left(0;u_3\right)+\frac{1}{2}G\left(\frac{1}{u_1},0,\frac{1}{u_3};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_1+u_3};1\right)H\left(0;u_3\right)+\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},0;1\right)H\left(0;u_3\right)+\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{u_3-1}{u_2+u_3-1};1\right)H\left(0;u_3\right)+\frac{1}{2}G\left(\frac{1}{u_2},0,\frac{1}{u_3};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_2+u_3};1\right)H\left(0;u_3\right)+\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H\left(0;u_3\right)-$$
$$\frac{3}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_1+u_3};1\right)H\left(0;u_3\right)-\frac{3}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_2+u_3};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H\left(0;u_3\right)+\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{2}G\left(\frac{1}{u_3},\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H\left(0;u_3\right)+\frac{1}{2}G\left(\frac{1}{u_3},\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{123};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{132};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{213};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{231};1\right)H\left(0;u_3\right)+$$

$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{312};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{321};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{1}{1-u_1};1\right)H\left(0;u_3\right)-\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{1}{u_3};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{1}{1-u_2};1\right)H\left(0;u_3\right)-\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{u_3-1}{u_2+u_3-1};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{2}\mathcal{G}\left(0,v_{123},\frac{1}{1-u_1};1\right)H\left(0;u_3\right)+\frac{1}{2}\mathcal{G}\left(0,v_{231},\frac{1}{1-u_2};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,v_{312},\frac{1}{1-u_3};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(0,v_{321},\frac{1}{1-u_3};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{123};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{132};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{123};1\right)H\left(0;u_3\right)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{132};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},1;1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{1}{1-u_1};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{1}{u_3};1\right)H\left(0;u_3\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},0;1\right)H\left(0;u_3\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},0;1\right)H\left(0;u_3\right)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1};1\right)H\left(0;u_3\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{213};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{231};1\right)H\left(0;u_3\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{213};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{231};1\right)H\left(0;u_3\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},0;1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{1}{1-u_2};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{u_3-1}{u_2+u_3-1};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},0;1\right)H\left(0;u_3\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},0;1\right)H\left(0;u_3\right)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{312};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{321};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{312};1\right)H\left(0;u_3\right)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{321};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},1;1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},1;1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3};1\right)H\left(0;u_3\right)-$$
