THE UNDERLYING EVENT

A new model for multiple parton scattering

P. Skands & T. Sjöstrand (Lund University).

1. Basic Phenomenology:
   Multiple Interactions — Lightning Review.

2. A New Model → PYTHIA 6.3:
   Flavour and Momentum Correlations.
   Beam Remnants.
   Colour Correlations and String Topologies.

3. Outlook.
The Underlying Event

✧ Need to understand correlations and fluctuations. Simple parametrizations not sufficient. From QCD point of view: many interesting questions remain unanswered.
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Lots of fresh data from Tevatron: → great topic for phenomenology right now!
Consider just perturbative QCD $2 \to 2$ scattering:

$\star$ dominated by $t$-channel gluon exchange: $\frac{d\tilde{\sigma}}{dp^2_\perp} \propto \frac{1}{p^4_\perp}$

Cross Section is Infrared Divergent:

$$\sigma_{2\to2}(p_{\perp\text{min}}) = \int_{p_{\perp\text{min}}}^{\sqrt{s}/2} \frac{d\sigma}{dp_\perp} dp_\perp \propto \frac{1}{p^2_{\perp\text{min}}}$$
Consider just perturbative QCD $2 \rightarrow 2$ scattering:

$$\sigma_{2\rightarrow 2(p_{\text{min}})} > \sigma_{\text{tot}} \text{ for } p_{\text{min}} \lesssim 5 \text{ GeV}$$
2 Reasons...

1. **Multiple interactions!**
   - Simple consequence of composite nature of hadrons. *Must* exist!
   - $\sigma_{\text{tot}}$: **hadron-hadron** collisions. $\sigma_{\text{tot}} = \sum_{n=0}^{\infty} \sigma_n$
   - $\sigma_{2\to2}$: **parton–parton** collisions. $\sigma_{2\to2} = \sum_{n=0}^{\infty} n \sigma_n$
   - $\sigma_{2\to2} > \sigma_{\text{tot}} \iff$ Many interactions / event: $\langle n \rangle > 1$

2. **Breakdown of perturbative QCD, colour screening.**
   - $\lambda \sim 1/p_\perp$
   - $p_{\perp 0} \sim 2$ GeV

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Direct Verifications

**Basic idea**: expect two pair-wise balancing jets in double parton scattering (DPS) but not in double bremsstrahlung emission.

- **AFS**: 4-jet events at $E_\perp > 4$ GeV in 1.8 units of $\eta$. Project out 2 pairs of jets and study imbalancing variable, $I = p_{\perp 1}^2 + p_{\perp 2}^2$. Excess of events with small $I$.  

- **CDF**: Extraction by comparing double parton scattering (DPS) to a mix of two separate scatterings. Event sample: 14000 $p\bar{p} \rightarrow \gamma/\pi^0 + 3j$ events. Strong signal observed, 53% DPS.
Indirect Verifications

**Basic idea**:  
- Hadronization alone produces roughly *Poissonian* fluctuations in multiplicity.  
- Additional soft interactions can ‘mess up’ colour flow → larger fluctuations.

**UA5**: (900 GeV)  
\[
\langle n_{\text{ch}} \rangle = 35.6, \\
\sigma_{n_{\text{ch}}} = 19.6.
\]

+ forward–backward correlations (UA5, E735)  
+ pedestal effect (UA1, CDF, H1), ...

![Charged multiplicity distribution at 900 GeV](image)
Why care?

**Multiple Interactions:**

- are guaranteed to exist (+ AFS, UA1, UA5, E735, H1, CDF).
- lead to correlations and fluctuations in activity for which no detailed physics model yet exists.
- even when soft, they can have drastic consequences, by affecting the colour flow.
- when (semi)hard they produce multiple (mini)jets.
- affect jet profiles and jet pedestals.
- give random as well as systematic shifts in jet energies.

precision physics involving jets or underlying events impossible without good understanding of multiple interactions.
This talk is about **PYTHIA 6.3**

“A solemn Hellenic assembly had met at Pytho, to celebrate the death of the Pythic serpent (v. 6.2), when Eunomos sang the reptile’s epitaph. Whether his ode was a hymn in praise of the serpent, or a dirge, I am not able to say.”

