From Traffic Jams to Transport Barriers - the Physics of Bottleneck Formation

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- Ozgur Gurcan; Ecole Polytechnique
- Yusuke Kosuga; Kyushu University
- Guilhem Dif-Pradalier; CEA, Cadarache

Some Relevant Publications

- Y. Kosuga, P.H. Diamond, O.D. Gurcan; Phys. Rev. Lett. 110, 105002 (2013)
- O.D. Gurcan, P.H. Diamond, et al, Phys. Plasmas 20, 022307 (2013)
- Y. Kosuga, P.H. Diamond, Phys. Plasmas, in press (2014)

also

- Y. Kosuga: Invited Talk, 2013 APS-DPP Meeting
- Z.B. Guo, P.H. Diamond, et al; Phys. Rev. E, in press (2014)
- Z.B. Guo, P.D. Diamond; in preparation (2014)

Outline

- Comments on the 'Big Picture'
- Brief Primer on Tokamak Transport → Avalanches in Flux-Driven Transport
- The ExB Staircase as a Heat Flux Jam
- Some Ongoing Work

Observations

- Fundamental concept of zonal flow formation is secondary mode in gas of drift waves, i.e. modulational instability
  - wave kinetics
  - envelope expansion
  - ...
- N.B. No clear scale separation, inverse cascade, Rhines mechanism ...
- Interest is driven by (favorable) impact of flows on confinement
- This drives a concern with feedback and the picture of co-existing, competing populations, etc.

Key Question:

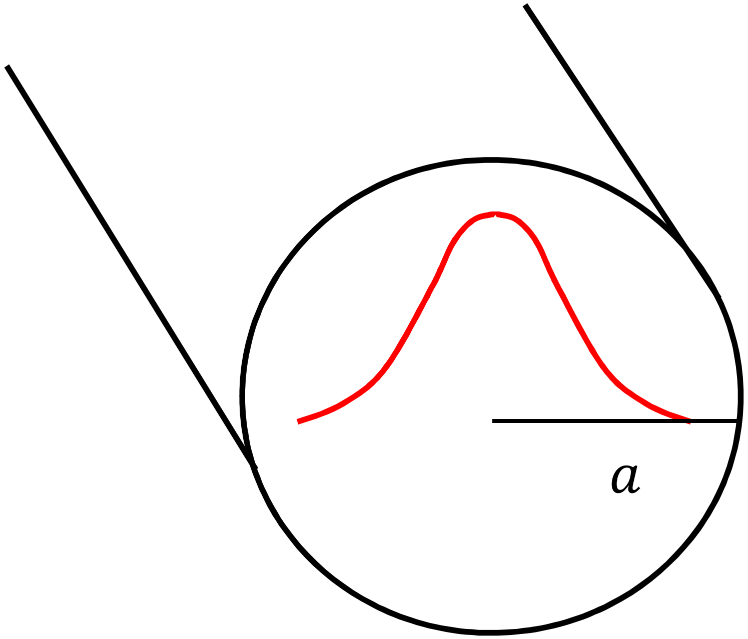
Is 'drift wave turbulence' in confined plasmas really wave turbulence?

Tokamak Turbulence and Transport

→ How do plasmas form a profile?

→ What limits gradients?

Primer on Turbulence in Tokamaks



- $\nabla T, \nabla n$, etc. driver
- Quasi-2D, elongated cells aligned with B_0
- Characteristic scale ~ few ρ_i
- Characteristic velocity $v_d \sim \rho_* c_s$

2 scales:

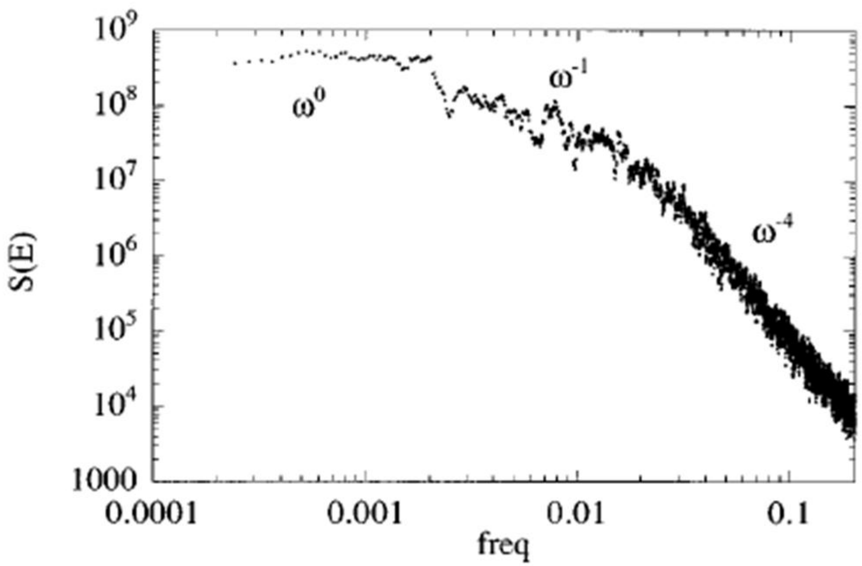
$\rho \equiv$ gyro-radius

$a \equiv$ cross-section

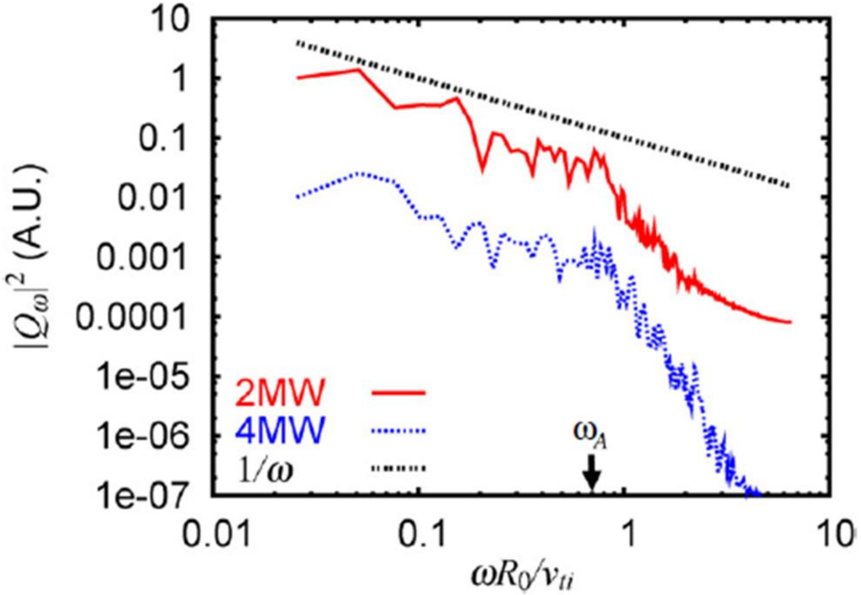
$\rho_* \equiv \rho/a$ ➔ key ratio

- Transport scaling: $D \sim \rho v_d \sim \rho_* D_B \sim D_{GB}$
- i.e. Bigger is better! ➔ sets profile scale via heat balance
- Reality: $D \sim \rho_*^{\alpha} D_B$, $\alpha < 1$ ➔ why??
- Key Issue: Is drift wave turbulence 'wave turbulence'?

