Hagedorn Inflation

Talk by
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Hagedorn Inflation

with Steve Abel, Ian Kogan

IDEA:
Open strings on branes
at high temperatures
(close to string scale)
drive inflation

n.b. NO potential required

How it works:
Negative pressure in bulk
cause our brane to inflate.
Why Inflation?

1) Smoothness of the Universe
   - homogeneity & isotropy on large scales

Microwave background radiation

\( R_{\text{then}} \approx \frac{10^{28} \text{ cm}}{1000} \approx 10 \text{ cm} \)

Surface of last scattering
\( t \approx 10^6 \text{ yrs} \)
\( e^p \rightarrow H \gamma \)
\( T \approx 4000 \text{ K} \)
\( \frac{\Delta T}{T} \approx 10^{-5} \)

Comoving size of observable universe today is
\( L_0 = \int \frac{dt}{a(t)} \)

Scale factor today
Decaying time of radiation decoupling

\( L_0 \leq L_n \)

Present observable universe was then \( \approx 10^5 \) causally distinct regions.
For $a > t^p, p < 1$

before $t_n$ and after $t_{dec},$

$L_0 \sim \frac{t_0}{a_0} \sim \frac{1}{Ho a_0} \quad (H = \frac{\dot{a}}{a})$

$L_n \sim \frac{t_n}{a_n} \sim \frac{1}{H_{nan}}$

causality condition becomes

$\frac{1}{an H_n} \geq \frac{1}{a_0 Ho} \Rightarrow \frac{1}{\dot{a}_n} > \frac{1}{\dot{a}_0}$

Inflation solves with $\ddot{a} > 0$ somewhere between $t_n \rightarrow t_0$

$\ddot{a} = -\frac{4\pi}{3mp} (\xi + 3p)$

e.g. vacuum energy with $p = -\xi$
The Hagedorn Phase

Fundamental strings have a large number of degrees of freedom. Many oscillator modes

\[ \text{Density of States} \quad w(\varepsilon) \propto e^{\beta_H \varepsilon} \]

where \( T_H = \frac{1}{\beta_H} = \text{Hagedorn Temperature} \)

and \( \varepsilon \) is energy and e.g. \( \langle E \rangle = \int d\varepsilon \, w(\varepsilon) e^{-\beta \varepsilon} \)

Physical quantities (\( F, \langle E \rangle, \ldots \))

\( \sim (\beta - \beta_H)^{-x} \)

typically diverge at \( T_H \).
Density of States for Open Strings on Branes at High Energy:

\[ \omega(\varepsilon)_{\text{open}} \propto \frac{V_{\parallel}}{V_{\perp}} \cdot e^{\beta_H \varepsilon} \]

obtained via
- microcanonical ensemble
  \[ E \to \Omega \to S = \ln \Omega \to T^{-1} = \frac{\Delta S}{\Delta E} \]
- random walk
  \[ \to \text{same answer} \]

Abel, Barbon, Kogan, Rabinovici
So Far:
Density of states of open strings attached to branes near $T_H$

Next:
find partition function $Z$

Bulk $T_{\mu\nu}$

Result: obtain negative pressure 
Put into Einstein's equations to find inflation of our brane.

Partition Function
\[
\log Z \sim \int d\varepsilon \omega(\varepsilon) e^{-\beta \varepsilon}
\]
\[
\sim 2 \frac{\nu_{11}^{2} \beta h}{\nu_{1} (\beta^{2} - \beta h)} + \text{nonsingular cut-off terms}
\]

Bulk Energy-Momentum Tensor
\[
\langle T_{\mu\nu} \rangle = \langle \frac{\delta S}{\delta g_{\mu\nu}} \rangle
\]
\[
\sim \frac{\delta \log Z}{\delta g_{\mu\nu}}
\]

for forces $T_{\mu\nu} = 0$

Assume: small changes in metric correspond to small changes in volume.

Eg: $\frac{\delta Z}{\delta g_{55}} = \int dx^5 \frac{\delta Z}{\delta V_1(x^1)} \frac{\delta V_1(x^1)}{55}$

for a single extra dim: $\frac{\delta V_1(x^1)}{55}$
Results for Bulk Energy-Momentum

\[ T^{\mu\nu} = \left( \frac{E}{\sqrt{v_i v_j}} \right) \delta^{\mu\nu} \]

\[ p \sim \frac{1}{\sqrt{v_l}} \rho^{3/2} \]

\[ (\rho = E/v_{ii}) \]

\[ \text{Negative pressure in Bulk} \]

Consider 5 dimensional universe:

\[ ds^2 = -n^2 dt^2 + a(t) (f_3 y)^2 d\xi^2 + b(t) (f_3 y)^2 dy^2 \]

scale factor of spatial 3 planes

scale factor of extra dimensions
Cosmological Equations in $D=5$

$$ds^2 = - n^2 dt^2 + a(t,y)^2 dx^2 + b(t,y)^2 dy^2$$

Scale factor of spatial 3-planes

Only one transverse dimension "y" that supports winding modes.

For simplicity, impose $Z_2$ symmetry under $y \to -y$.

Take 3-brane of observable universe to be at $y=0$.

Equations:
5-D bulk Einstein's eqns.

