Critical gravitational collapse
of a massive vector field

(Garfinkle, Mann and Vuille
gr-qc/0305014)

(1) critical gravitational collapse

(2) massive scalar field

(3) massive vector field paradox

(4) numerical simulations

(5) paradox resolved

Critical gravitational collapse

Choptuik’s discovery

collapse of a spherically symmetric scalar field $\phi$ initial data $\phi_p(0,r)$

for $p > p^*$ a black hole forms

for $p < p^*$ the field disperses

scaling of black hole mass:
for $p$ near $p^*$

$$ M = c(p - p^*)^\gamma $$

$\gamma \approx 0.374$
critical solution

The $p = p^*$ solution is discretely self-similar (DSS)

After a certain amount of time the profile of the scalar field repeats itself with the scale of space shrunk.

Massive scalar field

(Brady et. al PRD 56, 6057 (1997))

Choptuik solution is DSS This is a natural consequence of the scale invariance of the Einstein-scalar system.

What happens when we break scale invariance by treating a massive scalar field?
results of numerical simulations
two critical solutions
one periodic
one identical to that of the massless case.

How can the same solution be the critical solution for both the massless and massive cases?
terms in the stress-energy that depend on the mass depend on the amplitude of the field, while the other terms depend on the gradients of the field.
as the scale of space becomes small, the mass terms become negligible

Massive vector field paradox

What is the nature of critical collapse of a spherically symmetric massive vector field?

(1) the mass should be negligible so it should be the same as for a massless vector field.

(2) it can’t be the same as for a massless vector field (Maxwell field) because a spherically symmetric Maxwell field has no degrees of freedom
**Numerical simulations**

Three numerical methods

- Cauchy
- Characteristic
- Cauchy with AMR

**Cauchy codes**

use polar-radial coordinates

\[ ds^2 = -\alpha^2 dt^2 + a^2 dr^2 + r^2 d\Omega^2 \]

Initial data is \( X = (a/\alpha) A_t \) and \( A_r \).

solve constraints for field strength and metric components.
Use these to evolve \( X \) and \( A_r \).

use centered differences for spatial derivatives
use ICN for time evolution

use Kreiss-Oliger dissipation for added stability.

AMR Cauchy code is modified Choptuik EYM code
Characteristic code

use Christodoulou coordinates

\[ ds^2 = -e^{2\nu} du^2 - 2e^{\nu+\lambda} du dr + r^2 d\Omega^2 \]

Initial data on a null cone (u=const). is \( h = \partial_r (r\phi) \)
where \( \partial_r \phi = \mu A_r \)

integrate along the generators to find the other matter variables and the metric components.

evolve \( h \) along ingoing light rays. (essentially ODEs at each point)

points are lost when the corresponding light ray crosses the origin.
when half the points are lost replace them in between the remaining ones.
In this way resolution is maintained as the relevant distance scale shrinks.
Results

There are two critical solutions
one periodic and one DSS

The DSS solution is identical
to the Choptuik critical solution
for a massless scalar field

(effective scalar field $\phi$
defined by $\partial_r \phi = \mu A_r$
along outgoing null lines)
Paradox resolved

assume that

\[ A_a \to \frac{1}{\mu} P_a + \mu Q_a + \ldots \]

in the small \( \mu \) limit.

Then a (non-singular) limiting stress-energy requires that

\[ \nabla_{[a} P_{b]} = 0 \]

and therefore that \( P_a = \nabla_a \phi \)

for some scalar field \( \phi \).

The stress-energy of \( A_a \) then
goes over to the stress-energy of \( \phi \).

The \( \mu \to 0 \) limit of
a massive vector field
is a massless scalar field
(decoupling of longitudinal modes)