Theory and simulations of magnetic island evolution

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KITP Conference on Wave-Flow Interaction,
24 March 2014

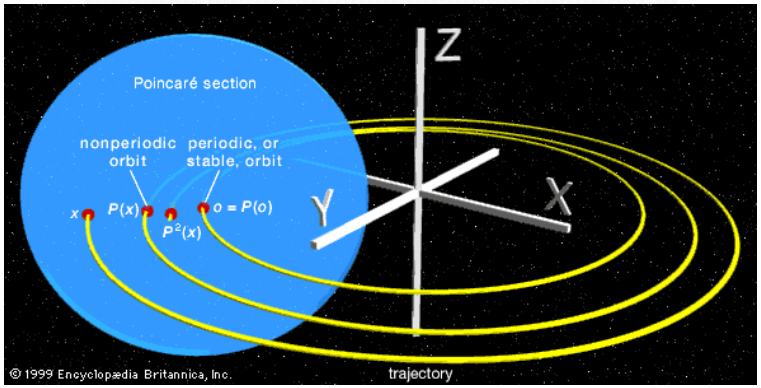
Outline

- Introduction
- General theory
- Suppressed islands
- Fulfilled islands
- Summary

Introduction

What are magnetic islands

The Magnetic geometry of fusion devices is described by Poincaré maps



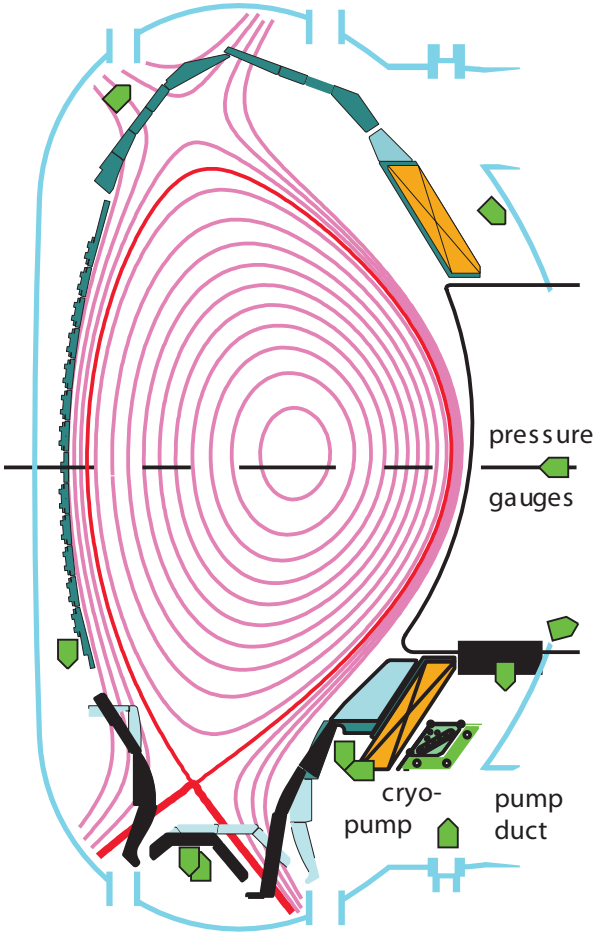
- Magnetic field lines constitute a Hamiltonian dynamical system

