Acknowledgements

  Alessandro Melchiorri, Laura Mersini, Carolina Odman & M.T.

- "Can the Dark Energy Equation of State Parameter be Less than -1?"
  Sean M. Carroll, Mark Hoffman & M.T.

- "Is Cosmic-Speed-Up Due to New Gravitational Physics?"
  Sean W. Carroll, Vikram Duvvuri, M.T. and Michael S. Turner.
  [astro-ph/0306439]

Most of the talk is based on

Our Problems

Really two problems associated with dark energy.

1. Its magnitude - we need to show why dark energy is the (ridiculous, as we'll soon see) size it is today.

2. The coincidence problem.

What Might Explain These?

So even if dark energy wasn't important in the past, it will dominate in the future. Why do we observe it at a time when it is comparable to matter - why now?

Remember, we could have lived easily with no dark energy.
Theoretical Description

Evolution of the universe governed by Einstein eqns

\[ H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi}{3M_p^2} \rho \]

The Friedmann equation

\[ \frac{\ddot{a}}{a} = -\frac{4\pi}{3M_p^2} (\rho + 3p) \]

The “acceleration” equation

Parameterize different types of matter by equations of state: \( p_i = w_i \rho_i \)

When evolution dominated by type i, obtain

\[ a(t) \propto t^{\frac{2}{3(1+w_i)}} \quad \rho(a) \propto a^{3(1+w_i)} \quad (w_i \neq -1) \]

Cosmic Acceleration

So, accelerating expansion means \( p < -\rho/3 \) or \( w_Q < -1/3 \)

Three Broad Possibilities

<table>
<thead>
<tr>
<th>Evolution of Energy Density</th>
<th>-1 &lt; w &lt; -1/3</th>
<th>w = -1</th>
<th>w &lt; -1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dilutes slower than any matter</td>
<td>Stays absolutely constant (( \Lambda ))</td>
<td>Increases with the expansion!!</td>
<td></td>
</tr>
<tr>
<td>Power-law quintessence</td>
<td>Exponential expansion</td>
<td>Infinite value in a finite time!!</td>
<td></td>
</tr>
</tbody>
</table>
Data on $w_Q$

Basically measuring (luminosity) distance as fn of redshift.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$w_Q$ Interval</th>
<th>$\Omega_m$ Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>CMB+HST</td>
<td>-1.65 &lt; $w_Q$ &lt; -0.54</td>
<td>0.19 &lt; $\Omega_m$ &lt; 0.43</td>
</tr>
<tr>
<td>CMB+HST+BBN</td>
<td>-1.61 &lt; $w_Q$ &lt; -0.57</td>
<td>0.20 &lt; $\Omega_m$ &lt; 0.42</td>
</tr>
<tr>
<td>CMB+HST+BBN +SNIa</td>
<td>-1.45 &lt; $w_Q$ &lt; -0.74</td>
<td>0.21 &lt; $\Omega_m$ &lt; 0.36</td>
</tr>
<tr>
<td>CMB+HST+BBN +SNIa+2dF</td>
<td>-1.38 &lt; $w_Q$ &lt; -0.82</td>
<td>0.22 &lt; $\Omega_m$ &lt; 0.35</td>
</tr>
</tbody>
</table>

[From: Melchiorri, Mersini, Odman and M.T. (2002)]

Restricting to $w_Q$ > -1

(this means imposing the null energy condition, for example, more about this soon)

This analysis gives $w_Q$ < -0.78

But as we’ve seen, Taking the data at face value leads to negative values allowed also

[From: Spergel et al. WMAP paper (2003)]
So, does it make sense to consider values of $w_Q$ that are less than $-1$?

Energy Conditions in GR

[From: Carroll, Hoffman and M.T. (2003)]
$w_Q < -1$?

What are theorists to make of the possibility of $w_Q < -1$ matter?

- Violates NEC (with positive energy):
  \[ T_{\mu\nu}N^\mu N^\nu < 0 \Rightarrow \rho_i + p_i < 0 \]
- One expects instabilities
- Are they there and are they fatal/irrelevant now in the universe?

Don’t expect an instability just for a single field uncoupled to others. To investigate an instability have to consider a coupled system.

