Quantum phases and critical points of correlated metals

Outline

I. Kondo lattice models
   Doniach’s phase diagram and its quantum critical point

II. A new phase: FL*
    Paramagnetic states of quantum antiferromagnets:
    (A) Bond order, (B) Topological order.

III. Lieb-Schultz-Mattis-Laughlin-Bonesteel-Affleck-Yamanaka-Oshikawa flux-piercing arguments

IV. Extended phase diagram and its critical points

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I. Doniach’s $T=0$ phase diagram for the Kondo lattice

\[ H = \sum_{i<j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_{\alpha} \left( J_K c_{i\alpha}^\dagger \vec{\tau} \cdot \vec{S}_{i\alpha} \cdot \vec{S}_{i\beta} \right) \]

$c_{i\alpha}$ → Conduction electrons;
\[ \vec{S}_{i\beta} \rightarrow \text{localized } f_{i\alpha} \text{ moments (assumed } S=1/2, \text{ for specificity)} \]

Local moments choose some static spin arrangement

"Heavy" Fermi liquid with moments Kondo screened by conduction electrons. Fermi surface obeys Luttinger’s theorem.

\[ J_{\text{RKKY}} = J_K^2 / t \quad T_K \sim \exp(-t / J_K) \]

SDW

FL

\[ J_K / t \]

Luttinger’s theorem on a $d$-dimensional lattice for the FL phase

Let $v_0$ be the volume of the unit cell of the ground state, $n_T$ be the total number density of electrons per volume $v_0$. (need not be an integer)

\[ n_T = n_f + n_c = 1 + n_c \]

\[ 2 \times \frac{v_0}{(2\pi)^d} \left( \text{Volume enclosed by Fermi surface} \right) \]

= $n_T \pmod{2}$

A "large" Fermi surface
Quantum phases and critical points of correlated metals

Arguments for the Fermi surface volume of the FL phase

Single ion Kondo effect implies $J_K \to \infty$ at low energies

\[
(c_{i\uparrow} f_{i\downarrow} - c_{i\downarrow} f_{i\uparrow}) |0\rangle \quad \text{and} \quad f_{i\downarrow}^\dagger |0\rangle, \ S=1/2 \text{ hole}
\]

Fermi liquid of $S=1/2$ holes with hard-core repulsion

Fermi surface volume \( = - (\text{density of holes}) \mod 2 \)
\( = -(1-n_f) = (1+n_f) \mod 2 \)

Alternatively:

Formulate Kondo lattice as the large $U$ limit of the Anderson model

\[
H = \sum_{i<j} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + \sum_i \left( V f_{i\sigma}^\dagger c_{i\sigma} + V f_{i\sigma} c_{i\sigma} + \epsilon_f \left( n_{f\uparrow} + n_{f\downarrow} \right) + U n_{f\uparrow} n_{f\downarrow} \right) + \cdots
\]

\[
n_T = n_f + n_c
\]

For small $U$, Fermi surface volume \( = (n_f + n_c) \mod 2 \).
This is adiabatically connected to the large $U$ limit where $n_f = 1$.
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**Quantum critical point between SDW and FL phases**

Spin fluctuations of renormalized $S=1/2$ fermionic quasiparticles, $h_\sigma$

*(loosely speaking, $T_\xi$ remains finite at the quantum critical point)*

**Gaussian** theory of paramagnon fluctuations: $\tilde{\phi} \sim h_\sigma^{1/2} \tau_\sigma h_\sigma$.

**Action:**

$$ S = \int \frac{d^d q d\omega}{(2\pi)^{d+1}} \left| \tilde{\phi}(q,\omega) \right|^2 \left( q^2 + |\omega| + \Gamma(\delta,T) \right) $$


Characteristic paramagnon energy at finite temperature $\Gamma(0,T) \sim T^p$ with $p > 1$.

Arises from non-universal *corrections* to scaling, generated by $\sim \frac{\phi}{\phi}$ term.


**Quantum critical point between SDW and FL phases**

Additional singular corrections to quasiparticle self energy in $d=2$


*A. Rosch Phys. Rev. B 64, 174407 (2001).*

Critical point *not* described by strongly-coupled critical theory with universal dynamic response functions dependent on $\hbar \omega/k_q T$

In such a theory, paramagnon scattering amplitude would be determined by $k_q T$ alone, and not by value of microscopic paramagnon interaction term.


*(Contrary opinions: P. Coleman, Q. Si...........)*
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Reconsider Doniach phase diagram

II. A new phase: FL*

This phase preserves spin rotation invariance, and has a Fermi surface of sharp electron-like quasiparticles.