- ‘Avalanches’ form! – flux drive + geometrical ‘pinning’



Newman PoP96 (sandpile)
(Autopower frequency spectrum of ‘flip’)



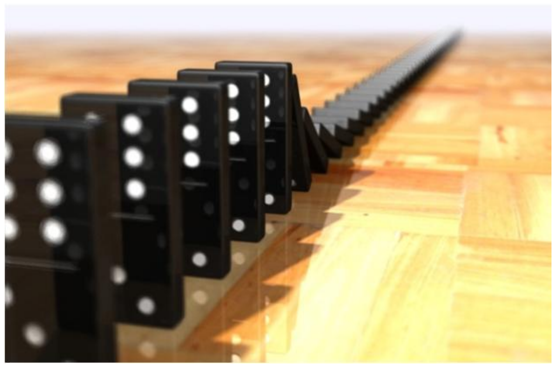
GK simulation also exhibits avalanching
(Heat Flux Spectrum) (Idomura NF09)

- Avalanching is a likely cause of ‘gyro-Bohm breaking’

➔ localized cells self-organize to form transient, extended transport events

- Akin domino toppling:
- Pattern competition

  with shear flows!



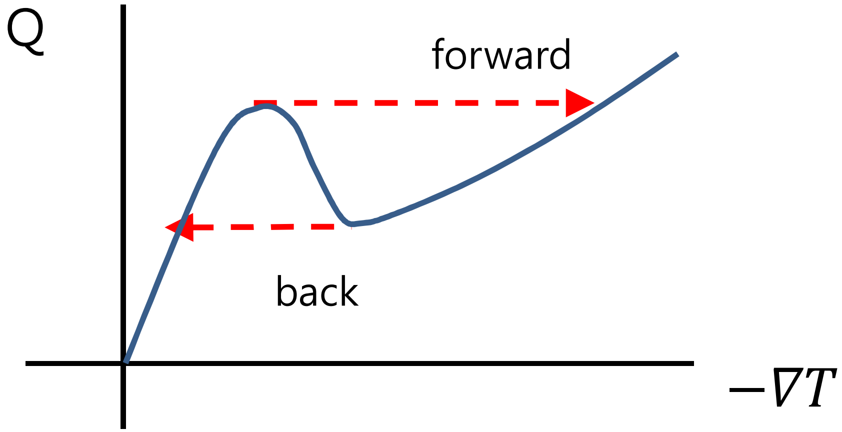
Toppling front can
penetrate beyond region
of local stability

Concept of a Transport Bifurcation
i.e. how generate the sheared flow??

N.B. Edge sheared flow / transport barrier ➔ L→H transition

→ First Theoretical Formulation of L→H Transition as an

- Transport Bifurcation
- $\langle E_r \rangle'$ Bifurcation



➔ Appearance of S-curve in a Physical Model of L→H Transition

➔ Formulation of Criticality Condition (Threshold) for Transport Bifurcation

→ Theoretical Ideas on Hysteresis, ELMs, Pedestal Width, …..

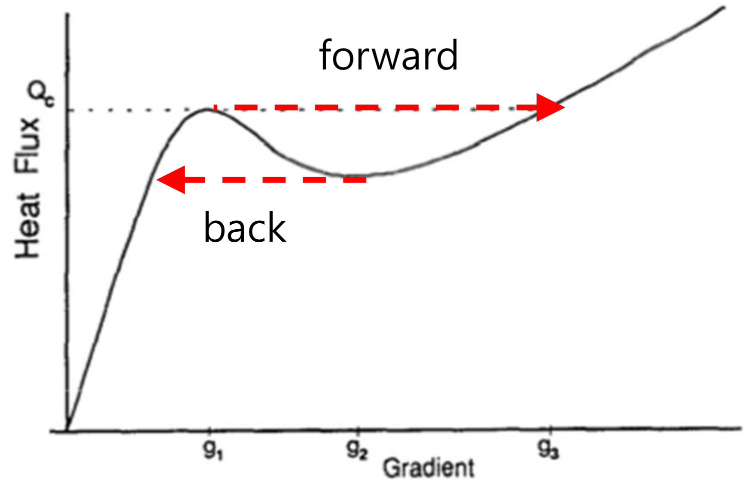
→ Coupling of Transport Bifurcation to turbulence, $\langle v_E \rangle'$ suppression

→ Non-linear Fick's Law, extension

$$Q = -\frac{\chi_T}{1+\alpha {v'_E}^2}\nabla T - \chi_{neo}\nabla T$$

Shearing feedback

$$v'_E = -\frac{\partial}{\partial r}\left(\frac{c}{eB}\frac{\nabla p}{n_0}\right) \quad p = n_0 T$$



Heat flux S-curve induced by profile-dependent shearing feedback

Profile Bifurcation

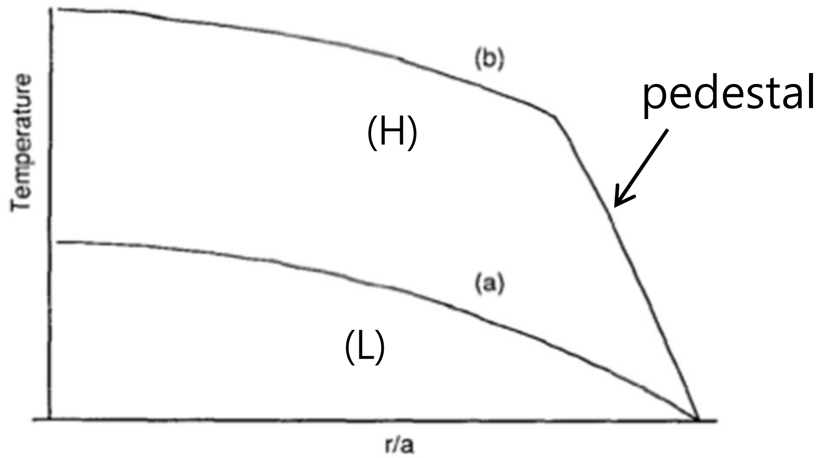


FIG. 2. Temperature profiles near the power threshold (arbitrary units): (a) $Q(a) = 0.99Q_c$; (b) $Q(a) = 1.01Q_c$.

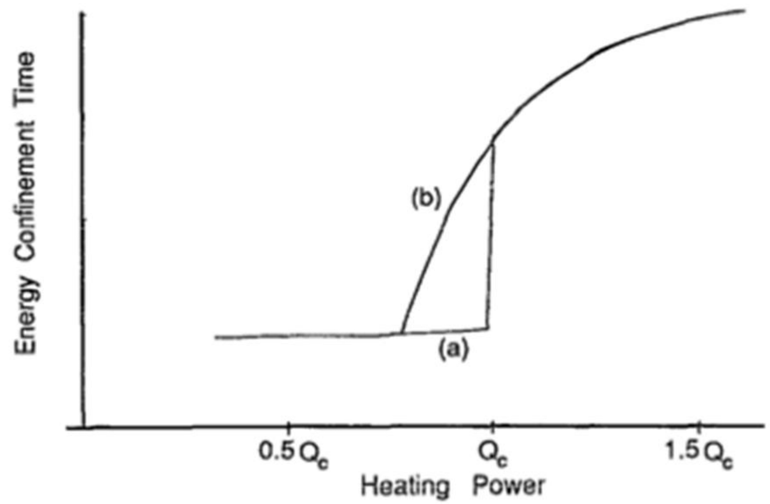


FIG. 4. Power hysteresis in the energy confinement time (arbitrary units): (a) increasing power; (b) decreasing power.