A Toy Model (uncoupled)

Flip the kinetic terms for a real scalar field

\[
L_\phi = -\frac{1}{2}(\partial_\mu \phi)\partial^\mu \phi - V(\phi) \quad \Rightarrow \quad w_\phi \equiv \frac{P_\phi}{\rho_\phi} = -\frac{\dot{\phi}^2 + 2V(\phi)}{-\dot{\phi}^2 + 2V(\phi)}
\]

Caldwell; Schulz & White; Carroll, Hoffman & M.T.
Related to k-essence work of Armendariz-Picon, Mukhanov & Steinhardt

So $w_\phi < -1$ if we keep $V(\phi) > 0$

Also note - for $w < -1$, dark energy density increases as universe expands.

To analyze properly need a model for $V(\phi)$:

\[
V(\phi) = V_0 e^{-\phi^2/\lambda^2} \quad \text{is convenient}
\]
Is Cosmic Acceleration Telling Us Something about Gravity?

Cosmological Evolution

\[ \frac{dV}{d\phi} = 0 \]

Only difference from usual field equation.

- Early Universe: Field is frozen; \( w = -1 \)
- Today: Field is moving; \( w < -1 \)
- Very Late Universe: Field is at maximum; \( w = -1 \)

Evolution of \( w(a) \)

Field wants to run up the potential!
A Coupled System

A Lagrangian for a toy model:

\[ L = \frac{1}{2} (\partial_\mu \phi) \partial^\nu \phi - \frac{1}{2} m^2 \phi^2 - \left[ \frac{1}{2} (\partial_\mu \psi) \partial^\nu \psi - \frac{1}{2} m^2 \psi^2 \right] - \lambda \phi^2 \psi^2 \]

Analyze behavior for different values of \( \lambda \) and different initial conditions \{\phi(0), \partial_t \phi(0), \psi(0), \partial_t \psi(0)\}

Plot energy in each field, approximately

\[ \rho_\phi = \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} m^2 \phi^2 \]
\[ \rho_\psi = -\left( \frac{1}{2} \dot{\psi}^2 + \frac{1}{2} m^2 \psi^2 \right) \]

Some Sample Results

![Graphs showing energy density \( \rho \) over time \( t \) for different values of \( \lambda \). The graphs illustrate oscillatory behavior and a peak for \( \lambda = 0.2 \) and a more pronounced peak for \( \lambda = 2.0 \).]
Doing Better - a Quantum Analysis

- Would like to ask about the decay rates of phantom particles to regular ones.
- Could forbid couplings to most other particles...
- ...but everything couples to gravity.
- So study decay rate to gravitons.
- It'll be useful first to say something about phantom kinematics

Phantom Kinematics

- Notation: ordinary particles $\psi$, phantoms $\phi$
- Reaction involving phantoms allowed if the equivalent reaction switching phantoms from left right and vice versa would conventionally occur.

**Simple Example**

$\psi_1 \rightarrow \psi_2 + \phi$ allowed if $\psi_1 + \phi \rightarrow \psi_2$ normally allowed

But this can happen even if mass of $\psi_2$ is greater than sum of masses of $\psi_1$ and $\phi$.

So ordinary particles can decay into heavier particles plus phantoms (negative energy!)
The Most Inevitable Decay

Ask me later for details, but the phantom decay involving least number of particles is:

\[ \phi_i \rightarrow h + \phi_1 + \phi_2 \]

Investigate this in our toy model with potential

\[ V(\phi) = V_0 e^{-\phi^2 \over M_P^2} \]

expanded around some background \( \phi_0 \sim M_P \)

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The Interaction Lagrangian

Need this to first order in \( h \) and third order in \( \phi \)

\[ L_I \sim \frac{1}{M_P^4} (M_P^2 h) \frac{1}{3!} V'''(\phi_0) \phi^3 \]

\[ = \lambda_{\text{eff}} (M_P^2 h) \phi^3 \]

But note that

\[ \lambda_{\text{eff}} \sim 2\phi \frac{V(\phi_0)}{M_P^5} \sim \frac{V(\phi_0)}{M_P^4} \sim 10^{-120} \]

Because of entirely independent cosmological constraints!
Dr. Mark Trodden, Syracuse U (KITP String Cosmology 10-08-03) Is Cosmic Acceleration Telling Us Something about Gravity?

**Phantom Decay Rate**

- At tree level: $\Gamma \sim \lambda_{\text{eff}}^2 \frac{\Lambda^2}{m_\phi}$
- Integral and approximate $E^{-p}$ (masses small)

**An Effective Theory**

- If upper limits on integrals are $\infty$, then decay rate is infinite! - Highly unstable!
- Can't treat as fundamental theory - so treat as effective theory, valid up to a cutoff $\Lambda$
- Crude approximation - just cutoff integrals at $\Lambda$

**So that**

- $H_0 \tau \sim 10^{120} \left( \frac{M_{\text{Pl}}}{\Lambda} \right)$

**Extremely weak constraint!**

(Unfortunately, there are other decay channels)

$\Lambda < 10^{10^6} M_{\text{Pl}}$
Other Couplings

Since we’re dealing with an effective theory, must include all possible nonrenormalizable interactions.