The state has “topological order” and associated neutral excitations. The topological order can be easily detected by the violation of Luttinger’s theorem. It can only appear in dimensions $d > 1$

$$2 \times \frac{V_0}{(2\pi)^d} \left( \text{Volume enclosed by Fermi surface} \right) = (n_T - 1) \pmod{2}$$

Quantum phases and critical points of correlated metals

It is more convenient to consider the Kondo-Heisenberg model:

\[ H = \sum_{i<j} t_{ij} \hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} + \sum_i \left( J_K \hat{c}_{i\sigma}^\dagger \hat{\tau}_{\sigma\sigma}^\dagger \hat{c}_{i\sigma} \cdot \hat{S}_{fi} \right) + \sum_{i<j} J_H (i, j) \hat{S}_{fi} \cdot \hat{S}_{fj} \]

_work in the regime \( J_H > J_g \)

Determine the ground state of the quantum antiferromagnet defined by \( J_H \), and then couple to conduction electrons by \( J_K \).

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**Ground states of quantum antiferromagnets**

Begin with magnetically ordered states, and consider quantum transitions which restore spin rotation invariance.

Two classes of ordered states:

(A) Collinear spins

(B) Non-collinear spins

\[ \langle \hat{S} (r) \rangle \propto \bar{N} \cos (Q \cdot r) \]

\( Q = (\pi, \pi) \); \( \bar{N}^z = 1 \)

\[ \langle \hat{S} (r) \rangle \propto \bar{N}_1 \cos (Q \cdot r) + \bar{N}_2 \sin (Q \cdot r) \]

\( Q = \left( \frac{4\pi}{3}, \frac{4\pi}{\sqrt{3}} \right) \); \( \bar{N}_1 = \bar{N}_2 = 1 \); \( \bar{N}_1 |\bar{N}_2 = 0 \)
Quantum phases and critical points of correlated metals

(A) Collinear spins, bond order, and confinement

\[
\langle \vec{S}(r) \rangle \propto \bar{N} \cos(Q \cdot \vec{r})
\]

\[
Q = (\pi, \pi); \quad \bar{N}^2 = 1
\]

Quantum phases and critical points of correlated metals

**State of conduction electrons**

At $J_K=0$ the conduction electrons form a Fermi surface on their own with volume determined by $n_c$.

Perturbation theory in $J_K$ is regular and so this state will be stable for finite $J_K$.

However, because $n_f=2$ (per unit cell of ground state) $n_f + n_c = n_f (\text{mod } 2)$, and Luttinger’s theorem is obeyed.

**FL state with bond order**

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**(B) Non-collinear spins, deconfined spinons, $Z_2$ gauge theory, and topological order**

$$\langle S(r) \rangle \propto \overline{N}_1 \cos(Qr) + \overline{N}_2 \sin(Qr)$$

$$Q = \left( \frac{4\pi}{3}, \frac{4\pi}{\sqrt{3}} \right) ; \overline{N}_1^2 = \overline{N}_2^2 = 1 ; \overline{N}_1 \parallel \overline{N}_2 = 0$$

Quantum transition restoring spin rotation invariance

RVB state with free spinons


Quantum phases and critical points of correlated metals

\[
\langle \hat{S}(r) \rangle \propto N_1 \cos(Q \cdot r) + N_2 \sin(Q \cdot r)
\]

\[
Q = \begin{pmatrix} 4\pi \\ 3 \end{pmatrix} \begin{pmatrix} 4\pi \\ \sqrt{3} \end{pmatrix} = N_1 = N_2 = 1; N_1 N_2 = 0
\]

Solve constraints by writing:

\[
N_1 + iN_2 = \epsilon_{a \sigma} z_a \sigma_{ab} z_b
\]

where \( z_{1,2} \) are two complex numbers with

\[
|z_1|^2 + |z_2|^2 = 1
\]

Order parameter space: \( S_3/Z_2 \)

Physical observables are invariant under the \( Z_2 \) gauge transformation \( z_a \rightarrow \pm z_a \)

Other approaches to a \( Z_2 \) gauge theory:

**Vortices associated with \( \pi_1(S_3/Z_2) = Z_2 \)**

Can also consider vortex excitation in phase without magnetic order, \( \langle \hat{S}(r) \rangle = 0 \): vison

A paramagnetic phase with vison excitations suppressed has topological order. Suppression of visons also allows \( z_\alpha \) quanta to propagate – these are the spinons.

State with spinons must have topological order
Quantum phases and critical points of correlated metals

State of conduction electrons
At $J_K = 0$ the conduction electrons form a Fermi surface on their own with volume determined by $n_c$

Perturbation theory in $J_K$ is regular, and topological order is robust, and so this state will be stable for finite $J_K$

So volume of Fermi surface is determined by $(n_T - 1) = n_A \pmod{2}$, and Luttinger's theorem is violated.

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Adiabatically insert flux $\Phi=2\pi$ (units $\hbar=c=1$) acting on $\uparrow$ electrons.
State changes from $|\Psi\rangle$ to $|\Psi'\rangle$, and $UH(U^{-1}=H(\Phi))$, where

$$U = \exp\left[\frac{2\pi i}{L_x} \sum_r x \hat{n}_{r\uparrow}\right].$$


Adiabatic process commutes with the translation operator $T_x$, so momentum $P_x$ is conserved.