Staircases and Traffic Jams

Single Barrier → Lattice of Shear Layers

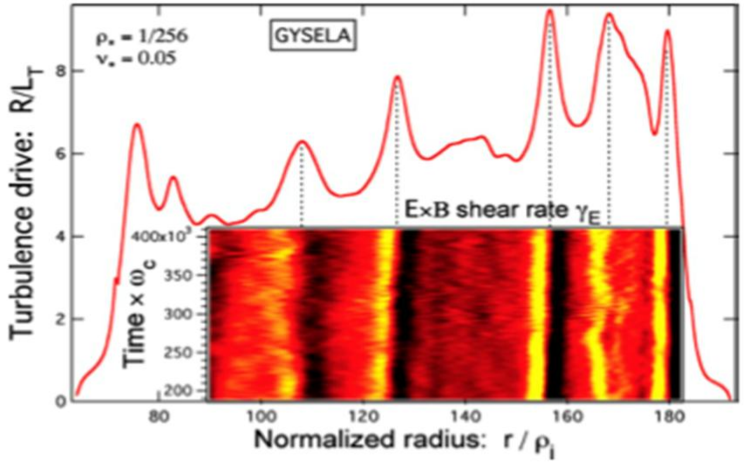
→ Jam Patterns

Highlights

Observation of ExB staircases

→ Failure of conventional theory

(emergence of particular scale???)



Model extension from Burgers to telegraph

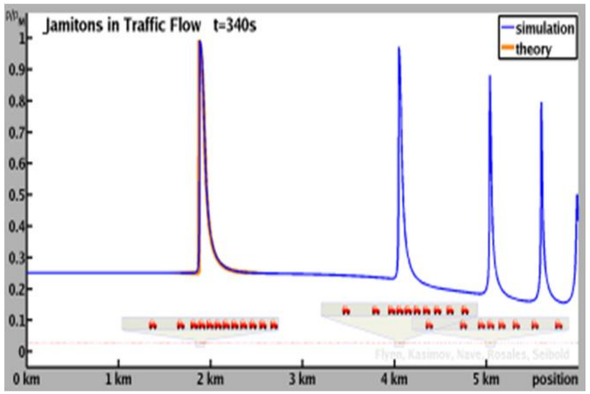
$$\partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T$$

$$\Rightarrow \tau \partial_t^2 \delta T + \partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T$$

finite response time → like drivers' response time in traffic

Analysis of telegraph eqn. predicts heat flux jam

- scale of jam comparable to staircase step

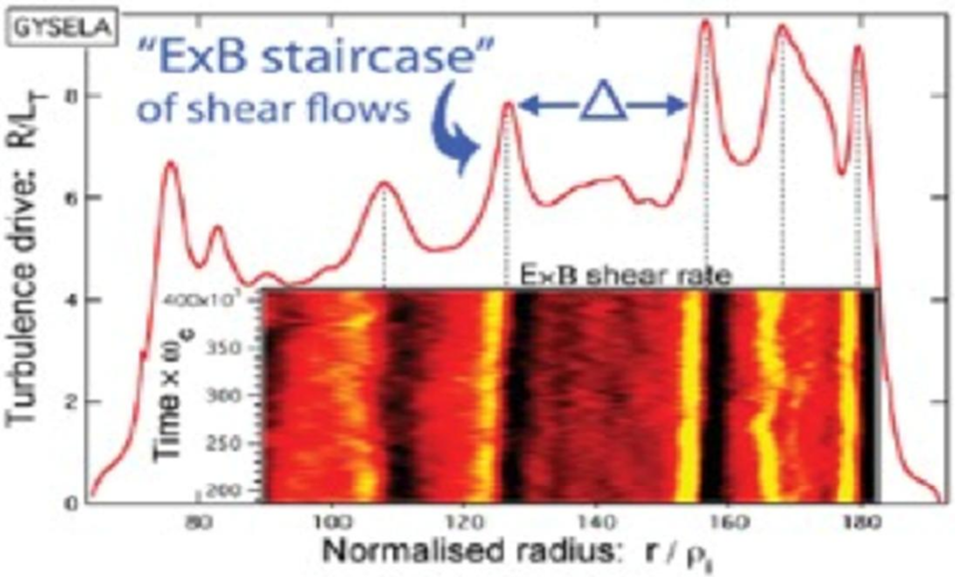


Motivation: ExB staircase formation (1)

- ExB flows often observed to self-organize in magnetized plasmas

  eg.) mean sheared flows, zonal flows, ...

- `ExB staircase' is observed to form (G. Dif-Pradalier, P.D. et al. Phys. Rev. E. '10)

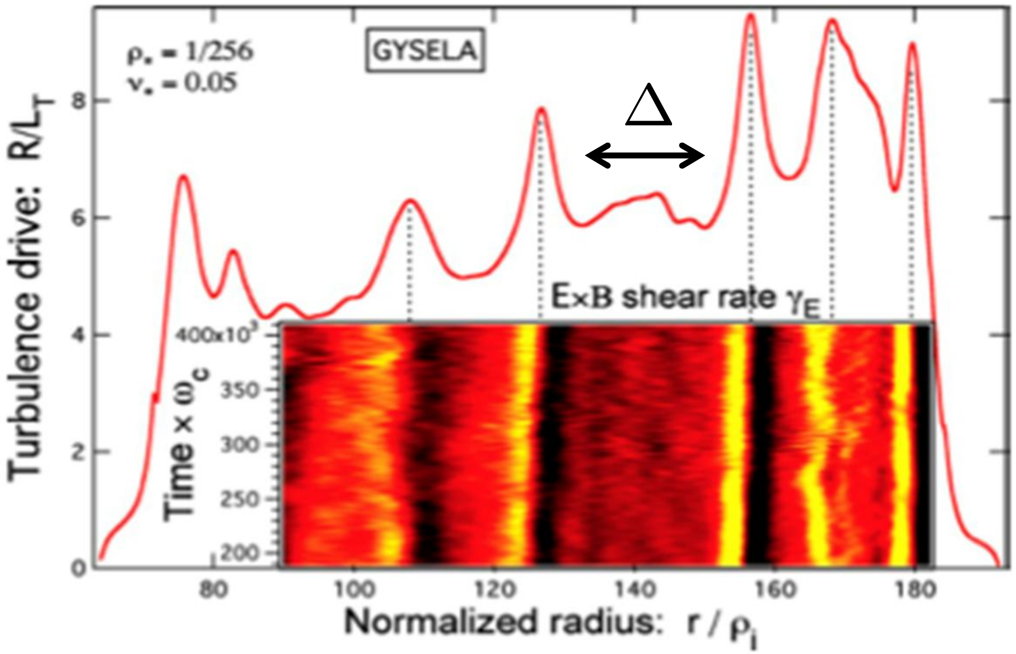


- flux driven, full f simulation
- Quasi-regular pattern of shear layers and profile corrugations
- Region of the extent $\Delta \gg \Delta_c$ interspersed by temp. corrugation/ExB jets

  → ExB staircases
- so-named after the analogy to PV staircases and atmospheric jets
- Step spacing → avalanche outer-scale

ExB Staircase (2)

- Important feature: co-existence of shear flows and avalanches



- Seem mutually exclusive ?!?

