Transplanckian Physics and the CMB

Discussion Session
KITP
September 16, 2003

Basic Question

- Can string/planck-scale physics leave an imprint on the CMB?
- Can observations of CMB give insight/information about string/planck-scale physics?
Why Think Possible?

- Expansion Factor at least $(e^{50})(10^{28}) = 10^{54}$
- Size of Universe = $10^{10}$ light-years = $10^6$ Planck Lengths
- Ripples we see began sub-Planckian. Leave Imprint?
- Only way to know for sure: Calculate perturbations standard way. See if string/quantum gravity modifies in significant way.

Outline

- Standard Calculation.
- Possible Modifications. Affect on CMB.
- Criticisms/Responses.
**Spectrum: Standard Results**

- Consider Tensor Modes:
\[ ds^2 = a^2(\eta)(d\eta^2 - (\delta_{ij} + h_{ij})dx^i dx^j) \]

- Expand in Traceless Symmetric Modes:
\[ h_{ij}(\eta, x) = h_+ e_{ij}^+ + h_\times e_{ij}^\times \]

- Equations of Motion:
\[ u_k'' + \left( k^2 - \frac{a''}{a} \right) u_k = 0 \]
\[ (u_k = a(\eta) h_k) \]

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**Spectrum: Standard Results**

- Quantum Mechanics:
\[ \hat{u}(\eta, x) = 1/(2\pi)^{3/2} \int d^3k [\hat{a}_k(\eta)e^{ikx} + \hat{a}_k^+(\eta)e^{-ikx}] \]
\[ \hat{a}_k(\eta) = \alpha_k(\eta)\hat{a}_k(\eta_i) + \beta_k(\eta)\hat{a}_{-k}^+(\eta_i) \]
\[ \hat{a}_k(\eta) \rightarrow (\alpha_k + \beta_k^*)(\eta)\hat{a}_k(\eta_i) = u_k(\eta)\hat{a}_k(\eta_i) \]

- Measure of Fluctuation: Power Spectrum/Two Point Fcn
\[ P(k) = \frac{k^3}{2\pi^2 a^2} \langle 0 \mid u_k(\eta)u_k^+(\eta) \mid 0 \rangle_{\text{LateTime}} = \frac{k^3}{2\pi^2} \left| \frac{u_k}{a} \right|^2 \]
Spectrum: Standard Results

Tensor Perturbations:

Power Spectrum:

Boundary Conditions: Early times/short scales—Minkowskian:

$$ u''_k + \left( k^2 - \frac{a''}{a} \right) u_k = 0 $$

$$ P^{1/2}(k) = \sqrt{\frac{k^3}{2\pi^2}} a_k $$

$$ u_k(\eta) \rightarrow \frac{1}{\sqrt{2k}} e^{-ik\eta} $$

Mode Solutions:

- Spectrum: Standard Results
Spectrum: Standard Results

- Tensor Perturbations:
  \[ u_k'' + \left( k^2 - \frac{a''}{a} \right) u_k = 0 \]

- Power Spectrum:
  \[ P^{1/2}(k) = \sqrt{\frac{k^3}{2\pi^2}} \left| \frac{u_k}{a} \right|_{k=aH} \]

- Boundary Conditions: Early times/short scales—Minkowskian:
  \[ u_k(\eta) \rightarrow \frac{1}{\sqrt{2k}} e^{-ik\eta} \]

- Assumption: Standard physics applies unmodified on arbitrarily short scales.

Short Scale Imprints?

- Modify Dynamics
- Modify Boundary Conditions
SHORT SCALE IMPRINTS

- Modify Dispersion Relations:

- Short Scale Noncommutativity:
  - Chu, BRG, Shiu hep-th/0011241
  - Lizzi, Mangano, Miele, Pelosi hep-th/0203119
  - Tsujikawa, Maartens, Brandenberger hep-th/0307016

- Introduce minimum length:
  - Kempf and Niemeyer astro-ph/0103225
  - Easther, BRG, Shiu, Kinney hep-th/0104102, hep-th/0110226

- Higher Order Operators in Dynamics:
  - Kaloper, Kleban, Lawrence, Shenker hep-th/0201158
  - Shiu, Wasserman hep-th/0203113

SHORT SCALE IMPRINTS

- Modify Short Scale Boundary Conditions:
  - U. Danielsson, hep-th/0203198
  - Easther, BRG, Kinney, Shiu hep-th/0204129
  - Goldstein, Lowe hep-th/0208167
  - Martin, Brandenberger hep-th/0305161

- Controlled Effective Field Theories:
  - Burgess, Cline, Lemieux, Holman hep-th/0210233
  - Burgess, Cline, Holman hep-th/0306079
  - Kaloper, Kaplinghat hep-th/0307016
Philosophy

- **Question:** How far do we need to deviate from conventional physics to yield a (potentially) observable imprint on the CMB?