$$\frac{3}{4}\mathcal{G}\left(v_{123},1,\frac{1}{1-u_1};1\right)H\left(0;u_3\right)-\frac{3}{4}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},1;1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{132},1,\frac{1}{1-u_1};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(v_{132},\frac{1}{1-u_1},1;1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}\mathcal{G}\left(v_{213},1,\frac{1}{1-u_2};1\right)H\left(0;u_3\right)-\frac{1}{4}\mathcal{G}\left(v_{213},\frac{1}{1-u_2},1;1\right)H\left(0;u_3\right)+$$

$$\frac{3}{4}\mathcal{G}\left(v_{231},1,\frac{1}{1-u_2};1\right)H\left(0;u_3\right)+\frac{3}{4}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},1;1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{312},1,\frac{1}{1-u_3};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},1;1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{321},1,\frac{1}{1-u_3};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(v_{321},\frac{1}{1-u_3},1;1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1};1\right)H\left(0;u_1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)H\left(0;u_3\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231};1\right)H\left(0;u_1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213};1\right)H\left(0;u_1\right)H\left(0;u_3\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231};1\right)H\left(0;u_1\right)H\left(0;u_3\right)+$$
$$\frac{5}{24}\pi^2H\left(0;u_1\right)H\left(0;u_3\right)+\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1};1\right)H\left(0;u_2\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H\left(0;u_2\right)H\left(0;u_3\right)+\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H\left(0;u_2\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123};1\right)H\left(0;u_2\right)H\left(0;u_3\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123};1\right)H\left(0;u_2\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132};1\right)H\left(0;u_2\right)H\left(0;u_3\right)+\frac{5}{24}\pi^2H\left(0;u_2\right)H\left(0;u_3\right)+$$
$$3H\left(0;u_2\right)H\left(0,0;u_1\right)H\left(0;u_3\right)+3H\left(0;u_1\right)H\left(0,0;u_2\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}H\left(0;u_2\right)H\left(0,1;\frac{u_1+u_2-1}{u_2-1}\right)H\left(0;u_3\right)+\frac{1}{2}H\left(0;u_1\right)H\left(0,1;\left(u_1+u_3\right)\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}H\left(0;u_1\right)H\left(0,1;\frac{u_2+u_3-1}{u_3-1}\right)H\left(0;u_3\right)+\frac{1}{2}H\left(0;u_2\right)H\left(0,1;\left(u_2+u_3\right)\right)H\left(0;u_3\right)+$$
$$\frac{3}{4}H\left(0;u_2\right)H\left(1,0;u_1\right)H\left(0;u_3\right)+\frac{3}{4}H\left(0;u_1\right)H\left(1,0;u_2\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213};1\right)H\left(0,0;u_1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231};1\right)H\left(0,0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312};1\right)H\left(0,0;u_1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321};1\right)H\left(0,0;u_1\right)-\frac{23}{24}\pi^2H\left(0,0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123};1\right)H\left(0,0;u_2\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132};1\right)H\left(0,0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312};1\right)H\left(0,0;u_2\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321};1\right)H\left(0,0;u_2\right)-$$
