Experiments on Slow Granular Shear Flow - JC Tsai and Jerry Gollub

- Response to shear is a key to understanding any granular flow.
- We show that slow shear can trigger internal ordering or crystallization of the material.
- The ordering substantially changes the flow properties of the material.
- This makes modeling substantially more difficult.

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Background

- We are concerned with granular materials in the regime of persistent contacts. Kinetic energy $\ll$ elastic energy.
- Structural and rheological changes due to shear are well known in
  » Colloidal systems (Ackerman et al.; Pusey et al.)
  » Emulsions (Pine’s group)
  » Soft and hard sphere simulations (Sierou and Brady; Silbert and Grest)
- Generally, the effect of ordering on granular flow has not been included in theories.
Many Experiments on Sheared Dense Flows

- Komatsu et al. – 2d avalanches
- Miller, Ohern, and Behringer – local forces
- Mueth et al. - MRI shear studies (noticed layering)
- Older studies using our geometry
  » Hanes and Innman
  » Savage and Sayed
- A novel feature here is the use of optical index matching to allow detailed studies of the interior of the flow.

Slowly Sheared Granular Flow (JC Tsai) (0.02-4 s⁻¹) - Annular Plane Couette Flow

- Transparent glass beads, d=0.6 mm
- Refraction-index-matching hydrocarbon fluid
- Fluorescent dye

Driven by a rough, transparent top ring (not shown)

Fixed glass walls (smooth)

Circumference ≈ 600d, channel width ≈ 25d, depth = 0 - 40d.

≈16cm
**Experimental Setup – Cross Section**

- d=0.6mm glass beads in index matched fluid.
- Constant speed driving at fixed normal load.
- Sensitive volume and force measurement.

![Diagram of experimental setup](image)

\[ \text{\~30d, \ (Circumference \sim 800d)} \]

**Normal load } W >> \text{beads' total weight \& fluid's viscous drag}**  

**Experimental Information**

- Various lower boundary conditions: flat; monolayer of particles; rough mixture.
- Weight is imposed on the particles, but the fluid volume adjusts as needed.
- Viscosity of the fluid is about 10 cS, but hydrodynamic forces are negligible (except for reducing friction).
- Particle centers are found accurately using convolution methods.
Outline

1. Crystallization transition
2. Role of boundary conditions and shear protocol (history); multiple final states.
3. Velocity profiles and rheology
4. Particle dynamics (local motion)

The Initial State (with driving speed = 8 d/s)
1. Crystallization Transition
   -- movies

XZ slice:

(9hrs total @ ~900X)

XY slice (before transition)

XY slice (after transition)

Crystallization Transition
   -- time-resolved measurements

The ordering transition results in step changes of granular volume (↓).

What happens to the shear Force and velocity??
Crystallization Transition
-- time-resolved measurements

The ordering transition results in step changes of granular volume (\(\downarrow\)), shear force (\(\downarrow\)), and mean particle speed (\(\downarrow\)) (stronger decay downwards).

Crystallization Timescales

(Driven at the same speed:)

(i) Dependence on layer thickness:

(ii) Dry particles:
\[\text{\(\rightarrow\)} \text{ Ordering transition occurs, but takes much longer!}\]
How long before the transition occurs?

- Long enough that the SLOWEST particles translate by several particle diameters.
- This might require a displacement of $10^5 d - 10^6 d$ at the upper boundary.
- Without the interstitial fluid, more time is required.

Does the ordering depend on polydispersity?

Our size range is about 4% (standard deviation).
- Polydisperse mixtures segregate.
- In a binary mixture, local crystallization still occurs after segregation.
2. Role of Boundaries

Quantization Effects

** Final volume:

* Final volume:

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Boundaries: Multiple Final States

Irreg. bottom; final state is path-dependent.

Regular bottom; always orders.

Volume quantization found to exist for flows as thick as 23-24 layers.
Summary: Role of Lower Boundary (deep)

- A monolayer crystalline lower boundary favors ordering in the bulk.
- A very rough boundary will suppress the crystalline state. However, AC shear can still induce order.

Stochastic selection of final states

(Number of oscillatory cycles applied prior to one-way shearing:
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10)

--- partial ordering
Multiple States or Attractors (cont.)

- AC shear can nucleate the crystalline state even when it is otherwise suppressed.
  - but only if the the layer has not already been compacted by long term DC shear.
- The ordered state is always stable once found.

