Discriminate MSSM Higgs Sector from its Alternative

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Introduction

- After the discovery of the Higgs bosons, we have to ask whether we have detected the Higgs bosons predicted by the MSSM (Minimal Supersymmetry Standard Model), not, say, the 2HDM (Two-Higgs-Doublet Model)?
  - (Both models predict $h^0$, $H^0$, $H^\pm$ and $A$.)
  - Check their Couplings and Masses.

- Measuring the Higgs self-coupling ($\lambda_{hhh}$)

- Testing the MSSM mass relation

\[
M_{H^+}^2 = M_A^2 + m_W^2
\]

via the associated production of $A$ and $H^+$. 

Based on collaborations with:
Introduction

LEP: $114\text{GeV} < m_h^{\text{SM}} < 196\text{ GeV}$ (95% CL)
Tevatron, LHC: Discovery of at least one Higgs boson

Once a Higgs boson ($h$) is found, precision study at Linear Colliders (LC's)

- Nature of EWSB $g_{hWW}, g_{hZZ}$
  $\sigma(e^+e^- \rightarrow Zh), \sigma(e^+e^- \rightarrow \bar{\nu}h) \Rightarrow \mathcal{O}(1\%)$

- Origin of fermion masses: Yukawa couplings $Y_f$
  $B(h \rightarrow \bar{f}f') \Rightarrow \mathcal{O}(1\%)$.

- Higgs potential: Higgs self-coupling $\lambda_{hhh}$

Double Higgs production

$\lambda_{hhh}$ can be measured in $\mathcal{O}(10-20\%)$ accuracy at LC: $\sqrt{s} = 0.5 - 1.5\text{ TeV}, \mathcal{L} = 1\text{ ab}^{-1}$

Radiative corrections

Precision Measurement at a LC

$\Delta^{\text{Exp}} g_{hVV}/g_{hVV} \sim 1\%$

$\Delta^{\text{Exp}} \lambda_{hhh}/\lambda_{hhh} \sim 10 - 20\%$

$\iff$ Radiative Corrections

Leading top-loop contribution

- $hVV$ coupling
  $g_{hZZ} \simeq \frac{2m_Z}{v} \left(1 - \frac{5N_cm_{t}^4}{96\pi^2v^2} + \cdots \right)$,
  loop effect $\sim 1\%$

- $hhh$ coupling
  $\lambda_{hhh} \simeq \frac{3m_h^2}{v} \left(1 - \frac{N_cm_{t}^4}{3\pi^2v^2m_h^2} + \cdots \right)$

$A m_t^4$ term appears in the 1-loop correction.
  $\text{loop effect} \sim 10\%$

The corrections are comparable to the measurement accuracy
$\Rightarrow$ How about new physics effects?
The two Higgs doublet model

A simplest extension of the SM Higgs sector for various physics motivations (MSSM, extra CP phases, topcolor, etc)

- THDM with a softly-broken discrete symmetry:
  \( \Phi_1 \rightarrow \Phi_1; \Phi_2 \rightarrow -\Phi_2 \)
  \( \Rightarrow \) Natural FCNC suppression

Yukawa interaction (Model II):
\[
\mathcal{L}_{II} = -y_D \bar{Q}_L \Phi_1 b_R - y_U \bar{t}_R \Phi_2^c Q_L + (h.c.)
\]

Higgs potential:
\[
\mathcal{V}_{\text{THDM}} = m_1^2 |\Phi_1|^2 + m_2^2 |\Phi_2|^2 - m_3^2 (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) + \frac{\lambda_1}{2} |\Phi_1|^4 + \frac{\lambda_2}{2} |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 + \lambda_5 \left[ (\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2 \right].
\]

\( \Phi_1 \) and \( \Phi_2 \Rightarrow h, H, A^0, H^\pm \oplus 3 \) Goldstone bosons

\( \uparrow \uparrow \uparrow \text{charged} \)