$$\frac{25}{4}H\left(0,0;u_1\right)H\left(0,0;u_2\right)-\frac{23}{24}\pi^2H\left(0,0;u_2\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123};1\right)H\left(0,0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132};1\right)H\left(0,0;u_3\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213};1\right)H\left(0,0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231};1\right)H\left(0,0;u_3\right)+3H\left(0;u_1\right)H\left(0;u_2\right)H\left(0,0;u_3\right)-$$
$$\frac{25}{4}H\left(0,0;u_1\right)H\left(0,0;u_3\right)-\frac{25}{4}H\left(0,0;u_2\right)H\left(0,0;u_3\right)-\frac{23}{24}\pi^2H\left(0,0;u_3\right)+\frac{1}{12}\pi^2H\left(0,1;u_1\right)+$$
$$\frac{1}{12}\pi^2H\left(0,1;u_2\right)-\frac{1}{24}\pi^2H\left(0,1;\frac{u_1+u_2-1}{u_2-1}\right)+\frac{1}{2}H\left(0;u_1\right)H\left(0;u_2\right)H\left(0,1;\left(u_1+u_2\right)\right)+$$

$$\frac{1}{12}\pi^2H\left(0,1;\left(u_1+u_2\right)\right)+\frac{1}{12}\pi^2H\left(0,1;u_3\right)+\frac{1}{4}H\left(0;u_1\right)H\left(0;u_2\right)H\left(0,1;\frac{u_1+u_3-1}{u_1-1}\right)-$$
$$\frac{1}{24}\pi^2H\left(0,1;\frac{u_1+u_3-1}{u_1-1}\right)+\frac{1}{12}\pi^2H\left(0,1;\left(u_1+u_3\right)\right)-\frac{1}{24}\pi^2H\left(0,1;\frac{u_2+u_3-1}{u_3-1}\right)+$$
$$\frac{1}{12}\pi^2H\left(0,1;\left(u_2+u_3\right)\right)-\frac{1}{2}G\left(0,\frac{1}{u_1+u_2};1\right)H\left(1,0;u_1\right)-$$
$$\frac{1}{2}G\left(0,\frac{1}{u_1+u_3};1\right)H\left(1,0;u_1\right)+\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H\left(1,0;u_1\right)+$$
$$\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H\left(1,0;u_1\right)+\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H\left(1,0;u_1\right)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1};1\right)H\left(1,0;u_1\right)+\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H\left(1,0;u_1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312};1\right)H\left(1,0;u_1\right)-\frac{3}{4}H\left(0,0;u_2\right)H\left(1,0;u_1\right)-\frac{3}{4}H\left(0,0;u_3\right)H\left(1,0;u_1\right)+$$
$$\frac{1}{4}H\left(0,1;\frac{u_1+u_3-1}{u_1-1}\right)H\left(1,0;u_1\right)-\frac{1}{3}\pi^2H\left(1,0;u_1\right)-\frac{1}{2}G\left(0,\frac{1}{u_1+u_2};1\right)H\left(1,0;u_2\right)-$$
$$\frac{1}{2}G\left(0,\frac{1}{u_2+u_3};1\right)H\left(1,0;u_2\right)+\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1};1\right)H\left(1,0;u_2\right)+$$
$$\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H\left(1,0;u_2\right)+\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H\left(1,0;u_2\right)+$$
$$\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H\left(1,0;u_2\right)+\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H\left(1,0;u_2\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123};1\right)H\left(1,0;u_2\right)-\frac{3}{4}H\left(0,0;u_1\right)H\left(1,0;u_2\right)-\frac{3}{4}H\left(0,0;u_3\right)H\left(1,0;u_2\right)+$$
$$\frac{1}{4}H\left(0,1;\frac{u_1+u_2-1}{u_2-1}\right)H\left(1,0;u_2\right)-\frac{1}{4}H\left(1,0;u_1\right)H\left(1,0;u_2\right)-\frac{1}{3}\pi^2H\left(1,0;u_2\right)-$$
$$\frac{1}{2}G\left(0,\frac{1}{u_1+u_3};1\right)H\left(1,0;u_3\right)-\frac{1}{2}G\left(0,\frac{1}{u_2+u_3};1\right)H\left(1,0;u_3\right)+$$
