\( \alpha_s \) from \( e^+e^- \) machines
(LEP & SLC)

Bill GARY

Department of Physics
U. California, Riverside

OPAL experiment at LEP

email: bill.gary@ucr.edu
Most LEP-2 QCD events:
Radiative returns: 
\( e^+ e^- \rightarrow Z^0 \gamma \)

\( e^- \quad \text{ISR} \gamma \quad \text{On-shell } Z^0 \)

\( e^+ \)

\( \rightarrow \) Cut on hadronic energy \( s' \)

For \( E_{\text{c.m.}} \gtrsim 160 \text{ GeV}, e^+ e^- \rightarrow W^+ W^- \) events contribute significant background, especially to the multi-jet (high thrust) region

Reduce WW background using cuts e.g. on QCD 4-jet matrix element, subtract residual background (\( \sim 10\% \)) using “4-fermion” MCs
Measurements of $\alpha_S$ from LEP/SLC

→ **Inclusive:** (LEP-1)
- \[ R_\ell = \frac{\Gamma(Z^0 \rightarrow \text{hadrons})}{\Gamma(Z^0 \rightarrow \ell^+\ell^-)} \quad \ell = e, \mu \text{ or } \tau \]
- \[ R_\tau = \frac{\Gamma(\tau \rightarrow \text{hadrons})}{\Gamma(\tau \rightarrow \ell^+\ell^-)} \]

→ **3-jet dominated:** (LEP-1/SLC or LEP-2)
- Event shapes: Thrust, Jet broadening, · · ·
- Jet rates
- Energy correlations

→ **Scaling violations:** (LEP-1 and LEP-2)
- “$Q^2$” evolution of fragmentation functions
\[ \alpha_S \text{ from } R_{\ell} = \frac{\Gamma(Z^0 \rightarrow \text{hadrons})}{\Gamma(Z^0 \rightarrow \ell^+\ell^-)} \]

\rightarrow \text{Ratio of the total hadronic to the single species (massless) leptonic branching ratios of the } Z^0: \ell = e, \mu \text{ or } \tau

\rightarrow \text{Based exclusively on } \text{event counting}

(experimentally, “only” need to understand the acceptance for the different types of events: Internal characteristics of the events are irrelevant)

\rightarrow \text{No correction for the effects of hadronization}

(but some hadronization uncertainty in the acceptance corrections)

\rightarrow \text{Complete } O(\alpha_S^3) \text{ (3 loop) calculation available}

(the only observable, along with } R_\tau, \text{ for which this is true)

\[ \alpha_S \text{ from } R_{\ell} \text{ has intrinsically small experimental and theoretical uncertainties!} \]
$Z^0 \rightarrow \ell^+ \ell^-$ and $Z^0 \rightarrow \text{hadrons}$
However, the dependence of $R_\ell$ on $\alpha_S$ is non-leading and therefore weak:

$$R_1 \sim Z^0 + Z^0 + Z^0$$

or $R_\ell = R_\ell^0 (1 + \delta_{QCD})$

with $R_\ell^0 = R_\ell (\alpha_S = 0) = 19.934$

and $\delta_{QCD} = 1.045 \left( \frac{\alpha_S(m_Z)}{\pi} \right) + 0.94 \left( \frac{\alpha_S(m_Z)}{\pi} \right)^2 - 15 \left( \frac{\alpha_S(m_Z)}{\pi} \right)^3 \approx 0.042$

An accurate determination of $\alpha_S$ from $R_\ell$ requires the total LEP-1 event statistics from the four experiments combined!

Total LEP-1 event statistics (combined):

$Z^0 \rightarrow \text{hadrons}: \sim 15$ million

$Z^0 \rightarrow \text{leptons}: \sim 2$ million

(all species)
\[ R_\ell = 20.767 \pm 0.025 \rightarrow \alpha_S(M_Z) = 0.1224 \pm 0.0038 \]


3\% precision \longrightarrow Uncertainty dominated by experimental systematics (e.g. acceptance for narrow 2-jet-like events near the beam axis) and statistics.

S. Bethke, J. Phys. G26 (2000) R27: \[ \alpha_S(M_Z) = 0.1184 \pm 0.0031 \]

Note the smallness of the \( \alpha_S \) result from \( \tau \) decays:

\[ \frac{\delta \alpha_S(M_Z)}{\alpha_S(M_Z)} \sim \frac{\alpha_S(M_Z)}{\alpha_S(Q)} < 1 \]

for \( Q < M_Z \)
\[ \alpha_S \text{ from Event Shapes} \]

\[ \rightarrow \text{ Internal momentum structure of an event, one entry "y" per event} \]

\[ \rightarrow \text{ 3-jet dominated quantities, leading terms } \sim \alpha_S \]

\[ y \sim z^0 + z^0 \]