$$B = \hat{z}B_0 + \hat{z} \times \nabla\psi(x,y,z)$$

$$\frac{dx}{dz} = \frac{\partial\psi}{\partial y}$$

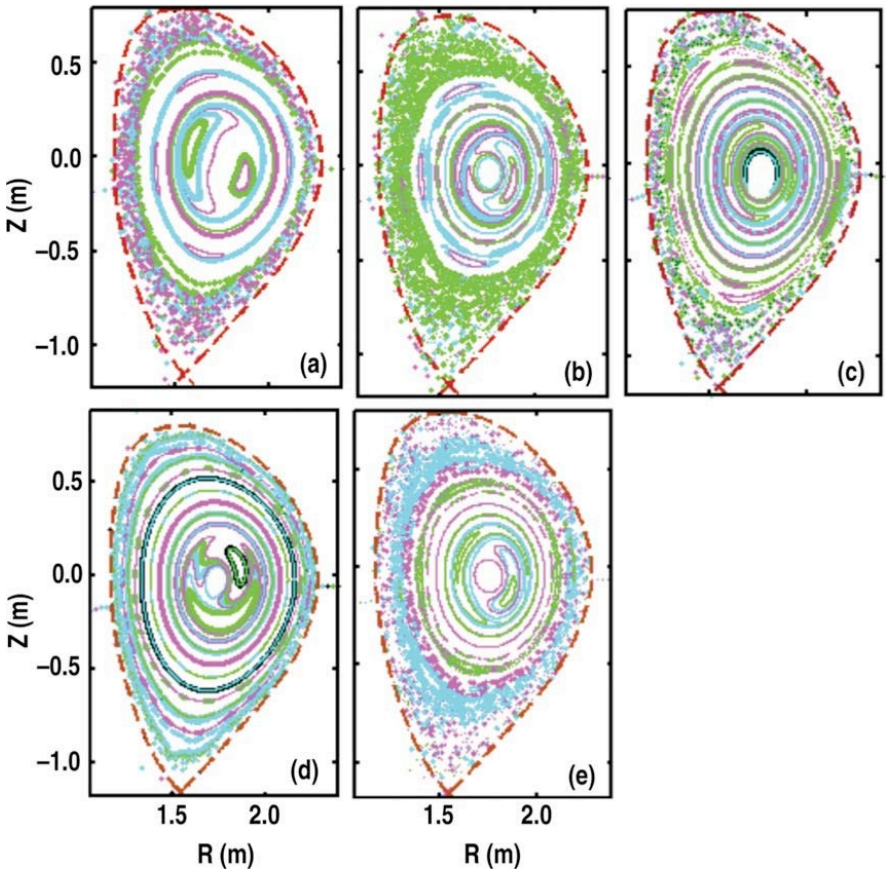
$$\frac{dy}{dz} = -\frac{\partial\psi}{\partial x}$$

Integrable systems have good confinement properties



- The simplest way to achieve integrability is with axisymmetry.
- Modern tokamak designs have one stable fixed point, called the magnetic axis, and one or two unstable fixed points called the X-point or the upper and lower nulls

Generically, perturbations lead to magnetic islands and chaos



- Consider the system:

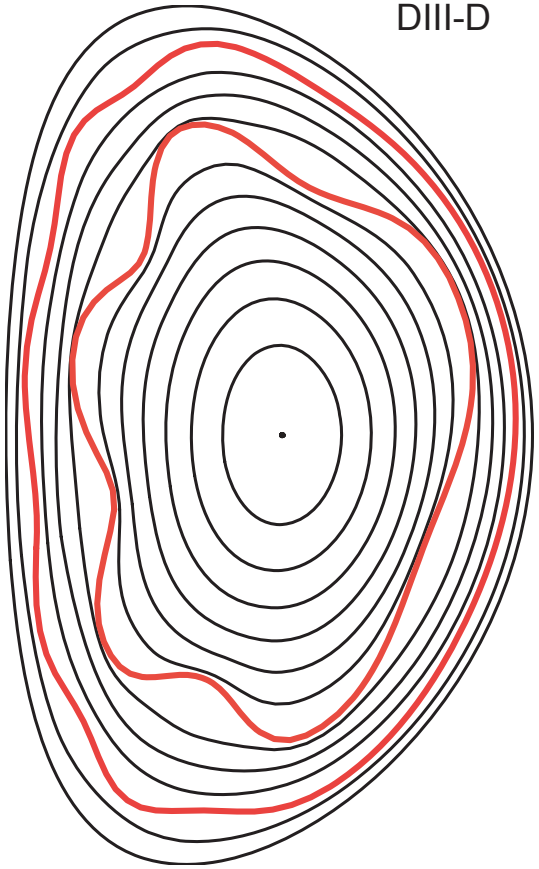
$$\psi(I,\theta) = \psi_0(I) + \varepsilon\psi_1(I,\theta).$$