→ strong ExB shear prohibits transport

→ avalanches smooth out corrugations

- Can co-exist by separating regions into:

1. avalanches of the size $\Delta \gg \Delta_c$

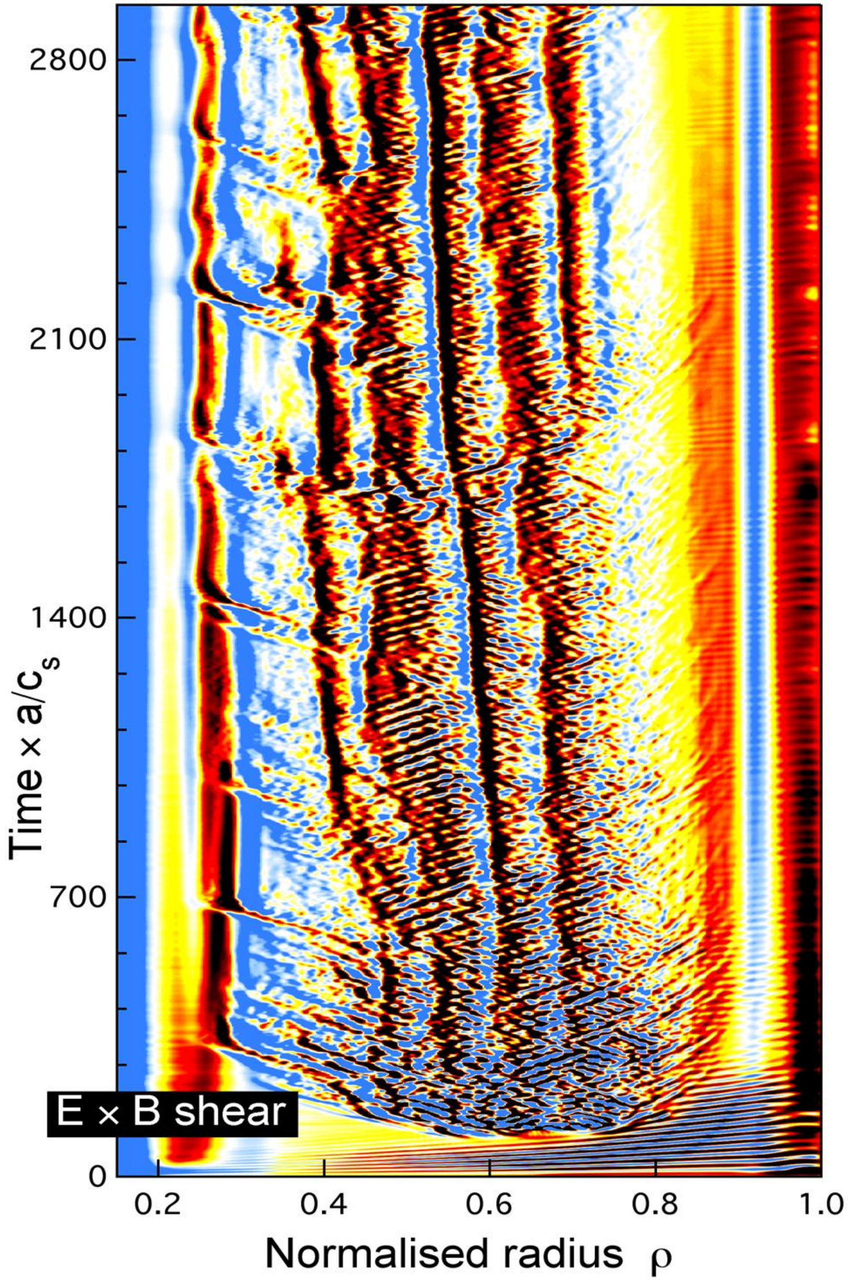
2. localized strong corrugations + jets

- How understand the formation of ExB staircase???

- What is process of self-organization linking avalanche scale to ExB step scale?

i.e. how explain the emergence of the step scale ???