However $U^{-1}T_xU = T_x \exp\left[\frac{2\pi i}{L_x} \sum_r \hat{n}_{r\uparrow}\right]$;
so shift in momentum $\Delta P_x$ between states $U|\Psi'\rangle$ and $|\Psi\rangle$ is

$$\Delta P_x = \frac{\pi L_x}{v_0} n_f \left(\text{mod} \frac{2\pi}{a_x}\right)$$

(1).

Alternatively, we can compute $\Delta P_x$ by assuming it is absorbed by quasiparticles of a Fermi liquid. Each quasiparticle has its momentum shifted by $2\pi/L_x$, and so

$$\Delta P_x = \frac{2\pi \left(\text{Volume enclosed by Fermi surface}\right)}{L_x} \left(\text{mod} \frac{2\pi}{a_x}\right)$$

(2).

From (1) and (2), same argument in $y$ direction, using coprime $L_x/a_x, L_y/a_y$:

$$2\times\frac{v_0}{(2\pi)^2} \left(\text{Volume enclosed by Fermi surface}\right) = n_f \left(\text{mod} 2\right)$$

Quantum phases and critical points of correlated metals

Effect of flux-piercing on a topologically ordered quantum paramagnet

N. E. Bonesteel,
G. Misguich, C. Lhuillier,
M. Mambrini, and P. Sindzingre,

\[ |D\rangle = \sum_{D} a_D |D\rangle \]

Effect of flux-piercing on a topologically ordered quantum paramagnet

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G. Misguich, C. Lhuillier,
M. Mambrini, and P. Sindzingre,

After flux insertion \[ |D\rangle \Rightarrow \]

Number of bonds \[ (-1)^{c}\]
cutting dashed line \[ |D\rangle ; \]

Equivalent to inserting a vison inside hole of the torus.

Vison carries momentum \[ \pi L_y / v_0 \]
Quantum phases and critical points of correlated metals

**Flux piercing argument in Kondo lattice**

Shift in momentum is carried by $n_T$ electrons, where

$$n_T = n_f + n_c$$

In topologically ordered, state, momentum associated with $n_f=1$ electron is absorbed by creation of vison. The remaining momentum is absorbed by Fermi surface quasiparticles, which enclose a volume associated with $n_c$ electrons.

**The FL* state.**

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V. **Conclusions**

Subir Sachdev, Yale University (KITP CEM Conference 11/21/02)
Quantum phases and critical points of correlated metals

IV. Extended $T=0$ phase diagram for the Kondo lattice

- $*$ phases have spinons with $Z_2$ ($d=2,3$) or $U(1)$ ($d=3$) gauge charges, and associated gauge fields.
- Fermi surface volume does not distinguish SDW and SDW* phases.

- Because of strong gauge fluctuations, $U(1)$-FL* may be unstable to $U(1)$-SDW* at low temperatures.
- Only phases at $T=0$: FL, SDW, $U(1)$-SDW*.

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- Because of strong gauge fluctuations, U(1)-FL* may be unstable to U(1)-SDW* at low temperatures.
- Only phases at $T=0$: FL, SDW, U(1)-SDW*.
- Quantum criticality dominated by a $T=0$ FL-FL* transition.

Non-trivial universal scaling function which is a property of a bulk $d$-dimensional quantum field theory describing “hidden” order parameter.

**Strongly coupled quantum criticality with a topological or spin-glass order parameter**

Order parameter does not couple directly to simple observables

Dynamic spin susceptibility

$$\chi(q, \omega) = \frac{1}{-i\gamma\omega + A(q - Q)^2 + B + T^\alpha \Phi \left[ \frac{\hbar \omega}{k_B T} \right] + T}$$
Quantum phases and critical points of correlated metals

\[ \text{Z}_2 \text{ fractionalization} \]

- Superconductivity is generic between FL and \( \text{Z}_2 \) FL* phases.

\[ \text{Magnetic frustration} \]

- Supercconductivity is generic between FL and Z_2 FL* phases.

\[ \text{SDW*} \]

- SDW*

\[ \text{SDW} \]

- SDW

- Hertz Gaussian paramagnon theory

\[ \text{FL*} \]

- FL*

\[ \text{FL} \]

- FL

\[ J_K / t \]

\[ \text{Mean-field phase diagram} \]

- Decoupled

- FL

- FL*

- Superconductor

Pairing of spinons in small Fermi surface state induces superconductivity at the confinement transition.

Small Fermi surface state can also exhibit a second-order metamagnetic transition in an applied magnetic field, associated with vanishing of a spinon gap.
Conclusions

- New phase diagram as a paradigm for clean metals with local moments.
- Topologically ordered (⋆) phases lead to novel quantum criticality.
- New FL* allows easy detection of topological order by Fermi surface volume.