3. Velocity Profiles and Rheology Driving Rate Independence

Velocity profile of the final state, normalized by driving speed, is insensitive to a change of driving speed by a factor of 100 (or more)
Profiles: Comparing Different States

- Disordered state (on a bumpy bottom)
- Crystallized state (on a mono-layer bottom)
- Crystallized state (on a bumpy bottom)

System Height Dependence

- Linear Curves $1 - \frac{z}{H}$ (for reference)
- Measured Velocity Profiles:
  - 5-Layer Flow
  - 8-Layer Flow
  - 12-Layer Flow
  - 18-Layer Flow
  - 24-Layer Flow
Particle Size Dependence (flat bottom BC)

Vertical profiles of velocity for different particle sizes & comparison to that of ordinary fluid flows (along the center of the channel)

Origin of Shear Banding?

- Stress balance
  \[
  \left[ \sigma_{\text{gran}}(0) - \sigma_{\text{gran}}(z) \right] W = 2 \sigma_{\text{wall}} z
  \]

- Weakly strain rate-dependent granular stress:
  \[
  \sigma_{\text{gran}} = \sigma_o (1 + \alpha \ln(\dot{\gamma}(z)/\dot{\gamma}(0))
  \]

- Resulting profile:
  \[
  \dot{\gamma}(z) = \dot{\gamma}(0) \exp(-\beta z)
  \]
3D Structure of the Velocity Field:
(a) disordered; (b) ordered.

Cross-channel profile of time-averaged velocity of a thick-layer flow, at the height $z = -12d \sim -H_0/2$ (Movie)

Disordered State:

Ordered State:

($y = \text{distance from outer wall}$)

3D Structure of the Disordered” State
(partial ordering at sidewalls)

After 2 weeks of shearing at a driving speed $12d \text{ s}^{-1}$:

Multiple vertical slices
($y = W_0/3 \rightarrow W_0/6$)

Multiple horizontal slices
($z = -H_0/2 \rightarrow -1d$)
Summary - Flow Structure

- Steeper velocity falloff in the crystalline state; slip between planes reduces vertical momentum transfer even though it is denser.

- Limiting velocity profile as a function of depth in deep layers; faster than exponential; and faster than a fluid in the same geometry.

- WHY? Because of internal friction that is almost independent of the velocity gradient, and a slightly velocity dependent shear stress.

- Cross channel flow properties dramatically changed by crystallization.

4. Particle Trajectories: Ordered State

\[ x_i(t) \]

\[ y_i(t) \]
**Particle Trajectories** $y_i(t)$: ordered vs. disordered states

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**Shear-induced vertical "diffusion"**
(disordered state)

Root-mean-square vertical displacement vs. non-dimensional time $\gamma_{local} \Delta t$:

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Looks roughly diffusive though probability distribution is non-Gaussian

1 pixel = $d / 40$
Local motion: Why does AC shear induce order?

$V_x(t)$: inst. mean velocity of $\sim 90$ grains at $z = -12d \sim -H_0/2$.

Disordered state

Particles move freely at the instant of reversal because stresses are eliminated. Anomalous mobility prior to the recreation of a new stress network.

Overall Summary

- Super-exponential decay of velocity profile; creep motion to apparently arbitrary depths.
- Shear-induced ordering with depth-dependent delay.
- Very long time required to reach a steady state.
- Multiple final states, sensitive to boundary conditions and history.
- Ordered state has distinct flow properties.
- Shear-induced diffusion and segregation.
Some Questions

- Why is the crystalline state always stable once reached?
- Is there a critical polydispersity that will suppress ordering?
- How will binary or polydisperse systems evolve?
- Diffusion and mixing properties?
- How can we incorporate local order into theories of dense flows?

Preview: steady shearing of binary mixture

Binary mixture:
(d=1.0 mm and 0.6 mm), (25% / 75%) by weight;
some of the 1.0 mm grains are painted black as tracers.

(~3000X Real time)
Clustering due to Hydrodynamic Interactions

\[ f = 50 \text{ Hz} \]

\[ \Gamma = 4.5 \]

Structures and Chaotic Fluctuations of Granular Clusters in a Vibrated Fluid Layer

- Fluid mediated interactions between particles lead to forces between them.
- A variety of tunable patterns are produced, including chaotic states with interesting statistical properties.
- A phenomenon involving both continuum and discrete dynamics that exhibits novel physics.

Jerry Gollub, Charles Thomas, Ben Bigger, Greg Voth
2D Granular Posieuille Flow - Inclined

- 2-Phase Flow (2PF)
- Accelerating Gaseous Flow (AGF)
More Information

- Movies and preprint at haverford.edu:
  http://www.haverford.edu/physics-astro/Gollub/internal_imaging

- Published papers at PRL or Phys. Rev. E:
  PRL 91, 064301 (2003)