So consider

\[ L = \frac{\beta}{M_{pl} \Lambda} \phi \left( M_{pl} h^{\mu \nu} \right) \partial_\mu \phi \partial_\nu \phi \]

Very similar decay calculation yields \( \Lambda \leq 10^{-3} \text{ eV} \)

Even if we impose discrete symmetries to forbid some terms, e.g. \( \phi \rightarrow \phi + \epsilon \), we can still show that the above bound holds.

Modifying Gravity

Since we’re on the subject of abandoning sacred principles...

Quintessence requires incredible fine-tuning, so much so that one really is considering incredibly unnatural matter to put on the RHS of Einstein’s equations.

Maybe we should look at the LHS - or more properly, the gravitational action.
GDP Braneworlds

(Dvali, Gabadadze, Parrati)

One way to do this is to include both a 5d and a 4d Einstein term in the action for a 3-brane in a 4+1 dimensional flat spacetime. Claim is that such a term will be induced anyway.

\[ S = M^3 \int d^5x \sqrt{-GR_5} + M_P^2 \int d^4x \sqrt{-g}R \]

Results in a theory that looks like Einstein theory (4d) at short distances, and shows 5d deviations at large distances.

Gravity ``leaking off the brane'' at large distances might mimic dark energy

(Deffayet, Dvali, Gabadadze)

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New Gravitational Physics

Carroll, Duvvuri, M.T. & Turner, astro-ph/0306438

Consider modifying the Einstein-Hilbert action

\[ S = \frac{M_P^2}{2} \int d^4x \sqrt{-g} \left( R - \frac{\mu^{2(n+1)}}{R^n} \right) + \int d^4x \sqrt{-g}L_M \]

(I'll focus on n=1 for most of this)

Field equation (n=1): (Recall, there are similar ways to get inflation)

\[ \left(1 + \frac{\mu^4}{R^2}\right)R_{\mu\nu} - \frac{1}{2} \left(1 - \frac{\mu^4}{R^2}\right)Rg_{\mu\nu} + \mu^4 \left[g_{\mu\nu} \nabla_\alpha \nabla^\alpha - \nabla_\rho \nabla^\rho \right] \left( \frac{1}{R^2} \right) = \frac{T^{\mu\nu}}{M_P^2} \]

With, for cosmology

\[ T^{\mu\nu}_{\mu\nu} = \left(\rho_M + P_M\right)U_\mu U_\nu + P_M g_{\mu\nu} \]
The Matter Frame

Can see immediately constant curvature vacuum solutions are de Sitter and anti-de Sitter

\[ R = \pm \sqrt{3} \mu^2 \]

Seems encouraging for dark energy, but - dS in unstable, with decay time \[ \tau \sim \mu^{-1} \] (Easy to see soon)

But cosmological evolution hard to see in this frame. E.g. Friedmann equation becomes

\[ \frac{H^4}{12(\dot{H} + 2H^2)^3} (2H\ddot{H} + 15H^2\dot{H} + 2\dot{H}^2 + 6H^4) = \frac{\rho_M}{M_p^2} \]

Fortunately can transform to an Einstein frame

The Einstein Frame

Make a conformal transformation

\[ p(\phi) = \exp\left(\frac{2\phi}{3M_p}\right) = 1 + \frac{\mu^4}{R^2} \]

\[ \tilde{g}_{\mu\nu} = p(\phi)g_{\mu\nu} \]

\[ \ddot{t} \equiv \sqrt{p} dt \]

\[ \tilde{a}(t) \equiv \sqrt{p} a(t) \]

\[ \tilde{U}_a \equiv \sqrt{p} U_a \]

\[ \tilde{\rho}_M = p^{-2} \rho_M \]

\[ \tilde{P}_M = p^{-2} P_M \]

