Heat budget & Thermal Evolution
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• Sources of heat in the Earth
• Transport of heat
• Aspects of thermal evolution
• Can a planetary perspective help?

Some powerful numbers...

Total global heat flow: 44 TW
Sun: Solar energy received (and re-radiated) 180,000 TW
Tides: Dissipation of Earth's rotational energy 3 TW
Earthquakes: Elastic wave energy released 0.03 TW
Humans: World power production 12 TW
Mantle Convection: Basic Version of Equations

Conservation of mass:
\[ \nabla \cdot \dot{u} = 0 \]

Momentum:
\[ -\nabla P + \nabla \cdot \tau + \frac{RaT}{k} = 0 \]

Heat flow:
\[ \frac{\partial T}{\partial t} + \dot{u} \cdot \nabla T = \nabla^2 T + \frac{Ra_H}{Ra} \]

Viscosity varies with temperature and depth:
\[ \mu = \mu_0 e^{\frac{RT}{\sigma}} \]

In this version, two parameters are of primary importance:
\[ Ra_H = \frac{\dot{p} \Delta \gamma d^3}{k \mu_0} \]
\[ Ra = \frac{P_c \Delta \gamma d^3}{k \mu_0} \]
Conservation of Mass Equation

Consider a particle of fluid

For an incompressible fluid, flow into the box balances the flow out
Conservation of Mass Equation

\[ \frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} = 0 \text{ that is, } \nabla \cdot \vec{u} = 0 \]
Conservation of Energy (heat flow)

\[ \frac{\partial T}{\partial \hat{t}} + \bar{u} \cdot \nabla T = \nabla^2 T + \frac{Ra_H}{Ra} \]

\[ \rho c u_x T \delta z \]

\[ q_x \delta z = -k \frac{\partial T}{\partial \hat{x}} \delta z \]

\[ \left( \rho c u_x T + \frac{\partial}{\partial \hat{x}} (\rho c u_x T) \delta \hat{x} \right) \delta \hat{z} \]

Momentum (force balance)

\[ -\nabla P + \nabla \cdot \tau + Ra T \hat{k} = 0 \]

\[ Ra = \frac{\rho_m g \alpha \Delta T \hat{d}^3}{\kappa \mu_0} \]

Pressure forces

\[ p(z) \delta \hat{x} \]

\[ p(x) \delta z \]

\[ \left( p(x) + \frac{\partial p}{\partial z} \delta z \right) \delta \hat{x} \]

\[ \left( p(z) + \frac{\partial p}{\partial z} \delta z \right) \delta \hat{x} \]
Momentum (force balance)

\[-\nabla P + \nabla \cdot \tau + Ra \hat{T} \hat{k} = 0\]

\[Ra = \frac{\rho_m g \alpha \Delta T d^3}{\kappa \mu_0}\]

Viscous forces

\[\tau_{xz}(z) \delta x\]

\[\tau_{xx}(x) \delta z\]

\[\tau_{xx}(x) \delta z\]

\[\tau_{xz}(z) \delta x\]

\[\left(\tau_{xz}(x) + \frac{\partial \tau_{xz}}{\partial x} \delta x\right) \delta z\]

\[\left(\tau_{xx}(x) + \frac{\partial \tau_{xx}}{\partial x} \delta x\right) \delta z\]

\[\left(\tau_{zz}(z) + \frac{\partial \tau_{zz}}{\partial z} \delta z\right) \delta x\]

\[\rho(T) = \rho_0(1 - \alpha(T - T_0))\]

Body forces (gravity)

\[g \rho \delta x \delta z\]
Mantle Convection: Basic Version of Equations

Conservation of mass: \( \nabla \cdot \vec{u} = 0 \)

Momentum: \(-\nabla P + \nabla \cdot \tau + Ra T \hat{k} = 0\)

Heat flow: \( \frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T = \nabla^2 T + Ra_H / Ra \)

Viscosity varies with temperature and depth: \( \mu = \mu_0 e^{\frac{-E^* + Vz}{RT}} \)

Two parameters are of primary importance:

\( Ra = \frac{\rho_m g \alpha \Delta T d^3}{\kappa \mu_0} \)

\( Ra_H = \frac{\rho_m^2 g \alpha H d^5}{k \kappa \mu_0} \)

Numerical methods discretize the region of interest

\( Ra = \frac{\rho_m g \alpha \Delta T d^3}{\kappa \mu_0} \)

\( Ra_H = \frac{\rho_m^2 g \alpha H d^5}{k \kappa \mu_0} \)

Top of mantle:
Traction-free surface
Constant temperature

Sides of mantle:
“Wrap around”