- Perturbation theory seeks a new set of action-angle variables I', θ' by solving the Hamilton-Jacobi equation for the generating function S

$$S = S_0 + \varepsilon S_1 + \varepsilon^2 S_2 + \ldots$$

$$S_1 = i\sum_m \frac{\psi_{1,m}(I')}{m\cdot\omega_0}\exp(i\,m\cdot\theta)$$

The growth of magnetic islands is forbidden by ideal MHD



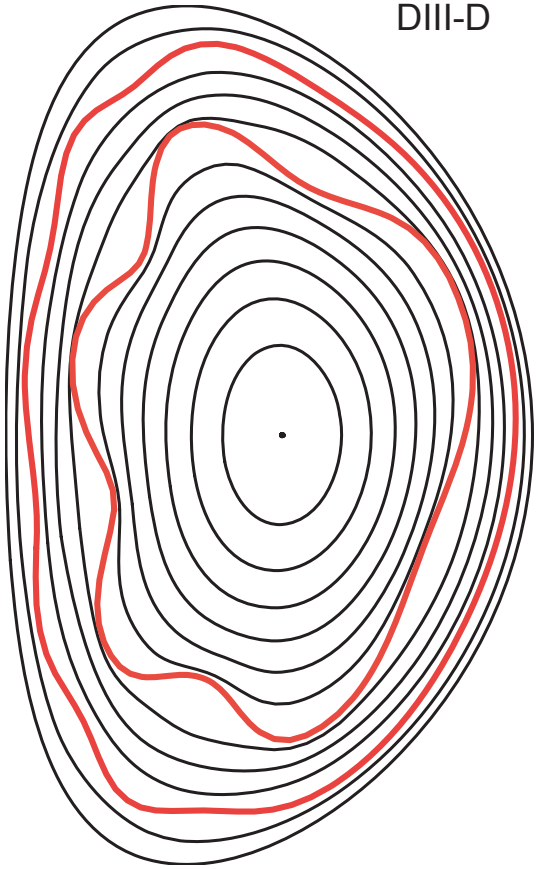
- Faraday's law and the ideal MHD "Ohm's law,"

$$\frac{\partial B}{\partial t} = -\nabla \times E = \nabla \times (V \times B)$$

Imply conservation of the flux through a material loop (analogous to Kelvin's circulation theorem)

- So islands can only be created through a non-ideal process, but Lundquist number S >> 1…

Non-axisymmetric equilibrium requires singular currents



- Force balance, $\mathbf{J}\times\mathbf{B} = \nabla p$, determines $\mathbf{J}_\perp$.
- There is a *diamagnetic current*

  $\mathbf{J}_\perp = (\mathbf{B}/B^2) \times \nabla p$

  (analogous to geostrophic flow)
- Charge conservation determines $J_{||}$:

  $$\mathbf{B}\cdot\nabla(J_{||}/B) = -\nabla\cdot\mathbf{J}_\perp$$
- The solution is

$$(J_{\|})_{mn} = \frac{\mu_0 p'}{B^2}\sum_{m,n}\frac{G_{mn}(q)}{(q - m/n)} + \Gamma_{mn}\delta(q - m/n)$$

- Current singularities also arise for $\nabla p=0$

Response to resonant perturbations analogous to planetary rings

- Rings are caused by resonances with satellites of saturn
- They are a collective phenomenon: collisions play a key role
- Ring formation involves transfer of momentum between the ice and satellites