CPeven CPodd

8 parameters: \( \Rightarrow \{ m_h, m_H, m_A, m_{H^\pm}, \alpha, \beta, v, M_{\text{soft}} \} \)

- Masses of physical Higgs bosons:
  \[
  m_h^2 = v^2 \left( \lambda_1 \cos^4 \beta + \lambda_2 \sin^4 \beta + \frac{\lambda_3}{2} \sin^2 2\beta \right) + \mathcal{O}(\frac{v^2}{M_{\text{soft}}^2}),
  \\
  m_H^2 = M_{\text{soft}}^2 + v^2 (\lambda_1 + \lambda_2 - 2\lambda) \sin^2 \beta \cos^2 \beta + \mathcal{O}(\frac{v^2}{M_{\text{soft}}^2}),
  \\
  m_{H^\pm}^2 = M_{\text{soft}}^2 - \frac{\lambda_4 + \lambda_5}{2} v^2,
  \\
  m_A^2 = M_{\text{soft}}^2 - \lambda_5 v^2.
  \\
  \text{M_{soft}}: \text{soft breaking scale of the discrete symmetry}
  \\
  \text{M_{soft}} \text{ determines decoupling/non-decoupling property of heavy Higgs bosons (} \Phi = A, H^\pm \text{ or } H \text{)}
  \\
  m_{\Phi}^2 = M_{\text{soft}}^2 + \lambda_i v^2,
  \\
  \text{Decoupling: for } m_{\Phi}^2 \sim M_{\text{soft}}^2 (M_{\text{soft}} \gg \lambda v^2)
  \\
  \text{Loop-effects of } H, A, H^\pm \text{ decouple (Decoupling Theorem)}
  \\
  \text{Non-Decoupling: for } m_{\Phi}^2 \sim \lambda_i v^2 (M_{\text{soft}} \ll \lambda v^2)
  \\
  \mathcal{O}(m_{\Phi}^2) \text{ terms in the low energy observables}
  \\
  \text{(similar to the top effects: } m_t^2 = y_t v^2 / 2)
The tree-level coupling constants in the THDM

\[ g_{hZZ}^{\text{tree}} = \frac{2m_Z^2}{v} \sin(\beta - \alpha) \]

\[ \lambda_{hhh}^{\text{tree}} = -\frac{3}{2v \sin 2\beta} \left[ \{\cos(3\alpha - \beta) + 3 \cos(\beta + \alpha)\} m_h^2 - 4 \cos^2(\alpha - \beta) \cos(\alpha + \beta) M_{\text{soft}}^2 \right] \]

- **\( \sin(\beta - \alpha) \approx 1 \) (Decoupling regime)**

  \[ g_{hZZ}^{\text{tree}} \approx \frac{2m_Z^2}{v} = g_{hZZ}^{\text{tree}}(\text{SM}) \]

  \[ \lambda_{hhh}^{\text{tree}} \approx \frac{3m_h^2}{v} = \lambda_{hhh}^{\text{tree}}(\text{SM}) \]

  Loop correction is essentially important!

- **\( \sin(\beta - \alpha) \neq 1 \)**

  The tree-level deviation from the SM prediction appears.

Tree-Level Deviation + Loop Correction

Radiative corrections to \( g_{hZZ} \) and \( \lambda_{hhh} \) in the THDM

One-loop calculation in the on-shell scheme

The 1-loop contribution for \( \sin(\beta - \alpha) \approx 1 \)

\[ g_{hZZ}^{\text{reno}} \approx \frac{2m_Z^2}{v} \left[ 1 - \frac{1}{16\pi^2} \left\{ \frac{5 N_c m_t^2}{6 v^2} + \frac{2 m_{\Phi}^2}{3 v^2} \left( 1 - \frac{M_{\text{soft}}^2}{m_{\Phi}^2} \right)^2 \right\} \right] \]

\[ \lambda_{hhh}^{\text{reno}} \approx -\frac{3m_h^2}{v} \left[ 1 + \frac{1}{16\pi^2} \left\{ \frac{16 N_c m_t^4}{3 v^2 m_h^2} + \frac{16}{3} \frac{m_{\Phi}^4}{m_h^2 v^2} \left( 1 - \frac{M_{\text{soft}}^2}{m_{\Phi}^2} \right)^3 \right\} \right] \]

