Ultracold Bosons and Fermions in Optical Lattices

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CQG people in Firenze

1) Rb BEC
Chiara FORT, Leonardo FALLANI, Francesco MINARDI, Jessica LYE, Francesco CATALIOTTI (also Ct), Jacopo CATANI, Luigi De SARLO

2) K – Rb Fermi-Bose MIXTURES
Giovanni MODUGNO, Giacomo ROATI, Herwig OTT, Francesca FERLAINO, Estefania de MIRANDES

Theory
Michele MODUGNO, Andrea SIMONI (now NIST)
Dynamical instability of a BEC
Energetic instability of a BEC
Transport properties of fermions and bosons in a lattice
(Role of collisions)
Atomic Interferometry with Fermions

Transport of BEC
in a 1D optical lattice
(Dynamical instability)
Ultrad cold bosons and fermions in optical lattices

Bose-Einstein condensates in optical lattices:
- Interference
- Quantum transport
- Strongly correlated systems
- Reduced dimensionality

Anderson & Kasevich, (1998), …
Cataliotti et al. (2001), …
Greiner et al. (2002), …
Florence, ETHZ, Munich, NIST

The interference pattern after expansion resembles that of an array of coherent dipole antennas

Ultracold bosons and fermions in optical lattices

**Transport in the lattice: dipolar oscillations**

Dipolar oscillations excited with a single kick

\[ \Delta x = 10 - 20 \mu m \]

**Array of Josephson junctions**

Collective dipole oscillation of a BEC in a harmonic trap + optical lattice

The oscillation frequency \( \omega^* \) is rescaled with an effective mass

\[ \omega^* = \frac{m}{m^e} \omega \]

dependent on the tunneling rate \( K \):

\[ m^e = \frac{2\lambda^2 m}{\hbar^2 K} \]

F. S. Cataliotti et al., Josephson junctions arrays with BECs, *Science*, 293, 843 (2001)
large amplitude dipole oscillations of a trapped BEC in presence of a 1D optical lattice:

The center-of-mass slowly moves towards the center of the magnetic potential

Loss of long range phase coherence

We observe a transition from a regime in which the wavepacket coherently oscillates in the array to another one in which the condensates stop in the optical potential sites and lose their relative phase coherence.


Dynamical instability of an array of condensates

\[ \Psi(\vec{r}, t) = \sqrt{N} \sum_{j} \psi_{j}(t) \phi_{j}(\vec{r} - \vec{r}_{j}) \]

GPE reduces to

Discrete Non-Linear Schrödinger Equation

\[ i\hbar \frac{\partial \psi_{n}}{\partial t} = -K \left( \psi_{n-1} + \psi_{n+1} \right) + \left( E_{n} + U |\psi_{n}|^{2} \right) \psi_{n} \]

\[ \omega_{q} = \frac{1}{m} \sin p \sin q + 2 \sin q \sqrt{\frac{1}{m^{2}} \cos^{2} p \sin^{2} q / 2 + \frac{1}{m} \frac{\partial}{\partial N} N \cos p} \]

When the Bogoliubov modes become imaginary \( \Rightarrow \) dephasing among different sites (no interference) \( \Rightarrow \) the wave suddenly stops (no oscillation)


Definitions and scales

**Optical lattice:** 
\[ V(x) = sE_{\alpha}\cos^2(kx) \]

- \( d = \frac{2}{2} = 0.39 \, \mu m \) 
  - lattice spacing
- \( E_{\alpha} = \frac{\hbar^2k^2}{2m} = \hbar \cdot 3.77 \, kHz \) 
  - recoil energy
- \( v_{B} = \frac{\hbar k}{m} = 5.80 \, mm/s \) 
  - Bragg velocity

**Bose-Einstein condensate of \(^{87}\text{Rb} \):**

- \( \omega = 2\pi \times 9 \, Hz \) 
  \( R_{z} = 75 \, \mu m \)
- \( \omega_{z} = 2\pi \times 90 \, Hz \) 
  \( R_{z} = 7.5 \, \mu m \)
- \( N \approx 10^5 \) atoms 
  \( |F = 1, m_F = -1 \rangle \)

A typical BEC extends on \(-10^3\) lattice sites:

\[ \Delta p \Delta z = \hbar \rightarrow \Delta p = \frac{\lambda}{2\pi \Delta z} = \frac{780 \, nm}{2\pi \cdot 150 \, \mu m} = 10^{-3} \]

The momentum spread of a BEC is a \( \delta \) in the momentum space.

