Design and realization of exotic quantum phases in atomic gases

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Atomic quantum gases

Bose-Einstein condensation
- Gross-Pitaevskii equation
- non-linear dynamics

Quantum degenerate \textit{dilute} atomic gases of fermions and bosons

Rotating condensates
- vortices
- fractional quantum Hall

Molecules
- Feshbach resonances
- BCS-BEC crossover
- dipolar gases

Optical lattices
- quantum information
- Hubbard models
- strong correlations
- exotic phases

control and tunability
Atomic gases in an optical lattice

Preparation
- lattice loading schemes
- controlled single particle manipulations (entanglement)
- decoherence of qubits

Thermodynamics
- Hubbard models
- design of Hamiltonians
- strongly correlated many-body systems

Measurement
- momentum distribution
- structure factor
- pairing gap
- ...

Ring exchange interaction

Exotic phases?
Bose-Hubbard tool box
Optical lattices

- AC Stark shift

\[ V(x) = V_0 \sin^2 kx + ... \]

- standing laser configuration

\[ E_r = \frac{\hbar^2 k^2}{2m} \sim 10 \text{kHz} \]

\[ V_0/E_r \sim 50 \]

- characteristic energies

- high stability of the optical lattice

1D, 2D, and 3D Lattice structures

Internal states

- spin dependent optical lattices

- alkaline earth atoms
Control of interaction

Interaction potential:
- effective range
  \[ r_0^3 n \ll 1 \]
- pseudo-potential approximation

Scattering properties
- scattering amplitude:
  \[ f(k) = -\frac{1}{1/a_s + ik} \]
- bound state energy \( a_s > 0 \):
  \[ E_M = -\frac{\hbar^2}{ma_s^2} \]

Tuning of scattering length
- changing the first “bound state” energy via an external parameter
  - magnetic Feshbach resonance
  - optical Feshbach resonance

\[ a_s \sim 10^2 a_0 \]
\[ \sim -\frac{c_6}{r^6} \quad \frac{r}{a_0} \]
Bohr radius
Microscopic Hamiltonian

\[ H = \int dx \, \psi^+(x) \left( -\frac{\hbar^2}{2m} \Delta + V(x) \right) \psi(x) + \frac{g}{2} \int dx \, \psi^+(x) \psi^+(x) \psi(x) \psi(x) \]

optical lattice

\[ g = \frac{4\pi \hbar^2 a_s}{m} \]

 interaction strength

- strong optical lattice \( V > E_r \)
- express the bosonic field operator in terms of Wannier functions
- restriction to lowest Bloch band (Jaksch et al PRL ‘98)

\[ \psi(x) = \sum_i w(x - x_i) b_i \]
Bose-Hubbard Model

Bose-Hubbard model (Fisher et al PRB '81)

\[ H_{BH} = -J \sum_{\langle i,j \rangle} b_i^+ b_j + U/2 \sum_i b_i^+ b_i^+ b_i b_i \]

- hopping energy
- interaction energy

Phase diagram

- Mott insulator
  - fixed particle number
  - incompressible
  - excitation gap
- superfluid
  - long-range order
  - finite superfluid stiffness
  - linear excitation spectrum

\[ U \sim E_r a_s / \lambda \]
\[ J \sim E_r e^{-2 \sqrt{V/E_r}} \]
Experiments

Long-range order:

Disappearance of coherence for strong optical lattices (Greiner et al. '02)

$\frac{V}{E_r} > 13$

Structure factor

Appearance of well defined two particle excitations

(Esslinger et al., 04)
Ring exchange interaction
Ring exchange

- bosons on a lattice

\[ H_{R-E} = K \left[ b_1^+ b_2 b_3^+ b_4 + b_1 b_2^+ b_3 b_4^+ \right] \]

Applications:

Dimer models

- spin liquids, VBS - phases
- topological protected quantum memory

2D spin systems

- Neel order versus VBS
- deconfined quantum critical points

Lattice gauge theories

- U(1) lattice gauge fields
- a model QED
Ring exchange

Toy model:
- bosons on a lattice
- resonant coupling to a molecular state via a Raman transition
  - molecule is trapped by a different optical lattice

Effective coupling Hamilton

\[
H = \nu m^+ m + g \sum_{i \neq j} c_{ij} \left[ m^+ b_i b_j + m b_i^+ b_j^+ \right]
\]
**Ring exchange**

First internal state

- Bosonic atoms in the corners of the square
- Bose-Hubbard model

Second internal state

- Trapped in the center of the square
- Quenched hopping
- Angular momentum $l = 0, \pm 1, 2$
- Interaction allows for a molecular state
Symmetries

- Hamilton is invariant under operations of the $C_{4v}$
- symmetries of single particle states $a_l$

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Energy levels

- design of optical lattice
- tune with the Raman transtition close to a $s$-wave molecule in the $d$-wave vibrational state
- $d$-wave symmetry for molecular state

$$m^+ = ca_2^+a_0^+ + d [a_1^+a_1^+ + a_{-1}^+a_{-1}^+] \ldots$$

- integrate out single-particle states $a_l$
Ring exchange

Toy model:
- bosons on a lattice
- resonant coupling to a molecular state via a Raman transition
  - molecule is trapped by a different optical lattice

Effective coupling Hamilton

\[ H = \nu m^+ m + g \sum_{i \neq j} c_{ij} \left[ m^+ b_i b_j + m b_i^+ b_j^+ \right] \]

\[ m^+ [b_1 b_3 - b_2 b_4] + c.c. \]
Ring exchange

Effective low energy Hamiltonian

$$H = \nu m^+ m + gm^+ [b_1 b_3 - b_2 b_4] + gm [b_1^+ b_3^+ - b_2^+ b_4^+]$$

Relation to Ring exchange

- integrating out the molecule

$$H = K \left[ b_1^+ b_2 b_3^+ b_4 + b_1 b_2^+ b_3 b_4^+ - n_1 n_3 - n_2 n_4 \right]$$

- perturbation theory

$$K = \frac{g^2}{\nu}$$
Hamiltonian on a lattice

- add hopping for the atoms
- half-filling for the bosons

\[
H = -J \sum_{\langle ij \rangle} b_i^+ b_j + \nu \sum_i m_i^+ m_i + g \sum_i m_i^+ [b_1 b_3 - b_2 b_4] + m_\square [b_1^+ b_3^+ - b_2^+ b_4^+]
\]

Superfluid

\[J \gg K\]
- superfluid of bosonic atoms
- long-ranger order

Molecules

\[J \ll K\]
- formation of molecules
- non-trivial structure due to d-wave symmetry

- intermediate regime
- quantum phase transition?
- exotic phases?
Lattice gauge theory

2D lattice gauge theory

- atoms on links with ring exchange and quenched hopping
- gauge transformation
  \[ b_{\langle nm \rangle} \rightarrow b_{\langle nm \rangle} e^{i[\chi(n) - \chi(m)]} \]
- represents a 2D dimer model

3D lattice gauge theory

- adding an additional dimension
- atoms on the links of the lattice
- molecules in the center of the faces
- pure U(1) lattice gauge theory exhibits a phase transition from the Coulomb phase to a confining phase
- presence of a Coulomb phase in the present model? (M. Hermele et al, PRB 2004)