Running Spectral Index, Inflation & Gravity Waves — an astrophysicist's perspective

Outline

- Is there evidence for a running spectral index?
  - a discussion on the Lyman-alpha forest.
- Is $\phi^4$ inflation potential ruled out?
  - a discussion on the number of e-folds.
- Gravity waves
Useful things to keep in mind:

1. CMB is not the whole story!

To measure $P(k)$ accurately, a large range in scales is naturally useful.

**Note:** $1$ pc = $3$ light yr

$10^3 \sim \ell^2$

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Useful things to keep in mind:

2. Your favorite theory might have large scale structure implications.

E.g. A. $W$ vs. $\tilde{W}$

B. Proposals of modifying gravity on large scales

C.f. Sean’s talk
Running or not?

Primordial $P(k) \propto k^{n_s-1}$

$\left( n_s(k) - 1 \right) = \left( n_s(k_0) - 1 \right) + \frac{dn_s}{dk} \frac{k_0}{k}$

From WMAP (Spergel et al.):

$k_0 = 0.05$ Mpc\(^{-1}\)

\[ n_s \]

\[ \frac{dn_s}{dk} \]

<table>
<thead>
<tr>
<th>WMAP</th>
<th>$0.93 \pm 0.07$</th>
<th>$-0.047 \pm 0.04$</th>
</tr>
</thead>
<tbody>
<tr>
<td>WMAP+ &amp; 2dF</td>
<td>$0.93 \pm 0.04$</td>
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<td>galaxies</td>
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<tr>
<td>WMAP+ &amp; 2dF &amp; Lyν forest</td>
<td>$0.93 \pm 0.03$</td>
<td>$-0.031 \pm 0.016$</td>
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</tbody>
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Cautionary note:

- When running is large, the whole expansion is suspect:
  i.e. one would like:
    \[ n_s(k_0) - 1 \gg \frac{dn_s}{dk} \frac{k_0}{k} \]
    \[ 0.07 \quad -0.031? \]

- Or see it this way:
  \[ n_s - 1 \sim O\left( \frac{V'}{V}^2, \frac{V''}{V} \right) \]
  \[ M_{Pl} = \]
  \[ \frac{dn_s}{dk} \sim O\left( \frac{V'}{V} \times \frac{V''}{V} \right) \]
  
  **usual assumption:** \[ \frac{dn_s}{dk} \sim (n_s - 1)^2 \]
Lyman $\alpha$ absorption physics

- $n_1 \to n_2$ transition
  
  $1216 \text{ Å}$

- $\delta t = n_{HI} \delta x \, dl$
  
  $e^{-t} = \text{probability of transmission}$
  
  $\propto \text{observed flux}$

- $t \sim \sigma^2$

The program:

1. measure $P(\nu)$ of $e^{-t}$
2. infer $P(\nu)$ of $\delta$. 

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Fig. 1.— High resolution (FWHM $\approx 6.6$ km s$^{-1}$) spectrum of the $\sigma_{56}$ QSO 1422+23 (V $= 16.5$), taken with the Keck HIRES (signal-to-noise ratio $\approx 150$ per resolution element, exposure time 25000 s). Data from Womble et al (1996).
Analysis of Lyman α forest from SDSS

Abazajian
Bernardi
Burles
Cen
Dodelson
Frieman
Hui
Lidz
McDonald
Schlegel
Seljak
Stebbins
Subramo
Weinberg
York
\[ P_{10}(k_\parallel) = \int P_3(k_\parallel, \vec{k}_\perp) \frac{d k_\perp}{(2\pi)^2} \]

\[ \Delta^2 (k_p = 0.008 \text{ s/km}, z=2.85) \]

\[ \Delta^2 = \frac{4\pi k^3 P}{(2\pi)^3} \]

\( P = \text{mass power spectrum (linear)} \)
Complications to keep in mind.

- The program of $P_{\text{flux}} \rightarrow P_{\text{mass}}$ involves several "nuisance" parameters, e.g. $T$, $\alpha$, $J$

- The ionizing background might introduce additional fluctuations.
  - Need for consistency checks
    - Linear growth
    - Hierarchy of correlations
      \[ \tilde{\xi}_2 \sim \tilde{\xi}_1^2 \]
      \[ \tilde{\xi}_N \sim \tilde{\xi}_1^{N-1} \]
Fig. 4. — The PDF of $F$ for the observations (histograms) and for the simulation with the continuum-fitting approximation (filled points) and without it (open points). The small number of points outside the displayed range of $F$ are included in the outermost bins. Error bars were generated by bootstrap resampling. The numerical simulation has $F$ fixed to agree with the observations. (a) shows $z = 3.89$, (b) shows $z = 3.90$, and (c) shows $z = 2.41$. 

Lidz et al.

McDonald et al. 99
Is a $\phi^4$ inflation potential ruled out? (chaotic inflation)

- Peiris et al. ruled out $\phi^4$ from WMAP data by setting $N=50$ ($k=0.002 \text{ Mpc}^{-1}$)
- Barger et al. found that $\phi^4$ can be made consistent with data by taking $N$ large enough (see also Kinney et al.)