Instabilities of a BEC in a moving optical lattice

We adiabatically switch on a moving optical lattice in order to load the trapped BEC in a state with well defined quasimomentum \( q \) and band index \( n \).

Since the time spent by the BEC in the lattice may be quite long (\( \approx 10 \) s) we use an RF-shield to remove the hottest atoms produced by heating of the sample.

After different evolution times in this potential we switch off both the magnetic trap and the lattice and measure the number of atoms in the BEC:

Exponential fit of number of atoms vs. time:
Dynamical instability of a BEC in a moving optical lattice

\[ \frac{i\hbar}{\partial t} \psi = \left( \frac{\hbar^2}{2m} \nabla^2 + sE_k \cos^2(kx) + s|\psi|^4 \right) \psi \]

Nonlinearities induced by atom-atom interactions cause the Bloch waves to be dynamically unstable. An exponential growth of perturbations may start, eventually leading to the destruction of the Bloch state.

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Dynamical instability of a BEC in a moving optical lattice

experiment
atom loss rate from the BEC

theory
growth rates of the most dynamically unstable modes

L.Fallani, L.DeSarlo, J.E.Lye, M.Modugno, R.Saers, C.Fort, M.Inguscio cond-mat 0404045
Dynamical instability of a BEC in a moving optical lattice

*Uniformity can be restored …*

![Image showing growth and relaxation of excitations](image)

Energetic instability of a BEC in a moving optical lattice

The presence of a small *thermal fraction* has a dramatic effect on the BEC lifetime in the moving lattice, causing the atomic sample to be completely destroyed in a much shorter time:

![Image showing pure BEC and 75% pure BEC](image)

We attribute this behavior to the onset of *energetic instability* (in an inhomogeneous system), occurring in the presence of dissipative processes, as those provided by the thermal fraction.
Energetic instability of a BEC in a moving optical lattice

In presence of a thermal fraction, as soon as the optical lattice starts to move, the number of condensed atoms surviving after long times strongly decreases.

A shallow optical lattice moving at low velocity works as a destructive detector of small thermal clouds surrounding the BEC.

Energetic instability timescale:
\[ \tau \approx 400 \text{ ms} \]
**Dynamics in a 1D optical lattice**

**FERMIOMS and THERMAL BOSONS**

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**Atomic quantum mixtures**


Ultracold bosons and fermions in optical lattices

**The Fermi-Bose system**

Fermi-Bose mixture in a magnetic trap

Numbers: \( N_f, N_b = 10^4\text{--}10^5 \)

Temperatures: \(< 0.5 \, T_c \quad 0.3 \, T_F \)

\[ T_c = \frac{\hbar \omega}{k_B} (N/1.2)^{1/3} = 150 \text{nK} \]

\[ T_F = \frac{\hbar \omega}{k_B} (6N)^{1/3} = 300 \text{nK} \]

Spin-polarized fermions: no collisions

Spin mixtures of fermions, bosons or mixtures of bosons and fermions: adjustable collisions

Roati, Riboli, Modugno, Inguscio, PRL 89, 150403 (2002).

**Fermions: shallow lattices**

The oscillation frequency is well described by the Bloch theory of motion in the lattice

\[ \omega_c = \omega \quad \frac{\Delta E}{\hbar^{2/3}} \]

Dephasing

Massimo Inguscio, U of Firenze & INFM (KITP Quantum Gases Conf 5/14/04)
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**Transport of thermal bosons**

Much larger damping in shallow lattices

\[ U = 0.08 \quad E_R \quad k_B T = 2 \quad E_R \]

Collisions can assist the dephasing mechanism.