(\( \Phi = H, A, H^+ \))

In \( \lambda_{hhh}^{\text{reno}} \), \( O(m_{\Phi}^4) \) terms appear with a suppression factor

\[ m_{\Phi}^4 \left( 1 - \frac{M_{\text{soft}}^2}{m_{\Phi}^2} \right)^3 \rightarrow \begin{cases} \frac{\lambda_i v^2}{m_{\Phi}^2}, & (m_{\Phi}^2 \sim M_{\text{soft}}^2), \quad \text{MSSM} \\ \text{decoupling for } m_{\Phi} \rightarrow \infty \\ m_{\Phi}^4, & (m_{\Phi}^2 \sim \lambda_i v^2), \\ \text{non decoupling effect} \\ \text{Dynamical SB, EW baryogen.} \end{cases} \]
Scan Analysis

Free parameters in the THDM:

\[ \tan \beta, m_{H}, m_{H \pm}, m_{A}, M_{\text{soft}} \]

for fixed \( m_{h} = 120 \text{ GeV} \), and each \( \delta \equiv 1 - \sin^2(\alpha - \beta) \).

Searching allowed region for the deviation in the \( hZZ \) and \( hhh \) vertices from the SM prediction.

Parameter constrained by

- LEP Precision Data:

  Constraint on the \((S,T)\) parameters

- Perturbative unitarity

  \[ |a^0(W^+_L W^-_L \rightarrow W^+_L W^-_L)| < \xi, \quad (\xi = 1/4) \]

  for 15 channels \( W^+_L W^-_L, Z_L Z_L, Z_L h, hh, hH, \ldots \)

  Kanemura, Kubota, Takasugi, Akeroyd, Arhrib, Naimi

- Vacuum stability

  \[ V_{\text{eff}}(\langle \Phi_1 \rangle, \langle \Phi_2 \rangle) \geq 0 \text{ for } \langle \Phi_i \rangle \rightarrow \infty. \]

  Deshpande, Ma; Sher

\[ \delta \equiv 1 - \sin^2(\alpha - \beta) \]

\[ M_{\text{soft}} = 0, \tan \beta = 1, m_A = m_H = m_{H \pm} \]
Discriminating MSSM Higgs Sector from its Alternative

Allowed Region ($M_{\text{soft}} = 0$ GeV)

Deviation of the $hZZ$ coupling

\[
\delta \equiv 1 - \sin^2(\alpha - \beta)
\]

\begin{figure}
\centering
\includegraphics[width=0.4\textwidth]{fig1}
\caption{hZZ coupling Deviation from the SM $\sim 1\%$}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=0.4\textwidth]{fig2}
\caption{hhh coupling Deviation from the SM $\sim 100\%$ due to the $m_A^4$ term}
\end{figure}

Allowed Region ($M_{\text{soft}} = m_A/2$)

Deviation of the $hZZ$ coupling

\[
\delta \equiv 1 - \sin^2(\alpha - \beta)
\]

\begin{figure}
\centering
\includegraphics[width=0.4\textwidth]{fig3}
\caption{hZZ coupling Deviation from the SM $\lesssim 1\%$}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=0.4\textwidth]{fig4}
\caption{hhh coupling Deviation from the SM $\lesssim 30\%-90\%$ due to the $m_A^4$ term}
\end{figure}
Summary

One-loop effective couplings of $hZZ$ and $hhh$ in the SM and the THDM

Scan Analysis:
- $hZZ$: deviation from the SM
  - One-loop contribution ($\sim 1\%$)
  - Tree-level difference when $\delta = 1 - \sin^2(\alpha - \beta) \neq 0$
- $hhh$: deviation from the SM
  - One-loop $O(m_h^3)$ contribution $\sim +30$ to $+100\%$
  - Tree-level difference when $\delta \neq 0$