Role of magnetic islands

Why do they matter

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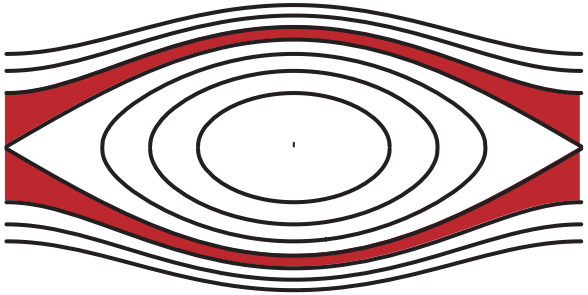
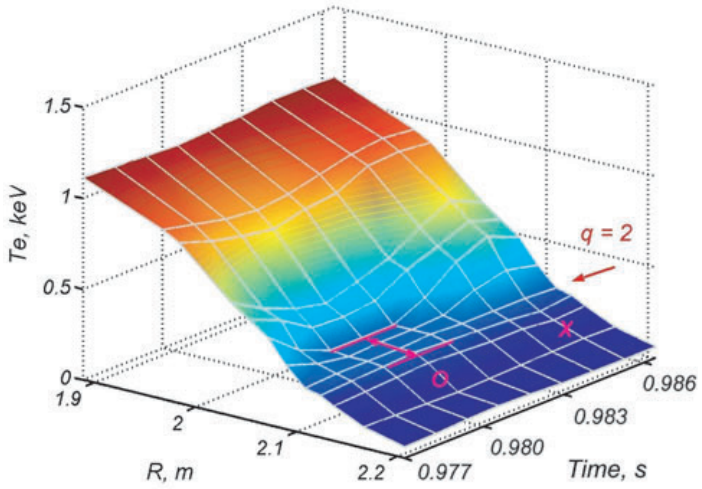
Magnetic islands lead to poor confinement and disruptions





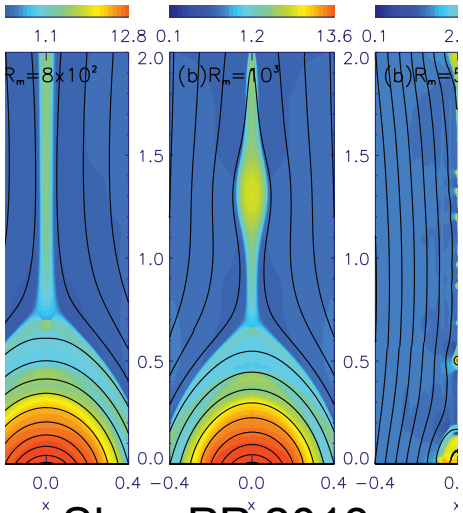
Udintsev et al.
PPCF 2003

- Rapid transport along the magnetic field leads to flattening of the temperature and density and inhibition of cross-island zonal flows
- Islands may be used deliberately to control profiles or to exhaust heat

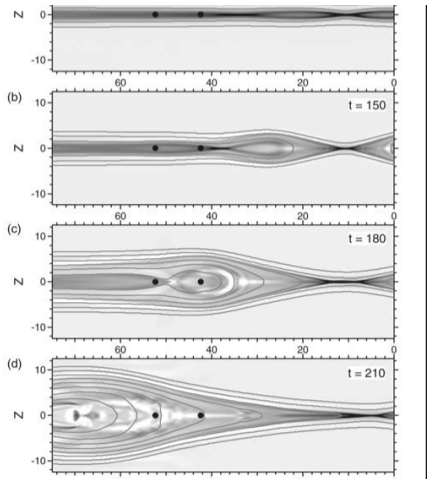
Islands play a role in "Non-Zonal Transition"

- Transport in low-pressure fusion devices is regulated by zonal flows
- As the pressure increases, the interchange-parity modes nonlinearly excite damped tearing modes, creating magnetic islands
- The magnetic islands short-out the radial electric field associated with the zonal flows
- This causes a transition to much more virulent turbulent transport.

