Surface waves

Guy Masters

Cider 2006
Standing waves and travelling waves

- Seismogram as a mode sum:

\[ s(t) = \sum_k A_k \cos(\omega_k t + \phi_k) e^{-\alpha_k t} \]

In epicentral coordinates, \( A_k \) includes both the source excitation and the geometrical mode behavior as a function of epicentral distance, \( \Delta \). This latter term is proportional to a spherical harmonic \( Y^m_l \). For surface waves, \( l \) is large and \( m \) is small, then

\[ Y^m_l \simeq \frac{1}{\pi \sin \Delta} \cos \left( l + \frac{1}{2} \right) \Delta - \frac{m}{4} + \frac{m \pi}{2} e^{im\phi} \]

Jeans formula is \( ka = l + \frac{1}{2} \) so with arc distance given by \( x = a \Delta \),

\[ Y^m_l \simeq \frac{1}{\pi \sin \Delta} \cos \left( kx - \frac{\pi}{4} + \frac{m \pi}{2} \right) e^{im\phi} \]

Seismogram of a single mode becomes proportional to a plane wave:

\[ s(t) \propto e^{i(\omega t - kx)} \]

Phase and group velocity

In general \( k = k(\omega) \) (dispersion). Mode sum becomes:

\[ s(x,t) = \int B(\omega) e^{i(\omega t - k(\omega)x)} \, d\omega \quad e(\omega) = \frac{\omega}{k(\omega)} \]

\( B(\omega) \) due to source – slowly varying. Consider phase, \( f \),

\[ f = \omega t - k(\omega)x \]

Main contribution to integral when phase is stationary (\( df/d\omega = 0 \)). If \( \omega_s \) is the point where the phase is stationary occurs, we have:

\[ t - \frac{dk}{d\omega}(\omega_s)x = 0 \]

\[ U = \frac{x}{t} = \frac{d\omega}{dk} \]

The energy associated with a particular frequency group centered on \( \omega_s \) travels with the “group velocity”, \( U(\omega_s) \).
Measuring phase

The phase of the Fourier transformed seismogram is just $kx$ (with a source term) but we need to be careful how we measure this to avoid bias. Measure relative phase between data and synthetic or measure phase difference between orbits. Transfer function approach.

Single spectral estimate:

$$ S_{obs}(\omega) = T(\omega)S_{syn}(\omega) $$

$$ T(\omega) = \frac{S_{s}(\omega)S_{o}(\omega)}{S_{s}(\omega)S_{o}(\omega)} = \frac{C_{12}(\omega)}{C_{11}(\omega)}. $$

Better to use a multi-taper estimate. Minimize

$$ \| S_{o}(\omega) - T \cdot S_{o}(\omega) \|^{2} $$

Solution is:

$$ T(\omega) = \frac{S_{o}(\omega) \cdot S_{s}(\omega)}{S_{o}(\omega) \cdot S_{s}(\omega)} $$

$T$ related to perturbation in (complex) wavenumber:

$$ T(\omega) = e^{-\delta_{s}(\omega)x} e^{-i\delta_{o}(\omega)x} $$

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ADK/VHZ 2001:26 (India)

Red = data, blue = fundamental mode synthetic
Phase and group velocity maps
Phase and group velocity maps

- Ray theory (great circle propagation):
  \[ e^{i k x} \longrightarrow e^{i k_x x} \]
  where \( k = \frac{1}{\gamma} \int k(x, \theta, \phi) \, dx \).

- Ray theory (great circle propagation):
  \[ c(\omega, \theta, \phi) = \frac{c_0(\omega)}{c_0} + \frac{\delta c(\omega, \theta, \phi)}{c_0} \]
  \[ \delta \Phi = \frac{\delta k_0}{\gamma} \int_0^1 \frac{1}{U} \, d\xi \]
  \[ t_g = \frac{1}{U} \int_0^1 \frac{d\xi}{U} \]
  Perturbation from reference group arrival time:
  \[ \delta t_g = \frac{\delta}{U_0} \int_0^1 \frac{d\xi}{U_0} \]
  (same form as phase velocity inversion)

Rayleigh phase at 50 sec
Undispersed 50 s Rayleigh waves

Note large number of cycle shifts possible
After computation of envelope functions

50 sec Rayleigh waves -- a typical example
Resolution of 1000km structure is global

Resolution of finer structure is more variable
Finite frequency kernels
Finite Frequency Kernels

- Born (single scattering) approximation

\[ s(\omega) \rightarrow s(\omega) + \delta s(\omega) \]

\[
\delta s(\omega) = \sum_{\sigma' \sigma''} \int \mathcal{S}' \times \left( \frac{e^{-i(k'\Delta' - n'\pi/2 + \pi/4)}}{\sqrt{8\pi k'} \sin \Delta'} \right) \times \sigma' \Omega_{\sigma''} \times \left( \frac{e^{-i(k''\Delta'' - n''\pi/2 + \pi/4)}}{\sqrt{8\pi k'' \sin \Delta''}} \right) \times \mathcal{R}'' \ dV
\]

Relation to measured transfer function:

\[ s(\omega) + \delta s(\omega) = T(\omega) s(\omega) \]

3D kernel:

\[ \delta \Psi(\omega) = \text{Im} \left( \frac{\delta s}{s} \right) = \int K_{\Psi} \delta m \ dV \]

\( K_{\Psi} \) is pretty ugly (see notes) but can be converted to a 2D kernel for a phase velocity perturbation with some approximations:

\[
K_{\Psi} = \frac{2k^2 \sin[k(\Delta' + \Delta'' - \Delta)] + \pi/4]}{\sqrt{8\pi k \sin \Delta \sin \Delta''}}
\]
Figure 4. 3-D sensitivity kernels $K_{P}^S$, $K_{A}^S$, $K_{R}^S$ for a 10 mile Love wave, excited by a 52 km deep strike-slip seismic source (S). Love wave radiation is maximized in the direction of the source-receiver geometrical ray (see hatched symbol). The epicentral distance to the receiver (R) is $\Delta = 80^\circ$. Sensitivity kernels are for 100 s cone-taper measurements, with the taper centered at the group travel time predicted by PREM. Top: Map view of kernels at the depth of approximately greatest sensitivity, 100 km. Middle: Slice view of cross-sections. All dotted lines are plotted at 100 km depth. Bottom: All cross-sections at a depth of 100 km; dotted lines indicate the width of the first Fresnel zone. Mode-coupling effects have been ignored, $\alpha = \gamma = 0$. 

2D Phase kernel: 50s Rayleigh Waves
Finite frequency kernels

- Expect effect to be greatest at longest periods

- Difficult to make direct comparison with great circle result since different damping is implied

- Comparisons with analyses of SEM synthetics are ambiguous but 2D kernels do not seem effective -- need to use 3D kernels (Zhou et al, 2004)?

- What do you find?
Inversion for crustal thickness

- Correct maps for the CRUST2 model
- Invert corrected maps with additional point constraints from receiver function studies for perturbation to CRUST2 moho depth
- Include only maps for periods shorter than 40 seconds to avoid mantle signal.
Observationally constrained locations

Crust-corrected
Crust2
Observed
Where do those pole phase shifts come from?

Comparison of X lm with asymptotic form for l=100

Rayleigh, 100s, s20RTS+CRUST2

Group

Phase