Even when the $hZZ$ measurement is consistent with the SM by $O(1)\%$, $hhh$ can deviate from the SM by $+30$ to $+100\%$ due to the non-decoupling loop effect of heavy Higgs bosons

Such deviation in the $hhh$ coupling is testable at a Linear Collider

Momentum dependences

Heavy Higgs effect on $\lambda^{THDM}_{hhh}$ (one-loop)

$\lambda^{THDM}_{hhh}$

$\lambda^{THDM}_{hhh} = \lambda^{THDM}(q^2)$

Case of $M_{soft} = 0$

$M_A = M_H = M_{H^*} = 450$ GeV

$M_A = M_H = M_{H^*} = 450$ GeV
Self-coupling $\lambda_{hhh}$ in the MSSM

The MSSM Higgs sector: a decoupling THDM ($\lambda_i \sim \mathcal{O}(g^2)$)

Relations among heavy Higgs bosons:

\[
\begin{align*}
M_{soft}^2 &= m_A^2, \\
m_{H+}^2 &= m_A^2 + m_W^2, \\
m_H^2 &\sim m_A^2, \quad (m_A \to \infty) \\
\sin^2(\alpha - \beta) &\sim 1, \quad (m_A \to \infty)
\end{align*}
\]

\[
\frac{\Delta \lambda_{hhh}^{MSSM}}{\lambda_{hhh}^{SM}} \propto m_{H+}^4 \left(1 - \frac{M_{soft}^2}{m_{H+}^2}\right)^3 \to \frac{m_W^6}{m_{H+}^2}. \quad \text{(Decoupling)}
\]

Loop corrections due to $H, H^+, A$

$\Rightarrow$ small for $\sin^2(\alpha - \beta) \simeq 1$ (less than 1%)

- Stop ($t$) loop contributions to $\lambda_{hhh}$

The stop mass matrix

\[
M_t^2 = \begin{pmatrix}
M_Q^2 + m_t^2 + \mathcal{O}(m_Z^2) & m_t X_t \\
m_t X_t & M_U^2 + m_t^2 + \mathcal{O}(m_Z^2)
\end{pmatrix},
\]

where $X_t = A_t + \mu \cot \beta$.

\[
\begin{pmatrix}
\tilde{t}_1 \\
\tilde{t}_2
\end{pmatrix} = \begin{pmatrix}
\cos \theta_t & -\sin \theta_t \\
\sin \theta_t & \cos \theta_t
\end{pmatrix} \begin{pmatrix}
\tilde{t}_L \\
\tilde{t}_R
\end{pmatrix}.
\]
The details of SUSY parameters.

Physics predictions depend strongly on

- Chargino have different decay distributions.
- e.g., a Higgsino and a gaugino type of
  SUSY parameters.
- In general, detection efficiency also depends
  on SUSY parameters.
- Where both $\phi$ and $BR$ depend on SUSY parameters:
  $\phi$ (production) × $BR$ (decay branching ratio).
- $\phi$ is expressed in terms of bounds on
  various physics reach of a process:
  e.g., $\tan \beta$ and stop mixings.
- e.g., the physics reach of $\phi$ depends on other SUSY
  parameters as well.
- e.g., $M_2$ and $m_A$

A typical SUSY phenomenology study depends

- MSSM
  Decoupling for $m_A \rightarrow \infty$, $M_2 \rightarrow \infty$.
  The stop-loop correction can reach to 5-8% for small $M_2$.
- $\Delta \chi^2_{THDM, heavy Higgs (H, A and H^+)}$ effects
  $\frac{\chi^2_{THDM}}{\chi^2_{SM}} \sim \frac{M_4}{16 \pi^2} \left( \frac{1}{M_2^2} \right)^3$.
  The correction can be $O(100%)$ for a light mass in $120-160 GeV$.