$$\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H\left(1,0;u_3\right)+\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1};1\right)H\left(1,0;u_3\right)+$$
$$\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H\left(1,0;u_3\right)-\frac{1}{3}\pi^2H\left(1,0;u_3\right)+\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H\left(1,0;u_3\right)+$$
$$\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H\left(1,0;u_3\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231};1\right)H\left(1,0;u_3\right)+$$
$$\frac{3}{4}H\left(0;u_1\right)H\left(0;u_2\right)H\left(1,0;u_3\right)-\frac{3}{4}H\left(0,0;u_1\right)H\left(1,0;u_3\right)-\frac{3}{4}H\left(0,0;u_2\right)H\left(1,0;u_3\right)+$$
$$\frac{1}{4}H\left(0,1;\frac{u_2+u_3-1}{u_3-1}\right)H\left(1,0;u_3\right)-\frac{1}{4}H\left(1,0;u_1\right)H\left(1,0;u_3\right)-\frac{1}{4}H\left(1,0;u_2\right)H\left(1,0;u_3\right)+$$
$$\frac{1}{24}\pi^2H\left(1,1;u_1\right)+\frac{1}{24}\pi^2H\left(1,1;u_2\right)+\frac{1}{24}\pi^2H\left(1,1;u_3\right)+\frac{1}{2}H\left(0;u_2\right)H\left(0,0,0;u_1\right)+$$
$$\frac{1}{2}H\left(0;u_3\right)H\left(0,0,0;u_2\right)+\frac{1}{2}H\left(0;u_1\right)H\left(0,0,0;u_3\right)-\frac{1}{2}H\left(0;u_2\right)H\left(0,0,1;\frac{u_1+u_2-1}{u_2-1}\right)-$$
$$\frac{1}{2}H\left(0;u_3\right)H\left(0,0,1;\frac{u_1+u_2-1}{u_2-1}\right)-H\left(0;u_1\right)H\left(0,0,1;\left(u_1+u_2\right)\right)-$$
$$H\left(0;u_2\right)H\left(0,0,1;\left(u_1+u_2\right)\right)-\frac{1}{2}H\left(0;u_1\right)H\left(0,0,1;\frac{u_1+u_3-1}{u_1-1}\right)-$$

$$\frac{1}{2}H\left(0;u_2\right)H\left(0,0,1;\frac{u_1+u_3-1}{u_1-1}\right)-H\left(0;u_1\right)H\left(0,0,1;\left(u_1+u_3\right)\right)-$$
$$H\left(0;u_3\right)H\left(0,0,1;\left(u_1+u_3\right)\right)-\frac{1}{2}H\left(0;u_1\right)H\left(0,0,1;\frac{u_2+u_3-1}{u_3-1}\right)-$$
$$\frac{1}{2}H\left(0;u_3\right)H\left(0,0,1;\frac{u_2+u_3-1}{u_3-1}\right)-H\left(0;u_2\right)H\left(0,0,1;\left(u_2+u_3\right)\right)-$$
$$H\left(0;u_3\right)H\left(0,0,1;\left(u_2+u_3\right)\right)-\frac{1}{2}H\left(0;u_2\right)H\left(0,1,0;u_1\right)-\frac{1}{2}H\left(0;u_3\right)H\left(0,1,0;u_2\right)-$$
$$\frac{1}{2}H\left(0;u_1\right)H\left(0,1,0;u_3\right)+\frac{1}{4}H\left(0;u_2\right)H\left(0,1,1;\frac{u_1+u_2-1}{u_2-1}\right)-$$
$$\frac{1}{4}H\left(0;u_3\right)H\left(0,1,1;\frac{u_1+u_2-1}{u_2-1}\right)+\frac{1}{4}H\left(0;u_1\right)H\left(0,1,1;\frac{u_1+u_3-1}{u_1-1}\right)-$$
$$\frac{1}{4}H\left(0;u_2\right)H\left(0,1,1;\frac{u_1+u_3-1}{u_1-1}\right)-\frac{1}{4}H\left(0;u_1\right)H\left(0,1,1;\frac{u_2+u_3-1}{u_3-1}\right)+$$
$$\frac{1}{4}H\left(0;u_3\right)H\left(0,1,1;\frac{u_2+u_3-1}{u_3-1}\right)+\frac{1}{2}H\left(0;u_2\right)H\left(1,0,0;u_1\right)-\frac{1}{2}H\left(0;u_3\right)H\left(1,0,0;u_1\right)-$$
$$\frac{1}{2}H\left(0;u_1\right)H\left(1,0,0;u_2\right)+\frac{1}{2}H\left(0;u_3\right)H\left(1,0,0;u_2\right)+\frac{1}{2}H\left(0;u_1\right)H\left(1,0,0;u_3\right)-$$
$$\frac{1}{2}H\left(0;u_2\right)H\left(1,0,0;u_3\right)-\frac{1}{4}H\left(0;u_3\right)H\left(1,0,1;\frac{u_1+u_2-1}{u_2-1}\right)-$$
$$\frac{1}{4}H\left(0;u_2\right)H\left(1,0,1;\frac{u_1+u_3-1}{u_1-1}\right)-\frac{1}{4}H\left(0;u_1\right)H\left(1,0,1;\frac{u_2+u_3-1}{u_3-1}\right)-$$
$$7H\left(0,0,0,0;u_1\right)-7H\left(0,0,0,0;u_2\right)-7H\left(0,0,0,0;u_3\right)+\frac{3}{2}H\left(0,0,0,1;\frac{u_1+u_2-1}{u_2-1}\right)+$$
$$3H\left(0,0,0,1;\left(u_1+u_2\right)\right)+\frac{3}{2}H\left(0,0,0,1;\frac{u_1+u_3-1}{u_1-1}\right)+3H\left(0,0,0,1;\left(u_1+u_3\right)\right)+$$