In the ``tilde’’ frame, this becomes a theory of Einstein gravity, minimally coupled to a scalar field, with a potential and which is nonminimally coupled to matter.
Einstein-Frame Dynamics

\[ \ddot{H} = \frac{1}{3M_p^2} \left( \rho + \bar{\rho}_M \right) \]

\[ \phi'' + 3H\phi' + \frac{dV}{d\phi} \frac{(1-3w)}{M_p \sqrt{6}} \bar{\rho}_M = 0 \]

\[ \rho_\phi = \frac{1}{2} \phi'^2 + V(\phi) \]

\[ \bar{\rho}_M = \frac{C}{a^{3(1+w)}} \exp \left[ \frac{-(1-3w)\phi}{\sqrt{6}M_p} \right] \]

\[ V(\phi) = \mu^2 M_p^2 \sqrt{\frac{p-1}{p^2}} \]

Our de Sitter solution - instability is now obvious

Some features of quintessence

Solutions-I

Clear in the matter frame - no Minkowski solution (can also be seen in the Einstein frame - c.f. usual case of a scalar coupled to gravity)

Solve equations in Einstein frame and transform back to physical frame.

1. \( \phi \) sits at maximum. Solution is de Sitter in both frames, but is unstable \( \tau \sim \mu^{-1} \). Nevertheless, if had a good reason to start there, dS acceleration might survive to today.
Solutions-II

2. $\phi$ begins to the left of the maximum, with insufficient velocity to get over the hump. N.B. matter helps!

$\phi'' + 3H \phi' + \frac{dV}{d\phi} - \frac{(1 - 3w)}{M_P \sqrt{6}} \bar{\rho}_M = 0$

Yields a future curvature singularity in physical frame, not a Minkowski vacuum!

Not helpful cosmologically.

Solutions-III

3. $\phi$ gets over the maximum (N.B. this can happen even if starts slightly to left of maximum with zero velocity!)

Then, $\phi$ rolls down potential and asymptotic soln is easy to find...
An Accelerating Universe!

Asymptotically, \( V(\phi) \sim \mu^2 M_p^2 \exp\left(-\frac{3}{2} \frac{\phi}{M_p}\right) \)

So can solve to get

\[ \ddot{a}(t) \propto t^{4/3} \]

in the Einstein frame, which yields

\[ a(t) \propto t^2 \]

in the matter frame

This is power-law acceleration! Recall, \( w \) is always inferred from the expansion, so this is like having an instantaneous equation of state parameter

\[ w_{\text{eff}} = -\frac{2}{3} \]

Facing the Data

Remember data yields \(-1.45 < w_{\text{eff}} < -0.74\)

but I was using \( n=1 \) for illustrative purposes.

More generally

\[ S = \frac{M_p^2}{2} \int d^4x \sqrt{-g} \left( R - \frac{\mu^{2(n+1)}}{R^n} \right) + \int d^4x \sqrt{-g} L_M \]

Analysis is very similar for \( n>1 \), with similar potential in Einstein frame. Yields, in matter frame

\[ a(t) \propto t^q \]

with

\[ q = \frac{(2n+1)(n+1)}{(n+2)} \]

Again, this is like

\[ w_{\text{eff}} = -1 + \frac{2(n+2)}{3(2n+1)(n+1)} \]

Easily fits data for many \( n \) (approaches dS for \( n \to \infty \))
Problems with the Simplest Models

Easy to see model has problems agreeing with GR on scales smaller than cosmology. Can map theory to

\[ S \propto \int d^4 x \sqrt{-g} \left[ \Phi R - \frac{\omega}{\Phi} \partial_\mu \Phi \partial^\mu \Phi - U(\Phi) \right] \]

i.e., a Brans-Dicke theory, with a potential that we may ignore, with \( \omega = 0 \)

But solar system measurements constrain \( \omega > 3500 \)

However, more complicated models seem to work OK

Phantom Conclusions

• If quantum decay into gravitons is to occur with a rate longer than age of universe then effective \( w_Q < -1 \) theory must be valid only below \( 10^{-3} \) eV in most optimistic case

• The onus is squarely on phantom model builders to show how any specific proposal avoids rapid vacuum decay.

• Either way, if \( w_Q < -1 \), implications for fundamental physics are profound.
Modified Gravity Status

- Have demonstrated that cosmic acceleration may arise from the gravitational sector.
- Simplest model fails solar system tests, but more complicated models seem to work OK
- Much more work in progress

Some Questions

- There are claims that one can get "phantom" behavior from string theory. Does this make sense?
- Does it make sense to think of purely gravitational effects as responsible for cosmic acceleration?
- What does it mean to have a theory in which corrections only appear at low energy?
- Any connection to the idea of UV/IR relationships?
- ...