Such large non-decoupling effects agree with the SM prediction.

In THDM, heavy Higgs (H, A and $H^+$) effects
Our task is to find a SUSY process

- whose tree level $\sigma$ (production) only depends on ONE SUSY parameter that can be determined by kinematic variable (e.g. invariant mass).
- that is not sensitive to the detailed SUSY parameters via radiative corrections.
- that can bound the SUSY models by (product of) $\text{Br}$ (decay branching ratio) without convoluting with $\sigma$ (production).
- that can be used to distinguish MSSM from its alternatives, e.g. 2HDM.
- whose final state particle kinematics can be properly modeled without specifying any SUSY parameters.
  $\Rightarrow$ The detection efficiency can be accurately determined.

Here is that promising process

$\bar{p}p, pp \rightarrow W^\pm \rightarrow AH^\pm$

- The vertex $W^\mu - A - H^+$ is determined by gauge interaction, which gives
  \[
  \frac{g}{2}(p_A - p_{H^+})^\mu.
  \]
  $\Rightarrow$
  No SUSY parameter.
- The production cross section $\sigma(AH^+)$ in general depends on two masses:
  $M_A$ and $M_{H^+}$
  e.g. in 2HDM.
  But, in MSSM,
  \[
  M_{H^+}^2 = M_A^2 + m_W^2.
  \]
  $\Rightarrow$ $\sigma(AH^+)$ only depends on $g$ and $M_A$.
  ($M_A$ can be determined from its decay kinematics, e.g. the invariant mass of $b\bar{b}$ in $A \rightarrow b\bar{b}$.)
Constraint on MSSM

Constraints on the product of branching ratios

\[ B(A \rightarrow b\bar{b}) \times B(H^+ \rightarrow \tau^+ \nu_\tau) \]

as a function of \( M_A \) for Case A and Case B, and

\[ B(A \rightarrow b\bar{b}) \times B(H^+ \rightarrow t\bar{t}) \]

for Case C, at the LHC, where \( \tau^+ \) decays into \( \pi^+ \nu_\tau \) channel.

![Graph showing branching ratios vs. M_A](image)

- The NLO QCD correction is about 20%.

- Uncertainty due to parton distribution is about 6% at Tevatron and 5% at LHC for \( M_A = 120 \) GeV, when applying the prescription given in hep-ph/0101032 (by Pumplin, Stump, Tung).

- The higher order QCD correction is estimated to be about 10% at Tevatron and less than 1% at LHC, when varying the factorization scale around the c.m. energy of \( q\bar{q} \rightarrow AH^\pm \) by a factor of 2.

Production rates

The LO (dotted lines) and NLO QCD (solid lines) cross sections of the \( AH^+ \) and \( AH^- \) pairs as a function of \( M_A \). The cross sections for \( AH^+ \) and \( AH^- \) coincide at the Tevatron for being a \( p\bar{p} \) collider.
Electroweak $K$-factor

The $K$-factor of $q\bar{q} \rightarrow H^+A$ for $M_A = 90$ GeV, as a function of the invariant mass $\sqrt{s}$ of $q\bar{q}$. The solid curves come from the top and bottom quark contributions. The squark-loop contributions are shown by dotted curves for those without stop mixing and by dashed curves for those with maximal stop mixing, respectively.

- The $K$-factor at the parton level:
  \[ K^{(1)}(q^2) = 2 \text{Re} F^{(1)}(q^2). \]

- The one-loop electroweak correction to the production rate of $pp, p\bar{p} \rightarrow AH^\pm$ is smaller than the PDF uncertainties.
Radiative correction to the MSSM mass relation

\[ M_{H^+}^2 = M_A^2 + m_W^2. \]

- In the on-shell scheme, after fixing \( M_A \) and \( \tan \beta \), \( M_{H^+} \) is determined by

\[
M_{H^+}^2 = M_A^2 + m_W^2 + \Pi_{AA}(M_A^2) - \Pi_{H^+H^-}(M_A^2 + m_W^2) + \Pi_{WW}(m_W^2),
\]

where \( \Pi(q^2) \) are the self-energies.