Fig. 1: The fluctuating density field has both long and short wavelength modes. The short wavelength modes in an overdense region caused by a long wavelength mode effectively evolve as if they are in a universe with a higher mean density, hence they evolve faster. The opposite is true for short wavelength modes in an underdense region. This effect creates a correlation between the small scale power and the large scale density fluctuations.
Some background.

- Scalar $P \sim H^2/\epsilon$
  \[ \epsilon \sim \left( \frac{\nu'}{\nu} \right)^2 \]
- Tensor $P \sim H^2$
- $n_s - 1 \sim O(\epsilon, \frac{\nu''}{\nu})$
- $n_T \sim \epsilon$
- Tensor/Scalar $\epsilon \approx r \approx \epsilon$

- Observations tell us $n_s - 1$ & $r$ are small.

- E.g. for $\phi^4$ chaotic inflation:
  \[ \epsilon \sim \frac{1}{\phi^2} \]
  \[ \uparrow \phi \Leftrightarrow \downarrow (n_s - 1) \text{ & } r. \]

- Alternatively, think of $n_s - 1 = -\frac{3}{N}$
  \[ r = \frac{1}{\sqrt{N}} \]

where $N =$ # of e-folds before end of inflation at which mode exits horizon (for $k = 0.002$ Mpc/sr).

$N_s$ = total # of e-folds of inflation.

Barger, Lee, Maartens 03
Limits on gravity wave (\(\sim \frac{H}{M_{pl}}\))

\[ H < 3 \times 10^{14} \text{ GeV} \]

\[ N \leq 60 \]

(for \(\phi^4\): \(N < 62\))

**Subtleties:**

\[ N < 60 + \frac{1}{2} \int_0^N E(N) \, dN' + \ln \left( \frac{0.002 \, M_{pl}'}{k} \right) \]

Bound weakens if \(g\) redshifts faster than radiation after inflation.

- Complications in reheating, change in d.o.f.
- late entropy production, etc

Liddle & Leach 23
Dodelson, LH 03

Interesting thought: we have some observational handle on the ‘desire’ between nucleosynthesis & inflation.
Upshot for $\phi^4$:

Barger et al.: $N$ has to $> 60$
to be consistent with data. (35)

Kinney et al.: $N$ has to be $> 66$
  to be consistent (35)

\textit{i.e. some tension}
  but perhaps not firmly
   ruled out.

\textbf{Lesson: Data are now good}
   enough for us to
   care about $N$ if a few.

\textbf{Actually, the real lesson:}
  we need better models
  to rule out (or confirm)!
Useful things to keep in mind:

3. Gravity waves are more robustly predicted \((H/M_p)\).

What is the energy scale of your favorite model?

In particular, standard models predict

\[ \text{tensor/scalar} = - \frac{n_T}{2} \]

known as the consistency relation.

(see review of Copeland et al.)

Short distance physics might break it.

Brandenberger & Martin
Niepomuc & Kof"pf
Danielson
Easther, Greene, Kunze, Shin
Kooper, Kleban, Lawrence, Shenker
LH, Kunze.
Work in progress on Detection of gravity waves. (with Jia Zhang)

"gradient" vector
E mode scalar flux.

"curl" B mode vector or tensor flux.

Gravitational lensing takes a polarization "vector" & displaces it stochastically.

i.e. pure E \rightarrow mixture of E & B modes.

Lensing introduces a fundamental limit to detection of gravity waves:

\text{Emit.} > 3 \times 10^{15} \text{ GeV}

Knox \& Songe
Cooray et al.
Dr. Lam Hui, Fermilab (KITP 9-03-03) Running Spectral Index Inflation and Gravity Waves -- an Astrophysicist’s Perspective

- Singular Core
- Defect Lines Can’t End
- Nematic is Greek for Thread

Topological charges in a polarization/shear map

\[ \text{Charge} = \frac{1}{2\pi} \quad \text{(net rotation of polar as you traverse a closed loop counterclockwise)} \]

Incremental counterclockwise rot. \( > 0 \)
\nIncremental clockwise rot. \( < 0 \)

Can combine:\n\[ \frac{1}{2} + \frac{1}{2} = 1 \]

Can cancel:\n\[ -\frac{1}{2} + \frac{1}{2} = 0 \]

Dolgov et al. 99
Vachaspati & Lue 03
LH Zhang 03

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J. Sethna 95
- Density of charges tells us about $C_E + C_B$ prelensing:
  \[ n_{\Delta E^2} \sim 0.2 \frac{\int d^2 E E^{E+B}}{\int d^2 E E^{E+B}} \]

- If over-sample, $|\text{charges}| > \frac{1}{2}$ unlikely.

- Information is limited:
  - Smoothing & lensing don't commute (except when $\mathcal{L}_{\text{smooth}} \gg 1000$)
  - An experiment with $f_{\text{sky}} \sim 0.8$,
    \[ \Delta \rho \sim 30 \text{ mil-arcmin} \]
    can measure $\#$ to $10^{-2}$.

- Counting charges is related to genus, but genus doesn't add info. unless
  $\Delta T/\Delta \rho$ non-Gaussian.

- More generally, one-pt. moments $\langle \Delta^m \rangle$ invariant under lensing.