Singular Corrections to the Fermi-liquid Behavior: 1D Physics in Higher Dimensions

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- Main subleading corrections to the Fermi liquid behavior are singular (non-analytic) functions of temperature, spatial scale
  Finite |Q|-singularity in the spin susceptibility changes the nature of the FM phase transition (→to integrate out or not …)

- Singularities result from 1D scattering processes embedded into D>1 space: link between D=1 and D>1.
  Precursors of the 1D—Luttinger-liquid-- behavior in D>1

- Naïve perturbation theory breaks down in 2D
  Re-summed perturbation theory:
  Fermi liquid survives but
  Non-Fermi-liquid features remain
  (non-Lorentzian spectral function)
Lowest energies:
Fermi liquid=Fermi gas with renormalized parameters

Fermi gas

\[ \frac{C}{T} = \gamma \]
\[ \chi_s(T = 0, Q = 0) = \chi^0_s \]

Fermi liquid

\[ \frac{C}{T} = \gamma^* \]
\[ \chi_s(T = 0, Q = 0) = \chi^*_s \]

What about not so low energies?
(next-order term in \( T / E_F \) )

\[ \frac{C}{T} = \gamma \left(1 - \frac{T^2}{E^2_F} \right) \]
\[ \chi_s(T, Q) = \chi^0_s \left(1 - \max \left\{ \frac{T^2}{Q^2}, \frac{Q^2}{E^2_F} \right\} \right) \]

\[ C = \gamma^* T + \ldots \]
\[ \chi_s = \chi^*_s + \ldots \]

Specific heat and spin susceptibility
are singular beyond the leading order

3D

\[ \frac{C(T)}{T} = \gamma - \Gamma T^2 \ln \left( \frac{E_F}{T} \right) \]
\[ \chi_s(Q) = \chi_0 + \beta Q^2 \ln \left( \frac{k_F}{|Q|} \right) \]

Eliashberg 63, Doniach & Englesberg 66
Amit, Kane, Wagner 68 ...
Belitz, Kirkpatrick, Vojta 97

2D

\[ \frac{C(T)}{T} = \gamma - \Gamma T \]
\[ \chi_s(Q) = \chi_0 + \beta \frac{Q}{|Q|} \]

Coffey & Bedell 93, Chubukov & DLM 03,
Das Sarma et al. 03
Belitz, Kirpatrick, Vojta '97
Chitov & Millis 01
Chubukov & DLM 03

1D

\[ \frac{C(T)}{T} \propto \ln T \]
\[ \chi_s(Q) \propto \ln |Q| \]

Dzyaloshinskii & Larkin 72
Japaridze & Nersesyan 83
Non-analytic term near the FM quantum critical point

Chubukov, Pepin, Reich 04

Spin- Fermion model, 2D

\[ \chi_s(q) = \frac{\chi_0}{q^2 - 0.17q^{3/2}p_F^{1/2}}. \]

3D: \[ \frac{C(T)}{T} = \gamma - \alpha T^2 \ln \left( \frac{W}{T} \right) \]

He 3

Abel et al. 66

“Paramagnon anomaly”

Stewart 84

UPt3
2D

\[ \frac{C}{T} = \gamma - \Gamma_{2D} T \]

He3 monolayer on graphite

**FIG. 4.** \( \Gamma_{2D} n \) vs \( m^*/m \), where \( \Gamma_{2D} \) is a coefficient of the \( T^2 \) term and \( n \) is the number of \(^3\)He atoms per unit area.

*Casey et al. 03*

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**Singularities in transport properties of disordered metals**

**Weak localization**

\[ \delta\sigma_{WL} \propto |\omega|^{1-D/2} \]

*Gor'kov, Larkin, Khmel'nitskii 79*

**Altshuler-Aronov effect:**
interaction corrections to tunneling density of states and conductivity

*Altshuler & Aronov 79*

...  
*Matveev, Glazman, Yue 94*  
*Aleiner, Glazman, Ruzin 97*  
*Zala, Narozhny, Aleiner 01*
Fermi Liquid:

\[ \text{Im} \Sigma \propto (\pi T)^2 + \omega^2 \]