$$\frac{3}{2}H\left(0,0,0,1;\frac{u_2+u_3-1}{u_3-1}\right)+3H\left(0,0,0,1;\left(u_2+u_3\right)\right)+\frac{9}{4}H\left(0,0,1,0;u_1\right)+$$
$$\frac{9}{4}H\left(0,0,1,0;u_2\right)+\frac{9}{4}H\left(0,0,1,0;u_3\right)-\frac{1}{2}H\left(0,1,0,0;u_1\right)-\frac{1}{2}H\left(0,1,0,0;u_2\right)-$$
$$\frac{1}{2}H\left(0,1,0,0;u_3\right)+\frac{1}{2}H\left(0,1,0,1;\frac{u_1+u_2-1}{u_2-1}\right)+\frac{1}{2}H\left(0,1,0,1;\frac{u_1+u_3-1}{u_1-1}\right)+$$
$$\frac{1}{2}H\left(0,1,0,1;\frac{u_2+u_3-1}{u_3-1}\right)+H\left(0,1,1,0;u_1\right)+H\left(0,1,1,0;u_2\right)+H\left(0,1,1,0;u_3\right)-$$
$$\frac{1}{4}H\left(0,1,1,1;\frac{u_1+u_2-1}{u_2-1}\right)-\frac{1}{4}H\left(0,1,1,1;\frac{u_1+u_3-1}{u_1-1}\right)-$$
$$\frac{1}{4}H\left(0,1,1,1;\frac{u_2+u_3-1}{u_3-1}\right)+H\left(1,0,0,1;\frac{u_1+u_2-1}{u_2-1}\right)+H\left(1,0,0,1;\frac{u_1+u_3-1}{u_1-1}\right)+$$
$$H\left(1,0,0,1;\frac{u_2+u_3-1}{u_3-1}\right)+2H\left(1,0,1,0;u_1\right)+2H\left(1,0,1,0;u_2\right)+2H\left(1,0,1,0;u_3\right)+$$
$$\frac{1}{4}H\left(1,1,0,1;\frac{u_1+u_2-1}{u_2-1}\right)+\frac{1}{4}H\left(1,1,0,1;\frac{u_1+u_3-1}{u_1-1}\right)+$$
$$\frac{1}{4}H\left(1,1,0,1;\frac{u_2+u_3-1}{u_3-1}\right)+\frac{1}{2}H\left(1,1,1,0;u_1\right)+\frac{1}{2}H\left(1,1,1,0;u_2\right)+\frac{1}{2}H\left(1,1,1,0;u_3\right)-$$
$$\frac{1}{24}\pi^2H\left(0;u_3\right)\mathcal{H}\left(1;\frac{1}{u_{123}}\right)-\frac{1}{24}\pi^2H\left(0;u_1\right)\mathcal{H}\left(1;\frac{1}{u_{231}}\right)-\frac{1}{24}\pi^2H\left(0;u_2\right)\mathcal{H}\left(1;\frac{1}{u_{312}}\right)+$$
$$\frac{1}{8}\pi^2H\left(0;u_2\right)\mathcal{H}\left(1;\frac{1}{v_{123}}\right)-\frac{1}{8}\pi^2H\left(0;u_3\right)\mathcal{H}\left(1;\frac{1}{v_{123}}\right)+\frac{1}{24}\pi^2H\left(0;u_2\right)\mathcal{H}\left(1;\frac{1}{v_{132}}\right)-$$

$$\frac{1}{24}\pi^2H\left(0;u_3\right)\mathcal{H}\left(1;\frac{1}{v_{132}}\right)-\frac{1}{24}\pi^2H\left(0;u_1\right)\mathcal{H}\left(1;\frac{1}{v_{213}}\right)+\frac{1}{24}\pi^2H\left(0;u_3\right)\mathcal{H}\left(1;\frac{1}{v_{213}}\right)-$$
$$\frac{1}{8}\pi^2H\left(0;u_1\right)\mathcal{H}\left(1;\frac{1}{v_{231}}\right)+\frac{1}{8}\pi^2H\left(0;u_3\right)\mathcal{H}\left(1;\frac{1}{v_{231}}\right)+\frac{1}{8}\pi^2H\left(0;u_1\right)\mathcal{H}\left(1;\frac{1}{v_{312}}\right)-$$
$$\frac{1}{8}\pi^2H\left(0;u_2\right)\mathcal{H}\left(1;\frac{1}{v_{312}}\right)+\frac{1}{24}\pi^2H\left(0;u_1\right)\mathcal{H}\left(1;\frac{1}{v_{321}}\right)-\frac{1}{24}\pi^2H\left(0;u_2\right)\mathcal{H}\left(1;\frac{1}{v_{321}}\right)-$$
$$\frac{1}{4}H\left(0;u_2\right)H\left(0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{u_{123}}\right)-\frac{1}{4}H\left(1,0;u_2\right)\mathcal{H}\left(0,1;\frac{1}{u_{123}}\right)+\frac{1}{24}\pi^2\mathcal{H}\left(0,1;\frac{1}{u_{123}}\right)+$$
$$\frac{1}{24}\pi^2\mathcal{H}\left(0,1;\frac{1}{u_{231}}\right)-\frac{1}{4}H\left(0;u_1\right)H\left(0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{u_{231}}\right)-\frac{1}{4}H\left(1,0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{u_{231}}\right)-$$
$$\frac{1}{4}H\left(0;u_1\right)H\left(0;u_2\right)\mathcal{H}\left(0,1;\frac{1}{u_{312}}\right)-\frac{1}{4}H\left(1,0;u_1\right)\mathcal{H}\left(0,1;\frac{1}{u_{312}}\right)+\frac{1}{24}\pi^2\mathcal{H}\left(0,1;\frac{1}{u_{312}}\right)-$$
$$\frac{1}{4}H\left(0;u_2\right)H\left(0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{v_{123}}\right)+\frac{1}{4}H\left(0,0;u_2\right)\mathcal{H}\left(0,1;\frac{1}{v_{123}}\right)+$$