- **Question:** Is the required deviation remotely sensible?

- **Related Approach:** Let CMB speak for itself.

Modify Initial State

- **Usual Case:**
  \[
  \tilde{u}(\eta, x) = \frac{1}{(2\pi)^{3/2}} \int d^3 k [\hat{a}_k(\eta)e^{ikx} + \hat{a}_k^+(\eta)e^{-ikx}]
  \]
  \[
  \hat{a}_k(\eta) = \alpha_k(\eta)\tilde{a}_k(\eta_i) + \beta_k(\eta)\tilde{a}_{-k}^+(\eta_i)
  \]
  \[
  \tilde{a}_k(\eta) \rightarrow (\alpha_k + \beta_k^*)(\eta)\tilde{a}_k(\eta_i) = u_k(\eta)\tilde{a}_k(\eta_i)
  \]

  **Require:**
  \[
  \beta_k(\eta) = 0, \text{ as } \eta \rightarrow -\infty \quad u_k(\eta) = \frac{1}{\sqrt{k}} e^{-ik\eta}
  \]

- **Modify:**
  \[
  \beta_k(\eta_k) = 0
  \]

  \[
  u_{k, \text{mod}}(\eta) = A_ku_k(\eta) + B_ku_k^*(\eta)
  \]
Short Scale Modifications to CMB

- **Vacuum**: For each $k$, set vacuum at critical time $\eta = \eta_k$ when $k/a = p = 1/L_{\text{pl}}$, $\eta_k = -1/(L_{\text{pl}}Hk)$

- **Condition**: $\beta_k = 0$ at $\eta = \eta_k$. (Danielsson)
  That is: $|A_k|^2 = 1/(1 - |b_k|^2)$ with $b_k = -1/(-1 + 2i \eta_k)$

- **Then**:

  \[
  P_k = \frac{k^3}{2\pi^2} \left\{ u_k \right\}^2 = \left( \frac{H}{2\pi} \right)^2 (1 + |b_k|^2 - b_k e^{-2i\eta_k} - b_k^* e^{2i\eta_k}) \frac{1}{1 - |b_k|^2} \\
  P_k = \left( \frac{H}{2\pi} \right)^2 (1 - \frac{H}{M_{\text{pl}}} \sin(\frac{2}{H/M_{\text{pl}}})) \quad \text{As } \eta_k \to -\infty \text{ we recover standard result}
  \]

- (Easther, BRG, Kinney, Shiu)
  If not in deSitter space, modulation in $k$. Effects are $O(H/M_{\text{pl}})$. 

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Short Scale Modifications to CMB

- $L_{\text{string}} = 100L_{\text{pl}}$, power law inflation, exponent = 500
- Roughly normalized to CMB
- Size of deviation: $O((H/M_{\text{string}})^1)$
Impact on Observations

- $\Delta C_l / C_l$

Criticisms/Responses

- Problems with $\alpha$-vacua.
  - Not relevant: only modify of sufficiently long wavelength.
  - Problems addressed (?)

- Large backreaction (Tanaka, hep-th/0112431; Goldstein, Lowe):
  - Issue: $E$ (fluctuations)—relative to BD vacuum—on order of $V$ (field).
  - Energy of modes with wavelengths less than Hubble radius, at end of inflation, relative to BD vacuum large—collapse.
  - Resolution: $(M_{\text{string}})^4 (H / M_{\text{string}})^2 < (M_{\text{planck}})^2 H^2$, i.e. $M_{\text{string}} < M_{\text{planck}}$
Criticisms/Responses

- Large backreaction II: Thinking Inside the Box (KKLSS)
  - What are modes doing "before" creation?
  - To be relevant, need to modify modes up to $e^{65} M_{\text{string}}$. Large backreaction.
  - Responses:
    - Only think outside the box.
    - Look for signature.