NOT an expansion in \( \omega^2 \)

3D: \( \text{Im} \Sigma(\omega) \propto \omega^2 + |\omega|^3 + \ldots \)

2D: \( \text{Im} \Sigma(\omega) \propto \omega^2 + \omega^2 \ln |\omega| \)

1D: \( \text{Im} \Sigma(\omega) \propto \omega^2 + |\omega| \)

**Analytic, “Fermi liquid” \( \omega^2 \) and singular terms come from different processes**

\[ \text{Im} \Sigma \propto \omega^2 : \]

\[ Q \sim \Lambda \]

\[ \Omega \sim \omega \]

Non-analytic part of \( \text{Im} \Sigma \)

\[ Q \sim |\omega| / v_F \]

\[ |Q - 2k_F| \sim |\omega| / v_F \]

\[ \Omega \sim \omega \]
\[ \text{Im} \Sigma_{\text{sing}} (\omega) \propto \int_0^\omega d\Omega \int_{\Omega / v_F} dQ Q^{D-1} \frac{\Omega}{Q^2} = \omega^2 \omega^{D-2} \]

<table>
<thead>
<tr>
<th>type</th>
<th>3D</th>
<th>2D</th>
</tr>
</thead>
<tbody>
<tr>
<td>![dynamic forward scattering icon]</td>
<td>Yes (``old'' paramagron term)</td>
<td>No</td>
</tr>
<tr>
<td>![1D dynamic forward or backscattering icon]</td>
<td>Yes (``new'' paramagron contribution)</td>
<td>Yes</td>
</tr>
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"any-angle" scattering event $\rightarrow$ Regular (FL) contribution
1D processes embedded into D>1 space

\[ \alpha \sim |\omega| / E_F \]

\( \text{“Infrared catastrophe” in D=2} \)
Breakdown of the naïve perturbation theory.
Features in single-particle properties (spectral function)

- Momentum transfer is either near 0 or 2k_F
- Total momentum is near 0

Singular terms in thermodynamics

Q=0 scattering

Regular part: \( k \) and \( p \) are not correlated

\[ \Pi(Q, \Omega) \propto \frac{\Omega}{Q} \]
comes from \( k \perp Q \)

Likewise, \( p \perp Q \)

Singular part: \( k \) and \( p \) are either parallel or anti-parallel

Parallel \( k \) and \( p \) do not contribute
\[ C(T) / T = \gamma \left(1 + F_1^s\right) - a \left[A_s^2(\pi) + 3A_a^2(\pi)\right] T \]

\[ A_s(\pi) = \sum_n (-)^n \frac{F_n^s}{1 + F_n^s} < 1 \]

\[ \chi_s(Q, T) = \frac{\chi_0(1 + F_1^s)}{1 + F_0^a} + bA_a^2(\pi) \max \left\{v_F \mid Q \mid, T\right\} \]
Infrared catastrophe: 1D
Bychkov, Gor’kov & Dzyaloshinskii 66

1D + linearized spectrum →
on-shell fermion can emit an infinite number of soft bosons:
  charge and spin density fluctuations

\[ \omega, k \quad \text{On-shell: energy and momentum conservations are the same} \]

\[ \Omega = v_F Q \]

\[ \sum \sim \frac{\omega^2}{\omega - v_F (k - k_F) + i\delta} \]

Does not renormalize the wave function but leads to spin-charge separation

Infrared catastrophe: 2D

\[ \text{Im} \Sigma(\omega, k = k_F) \sim g^2 \omega^2 \ln |\omega| \]

Linearized dispersion:

\[ \varepsilon_k = v_F (k - k_F) \]

Mass-shell singularity

\[ \text{Im} \Sigma(\omega, k) \sim g^2 \left[ \omega^2 \ln |\omega - \varepsilon_k| + \omega^2 \ln |\omega + \varepsilon_k| \right] \]

Castellani, di Castro, Metzner 94
Metzner 98
Chubukov and DLM 03

Finite curvature of the spectrum → finite mass

Singularity is regularized at \[ |\omega - \varepsilon_k| \sim \omega^2 / E_F \]

\[ \text{Im} \Sigma \sim g^2 \omega^2 \ln |\omega| \]
3rd order and beyond ...