$$\frac{1}{4}H\left(0,0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{v_{123}}\right)+\frac{1}{6}\pi^2\mathcal{H}\left(0,1;\frac{1}{v_{123}}\right)-\frac{1}{4}H\left(0;u_2\right)H\left(0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{v_{132}}\right)+$$
$$\frac{1}{4}H\left(0,0;u_2\right)\mathcal{H}\left(0,1;\frac{1}{v_{132}}\right)+\frac{1}{4}H\left(0,0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{v_{132}}\right)+\frac{1}{6}\pi^2\mathcal{H}\left(0,1;\frac{1}{v_{132}}\right)-$$
$$\frac{1}{4}H\left(0;u_1\right)H\left(0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{v_{213}}\right)+\frac{1}{4}H\left(0,0;u_1\right)\mathcal{H}\left(0,1;\frac{1}{v_{213}}\right)+$$
$$\frac{1}{4}H\left(0,0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{v_{213}}\right)+\frac{1}{6}\pi^2\mathcal{H}\left(0,1;\frac{1}{v_{213}}\right)-\frac{1}{4}H\left(0;u_1\right)H\left(0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{v_{231}}\right)+$$
$$\frac{1}{4}H\left(0,0;u_1\right)\mathcal{H}\left(0,1;\frac{1}{v_{231}}\right)+\frac{1}{4}H\left(0,0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{v_{231}}\right)+\frac{1}{6}\pi^2\mathcal{H}\left(0,1;\frac{1}{v_{231}}\right)-$$
$$\frac{1}{4}H\left(0;u_1\right)H\left(0;u_2\right)\mathcal{H}\left(0,1;\frac{1}{v_{312}}\right)+\frac{1}{4}H\left(0,0;u_1\right)\mathcal{H}\left(0,1;\frac{1}{v_{312}}\right)+$$
$$\frac{1}{4}H\left(0,0;u_2\right)\mathcal{H}\left(0,1;\frac{1}{v_{312}}\right)+\frac{1}{6}\pi^2\mathcal{H}\left(0,1;\frac{1}{v_{312}}\right)-\frac{1}{4}H\left(0;u_1\right)H\left(0;u_2\right)\mathcal{H}\left(0,1;\frac{1}{v_{321}}\right)+$$
$$\frac{1}{4}H\left(0,0;u_1\right)\mathcal{H}\left(0,1;\frac{1}{v_{321}}\right)+\frac{1}{4}H\left(0,0;u_2\right)\mathcal{H}\left(0,1;\frac{1}{v_{321}}\right)+\frac{1}{6}\pi^2\mathcal{H}\left(0,1;\frac{1}{v_{321}}\right)-$$
$$\frac{1}{2}H\left(0;u_2\right)H\left(0;u_3\right)\mathcal{H}\left(1,1;\frac{1}{v_{123}}\right)+\frac{1}{2}H\left(0,0;u_2\right)\mathcal{H}\left(1,1;\frac{1}{v_{123}}\right)+$$
$$\frac{1}{2}H\left(0,0;u_3\right)\mathcal{H}\left(1,1;\frac{1}{v_{123}}\right)+\frac{11}{24}\pi^2\mathcal{H}\left(1,1;\frac{1}{v_{123}}\right)-\frac{1}{24}\pi^2\mathcal{H}\left(1,1;\frac{1}{v_{132}}\right)-$$
$$\frac{1}{24}\pi^2\mathcal{H}\left(1,1;\frac{1}{v_{213}}\right)-\frac{1}{2}H\left(0;u_1\right)H\left(0;u_3\right)\mathcal{H}\left(1,1;\frac{1}{v_{231}}\right)+\frac{1}{2}H\left(0,0;u_1\right)\mathcal{H}\left(1,1;\frac{1}{v_{231}}\right)+$$
$$\frac{1}{2}H\left(0,0;u_3\right)\mathcal{H}\left(1,1;\frac{1}{v_{231}}\right)+\frac{11}{24}\pi^2\mathcal{H}\left(1,1;\frac{1}{v_{231}}\right)-\frac{1}{2}H\left(0;u_1\right)H\left(0;u_2\right)\mathcal{H}\left(1,1;\frac{1}{v_{312}}\right)+$$
$$\frac{1}{2}H\left(0,0;u_1\right)\mathcal{H}\left(1,1;\frac{1}{v_{312}}\right)+\frac{1}{2}H\left(0,0;u_2\right)\mathcal{H}\left(1,1;\frac{1}{v_{312}}\right)+\frac{11}{24}\pi^2\mathcal{H}\left(1,1;\frac{1}{v_{312}}\right)-$$
$$\frac{1}{24}\pi^2\mathcal{H}\left(1,1;\frac{1}{v_{321}}\right)+\frac{1}{2}H\left(0;u_2\right)\mathcal{H}\left(0,0,1;\frac{1}{u_{123}}\right)+\frac{1}{2}H\left(0;u_3\right)\mathcal{H}\left(0,0,1;\frac{1}{u_{123}}\right)+$$
$$\frac{1}{2}H\left(0;u_1\right)\mathcal{H}\left(0,0,1;\frac{1}{u_{231}}\right)+\frac{1}{2}H\left(0;u_3\right)\mathcal{H}\left(0,0,1;\frac{1}{u_{231}}\right)+\frac{1}{2}H\left(0;u_1\right)\mathcal{H}\left(0,0,1;\frac{1}{u_{312}}\right)+$$