Staircases build up from the edge

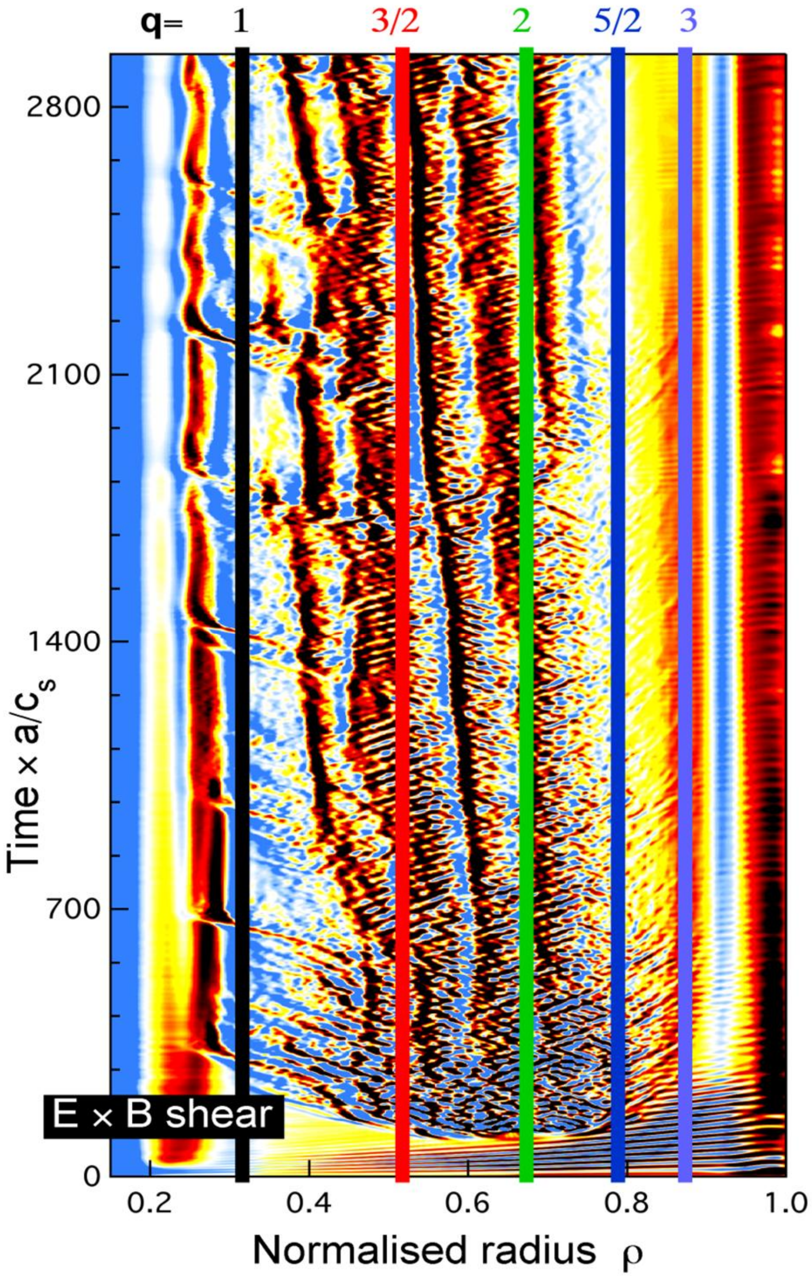


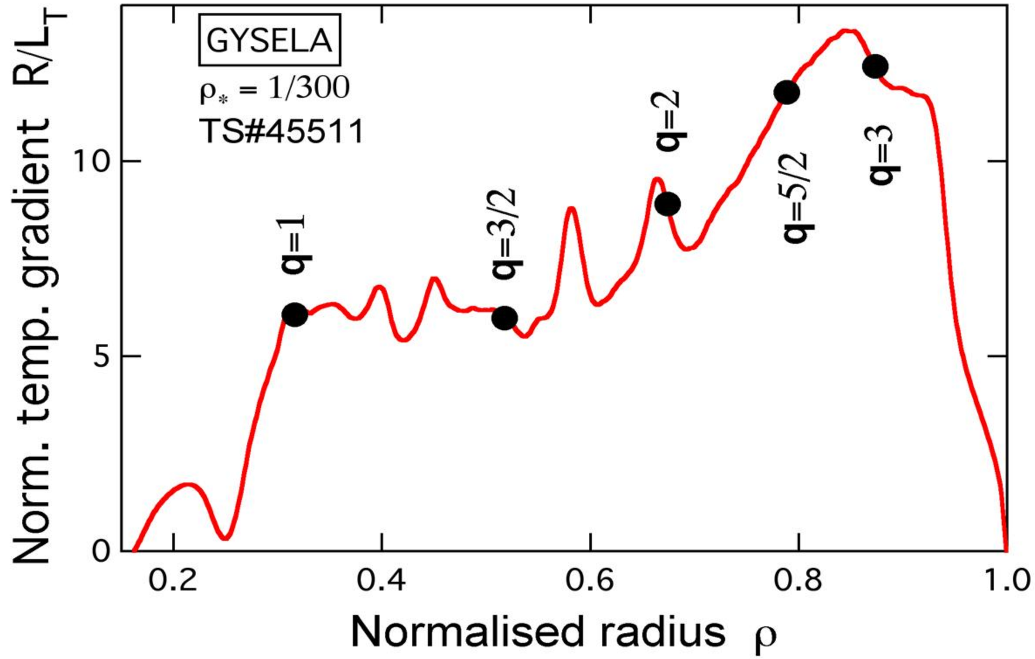
→ staircases may not be related to zonal flow eigenfunctions

→ How describe generation mechanism??

(GYSELA simulation)

Corrugation points and rational surfaces – no relation!





Step location not tied to magnetic geometry structure in a simple way

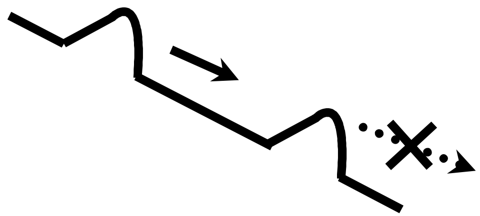
Towards a model

• How do we understand quasi-regular pattern of ExB staircase, generated from stochastic heat avalanche???

• An idea: jam of heat avalanche

corrugated profile ⟷ ExB staircase

→ corrugation of profile occurs by 'jam' of heat avalanche flux



→ accumulation of heat increment
→ stationary corrugated profile

* → time delay between $Q[\delta T]$ and δT is crucial element

like drivers' response time in traffic

• How do we actually model heat avalanche 'jam' ??? → origin in dynamics?

Traffic jam dynamics: ‘jamiton’

- A model for Traffic jam dynamics → Whitham

$$\rho_t + (\rho v)_x = 0$$

$$v_t + v v_x = -\frac{1}{\tau}\left\{ v - V(\rho) + \frac{\nu}{\rho}\rho_x \right\}$$

ρ → car density

v → traffic flow velocity

$V(\rho) - \frac{\nu}{\rho}\rho_x$ → an equilibrium traffic flow

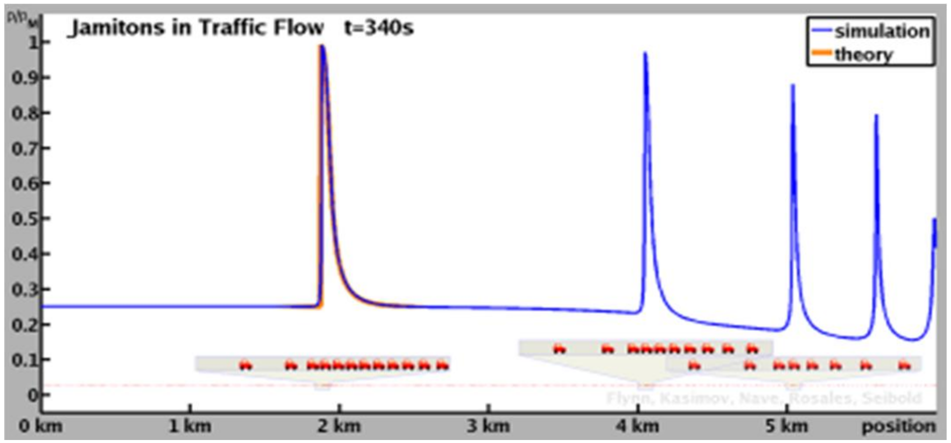
τ → driver’s response time

→ Instability occurs when $\tau > \nu/(\rho_0^2 V_0'^2)$

$D_{eff} = \nu - \tau\rho_0^2 V_0'^2 < 0$ → clustering instability

→ Indicative of jam formation

- Simulation of traffic jam formation



http://math.mit.edu/projects/traffic/

→ Jamitons (Flynn, et.al., ’08)

n.b. I.V.P. → decay study

Heat avalanche dynamics model (`the usual')

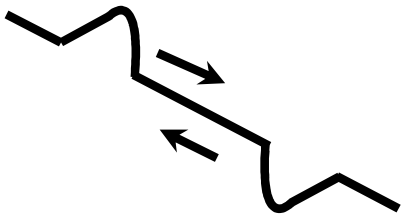
Hwa+Kardar ' 92, P.D. + Hahm ' 95, Carreras, et al. ' 96, ... GK simulation, ... Dif-Pradalier ' 10

- δT :deviation from marginal profile → conserved order parameter

- Heat Balance Eq.: $\partial_t \delta T + \partial_x Q[\delta T] = 0$ → up to source and noise

- Heat Flux $Q[\delta T]$? → utilize symmetry argument, ala' Ginzburg-Landau

  - Usual: → joint reflectional symmetry (Hwa+Kardar'92, Diamond+Hahm '95)



$$\delta T \leftrightarrow -\delta T$$
$$x \leftrightarrow -x$$

$$Q = Q_0(\delta T) = \frac{\lambda}{2}\delta T^2 - \chi_2 \partial_x \delta T + \chi_4 \partial_x^3 \delta T$$

hyperdiffusion

lowest order → Burgers equation

$$\partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T$$

An extension of the heat avalanche dynamics

- An extension: a finite time of relaxation of Q toward SOC flux state

$$\partial_t Q = -\frac{1}{\tau}\left(Q - Q_0(\delta T)\right)$$

$$Q_0[\delta T] = \frac{\lambda}{2}\delta T^2 - \chi_2 \partial_x \delta T + \chi_4 \partial_x^3 \delta T$$

→ In principle $\tau(\delta T, Q_0)$ ⟷ large near criticality (∼ critical slowing down)

i.e. enforces time delay between δT and heat flux

- Dynamics of heat avalanche:

$$\partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T - \chi_4 \partial_x^4 \delta T - \tau \partial_t^2 \delta T$$

→ Burgers
(P.D. + T.S.H. ’95)

New: finite response time

→ Telegraph equation

n.b. model for heat evolution

diffusion → Burgers → Telegraph

Relaxation time: the idea

- What is 'τ ' physically? → Learn from traffic jam dynamics
- A useful analogy:

heat avalanche dynamics	traffic flow dynamics
temp. deviation from marginal profile	local car density
heat flux	traffic flow
mean SOC flux (ala joint relflection symmetry)	equilibrium, steady traffic flow
heat flux relaxation time	driver's response time

- driver's response can induce traffic jam
- jam in avalanche → profile corrugation → staircase?!?
- Key: instantaneous flux vs. mean flux

Heat flux dynamics: when important?