Israel conditions for boundary

$$3 \left[ \frac{a'}{a} \right]_0 = - \frac{8 \pi}{M_5^3} b_0 \delta_{br}$$

$$3 \left[ \frac{n'}{n} \right]_0 = \frac{8 \pi}{M_5^3} b_0 (2 \delta_{br} + 3 \delta_{pr})$$

5-D bulk Einstein eqns:

$$G_{00} = 3 \left( \frac{\dot{a}}{a} + \frac{b'}{b} \right) - \frac{n^2}{b^2} \left[ \frac{a''}{a} + \frac{a' b'}{b} - \frac{b''}{b} \right]^2 = \kappa^2 T_{00}$$

$$G_{ii} = \frac{a^2}{b^2} \left( \frac{a^2}{a} + 2 \frac{n^2}{n} \right) + \frac{b'}{b} \left( \frac{n^2}{n^2} + 2 \frac{a'}{a} \right) + 2 \frac{a''}{a} + \frac{n''}{n}$$

$$+ \frac{a^2}{n^2} \left( - \frac{\dot{a}}{a} + 2 \frac{n'}{n} \right) - 2 \frac{a'}{a} + \frac{b'}{b} \left( - \frac{\dot{a}}{a} + \frac{n'}{n} \right) - \frac{\dot{b}^2}{b^2} = \kappa^2 T_{ii}$$

$$G_{05} = 3 \left( \frac{\dot{a}}{a} + \frac{a'}{a} \frac{b}{b} - \frac{\dot{b}}{b} \right) = \kappa^2 T_{05}$$

$$G_{55} = 3 \left( \frac{a'}{a} + \frac{n'}{n} \right) - \frac{\dot{a}^2}{a^2} - \frac{\dot{b}^2}{b^2} - \frac{\dot{b}^2}{b^2} = \kappa^2 T_{55}$$

where $\kappa^2 = \frac{8 \pi}{M_5^3}$

prime $\to \frac{1}{y}$ and dot $\to \frac{1}{t}$

Use $T_{00}$ appropriate to Hagedorn regime.

Use negative bulk pressure.
Israel Conditions for Boundary

similar to electrostatics

\[ \Delta E_1 = 4\pi \sigma \]

charge/area in braneworlds

\[ a' = \frac{da}{dy} \]

\[ \Delta a' \propto s_{br} \]

Relates energy-momentum on our brane to its extrinsic curvature, i.e., to the way it's embedded in the bulk.

Results: Behavior of Scale Factors \( a(t) \) and \( b(t) \) at \( y = 0 \)

Case: \( \lambda_{br} = \lambda_{bulk} = b = 0 \)

Look at 55 equation:

1) In high energy limit of windup in all transverse dimensions, \( T_{S} \propto \sqrt{E} \propto a^{-p/2} \)

\[ \Rightarrow a t^{4/p}, \quad p = \# \text{ large parallel dimensions} \]

n.b. true for any number of extra dimensions

For our observable universe, \( p = 3 \Rightarrow a t^{4/3} \)

SUPERLUMINAL EXPANSION

Amusing result: inflation requires \( p \leq 3 \) regardless of total \# dimensions

2) For 2 dimensions w/o windups:

Exponential expansion

n.b. Results for \( a(t) \) assume \( E \& S = \text{constant} \)
Define: \[ \delta = \frac{d_0}{2} - 1 \]

where \(d_0\) is the number of dimensions transverse to the brane with no windings.

Standards high-energy case:
\[ d_0 = 0 \Rightarrow \delta = -1 \]

Results for "\(p\)" large parallel dimensions:

\begin{itemize}
  \item \(d_0 = 0\) (High Energy)
    - \(S = E / V_{11} \)
    - \(a(t) / a(0) = t^{4p}\)
  \item \(d_0 = 1\)
    - \((\beta - \beta_H)^{-3/2}\)
    - \(S = t^{4p}\)
  \item \(d_0 = 2\)
    - \((\beta - \beta_H)^{-1}\)
    - \(S = \log t \exp(-Ct^2 + Dt)\)
  \item \(d_0 = 3\)
    - \((\beta - \beta_H)^{-1/2}\)
    - \(S = t^{-2p}\)
  \item \(d_0 = 4\)
    - \(-\log(\beta - \beta_H) e^{-S}\)
\end{itemize}

Explanation for 3 large dimensions?

In high energy regime, superluminal expansion requires a brane with \(\rho \leq 3\). Our brane has \(\rho = 3\).

Case: 'net' cosmological constant = 0
\[ - \frac{\kappa^2}{12} \Lambda_{br}^2 + \Lambda_{bulk} = 0 \]

Take \(b \neq 0\).

Family of power law solutions
\[ a_0(t) \sim A t^q, q = \frac{\delta - 1}{2} \left( \frac{4}{3} - r \right) \]

For high energy \(\delta = -1\), find superluminal \(q > 1\) for \(r < 1/3\)

\(e.g.\): \(r < 0\) (shrinking)

We also find a family of hyperbolic solutions.
\[ a(t) = \left( \frac{\sinh 2C (t + t_1)}{\sinh 2C t} \right)^{1/2} \]

Can be exponentially expanding.
Which, if any, of these solutions is appropriate depends on initial value for \(d_0\).
Sustaining Inflation

We find: inflation begins.

We know: once $T$ drops out of Hagedorn regime, inflation ends.

We cannot calculate in between.

Once inflation begins, our calculations break down [our interpretation of $\frac{1}{g} \to \frac{1}{S} V$ fails].

We can speculate de Sitter.

1) Turok: strings in AdS sustain inflation.
   
   Our work leads to this setup. Present work provides explanation for how universe enters de Sitter.

2) Bath of branes + bulk strings
   
   (e.g. branes smashing into each other)
   
   keep system hot $\Leftarrow$ TO DO

3) Generalized 2nd law
   
   $S = S_{\text{bulk}} + S_{\text{brane}} = S_{\text{brane}} + \frac{1}{4} \alpha' \text{Aosenon}
   
   $dS > 0$ drives $T \to T_H$


Conclusion

Hagedorn inflation:

open strings on branes near $T_H$

$\downarrow$

negative pressure in bulk

$\downarrow$

drives our brane to inflate