$$\frac{1}{2}H\left(0;u_2\right)\mathcal{H}\left(0,0,1;\frac{1}{u_{312}}\right)+\frac{1}{4}H\left(0;u_3\right)\mathcal{H}\left(0,1,1;\frac{1}{u_{123}}\right)+\frac{1}{4}H\left(0;u_1\right)\mathcal{H}\left(0,1,1;\frac{1}{u_{231}}\right)+$$

$$\frac{1}{4}H\left(0;u_2\right)\mathcal{H}\left(0,1,1;\frac{1}{u_{312}}\right)+\frac{1}{4}H\left(0;u_2\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{123}}\right)-\frac{1}{4}H\left(0;u_3\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{123}}\right)-$$
$$\frac{1}{4}H\left(0;u_2\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{132}}\right)+\frac{1}{4}H\left(0;u_3\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{132}}\right)+\frac{1}{4}H\left(0;u_1\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{213}}\right)-$$
$$\frac{1}{4}H\left(0;u_3\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{213}}\right)-\frac{1}{4}H\left(0;u_1\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{231}}\right)+\frac{1}{4}H\left(0;u_3\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{231}}\right)+$$
$$\frac{1}{4}H\left(0;u_1\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{312}}\right)-\frac{1}{4}H\left(0;u_2\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{312}}\right)-\frac{1}{4}H\left(0;u_1\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{321}}\right)+$$
$$\frac{1}{4}H\left(0;u_2\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{321}}\right)+\frac{1}{4}H\left(0;u_3\right)\mathcal{H}\left(1,0,1;\frac{1}{u_{123}}\right)+\frac{1}{4}H\left(0;u_1\right)\mathcal{H}\left(1,0,1;\frac{1}{u_{231}}\right)+$$
$$\frac{1}{4}H\left(0;u_2\right)\mathcal{H}\left(1,0,1;\frac{1}{u_{312}}\right)+\frac{1}{4}H\left(0;u_2\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{123}}\right)-\frac{1}{4}H\left(0;u_3\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{123}}\right)-$$
$$\frac{1}{4}H\left(0;u_2\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{132}}\right)+\frac{1}{4}H\left(0;u_3\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{132}}\right)+\frac{1}{4}H\left(0;u_1\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{213}}\right)-$$
$$\frac{1}{4}H\left(0;u_3\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{213}}\right)-\frac{1}{4}H\left(0;u_1\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{231}}\right)+\frac{1}{4}H\left(0;u_3\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{231}}\right)+$$
$$\frac{1}{4}H\left(0;u_1\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{312}}\right)-\frac{1}{4}H\left(0;u_2\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{312}}\right)-\frac{1}{4}H\left(0;u_1\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{321}}\right)+$$
$$\frac{1}{4}H\left(0;u_2\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{321}}\right)+H\left(0;u_2\right)\mathcal{H}\left(1,1,1;\frac{1}{v_{123}}\right)-H\left(0;u_3\right)\mathcal{H}\left(1,1,1;\frac{1}{v_{123}}\right)-$$
$$H\left(0;u_1\right)\mathcal{H}\left(1,1,1;\frac{1}{v_{231}}\right)+H\left(0;u_3\right)\mathcal{H}\left(1,1,1;\frac{1}{v_{231}}\right)+H\left(0;u_1\right)\mathcal{H}\left(1,1,1;\frac{1}{v_{312}}\right)-$$
$$H\left(0;u_2\right)\mathcal{H}\left(1,1,1;\frac{1}{v_{312}}\right)-\frac{3}{2}\mathcal{H}\left(0,0,0,1;\frac{1}{u_{123}}\right)-\frac{3}{2}\mathcal{H}\left(0,0,0,1;\frac{1}{u_{231}}\right)-$$
$$\frac{3}{2}\mathcal{H}\left(0,0,0,1;\frac{1}{u_{312}}\right)-3\mathcal{H}\left(0,0,0,1;\frac{1}{v_{132}}\right)-3\mathcal{H}\left(0,0,0,1;\frac{1}{v_{213}}\right)-3\mathcal{H}\left(0,0,0,1;\frac{1}{v_{321}}\right)-$$