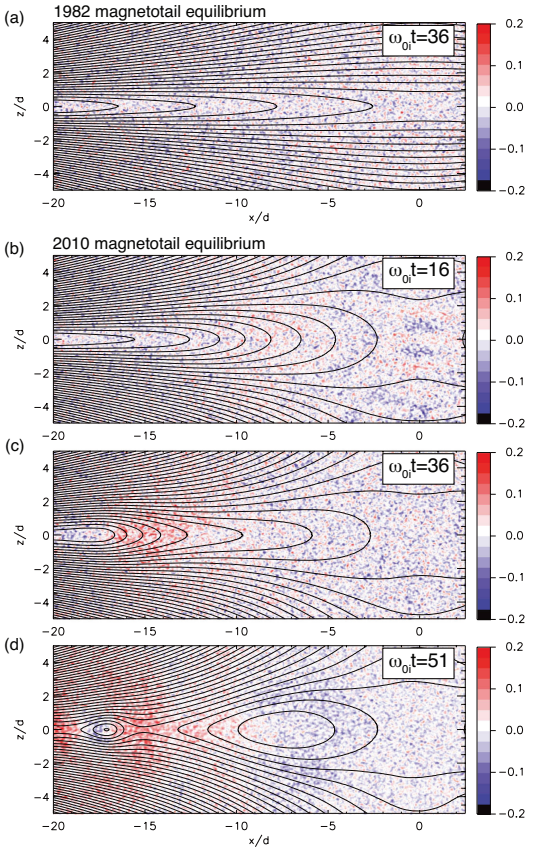
Islands are also of interest in space, solar, astrophys. and laboratory plasma



Shen PP 2013

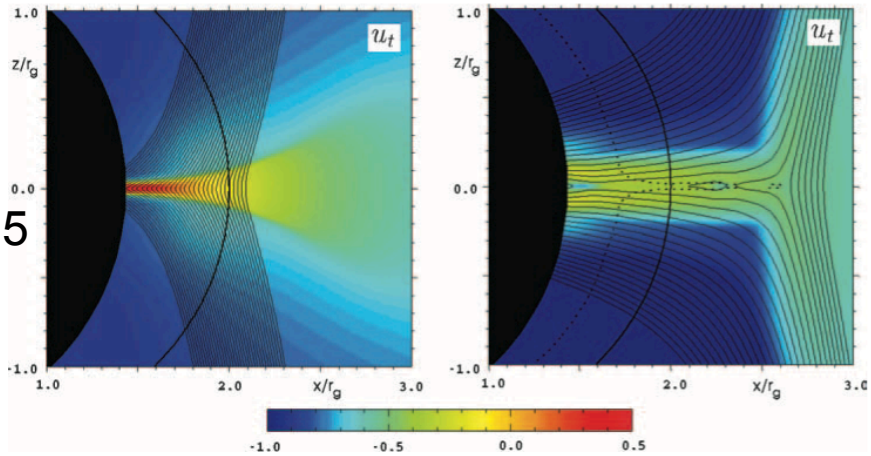


Ohtani JGR 2004



Sitnov GRL 2013

Komissarov
MNRAS 2015



General theory of magnetic islands

Response of the plasma to resonant perturbations

Lift and drag in the evolution equations

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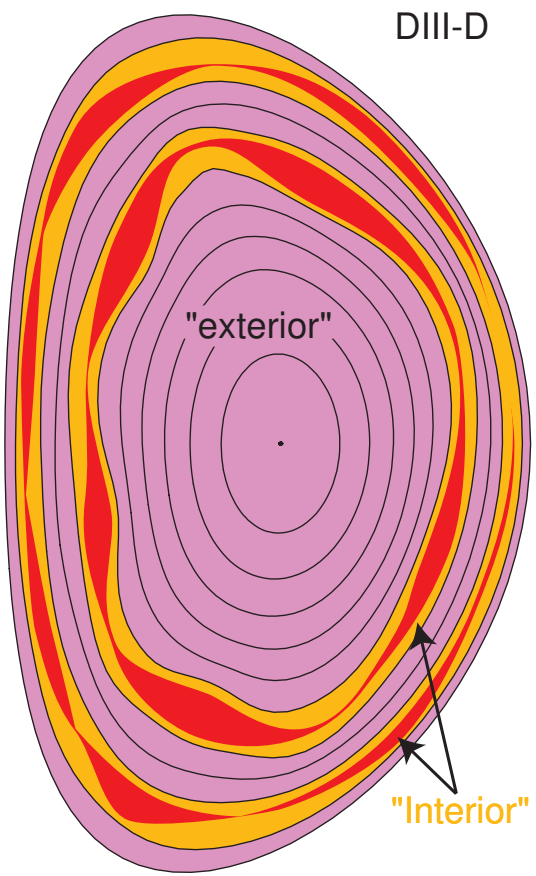
Island evolution is described by boundary layer analysis



