The Ongoing Saga Surrounding the Velocity Fluctuations in Sedimentation

Michael P. Brenner
Division of Engineering and Applied Sciences
Harvard University

Collaborators:

Theoretical:

Peter Mucha (Georgia Tech)

Experiments:

Dave Weitz Harvard
Shang Tee
Suliana Manley
Luca Cipelletti

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\[ \mathcal{N} a U = F \]
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\[
\langle U \rangle = U_{\text{stokes}}(1 - \frac{6.55}{f}) \quad \text{Batchelor (1972)}
\]

\[
\text{Kermack (1929)}
\]

**Assumptions**

- Vanishing Reynolds Number
- Monodisperse
- Dilute
- No Brownian Motion
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E. Guazzelli et al., Phys Fluids, 1995

\[ F \propto a U = F \]
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\[ \Box_x^2 u \Box_x p = F \Box_x(x) \hat{z}, \quad \Box_x u = 0 \]

\[ u(x) = \frac{F}{8\pi} \frac{I}{x \cdot x'} + \frac{(x \cdot x')(x \cdot x')}{|x \cdot x'|^3} + O\left(\frac{a^*}{|x \cdot x'|^2}\right) \]

Corrections to satisfy B.C.

\[ S(x-x') \]

\[ \frac{dx_i}{dt} = U_{stokes} + S(x_j \Box x_i) \quad \text{for} \quad j \neq i \]
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\[ \frac{dx_i}{dt} = U_{\text{stokes}} + \sum_{j \neq i} S(x_j - x_i) \]

\[ \sim U_{\text{stokes}}^2 \frac{d^3 r}{a^3} S(r)^2 \]

\[ \sim U_{\text{stokes}}^2 \frac{d^3 r}{a^3} \sim U_{\text{stokes}}^2 \frac{r}{a} \]

Physical Mechanism:

\[ N \sim \pm \sqrt{N} \sim \sqrt{R^3/a^3} \]

Number Fluctuation

\[ U \sim U_{\text{stokes}} \frac{\sqrt{N}}{R} \sim U_{\text{stokes}} \sqrt{R/a} \]


E. J. Hinch

Dr. Michael Brenner, Harvard University (KITP Colloquium 5/07/03)
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Two Views:

(a) **The argument is wrong.** No way the velocity fluctuations depend on system size. Some type of “screening mechanism” exists.

(b) **The arguments are correct.** Diffusion in a sediment qualitatively different than normal diffusion.

Experiments

Lei, Tong & Ackerson PRL 2001
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**Experiments:**
Velocity Fluctuations independent of system size

(1) Ham and Homsy (1988);
(2) Nicolai and Guazzelli (1994)

(3) **Segre’, Herboltzheimer and Chaikin**, 1997, PIV:
(4) Guazzelli (Phys. Fluids, 2001) (larger cells)

(5) Lei, Tong and Ackerson (Phys. Rev. Lett. 2001)

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\[ V \sim U_{\text{stokes}}^{1/3} \]
\[ \ell \sim a^{1/3} \]

The Fluctuations are universal, independent of cell size.
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Change in Character of Number Fluctuations

X. Lei, B. Ackerson and P. Tong, Phys Rev Lett, 2001

Theories and Simulations

(1) Koch and Shaqfeh, 1991
   special particle distribution

   Renormalization arguments $\Rightarrow$ screening


(2) Koch, 1994: point particle simulations
   No evidence of screening in periodic box

   particle number? $\sim$ 30000
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Outline:

1. The Dilemma

2. Finite Cells: Expectations and Simulations

3. Wide Cell experiments (Weitz)

4. A mystery.

5. A resolution.

If there is time: A short story
  Elastic Instability of a Growing Tissue

All theories and simulations of fluctuations assumed that system is:

(a) infinite
(b) homogeneous.

Experiments are definitely not infinite.

D/a < 100
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“Universal” Fluctuations, independent of cell size.

D/a ~ 100 (fixed)

(b) $12 \times (V_t - V_{sed})$

(e) $2 \times (V_t - V_{sed})$

Segre et al.
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Wall Effects.

\[
\Box U_{\parallel}^2(x) = \int \frac{d^3 x'}{a^3} u_{\parallel}(x \Box x')^2
\]

Integral is convergent when wall effects are taken into account!

\[
\frac{\Box U_{\parallel}^2}{U_{\text{stokes}}^2} = c(\text{geometry}) \frac{d \Box}{a} \ell \sim \frac{d}{2}
\]

How does it compare to experiments?
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\[ \nabla V_{\parallel} = V_{\text{stokes}} \sqrt{\frac{d}{a}} \]

Perhaps the uniform distribution is destabilized by boundaries?

Developed method for solving

\[ \frac{dx_j}{dt} = \nabla S(x_j | \nabla x_i) \]

With bounding walls in O(N log(N)) operations.