\[ \text{Perturbation theory must be re-summed even for } u \to 0 \]

Finite curvature does not help: series diverges in the infrared

\[ \omega < u^2 E_F \]

Re-summmed perturbation theory: (asymptotically) exact

\[ \Sigma_F (p) = \frac{1}{2} \int_q G(p-q) \times \left[ 4U - 2U^2 \Pi(q) + \frac{U}{1-U \Pi(q)} - \frac{3U}{1+U \Pi(q)} \right] \]

\[ \Omega = cQ \]

\[ \Omega = v_F Q \]

\[ c = v_F (1 + U^2 / 2 + ...) \]
Main subleading corrections to the Fermi liquid behavior are singular (non-analytic) functions of temperature, spatial scale ...

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A. V. Chubukov, DLM, S. Gangadharaiah cond-mat/0412283, 0501013
Long-range dynamic interaction

\[ V_{\text{eff}}(\Omega, Q) \sim g^2 \Pi(Q, \Omega) \]

Singularity near \( Q=0 \) \( \rightarrow \) long-range dynamic interaction

\[ V_{\text{eff}}(\Omega, r) \sim \frac{\Omega}{r^{D-1}} \]

Singularity near \( Q=2k_F \) \( \rightarrow \) dynamic Friedel oscillation

\[ V_{2k_F}^{\text{eff}}(\Omega, r) \sim \frac{\Omega}{r^{D-1}} \sin 2k_F r \]

Non-analytic term near the quantum critical point

*Chubukov, Pepin, Reich 04*

Spin-Fermion model

\[ H = \sum_{p, \alpha} v_F (p - p_F) c_{p, \alpha}^\dagger c_{p, \alpha} + \sum_q X_{x_0}^{-1}(q) S_q S_{-q} + g \sum_{p, q, \alpha, \beta} \sigma_{\alpha, \beta} c_{p+q, \alpha}^\dagger c_{p, \beta} S_q. \]

\[ X_x(q) = \frac{X_0}{q^2 - 0.17 |q|^{3/2} p_F^{1/2}}. \]

\[ q < (0.17)^2 p_F \]

\[ q \propto \omega^{1/3} \sim T^{1/3} \]

\[ T < (0.17)^6 E_F \]
Sr$_2$RuO$_4$: $\chi_s(T) = \chi_s(0) + AT$

$\Delta_z \ll 1/\tau_{\text{imp}}$

$\Delta_z \gg 1/\tau_{\text{imp}}$

Figure 7.5: Typical dependence of the spin susceptibility on the in-plane magnetic field, measured for n-Si-MOS sample at $T \approx 0.15K$. Density is given in units of $10^{14}$ cm$^{-2}$.

Data: Gershenson, Kojima & Pudalov 04

in ALL dimensions, $\chi_s(Q)$

1) non-analytic in $Q$

b) peaked at finite $Q$

$1/|\ln Q|^{1D}$

$Q^2 |\ln Q|^{2D}$

$|Q|^{3D}$

$\chi_s$
In 3D

\[ \delta C(T) \propto T^3 \log T, \quad \bar{a} / (T) \propto T^2, \quad \bar{a} / (Q) \propto Q^2 \log Q \]

This is not the same log as in paramagnon theory

**Paramagnon story**

\[ \Omega \propto T \sum \int d^3 q \Pi^2 (q, \omega) \]

\[ \Pi \propto \frac{\omega}{q}, \quad \Omega \propto T \sum \omega^3 \log \omega \]

**Our story**

\[ \Omega \propto T \sum \int d^3 q \Pi^2 (q, \omega) \]

\[ \Pi \propto \frac{\omega}{q} + \left( \frac{\omega}{q} \right)^2, \quad \Omega \propto T \sum \omega^3 \log \omega \]

Doniach et al,
Larkin et al, ...

\[ -k \quad -k \quad \frac{q}{2} \approx 2k_F \quad \frac{k}{2} \]

\[ \frac{k}{2} \quad \frac{k}{2} \]

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