Core-mantle boundary:
Traction-free surface
Constant temperature (or constant heat flux)
A simple plume
Heat sources from the Earth’s interior

<table>
<thead>
<tr>
<th>Parent</th>
<th>Daughter</th>
<th>half-life</th>
<th>heat production</th>
</tr>
</thead>
<tbody>
<tr>
<td>238 U</td>
<td>206 Pb</td>
<td>4.49 By</td>
<td>94 W/kg of U</td>
</tr>
<tr>
<td>235 U</td>
<td>207 Pb</td>
<td>0.704 By</td>
<td>570</td>
</tr>
<tr>
<td>232 Th</td>
<td>208 Pb</td>
<td>14.0 By</td>
<td>26.6</td>
</tr>
<tr>
<td>40 K</td>
<td>40 Ar</td>
<td>1.25 By</td>
<td>27.9</td>
</tr>
</tbody>
</table>

Heat budget of the Earth
(all values given in terawatts)

Total global heat flow: 44 TW

Stein, 1995
Heat budget of the Earth
(all values given in terawatts)

Total global heat flow: 44 TW

Total BSE Heat production: 20 TW
OR HIGHER

Continental crust produces: 5.6 to 12 TW

McDonough & Sun, 1995
Van Schmus 1995
Turcotte et al. 2001

Rudnick & Fountain, 1995
Taylor & McLennan, 1995
Gao et al., 1997
Wedepohl, 1995
Heat budget of the Earth
(all values given in terawatts)

- Total global heat flow: 44 TW
- Total BSE Heat production: 20 TW OR HIGHER
- Continental crust produces: 5.6 to 12 TW
- A depleted, whole mantle can produce: 2 – 7 TW

e.g. Stacey, 1992
Zindler & Hart 1986

Heat budget of the Earth
(all values given in terawatts)

- Total global heat flow: 44 TW
- Total BSE Heat production: 20 TW OR HIGHER
- Continental crust produces: 5.6 to 12 TW
- A depleted, whole mantle can produce: 5 – 7 TW
- Requires (AT LEAST)1 to 9.4 TW produced elsewhere (mantle or core)
Mantle viscosity is self-regulating

\[ \frac{dH}{dt} = \frac{MC}{\mu} \frac{dT}{dt} = MH - Aq \]

\[ H = H_0 e^{-\lambda \tau} \]

\[ \lambda = \frac{E^* + V_z}{RT} \]

- As the mantle heats up, the viscosity drops
- More rapid convection cools the mantle
- Viscosity increases in a cooler mantle

\[ \frac{dT}{dt} = -Aq/MC(1 - Ur) \]

Where Ur, the Urey ratio, is MH/Aq


Changes in Convective Style (existence or extent of plate tectonics) can have big effects on Thermal History

Images courtesy of D. Stevenson


Ferrachat & Kellogg

A menagerie of models
Consider the Earth's mantle as a non-linear, chaotic system.

- Conservation of mass: $\nabla \cdot u = 0$
- Momentum: $\nabla^2 u + \tau + Ra/T_0 = 0$ where $T = \mu c$
- Heat flow: $\frac{dT}{dt} + u \cdot \nabla T = V(k \nabla T)$

Rayleigh Number - $Ra = \frac{\rho c_p \alpha \Delta T d^3}{\mu \nu^2}$, $\alpha = 10^{-5}$ - Referenced to Earth's near surface values

Flow is driven by basal heating ($T = 1$) and surface cooling ($T = 0$)

See for example Stewart & Turcotte JGR, 94, 13707-13717 (1999)

Thermal and dynamical evolution of the Earth?

E. M. Moores et al.
We use passive “tracers” to track trace elements (such as strontium isotopes).

Example of mixing in 2-D internally heated convection

300,000 particles
Starting position

Numerical method: the finite element method
Hunt and Kellogg 2000

Mixing in 2-D with particles
- Added at subduction zones
- Removed at mid-ocean ridges

\[ T = 0 \]
\[ \frac{dT}{dz} = 0 \]

Hunt & Kellogg - effect of viscosity on mixing

Constant viscosity

Pressure-dependent viscosity: smooth increase

Transition zone viscosity: Jump at 670 km
Initial location of particles (Hunt and Kellogg model)

a. Constant viscosity

b. Pressure-dependent viscosity

c. Transition zone

no particles present

Ferrachat & Ricard, Mixing in 3-D plate driven flows

Chaotic trajectories occur even in steady-state flows
A planetary perspective on mantle flow

See, for example, work by Solomatov, Moresi, Lenardic, Stevenson, others