N < 4 \times 10^6 particles
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![Diagram](image)

- Experiments (S. Tee)
- Simulations (P. Mucha)

**Diagram**: Velocity Fluctuations relax to Poisson Predictions

- 64,000 particles
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\[ V_{||} = V_{stokes} \cdot 0.864 \cdot \sqrt{\frac{d}{a}} \]
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Experiments

Shang Tee, Suliana Manley, Luca Cipelletti
Dave Weitz

Increase the cell depth.

Do the velocity fluctuations increase or not?

Two techniques:

- Particle Imaging Velocimetry (~25 micron particles)
- Light Scattering (~2.5 micron particles)

Cell Sizes: (D/a, W/a, H/a) ~ (1000,5000,15000)
Simulations do not produce decay
There must be an additional physical effect.

Possible Effects:
(1) Polydispersity
(2) Inertia
(3) Boycott Effect (Cell is tilted?)
(4) Thermal Convection?
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\[ \frac{U}{U_{\text{stokes}}} < 1 \]

\[ \frac{U}{U_{\text{stokes}}} > 1 \]
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The Mechanism

\[ U \sim U_{stokes} \frac{\sqrt{N}}{R} \sim U_{stokes} \sqrt{\frac{R}{a}} \]
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Stratification

\[ \theta = \theta_0 (1 \theta z) \]

If

\[ R > \frac{\theta}{\theta} \]

Density fluctuations are suppressed

Stratification

\[ \theta = \theta_0 (1 \theta z) \]

If

\[ R > \frac{\sqrt{a^3}}{R^3} \]

Density fluctuations are suppressed, (Luke, 2002)
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\[ R^* \sim a \left( \frac{d}{a} \right)^{1/5} \left( \frac{d}{a} \right)^{2/5} \]

\[ \Box V \sim V_{sed} \left( \frac{d}{a} \right)^{2/5} \left( \frac{d}{a} \right)^{1/5} \]

Critical Stratification! \[ R^* \sim d \]

\[ \Box_{crit} d \sim \frac{1}{\sqrt{N_d}} = \frac{1}{\sqrt{\Box (d/a)^3}} \]

\[ (\Box d) \sim (10^3)^{3/2} 10^{2 \Box 1/2} \sim 3 \Box 10^4 \]
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Simulations Show Critical Stratification

\[ \frac{\Delta V_z^2}{\Delta V_{\text{Poisson}}} \]

\[ \beta / \beta_{\text{crit}} \]

A theoretical Issue
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**A theoretical Issue**

**Particle Dynamics**

\[
\frac{dx_i}{dt} = U_{\text{stokes}} + \sum_{j \neq i} S(x_j - x_i)
\]

**Continuum Model:**

\[
\partial_t \phi + \nabla \cdot [u(r, t)\phi] - D \cdot \nabla \phi + \xi(r, t) = 0
\]

- average vel.
- large scale fluctuations
- short wavelength noise
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A theoretical Issue

Particle Dynamics

Continuum Model:

\[ \frac{d x_i}{d t} = U_{stokes} + \sum_{j \neq i} S(x_j \bigotimes x_i) \]

\[
\partial_t \phi + \nabla \cdot [u(r, t) \phi] - D \cdot \nabla \phi + \xi(r, t) = 0
\]

average vel. \hspace{1cm} large scale \hspace{1cm} short wavelength
fluctuations \hspace{1cm} noise

\[
u = \int (x \bigotimes x') \bigotimes (x') \, dx'
\]

\[
D = \bigotimes V \ell
\]

Including \( \bigotimes \) (or not including it) is important:

Without \( \bigotimes \), noise is only from initial condition.

A constant stratification would decay cause fluctuations to decay
continuously in time.

But, no fluctuation dissipation theorem (just self consistency argument).
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**Experimental Tests**

(0) **Experiments with Constant Stratification**

(1) **Time dependent stratification**

(2) Fluctuation decay rate is cell-height dependent.

(3) Suppression of number density fluctuations (Tong and Ackerson)

(4) \( j^{1/3} \) law

(5) Calculation of structure factor

Impose a salt gradient
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Experimental Tests

(0) Experiments with Constant Stratification

(1) **Time dependent stratification**

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**Simulations**

![Graph showing simulations](image)

**Predict the gradient:**
**Diffusion Model**

\[
\partial_t n + V_{sed} \partial_z n = D \partial_z^2 n
\]

D taken from simulations (cell size dependent!)