- Heat flux evolution: $$\partial_t Q = -\frac{1}{\tau_{mix}}(Q - Q_0)$$ → time delay, when important?

Conventional Transport Analysis

$\tau_{mix} \ll$ time scale of interest

→ Heat flux relaxes to the mean value immediately

$$Q = Q_0$$

→ Profile evolves via the mean flux

$$\partial_t T + \partial_x Q_0 = 0$$

then

diff. $$\partial_t T = \chi \partial_x^2 T$$

Burgers $$\partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T$$

New approach for transport analysis

→ mixing time can be long, so

$\tau_{mix} \sim$ time scale of interest

→ Heat evo. and Profile evo. must be treated self-consistently

$$\begin{cases} \partial_t Q = -\frac{1}{\tau}(Q - Q_0) \\ \partial_t \delta T + \partial_x Q[\delta T] = 0 \end{cases}$$

then telegraph equation:

$$\partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T - \tau \partial_t^2 \delta T$$

Time delay: microscopic foundation?

- Relaxation by plasma turbulence = mixing of phase space density

$$\frac{df}{dt} = 0 \Rightarrow \partial_t \langle \delta f(1) \delta f(2) \rangle + \frac{1}{\tau_{mix}} \langle \delta f(1) \delta f(2) \rangle = - \langle \tilde{v}_r \delta f \rangle \langle f \rangle'$$

phase space density correlation =
'phasetrophy'

turbulent mixing
↓
i.e. PV mixing time sets delay

production
due gradient relaxation

- Energy moment leads to heat flux evolution equation (Gurcan '13)

$$\partial_t Q = - \frac{1}{\tau_{mix}} (Q - Q_0) \qquad Q_0 = - \chi_{turb} \nabla T$$

→ Heat flux relaxes toward the mean value, in the mixing time

The delay time is a natural consequence of phase space density mixing. The delay time is typically in the order of mixing time.

Brief summary on model extension

	Heat Flux	Profile evo.	
Usual:	$Q = Q_0[\delta T]$	$\partial_t T = \chi \partial_x^2 T$	Diffusion
		$\partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T$	Burgers
Extended:	$\partial_t Q = -\frac{1}{\tau}(Q - Q_0)$	$\tau \partial_t^2 \delta T + \partial_t \delta T + \lambda \delta T \partial_x \delta T = \chi_2 \partial_x^2 \delta T$	telegraph

finite response time

- Physical idea: analogy to traffic dynamics, drivers' response time

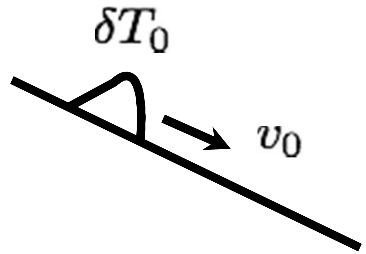
- Microscopic foundation: mixing of phase space density

- Finite response time → Heat dynamics described by telegraph eqn.

  → Wavy feature, speed determined by $\sqrt{\chi_2/\tau}$

Analysis of heat avalanche dynamics via telegraph

- How do heat avalanches jam?



- Consider an initial avalanche, with amplitude δT_0, propagating at the speed $v_0 = \lambda \delta T_0$

→ turbulence model dependent

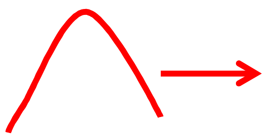
- Dynamics:

$$\partial_t \widetilde{\delta T} + v_0 \partial_x \widetilde{\delta T} = \chi_2 \partial_x^2 \widetilde{\delta T} - \chi_4 \partial_x^4 \widetilde{\delta T} - \tau \partial_t^2 \widetilde{\delta T}$$

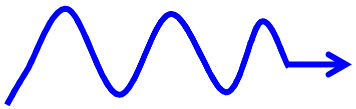
pulse

'Heat flux wave': $\sqrt{\frac{\chi_2}{\tau}}$
telegraph → wavy feature

two characteristic propagation speeds



v_0



$\sqrt{\frac{\chi_2}{\tau}}$

→ In short response time (usual) heat flux wave propagates faster

→ In long response time, heat flux wave becomes slower and pulse starts overtaking.
What happens???

Analysis of heat avalanche jam dynamics

- In large tau limit, what happens? → Heat flux jams!!

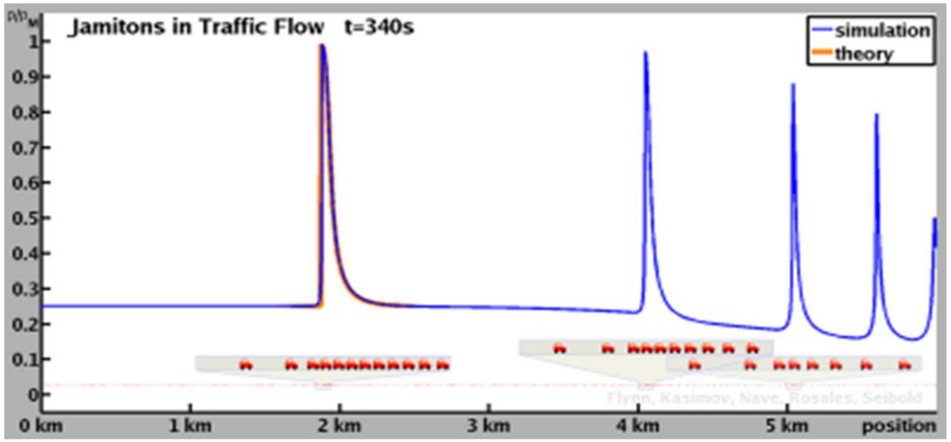
- Recall plasma response time akin to driver's response time in traffic dynamics

- negative heat conduction instability occurs (as in clustering instability in traffic jam dynamics)

$$\partial_t \widetilde{\delta T} + v_0 \partial_x \widetilde{\delta T} = \chi_2 \partial_x^2 \widetilde{\delta T} - \chi_4 \partial_x^4 \widetilde{\delta T} - \tau \partial_t^2 \widetilde{\delta T}$$
$$\rightarrow \underline{(\chi_2 - v_0^2 \tau)} \partial_x^2 \widetilde{\delta T} - \chi_4 \partial_x^4 \widetilde{\delta T}$$