- There are 7 parameters in the Higgs sector of the MSSM. They are \( g', g, \nu_1, \nu_2, m_1, m_2, \) and \( m_3 \).
  Beyond the Born level, the wavefunction renormalization factors \( Z_H \) and \( Z_{H^*} \) are also needed.

- The standard model parameters are fixed by defining \( \alpha_{em}, m_W \) and \( m_Z \), and the additional SUSY parameters in the Higgs sector are fixed by the following renormalization conditions:
  - the tadpole contributions \((T_H = 0, T_{H^*} = 0)\),
  - the on-shell condition for the mass of \( A \),
  - the on-shell condition for the wavefunction of \( A \),
  - a renormalization condition on \( \tan \beta \) (which requires \( \delta \nu_1/\nu_1 = \delta \nu_2/\nu_2 \)), and
  - a vanishing \( A - Z \) mixing for an on-shell \( A \).

Detecting the Signal Event

- Signal

- Backgrounds

- Veto additional lepton and jet from the parton level background events that satisfy

\[ p_T(\text{lepton}) > 10 \text{ GeV}, \text{ and } |\eta(\text{lepton})| < 3. \]

\[ p_T(\text{jet}) > 10 \text{ GeV}, \text{ and } |\eta(\text{jet})| < 3.5. \]
The model parameters, production rates and decay branch ratios

<table>
<thead>
<tr>
<th>Sets</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_A/\Gamma_A$</td>
<td>101/3.7</td>
<td>165.7/5.6</td>
<td>250/7.9</td>
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<td>$m_h/\Gamma_h$</td>
<td>96.8/3.3</td>
<td>112/0.04</td>
<td>112/0.01</td>
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<td>$m_H/\Gamma_H$</td>
<td>113/0.38</td>
<td>163/5.5</td>
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<tr>
<td>$m_{H^+}/\Gamma_{H^+}$</td>
<td>126/0.43</td>
<td>182/0.68</td>
<td>261.4/4.2</td>
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</table>

\[ \sigma(AH^+) \text{ [fb]} = 164/36 = 5.4 \]
\[ \sigma(HH^+) \text{ [fb]} = 137.4/37.4 = 5.4 \]
\[ Br(A \rightarrow bb) = 0.91/0.90 = 0.89 \]
\[ Br(H \rightarrow bb) = 0.90/0.90 = 0.89 \]
\[ Br(H^+ \rightarrow \tau^+\nu) = 0.98/0.90 = 0.00 \]
\[ Br(B \rightarrow bb) = 0.90/0.90 = 0.00 \]
\[ Br(H^+ \rightarrow \tau^+\nu) = 0.11/0.11 = 0.11 \]

where tan $\beta = 40, \mu = M = 500$GeV.

Imposing the following Basic Cuts:

\[ P_T(b, \overline{b}, \pi^+) > 15 \text{ GeV}, \]
\[ |\eta(b, \overline{b}, \pi^+)| < 3.5, \]
\[ \Delta R(b, \overline{b}, \pi^+) > 0.4. \]

**Set A ($m_A = 101$ GeV)**

- Numbers of signal and background events at the LHC with 100 fb$^{-1}$. The $b$-tagging efficiency (50%, for tagging both $b$ and $\overline{b}$ jets) is included, and the kinematic cuts listed in each column are applied sequentially.