- **Thrust T**: \[ T = \max \left( \frac{\sum_i \vec{p}_i \cdot \hat{n}}{\sum |\vec{p}_i|} \right) \quad i = \text{particles} \]

  resulting \( \hat{n} = \hat{n}_T \rightarrow \text{the thrust axis} \)

- **Jet broadening variables** \( B_T \) and \( B_W \):
  \[ B_k = \frac{\sum_{i \text{ in hemis. } k} |\vec{p}_i \times \hat{n}_T|}{\sum |\vec{p}_i|} \quad k = 1, 2 \quad \text{(hemispheres)} \]

  \[ B_T = B_1 + B_2 \quad \text{Total} \quad B_W = \max (B_1, B_2) \quad \text{Wide} \]
• plus many others: C parameter, jet rate “flip” value $y_{23}, \cdots$

Many of these variables are equivalent to each other at LO but have different higher order corrections.

Unlike $R_\ell$, event shape distributions are based on the internal characteristics of events.

A hadronization correction ($\sim 10\%$) is usually applied to the data before being fitted by theoretical expressions.

The hadronization correction is determined by the ratio of the Monte Carlo predictions at the parton & hadron levels.

Use Jetset versus Herwig, parton shower versus fixed order $\alpha_s^2$, etc.
QCD predictions for event shape variables

- Exact $\mathcal{O}(\alpha_S^2)$ expressions:

\[
\frac{1}{\sigma_0} \frac{d\sigma}{dy} = A(y) \frac{\alpha_S(\mu)}{2\pi} + \left[ B(y) + A(y)2\pi c_0 \log \left( \frac{\mu^2}{s} \right) \right] \left( \frac{\alpha_S(\mu)}{2\pi} \right)^2
\]

\[
c_0 = \frac{(33 - 2n_f)}{12\pi}
\]

→ The renormalization scale $\mu$ is an unphysical parameter

→ If the calculation were available to all orders in perturbation theory, there would be no dependence on $\mu$

→ For finite orders, a residual dependence $\sim \left( \log \frac{\mu^2}{s} \right)^n$ is present

→ Need $\mu \approx \sqrt{s}$ for the effects of higher order terms to be negligible

→ Two parameter fits of $\Lambda_{\overline{MS}}$ and $\mu$ to the hadronization corrected data typically yield $\mu \approx \sqrt{s}/20$, indicating the importance of the missing higher order terms

→ Theory uncertainties due to the missing higher orders (renormalization scale dependence) dominate the total uncertainty of $\alpha_S$ from event shapes
Importance of NLO terms (LEP-1 data)

[N. Magnoli, P. Nason, R. Rattazzi, PL B252 (1990) 271]

$$\alpha_S(M_Z)$$ from Thrust, Oblateness, Major, C & N3 (Jade jet finder, $y_{cut} = 0.08$)

(a) LO term only, no hadronization correction

$$\rightarrow$$ Result from Oblateness strongly inconsistent with the others

(b) NLO term also, no hadronization correction

$$\rightarrow$$ Results more consistent, smaller uncertainties because of a reduction in the scale dependence (vary $\mu$ between $M_Z$ and $M_Z/4$)

(c) and (d) NLO theory fitted to hadronization-corrected data

$$\rightarrow$$ At LEP, the effect of the NLO perturbative terms is more important than hadronization effects (this had not been the case at PEP & PETRA)
\[ \mathcal{O}(\alpha_s^2) + \text{NLLA expressions:} \]

\[ \rightarrow \text{Perturbative expansion for the cumulative event shape: } R(y) = \int_0^y \frac{1}{\sigma} \frac{d\sigma}{dy'} dy' \]

\[ \rightarrow \text{Expressed as a series in } L = \ln(1/y) \]

\[ \rightarrow \text{Most singular (largest) terms are in the 2-jet region, } y \to 0 \]

\[ \rightarrow \text{Leading and next-to-leading logarithmic terms have been summed to all orders of } \alpha_s \text{ for a number of event shape variables: NLLA} \]
→ Terms up to $\mathcal{O}(\alpha_s^2)$ in the NLLA expression are replaced by the exact $\mathcal{O}(\alpha_s^2)$ results $\longrightarrow \mathcal{O}(\alpha_s^2) + \text{NLLA}$ [ln(R) matching]

→ The most complete analytic description of event shapes currently available

→ Fits of the $\mathcal{O}(\alpha_s^2) + \text{NLLA}$ expressions to data yield results for $\mu$ much closer to the physical scale $\sqrt{s}$ than the pure $\mathcal{O}(\alpha_s^2)$ expressions

→ The NLLA terms reduce the sensitivity of the $\alpha_s$ result to the choice of $\mu$

→ The perturbative description of the data is more sensible

→ The description of the 2-jet region is improved (important for LEP-2, where statistics are limited, and where the multi-jet region is contaminated by $e^+e^- \longrightarrow WW$ events)