<0 when overtaking

→ clustering instability



n.b. akin to negative viscosity instability of ZF in DW turbulence

instead ZF as secondary mode in the gas of primary DW

➔ Heat flux 'jamiton' as secondary mode in the gas of primary avalanches

Analysis of heat avalanche jam dynamics

- Growth rate of the jamiton instability

$$\gamma = -\frac{1}{2\tau} + \frac{1}{2\tau}\sqrt{\frac{r+1}{2} - 2\tau\chi_2 k^2\left(1+\frac{\chi_4 k^2}{\chi_2}\right)} \qquad r = \sqrt{\left\{4\tau\chi_2 k^2\left(1+\frac{\chi_4 k^2}{\chi_2}\right)-1\right\}^2 + 16 v_0^2 k^2 \tau^2}$$

- Threshold for instability

$$\tau > \frac{\chi_2}{v_0^2}\left(1+\frac{\chi_4 k^2}{\chi_2}\right)$$

→ critical minimal delay time

n.b. $1/\tau = 1/\tau[\mathcal{E}]$

→ clustering instability strongest near criticality

- Scale for maximum growth

$$k^2 \cong \frac{\chi_2}{\chi_4}\sqrt{\frac{\chi_4 v_0^2}{4\chi_2^3}}$$

from

$$\frac{\partial\gamma}{\partial k^2} = 0 \quad \Longrightarrow \quad 8\tau\frac{\chi_4^2}{\chi_2}k^6 + 4\tau\chi_4 k^4 + 2\frac{\chi_4}{\chi_2}k^2 + 1 - \frac{v_0^2\tau}{\chi_2} = 0$$

→ staircase size, $\Delta_{stair}^2(\delta T)$, δT from saturation: consider shearing

Scaling of characteristic jam scale

- Saturation: Shearing strength to suppress clustering instability

Jam growth → profile corrugation → ExB staircase → $v'_{E\times B}$

→ estimate, only

→ saturated amplitude: $$\frac{\delta T}{T_i} \sim \frac{1}{v_{thi}\rho_i}\sqrt{\frac{\chi_4}{\tau}}$$