- Outside the boundary layer, linear ideal MHD applies
- Inside the boundary layer, resistivity, diamagnetic effects, and nonlinearity are all important
- The electron and ion species both experience diamagnetic drifts:

$$\mathbf{V}_{e,i} = (\mathbf{B}/neB^2) \times \nabla p_{e,i}$$

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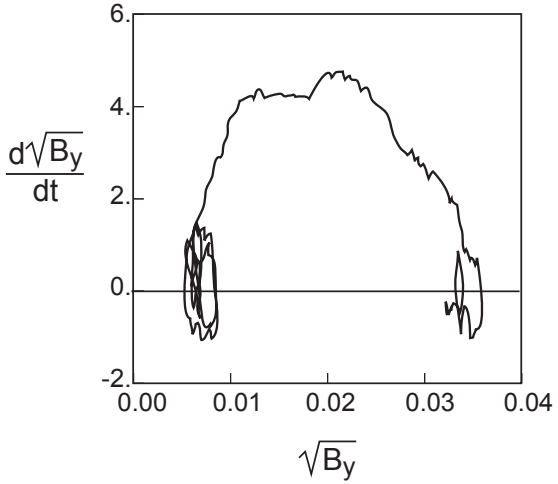
The magnetic island is similar to a sailboat

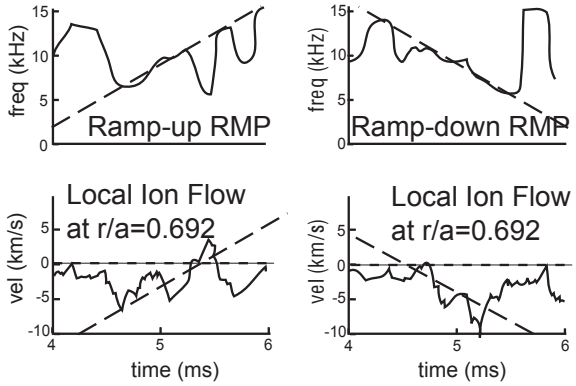
The sailboat couples two waves: the sail's shadow and the gravity wave excited by the keel.

- Electrons and ions play the role of the wind and water
- The goal of theory is to provide expressions for the lift and drag from the combination of keel and sail
- Kelvin's circulation theorem plays a role analogous to ideal MHD's flux conservation

The "lift" and "drag" can sometimes be measured directly



R. Buttery et al., Nucl. Fusion 2001



E.D. Taylor et al., Phys Plasmas 2002.