→ The theory uncertainty due to missing higher order terms remains the dominant uncertainty, however
Example: $x_\mu \equiv \frac{\mu}{E_{c.m.}}$ for $O(\alpha_s^2)$ and $O(\alpha_s^2) + \text{NLLA}$

[Dashed curves: $\chi^2$ of fit of the theory curve to the data for different choices of $\mu$]

$O(\alpha_s^2)$ (left plot): Minimum $\chi^2$ occurs for $x_\mu \approx 0.06$

$O(\alpha_s^2) + \text{NLLA}$ (right plot): Reasonable $\chi^2$, flat $\chi^2$ curve for $x_\mu \approx 1$
SLD

- $\mathcal{O}(\alpha_s^2)$
- Resummed $+\mathcal{O}(\alpha_s^2)$
- Data

$C_H$ = Hadronization correction

$C_D$ = Correction for detector acceptance & resolution

Fit Range

Bill GARY / U. California, Riverside

KITP, March 25, 2004
\[ \tau = 1 - T \]
\[ \rho = \frac{M_H^2}{E_{\text{vis}}}, \]
\[ M_H = \max(M_1, M_2) \]

- Solid bars \( \rightarrow \) Experimental uncertainties
- Dashed bars \( \rightarrow \) Experimental & theory uncertainties
- Top section \( \rightarrow \) \( \mathcal{O}(\alpha_S^2) \) results
- Bottom section \( \rightarrow \) \( \mathcal{O}(\alpha_S^2) \) + NLLA results (Note the reduced theoretical uncertainties compared to the \( \mathcal{O}(\alpha_S^2) \) results)
- Shaded regions \( \rightarrow \) Average \( \alpha_S \) value and total uncertainty
Hadronization correction can still be \( \sim 10\% \), even at the highest LEP-2 energies.
Combine $\alpha_S$ results based on six event shapes ($T, -\ln y_{23}, \rho, B_W, B_T, C$) and $O(\alpha_S^2) + \text{NLLA}$ calculations.

Inner error bars in plot on right exclude the perturbative uncertainty:

$\rightarrow$ Choice of $\mu$: $0.5 \leq \mu/M_Z \leq 2$

$\rightarrow$ Arbitrariness in the definition of logarithms to be summed
[e.g. whether to sum powers of $\alpha_S \ln(y_0/y)$ or $\alpha_S \ln(y_0/2y)$; $y_0 = \text{constant depending on the event shape variable}$]

$\rightarrow$ Sum powers of $\alpha_S \ln[y_0/(x_L y)]$, $\frac{2}{3} \leq x_L \leq \frac{3}{2}$
[see R.W.L. Jones et al., JHEP0312 (2003) 007]
\[ \alpha_s(M_Z) = 0.1214 \pm 0.0045 \text{(perturbative)} \pm 0.0018 \]

(combination of the results of six event shape measurements at eight energies)

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\(\rightarrow\) 4\% precision

The perturbative uncertainty decreases with increasing energy, faster than \(\alpha_s\) itself (16\% versus 1.5\%), but it remains the dominant uncertainty of the measurement
\( \alpha_S \) using NLO corrections to 4-jet observables


Fit of NLO predictions for the 4-jet rate (Durham jet finder) to the hadronization & detector corrected data

NLO + resummed 4-jet predictions from DEBRECAN MC, Z. Trócsányi

\[ \alpha_S(M_Z) = 0.1170 \pm 0.0001(\text{stat.}) \pm 0.0013(\text{syst.}) \]

(1% uncertainty, Bayesian method: systematic variations which result in larger \( \chi^2 \) are given less weight)

\[ \alpha_S(M_Z) = 0.1170 \pm 0.0001(\text{stat.}) \pm 0.0022(\text{syst.}) \]

(2% uncertainty, add systematic uncertainties in quadrature)

Include color factors \( C_A \) and \( C_F \) in the fit:

\[ \alpha_S(M_Z) = 0.119 \pm 0.006(\text{stat.}) \pm 0.026(\text{syst.}) \] (ALEPH 2003)

\[ \alpha_S(M_Z) = 0.120 \pm 0.011(\text{stat.}) \pm 0.020(\text{syst.}) \] [OPAL Eur.Phys.J.C20 (2001) 601]
The basic experimental situation with respect to measuring $\alpha_S$ at $e^+e^-$ colliders hasn't much changed in the past 10 years!!

- Availability of $\mathcal{O}(\alpha^2_S)$ + NLLA expressions

- Uncertainties dominated by the lack of higher orders

Improvements in the experimental situation (higher energies of LEP-2) haven’t had too much impact on the overall uncertainty attributed to $\alpha_S$

We need improvements in the theory!

- NNLO [order $\mathcal{O}(\alpha^3_S)$] calculations of event shapes should lead to a reduction in the dependence of the result on the choice of the renormalization scale

- Re-analysis of LEP/SLD data

For the two-jet region, we need NNLLA resummed calculations ??