An update on Cauchy-Characteristic Matching in General Relativity

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Cauchy Characteristic Matching (CCM)

What is it?

In this approach one covers space-time by a Cauchy and a Characteristic coordinate-patch.
Well-posedness of CCM

Does it make sense?

_Hopefully._
Well-posedness of CCM

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Well-posedness of CCM

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Does it make sense?

Hopefully.

One of the issues is the definition of the matching boundaries.
Well-posedness of CCM

Does it make sense?

Hopefully.

Another issue is that of the constraints of the two formulations.

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Well-posedness of CCM

Does it make sense?

Hopefully.

Yet another issue is that of consistent initialization.
Successful CCM schemes

Can it work?

Apparently.

Examples:

* 3D non-linear scalar evolution on a fixed Euclidean background

* 1D (non-linear) GR evolution, with Black Hole excision
  Gomez et al, PRD 56, 6310 (1997)

* [2D (non-linear) GR evolution – Southampton group]

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Can it work?

*It also works for 3D linearized GR.*

CCM in linearized GR

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CCM in linearized GR

**Details:**

**Cauchy evol:** linearized harmonic

**Cauchy bdry:** enforce 4 constraints

**Characteristic evol:**
- linearized Bondi
- Gauge evolution – this allows knowledge of the Bondi -> Cartesian Jacobian

**Extraction:** polynomial interp & lots of algebra

**Injection:** “Sommerfeld”-style, normal to faces of the cube

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CCM in linearized GR

Result:

\[ \phi (t = 0) \]
Result:
CCM in linearized GR

Result:

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Lessons learned so far:

- Need working Cauchy (and Characteristic) initial-boundary code(s)
- Need constraint preserving boundary conditions (Cauchy boundary equations)
- Need finite distance between Injection and Extraction (?)
- Need Sommerfeld style Injection
- Need parallelization (Yosef is getting there)
Status of the non-linear Cauchy code

**Numerical evolution scheme:**

- 1\(^{st}\) differential order in time, 2\(^{nd}\) differential order in space (Does anybody know how to keep it fully 2\(^{nd}\) differential order?)

- Discretization: finite differencing (2\(^{nd}\) or 4\(^{th}\) order accurate)

- Time-integration:
  - Iterative Crank-Nicholson
  - 3\(^{rd}\) and 4\(^{th}\) order accurate predictor-corrector schemes
  - 4\(^{th}\) order Runge Kutta

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Status of the non-linear Cauchy code

Numerical boundary scheme:

- A first major issue is the need of a good Neumann algorithm
  1) Sideways algorithm:

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Status of the non-linear Cauchy code

Numerical boundary scheme:

* A first major issue is the need of a good Neumann algorithm

1) Sideways algorithm:
   - works (too) well in 1D
   - uses future time-levels
   - no obvious (smooth) extension to edges
Status of the non-linear Cauchy code

Numerical boundary scheme:

* A first major issue is the need of a good Neumann algorithm
  1) Sideways algorithm
  2) “Evolution-based” algorithm:
     - simple stencil
     - good for edges
     - provides no boundary value for evolution variables

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Status of the non-linear Cauchy code

Numerical boundary scheme:

- A first major issue is the need of a good Neumann algorithm
  1) Sideways algorithm
  2) “Evolution-based” algorithm
  3) Characteristic algorithm:
     - no future levels
     - Generalization of 1\textsuperscript{st} diff. order schemes
     - no good edge algorithm

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Status of the non-linear Cauchy code

Numerical boundary scheme:

* A first major issue is the need of a good Neumann algorithm
  1) Sideways algorithm
  2) “Evolution-based” algorithm
  3) Characteristic algorithm

* The boundary equations ask for both the metric components and their Neumann derivatives at the boundary point. As a result, we plan to use a combination of the 2\textsuperscript{nd} and 3\textsuperscript{rd} algorithms.
Conclusion

- CCM works in linearized (harmonic) GR
- Implementation of a non-linear (harmonic) Cauchy evolution-boundary algorithm is well under way
- We already have:
  - A non-linear characteristic code
  - A non-linear extraction module (on 1 CPU)
- Still need to work out a non-linear injection module (in it's current setup)