<table>
<thead>
<tr>
<th>Signal: $AH^+$</th>
<th>Basic Cuts</th>
<th>$E_T &gt; 50$</th>
<th>$P_T &gt; 40$</th>
<th>$90 &lt; M_H &lt; 110$ [GeV]</th>
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<td>$AH^+$</td>
<td>507</td>
<td>391</td>
<td>241</td>
<td>216</td>
</tr>
<tr>
<td>$Wb\bar{b}$</td>
<td>11555</td>
<td>3111</td>
<td>864</td>
<td>67</td>
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<tr>
<td>$t\overline{t}$</td>
<td>1228</td>
<td>614</td>
<td>163</td>
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<tr>
<td>$Wq$</td>
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<td>236</td>
<td>68</td>
<td>11</td>
</tr>
<tr>
<td>$t\overline{t}$</td>
<td>110</td>
<td>80</td>
<td>17</td>
<td>2</td>
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<tr>
<td>Signal (S)</td>
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<td>391</td>
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<td>216</td>
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<td>Bckg (B)</td>
<td>13507</td>
<td>4078</td>
<td>1135</td>
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<td>38</td>
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<td>Bckg (B)</td>
<td>13966</td>
<td>4431</td>
<td>1352</td>
<td>101</td>
</tr>
</tbody>
</table>

| $S/B$ | 0.003 | 0.008 | 0.018 | 0.22 |
| $S/\sqrt{B}$ | 0.41 | 0.57 | 0.65 | 2.26 |
| $\sqrt{S+B}/S$ | 2.47 | 1.76 | 1.55 | 0.49 |
- The transverse mass of $\pi^+$ and $E_T$, i.e. of $H^+$ in the signal event, after imposing the additional cuts:

$$AH^+: |M(b\bar{b}) - 100| < 10 \text{ GeV}$$

or

$$HH^+: |M(b\bar{b}) - 115| < 10 \text{ GeV}$$

- $\Delta\phi$ is the azimuthal angle between $\pi^+$ and $E_T$, the transverse mass

$$M_T = \sqrt{2 p_T(\pi) E_T (1 - \cos \Delta\phi)}$$
Summary

- Assuming the double $b$-tagging efficiency to be 50%, there will be about 190 signal events, $pp \rightarrow A(\rightarrow b\bar{b}) H^+ (\rightarrow \tau^+ (\rightarrow \pi^+ \nu) \nu)$ detected at the LHC (with a 100 fb$^{-1}$ integrated luminosity).

- The total number of background events is about half of the signal event.

- Including the negative charge of $\pi^-$ increases the signal and the background rate by about 50%.

- One can also include the decay mode $\tau^\pm \rightarrow \rho^\pm \nu$ whose Br is about 22%. Hence, the event rate will roughly be tripled.

$\Rightarrow$ The observed signal event rate can be at the order of 500 to 1000 per LHC year.

$\Rightarrow$

The $AH^\pm$ signal is a promising one, indeed.

Conclusion

- If a signal is not found, studying the $AH^\pm$ associated production process can provide an upper bound on the product of the decay branching ratios of $A$ and $H^\pm$ as a function of the only one SUSY parameter $M_A$.

- In case that a signal is found, the analysis is slightly more complicated.
  
  - For $M_A \gtrsim 120$ GeV and $\tan \beta \gtrsim 10$, $M_H \sim M_A$ (less than about 10 GeV).
  
  - For $M_A \gtrsim 190$ GeV and $\tan \beta \gtrsim 10$, $\sin^2(\alpha - \beta) \simeq 1$ and $\sigma(qq' \rightarrow HH^\pm) \sim \sigma(qq' \rightarrow AH^\pm)$.

  [Generally, the coupling of $W^\pm HH^\mp$ depends on $g$ and $\sin(\alpha - \beta)$.

  - Studying different decay channels can help to separate these two production modes. For instance, a heavy $H$ can decay into a $ZZ$ pair at the Born level, but $A$ cannot.

- In conclusion, if no signal is found experimentally, a conservative bound on the product of the decay branching ratios of $A$ and $H^\pm$ can be derived for a CP-conserving model. This is because in a CP-conserving model, the $AH^+$ and $HH^+$ production modes do not interfere even if the masses of $A$ and $H$ are about the same. ($A$ is a CP-odd scalar, while $H$ is CP-even.)