- The equations of motion for the island are

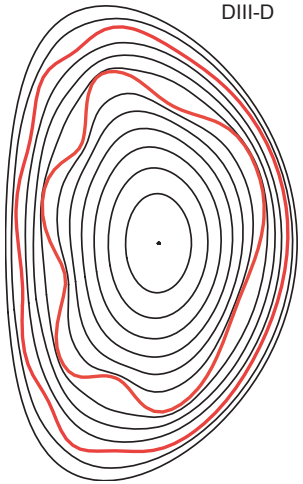
$$dW/dt = D(W,\omega)$$

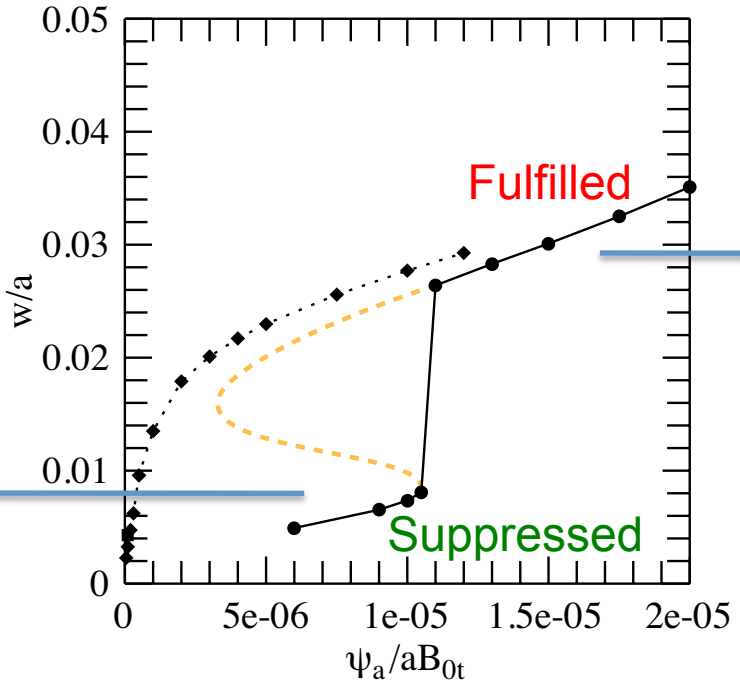
$$d\omega/dt = F(W, \omega)$$

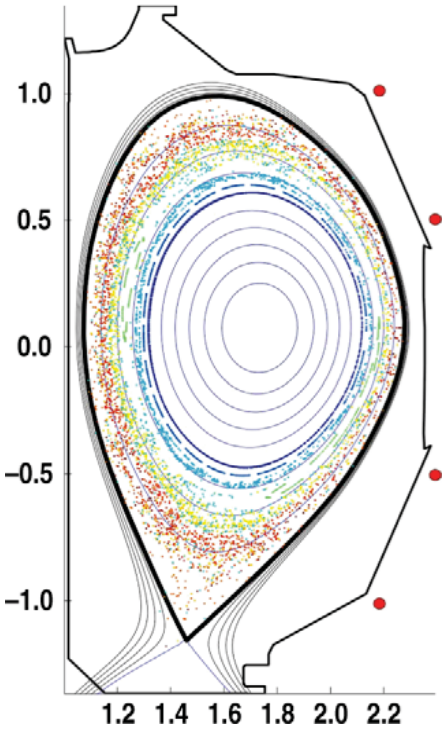
- The growth rate for the island is thus a measure of the lift
- The acceleration of ions when the island velocity is swept gives a measure of the drag

There are two or more regimes

Garabedian,
Proc N. Acad. Sci. '06







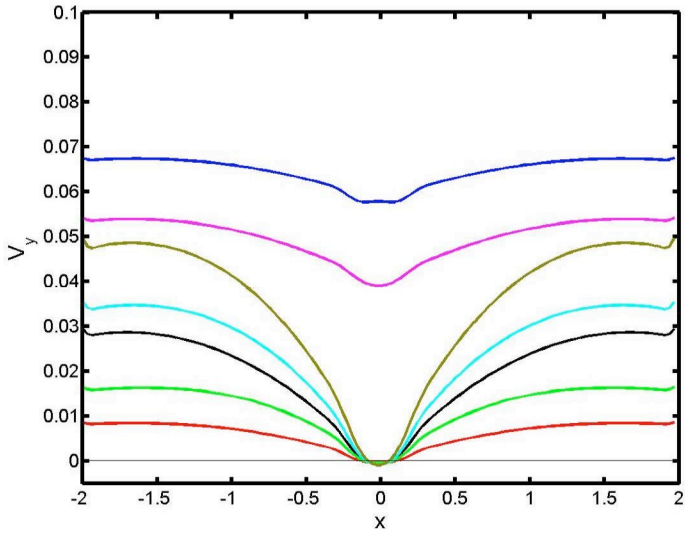
Izzo & Joseph, NF '08

Suppressed island
Regime: simply nested
Flux surfaces

X. Yu et al, NF 2008

Fulfilled magnetic
Islands; chaos

The mode-penetration bifurcation is analogous to the surfing bifurcation



- At constant field strength, locking occurs when the driving velocity drops below a threshold (here 0.05 v_A)
- The "suppressed" island regime is similar to the ship climbing on top of its bow wave and "surfing" above the water