[Clement of Alexandria (~ 200 AD): “Exhortation to the Heathen”]
How are the hard scattering initiators and beam remnant partons correlated?
Towards a realistic model

How are the hard scattering initiators and beam remnant partons correlated?

- In impact parameter?
- In flavour?
- In longitudinal momentum?
- In colour?
- In (primordial) transverse momentum?

(How) are the showers correlated / intertwined?
Correlations in flavour and $x_i$

Consider a hadron:

PDF for finding flavours $i_1 \ldots i_n$ with momenta $x_1 \ldots x_n$ in a hadron $H$ probed at scales $Q_1 \ldots Q_n$:

$$f_{i_1 \ldots i_n}/H(x_1 \ldots x_n, Q_1^2 \ldots Q_n^2)$$

But experimentally, all we got is $n = 1$. 
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Help: scatterings ordered in $p_\perp$!
Q: What are the pdf’s for a proton with 1 valence quark, 2 sea quarks, and 5 gluons knocked out of it?

1. Overall momentum conservation (old):

Starting point: simple scaling ansatz in $x$.

For the $n$’th scattering:

$$x \in [0, X] ; \quad X = 1 - \sum_{i}^{n-1} x_i \implies f_n(x) \sim \frac{1}{X} f_0 \left( \frac{x}{X} \right)$$
Correlations in flavour and $x_i$

**Q:** What are the pdf’s for a proton with 1 valence quark, 2 sea quarks, and 5 gluons knocked out of it?

Normalization and shape:

✧ If valence quark knocked out.

→ Impose valence counting rule: $\int_0^X q_{f_n}^{\text{val}}(x, Q^2) \, dx = N_{f_n}^{\text{val}}$.

✧ If sea quark knocked out.

→ Postulate “companion antiquark”: $\int_0^{1-x_s} q_{f}^{\text{cmp}}(x; x_s) \, dx = 1$.

✧ But then momentum sum rule is violated:

$$\int_0^X x \left( \sum_f q_{f_n}(x, Q^2) + g_n(x, Q^2) \right) dx \neq X$$

→ Assume sea+gluon fluctuates up when a valence quark is removed and down when a companion quark is added.
Correlations in flavour and $x_i$

Remnant PDFs

$$q_{fn}(x) = \frac{1}{X} \left[ \frac{N_{fn}^{val}}{N_{f0}^{val}} q_{f0}^{val} \left( \frac{x}{X}, Q^2 \right) + a q_{f0}^{sea} \left( \frac{x}{X}, Q^2 \right) + \sum_j q_{f0}^{cmpj} \left( \frac{x}{X}; x_{s_j} \right) \right]$$

$$q_{f0}^{cmp} (x; x_s) = C \tilde{g}(x + x_s) \frac{1}{x + x_s} P_{g \rightarrow q_{f0}} \left( \frac{x_s}{x + x_s} \right); \left( \int_0^1 x_s q_{f0}^{cmp}(x; x_s) \, dx = 1 \right)$$

$$g_{n}(x) = \frac{a}{X} g_0 \left( \frac{x}{X}, Q^2 \right)$$

$$a = \frac{1 - \sum_f N_{fn}^{val} \langle x_{f0}^{val} \rangle - \sum_{f,j} \langle x_{f0}^{cmpj} \rangle}{1 - \sum_f N_{f0}^{val} \langle x_{f0}^{val} \rangle}$$

Used to select a $p_\perp$-ordered sequence of hard $2 \rightarrow 2$ scatterings, and to perform backwards DGLAP shower evolution.
Intermezzo 1: exit perturbation theory

Perturbation theory got us:

- A set of interactions, with showers, starting from $k_\perp = 0$ initiator partons.
- A set of partons left behind in the beam remnants, with only flavours known at this point (by flavour conservation).
- A total $1 - X$ of longitudinal momentum has been removed from each beam remnant.