**Fermions in two spin states**

Add some \( m_F = 7/2 \) atoms: two colliding Fermi gases

\[ N_{9/2} = 2 \times 10^4 \]

\[ s = 0.6 \]

\[ T/T_F = 0.5 \]

Larger damping due to the collisions as for thermal bosons.
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Collision-induced transport in periodic potentials

Dynamical instability of a BEC
Energetic instability of a BEC

Transport properties of fermions and bosons in a lattice
(Role of collisions)

**Atomic Interferometry with Fermions**

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**Interferometry with optical lattices**

- High vacuum cell
- Mirror
- Ultracold atoms
- Non resonant lattice
- CCD
- Resonant laser (imaging)
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**Wannier-Stark states**

Energy spectrum in presence of gravity:

\[ \Delta E = \frac{mg\lambda}{2} \]

\[ \Delta \phi = \frac{\Delta E}{\hbar} \]

Semiclassical picture: Bloch oscillations

\[ \dot{q} = -mg \]

\[ \omega_B = mg\lambda/2\hbar \]

**Linear external potential**

Vertical lattice plus gravitational force: Bloch oscillations

Landau-Zener tunnelling

\[ P = \exp\left( -\frac{\lambda e^2}{8\hbar^2 g} \right) \]

Bloch oscillations period

\[ T_B = \frac{2\hbar}{mg\lambda} \]
Ultracold bosons and fermions in optical lattices

**Shallow lattice: Zener tunnelling**

- \( P = \exp\left(-\frac{\lambda e^2}{8\hbar^2 g}\right) \)
- \( U_0 = 1.5E_g \)
- \( P = 0.1 \)

Spatially resolved Bloch oscillations of a BEC


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**Fermi vs Bose**

fermions

0 ms

7 ms

0 ms

2 ms
Time resolved Bloch oscillations

By adiabatically switching off the lattice, it is possible to map out the quasi-momentum states to momentum states: Bloch oscillations

LONG LIVE FERMIONS!
Ultracold bosons and fermions in optical lattices

**Bloch oscillations with fermions**


We can measure the external force exerted on the atoms in our case, GRAVITY.

Absence of collisions → Long lifetime of Bloch oscillations

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**Decoherence: Fermi vs Bose**

Absence of the clock shift for fermions

“Expected” splitting of the blue and the red line:

\[ 2\Delta \nu = \frac{4\hbar}{m} n a_{12} \sim 10 \text{kHz} \]

\[ |2\rangle \quad \text{blue shift} \quad |2\rangle \quad \text{red shift}\]

\[ |1\rangle \quad \text{blue shift} \quad |1\rangle \quad \text{red shift} \quad (a_{12} \geq 0) \]

\[ \text{Number of Atoms in Final State (arb)} \]


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**A high spatial resolution interferometer**

**Trapped samples:** high spatial resolution

In principle limited just by the extension of Wannier-Stark states:

For K at \( \lambda = 830 \text{ nm} \) \( \Delta z = 2\delta / F < 4 \mu \text{m} \), and decreases exponentially with increasing \( U \).

**Long-lived oscillations of fermions:** high sensitivity

Presently limited by a broadening of the momentum distribution to \( 10^{-4}\text{g} \) over 100 oscillations: it can be improved

**High accuracy:**

Only \( h \) and \( m \) are in principle involved in the measurement of gravitational forces, but the trap might affect the measure

**Possible applications:**

Forces close to surfaces, Casimir, gravity at small length scales, …
Non-newtonian forces

FIG. 1. Experimental bounds and theoretical expectations on new forces from potentials of the form $F(r) = -k/r^2 (1 + ax^{-2})$ below 1 cm. The projected reach of the first-round BEC experiments is shown as the upper "BEC Experiments" line. The lower line indicates the reach with an improved sensitivity of $10^{-11}$. Experimental data are from Refs. [8-10,14,15]. The shown theoretical expectations are discussed in the text.


“Quantum gases” people in Firenze