$$\frac{1}{2}\mathcal{H}\left(0,0,1,1;\frac{1}{u_{123}}\right)-\frac{1}{2}\mathcal{H}\left(0,0,1,1;\frac{1}{u_{231}}\right)-\frac{1}{2}\mathcal{H}\left(0,0,1,1;\frac{1}{u_{312}}\right)-$$
$$\frac{1}{2}\mathcal{H}\left(0,1,0,1;\frac{1}{u_{123}}\right)-\frac{1}{2}\mathcal{H}\left(0,1,0,1;\frac{1}{u_{231}}\right)-\frac{1}{2}\mathcal{H}\left(0,1,0,1;\frac{1}{u_{312}}\right)+$$
$$\frac{1}{4}\mathcal{H}\left(0,1,1,1;\frac{1}{v_{123}}\right)+\frac{1}{4}\mathcal{H}\left(0,1,1,1;\frac{1}{v_{132}}\right)+\zeta_3 H\left(0;u_1\right)+\zeta_3 H\left(0;u_2\right)+\zeta_3 H\left(0;u_3\right)+$$
$$\frac{5}{2}\zeta_3 H\left(1;u_1\right)+\frac{5}{2}\zeta_3 H\left(1;u_2\right)+\frac{5}{2}\zeta_3 H\left(1;u_3\right)+\frac{1}{2}\zeta_3\mathcal{H}\left(1;\frac{1}{u_{123}}\right)+\frac{1}{2}\zeta_3\mathcal{H}\left(1;\frac{1}{u_{231}}\right)+$$
$$\frac{1}{2}\zeta_3\mathcal{H}\left(1;\frac{1}{u_{312}}\right)-\frac{1}{2}\mathcal{H}\left(1,0,0,1;\frac{1}{u_{123}}\right)-\frac{1}{2}\mathcal{H}\left(1,0,0,1;\frac{1}{u_{231}}\right)-\frac{1}{2}\mathcal{H}\left(1,0,0,1;\frac{1}{u_{312}}\right)+$$
$$\frac{1}{4}\zeta_3\mathcal{H}\left(1;\frac{1}{v_{123}}\right)+\frac{1}{4}\zeta_3\mathcal{H}\left(1;\frac{1}{v_{132}}\right)+\frac{1}{4}\zeta_3\mathcal{H}\left(1;\frac{1}{v_{213}}\right)+\frac{1}{4}\zeta_3\mathcal{H}\left(1;\frac{1}{v_{231}}\right)+\frac{1}{4}\zeta_3\mathcal{H}\left(1;\frac{1}{v_{312}}\right)+$$
$$\frac{1}{4}\zeta_3\mathcal{H}\left(1;\frac{1}{v_{321}}\right)+\frac{1}{4}\mathcal{H}\left(0,1,1,1;\frac{1}{v_{213}}\right)+\frac{1}{4}\mathcal{H}\left(0,1,1,1;\frac{1}{v_{231}}\right)+\frac{1}{4}\mathcal{H}\left(0,1,1,1;\frac{1}{v_{312}}\right)+$$
$$\frac{1}{4}\mathcal{H}\left(0,1,1,1;\frac{1}{v_{321}}\right)+\frac{1}{4}\mathcal{H}\left(1,0,1,1;\frac{1}{v_{123}}\right)+\frac{1}{4}\mathcal{H}\left(1,0,1,1;\frac{1}{v_{132}}\right)+\frac{1}{4}\mathcal{H}\left(1,0,1,1;\frac{1}{v_{213}}\right)+$$
$$\frac{1}{4}\mathcal{H}\left(1,0,1,1;\frac{1}{v_{231}}\right)+\frac{1}{4}\mathcal{H}\left(1,0,1,1;\frac{1}{v_{312}}\right)+\frac{1}{4}\mathcal{H}\left(1,0,1,1;\frac{1}{v_{321}}\right)+\frac{1}{4}\mathcal{H}\left(1,1,0,1;\frac{1}{v_{123}}\right)+$$
$$\frac{1}{4}\mathcal{H}\left(1,1,0,1;\frac{1}{v_{132}}\right)+\frac{1}{4}\mathcal{H}\left(1,1,0,1;\frac{1}{v_{213}}\right)+\frac{1}{4}\mathcal{H}\left(1,1,0,1;\frac{1}{v_{231}}\right)+\frac{1}{4}\mathcal{H}\left(1,1,0,1;\frac{1}{v_{312}}\right)+$$

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Goncharov, Vergu, Volovich and I used some mathematical magic called the symbol of a transcendental function to simplify this expression down to a single line, involving nothing more complicated than the "standard" Li_n polylogarithm functions.

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Rather, it is important because it provides hope, where previously was none, that we might really unlock the secrets to all orders...

Looking Forward

We do not yet know the full symbol of more general amplitudes, but data is starting to accumulate (Gaiotto, Maldacena, Sever, Vieira; Arkani-Hamed, Caron-Huot, ...)

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2. This is an experimental science.
   (Get the answer first, by any means necessary, then analyze it.)
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If you find these principles compelling, please join us for a fun ride over the next three months @ the KITP!