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**Experimental Tests**

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Number Fluctuations

![Graph](image)

Data from: X. Lei, B. Ackerson and P. Tong, Phys Rev Lett, 2001

Simulations

![Graph](image)

(Dashed lines: Theoretical Model with no fitting parameters)
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**Experimental Tests**

(0) Experiments with Constant Stratification

(1) Time dependent stratification

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<table>
<thead>
<tr>
<th>φ</th>
<th>Observed $\Delta V_i/V_0$</th>
<th>Poisson (2.1)</th>
<th>$h/n$</th>
<th>$t_{\text{crit}}$</th>
<th>Obs. $\Delta V_i$/Poisson</th>
<th>$t_{\text{crit}}/t_{\text{exp}}$</th>
<th>$I/I_{\text{exp}}$</th>
</tr>
</thead>
</table>
| Nicolai & Guazzelli (1995):  
  $5 \times 10^{-2}$ | 0.67          | 1.15         | 1269  | 363               | 0.58                      | 0.43                         | 0.48              |
  $5 \times 10^{-2}$ | 0.62          | 1.62         | 1269  | 162               | 0.38                      | 0.19                         | 0.22              |
  $5 \times 10^{-2}$ | 0.60          | 1.99         | 1269  | 88                | 0.30                      | 0.10                         | 0.12              |
  $5 \times 10^{-2}$ | 0.73          | 2.29         | 1269  | 55                | 0.32                      | 0.065                        | 0.072             |

| Segro et al. (1997):  
  $1 \times 10^{-4}$ | 0.13          | 0.13      | 39077 | 23708             | 0.94                      | 0.91                         | 0.93              |
  $2 \times 10^{-4}$ | 0.12          | 0.13      | 12821 | 7725              | 0.87                      | 0.90                         | 0.93              |
  $3 \times 10^{-4}$ | 0.28          | 0.37      | 12821 | 7410              | 0.75                      | 0.86                         | 0.89              |
  $3 \times 10^{-4}$ | 0.56          | 0.77      | 6410  | 3616              | 0.72                      | 0.85                         | 0.92              |
  $6 \times 10^{-4}$ | 0.167         | 0.24      | 12821 | 7387              | 0.70                      | 0.86                         | 0.89              |
  $3 \times 10^{-4}$ | 0.75          | 1.18      | 12821 | 6411              | 0.64                      | 0.72                         | 0.76              |
  $6 \times 10^{-4}$ | 0.19          | 0.33      | 39077 | 22097             | 0.56                      | 0.85                         | 0.87              |
  $2 \times 10^{-4}$ | 0.53          | 0.965     | 12821 | 6390              | 0.55                      | 0.75                         | 0.79              |
  $3 \times 10^{-4}$ | 0.25          | 0.536     | 12821 | 6698              | 0.47                      | 0.78                         | 0.82              |
  $1 \times 10^{-4}$ | 0.44          | 0.979     | 12821 | 6601              | 0.45                      | 0.70                         | 0.74              |
  $2 \times 10^{-4}$ | 0.556         | 1.39      | 12821 | 5523              | 0.40                      | 0.65                         | 0.69              |
  $4 \times 10^{-4}$ | 0.244         | 0.619     | 12821 | 6547              | 0.39                      | 0.77                         | 0.80              |
  $3 \times 10^{-4}$ | 0.589         | 1.70      | 12821 | 5220              | 0.35                      | 0.61                         | 0.65              |
  $5.5 \times 10^{-2}$ | 0.75          | 2.30      | 12821 | 4735              | 0.33                      | 0.55                         | 0.60              |
  $6 \times 10^{-4}$ | 0.16          | 0.759     | 25641 | 6208              | 0.211                     | 0.36                         | 0.39              |

| Guazzelli (2001):  
  $5 \times 10^{-4}$ | 0.35          | 0.50      | 2703  | 159               | 0.70                      | 0.088                        | 0.095             |
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Experimental Tests

(1) Time dependent stratification

(2) Fluctuation decay rate is cell-height dependent.

(3) Suppression of number density fluctuations (Tong and Ackerson)

(4) $\Delta^{1/3}$ law

(5) Measure structure factor

(a) Tony Ladd, Phys. Rev. Lett. 2002 (Lattice Boltzman, Re ~1)
(b) Sarah Dance and Martin Maxey, APS FE2
Summary

(1) Velocity fluctuations in sedimentation highly nonuniversal. Depend on container dimension + small inhomogeneities

(2) Fluctuations are sensitive to very small physical effects.

\[ U \sim \sqrt{\text{Volume}} \]

Most physical effects \( \sim \) volume

(3) Diffusivity of a sediment:

\[
D \left( \phi, \frac{\partial \phi}{\partial z} \right) = \begin{cases} 
CdV_0 \sqrt{\phi d/a} & \text{for } \beta \leq \beta_{\text{crit}} \\
CB^{3/5}aV_0d^4/5 \left| \frac{\partial^2 \phi}{\partial z^2} \right|^{-3/5} & \text{for } \beta \geq \beta_{\text{crit}} 
\end{cases}
\]
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- Higher Volume Fractions
- Nonzero Reynolds Numbers
- Polydispersity
- Etc.

\[ \Box r^2 \]

Verifies scalings…

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**Issues**

- Fourier Transform $\parallel$ to side walls on course mesh
- Explicitly compute flows perpendicular sums
- FFT for summing $\parallel O(N \log N)$
- Clever organizational tricks for perpendicular sums
- Near field corrections…
- Back flow

- Can do $10^6$ particles; *c.f. previous 50000 periodic*