Hurdles remaining:

- Confinement effects $\rightarrow$ primordial $k_\perp$. How much? Recoils?
- What is the momentum sharing in the remnants?
- How are initiator and remnant partons correlated in colour?
- How do the remnant systems hadronize?
Confinement and primordial $k_\perp$

- Confined wavefunctions $\implies k_\perp = \hbar / r_p \sim \Lambda_{QCD}$.
- Empirically, one notes a need for larger values.

$$\frac{d^2 N}{dk_x \, dk_y} \propto e^{-k_\perp^2 / \sigma^2(Q)}$$

- $\sigma(1 \text{ GeV}) \approx 0.36 \text{ GeV (hadr.)}$
- $\sigma(10 \text{ GeV}) \approx 1 \text{ GeV (EMC)}$
- $\sigma(m_Z) \approx 2 \text{ GeV (Tevatron)}$

Recoils: along colour neighbours (or chain of neighbours) or onto all initiators and beam remnant partons equally. ($k_z$ rescaled to maintain energy conservation.)

\begin{align*}
\text{Solid:} & \quad \frac{2.1Q}{7 + Q} \quad \text{(hardcoded default)} \\
\text{Dashed:} & \quad \frac{4\sqrt{Q}}{10 + \sqrt{Q}} \\
\text{Dotted:} & \quad \frac{3\sqrt{Q}}{5 + \sqrt{Q}} \\
\text{Dot-dashed:} & \quad \frac{2.5\sqrt{Q}}{2.5 + \sqrt{Q}}
\end{align*}
Each hard scattering subsystem has light-cone momenta:

\[ p_+ = \gamma (E_1^{CM(z)} + E_2^{CM(z)}) + \gamma \beta (E_1^{CMz} + E_2^{CMz}) \]
\[ = \sqrt{\frac{1+\beta}{1-\beta}} \left( \hat{s} + (\hat{p}_{1\perp} + \hat{p}_{2\perp})^2 \right) \]
\[ = \sqrt{\frac{x_1}{x_2}} \sqrt{\hat{s}_\perp} \]

\[ p_- = \gamma (1 - \beta)(E_1^{CM(z)} + E_2^{CM(z)}) = \sqrt{\frac{x_2}{x_1}} \sqrt{\hat{s}_\perp} \]

Remaining light-cone momenta available for BR:

\[ p_{rem}^+ = \sqrt{s} - \sum_i \sqrt{\frac{x_i^{(+)}}{x_i^{(-)}}} \left( \hat{s}_i + (\hat{p}_{1\perp i}^{(+)}) + (\hat{p}_{1\perp i}^{(-)}) \right)^2 \]
\[ p_{rem}^- = \sqrt{s} - \sum_i \sqrt{\frac{x_i^{(-)}}{x_i^{(+)}}} \left( \hat{s}_i + (\hat{p}_{1\perp i}^{(+)}) + (\hat{p}_{1\perp i}^{(-)}) \right)^2 \]

Def: “±” side BR partons have fractions \( x_{j/k} \) of \( p_{rem}^\pm \).

✧ Assume \( x_{j,k} \) distributed according to ‘remnant’ pdf’s and fragmentation functions (with \((E, p)\) conserved).

✧ NB: composite BR systems (w. pion/gluon clouds?) \( \rightarrow \) larger \( x \)?
We have arrived at:

- A set of $p_\perp$-ordered interactions, with showers, taking into account non-zero primordial $k_\perp$ effects.
- A set of partons left behind in the beam remnants, whose flavours are known and whose kinematics have been worked out (i.e. $x$ and $\vec{k}_\perp$).

But life grants nothing to us mortals without hard work

- How are initiator and remnant partons correlated in colour?
- How do remnant systems hadronize?
Imagine placing a stick o’ dynamite inside a proton, imparting the 3 valence quarks with large momenta relative to each other.