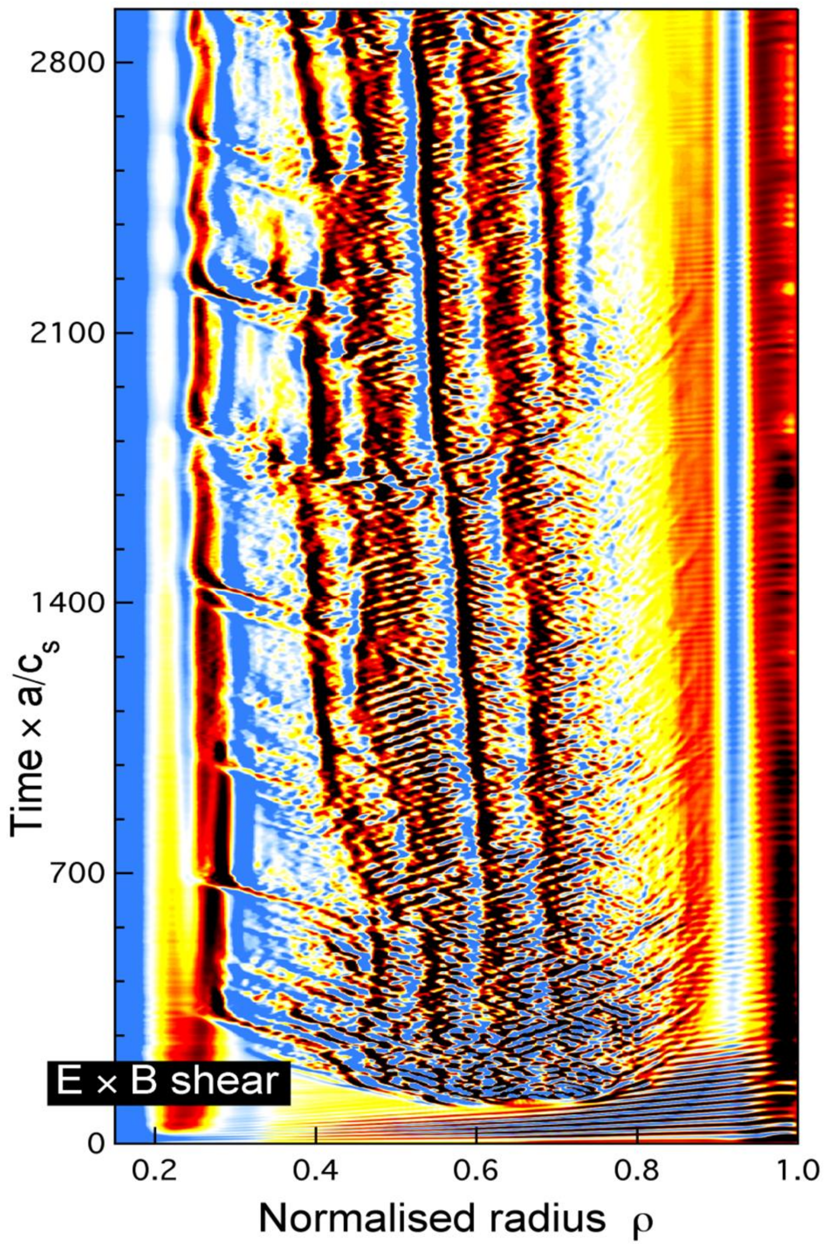
- Characteristic scale

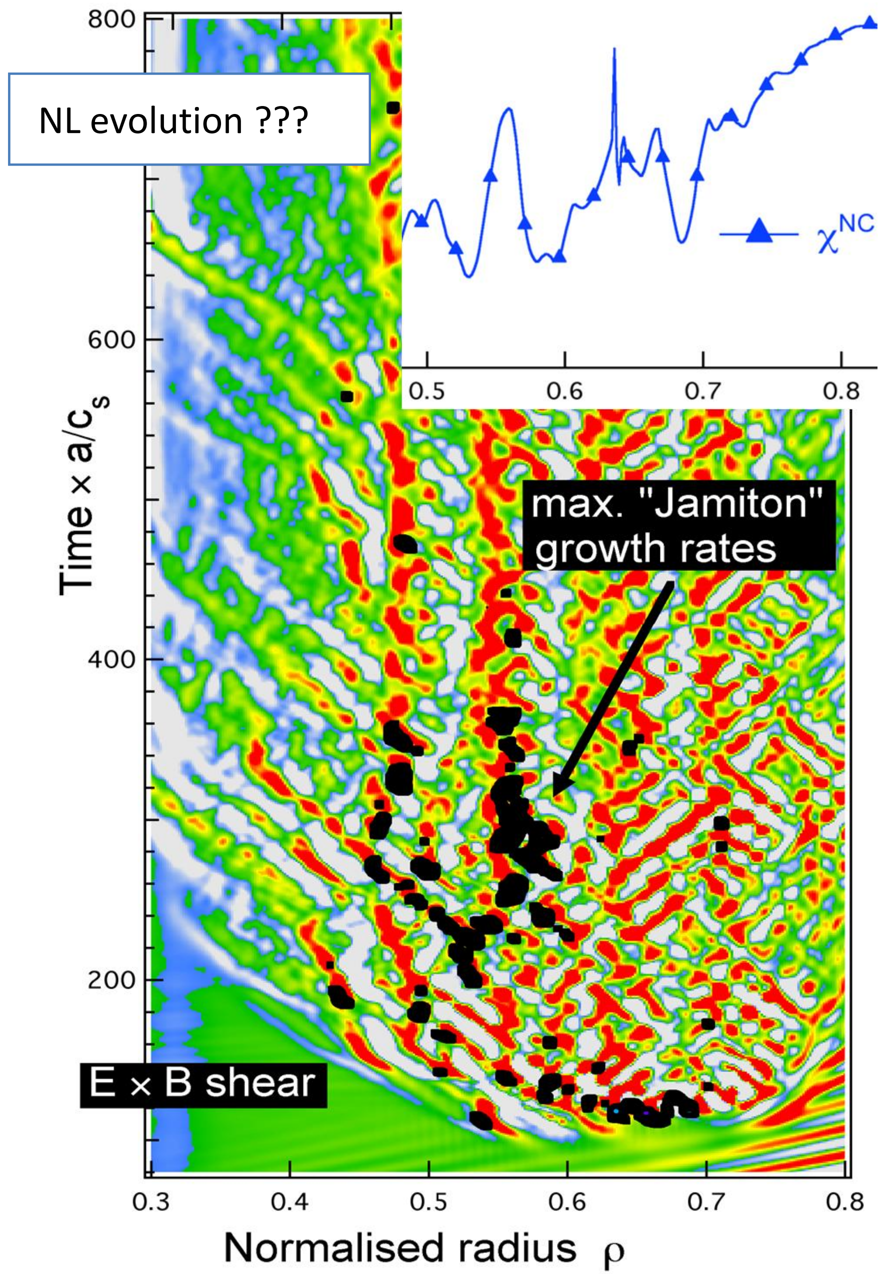
$$\Delta^2 \sim k^{-2}(\delta T) \sim \frac{2v_{thi}}{\lambda T_i}\rho_i\sqrt{\chi_2\tau} \qquad \chi_2 \sim \chi_{neo}$$

- Geometric mean of ρ_i and $\sqrt{\chi_2\tau}$: ambient diffusion length in 1 relaxation time

- ‘standard’ parameters: $\Delta \sim 10\Delta_c$

Jam growth qualitatively consistent with staircase formation





outer radius:
large chi
→ smear out instability
or
→ heat flux waves propagate faster
→ harder to overtake, jam

good agreement in early stage

Dif-Pradalier ’13 caveat: based on model with compressional waves

Aside: FYI – Historical Note

- –→ Collective Dynamics of Turbulent Eddy
- 'Aether' I – First Quasi-Particle Model of Transport?!

– Kelvin, 1887

XLV. *On the Propagation of Laminar Motion through a turbulently moving Inviscid Liquid. By* Sir WILLIAM THOMSON, *LL.D., F.R.S.**

1. IN endeavouring to investigate turbulent motion of water between two fixed planes, for a promised communication to Section A of the British Association at its coming Meeting in Manchester, I have found something seemingly towards a solution (many times tried for within the last twenty years) of the problem to construct, by giving vortex motion to an incompressible inviscid fluid, a medium which shall transmit waves of laminar motion as the luminiferous æther transmits waves of light.

2. Let the fluid be unbounded on all sides, and let u, v, w be the velocity-components, and p the pressure at (x, y, z, t). We have

$$\frac{du}{dx}+\frac{dv}{dy}+\frac{dw}{dz}=0 \quad . \quad . \quad . \quad . \quad . \quad (1),$$

* Communicated by the Author, having been read before Section A of the British Association at its recent Meeting in Manchester.

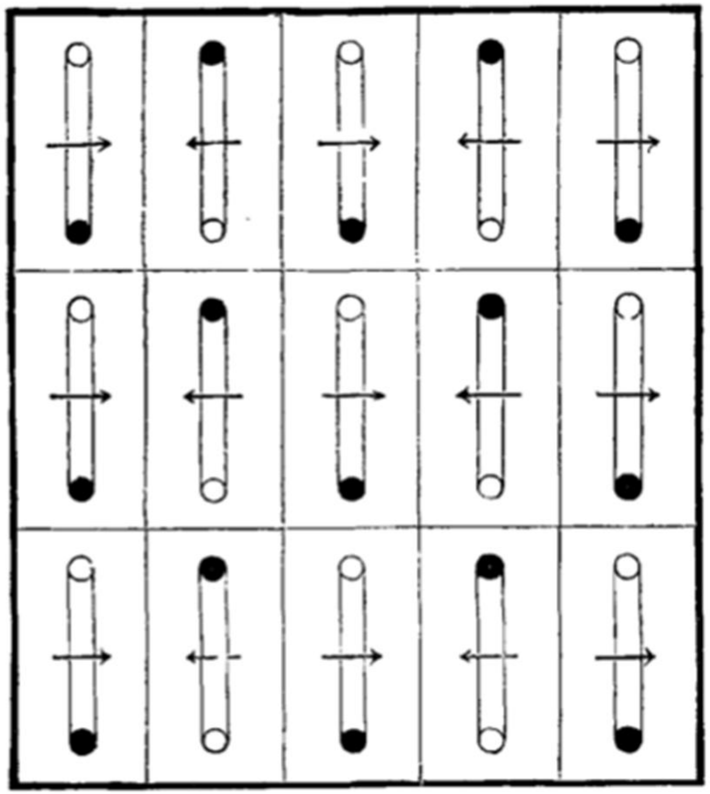
21. Eliminating the first member from this equation, by (34), we find

$$\frac{d^2 f}{dt^2} = \frac{2}{9} R^2 \frac{d^2 f}{dy^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (51).$$

$R^2 \sim \langle \tilde{v}^2 \rangle$

Thus we have the very remarkable result that laminar disturbance is propagated according to the well-known mode of waves of distortion in a homogeneous elastic solid; and that the velocity of propagation is $\frac{\sqrt{2}}{3}$R, or about ·47 of the average velocity of the turbulent motion of the fluid.

Fig. 1.



→ time delay between Reynolds stress and wave shear introduced

→ converts diffusion equation to wave equation

→ describes wave in ensemble of vortex quasi-particles

– c.f. "Worlds of Flow", O. Darrigol

Summary

- A model for ExB staircase formation

  - Heat avalanche jam → profile corrugation → ExB staircase

  - model developed based on analogy to traffic dynamics → telegraph eqn.

- Analysis of heat flux jam dynamics

  - Negative conduction instability as onset of jam formation

  - Growth rate, threshold, scale for maximal growth

  - Qualitative estimate: scale for maximal growth $\Delta \sim 10\Delta_c$

    → comparable to staircase step size

Ongoing Work

- This analysis ↔ set in context of heat transport
- Implications for momentum transport? ➔
  - consider system of flow, wave population, wave momentum flux
  - time delay set by decay of wave population correlation due ray stochastization → elasticity
  - flux limited PV transport allows closure of system

Results:

- Propagating (radially) zonal shear waves predicted, as well as vortex mode
- For τ_{delay} larger, Z.F. state transitions to LCO, rather than fixed point
- τ_{delay} due elastization necessarily impacts dynamics of L→I→H transition