‘Ordinary’ colour topology

\[ Z^0 \rightarrow q\bar{q} \]:

‘Baryonic’ colour topology

\[ \text{additional diagram} \]:

Need to extend string model to handle baryonic topology.
String Junctions

- Fundamental properties of QCD vacuum suggest string picture still applicable.
- Baryon wavefunction building and string energy minimization → picture of 3 string pieces meeting at a ‘string junction’.

(Warning: This picture was drawn in a “pedagogical projection” where distances close to the center are greatly exaggerated!)
How does the junction move?

- A junction is a topological feature of the string confinement field: $V(r) = \kappa r$. Each string piece acts on the other two with a constant force, $\kappa \vec{e}_r$.

- In junction rest frame (JRF) the angle is 120° between the string pieces.

- Or better, ‘pull vectors’ lie at 120°:

$$ p_{\text{pull}}^\mu = \sum_{i=1,N} p_i^\mu e^{-\sum_{j=1}^{i-1} \frac{E_j}{\kappa}} $$

(since soft gluons ‘eaten’ by string)

- Note: the junction motion also determines the baryon number flow!)
How does the system fragment?

First 2 pieces fragmented outwards–in, junction baryon formed around junction, last string piece fragmented as ordinary $q\bar{q}$ string.

NB: Other topologies also possible (junction–junction strings, junction–junction annihilation).
But how to draw the strings? How are initiator and beam remnant partons colour connected to each other?

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What is the colour flow?
What is the Colour Flow?

Possible ordering mechanisms:

- Always require **physical colour flow** (e.g. no singlet $g$).
- Simplest ordering is random, but gives *very* large multiplicity increase per interaction *and* large baryon number stopping.
- Tune A indicates that nature favours small increases in string length over large ones $\rightarrow$ try ‘smarter’ ways of connecting initial state colours.

1. **Random** (but with suppression of remnant breakups)
2. Ordering of connections by rapidity, $\Delta y$.
3. Ordering by approximate string length, $\Delta \lambda$. 

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Testing the Colour Correlations

- A variable that we have found to be very sensitive is the average transverse momentum (per charged particle) as a function of $n_{\text{ch}}$, $\langle p_\perp \rangle (n_{\text{ch}})$.

- At present, we cannot describe it.

![Graph showing Tevatron $\langle p_\perp \rangle (N_{\text{ch}})$ comparison between Old MI: Tune A ($p_\perp = 2.0$ GeV), New MI: random ($p_\perp = 3.15$ GeV), and New MI: $\Delta y$ ordered ($p_\perp = 3.10$ GeV), and New MI: $\Delta \lambda$ ordered ($p_\perp = 2.95$ GeV).]
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→ intertwined showers and/or FS reconnections?
Overwhelming amount of data confirms basic idea.

(AF5, UA1, UA5, E735, H1, CDF)

★ Overwhelming amount of data confirms basic idea.

(AF5, UA1, UA5, E735, H1, CDF)

★ $p_{\text{min}}/p_{\perp 0}$ cutoff.
★ Impact parameter dependence.
★ Energy dependence.
★ Multiparton densities in incoming hadrons.
★ Colour correlations and colour reconnections.
★ Interferences between showers.

Important to understand for hadronic collisions.
(+ extensions to diffractive topologies, baryon flow in heavy ion collisions, and to meson/photon beams are imaginable.)

A new physical model for detailed studies has been developed; available in PYTHIA 6.3. Right now, we’re concentrating on figuring out how to hook up those colour strings...
Summary & Outlook — Multiple Interactions

Overwhelming amount of data confirms basic idea. (AFS, UA1, UA5, E735, H1, CDF)

Much remains uncertain!

- Cutoff.
- Impact parameter dependence.
- Energy dependence.
- Multiparton densities in incoming hadrons.
- Colour correlations and colour reconnections.
- Interferences between showers.

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