Suppressed islands

With application to error fields and locked modes

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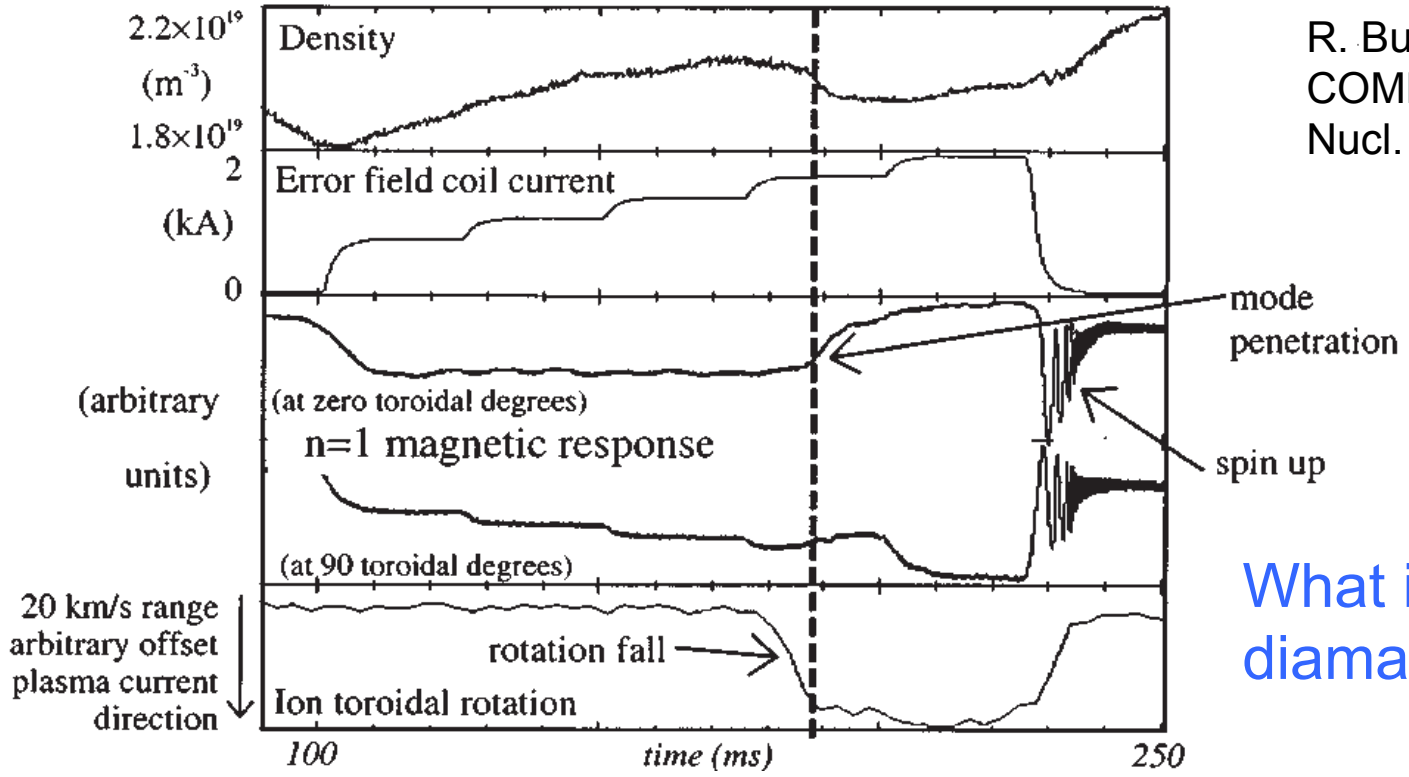
The bifurcation theory is in qualitative agreement with observations

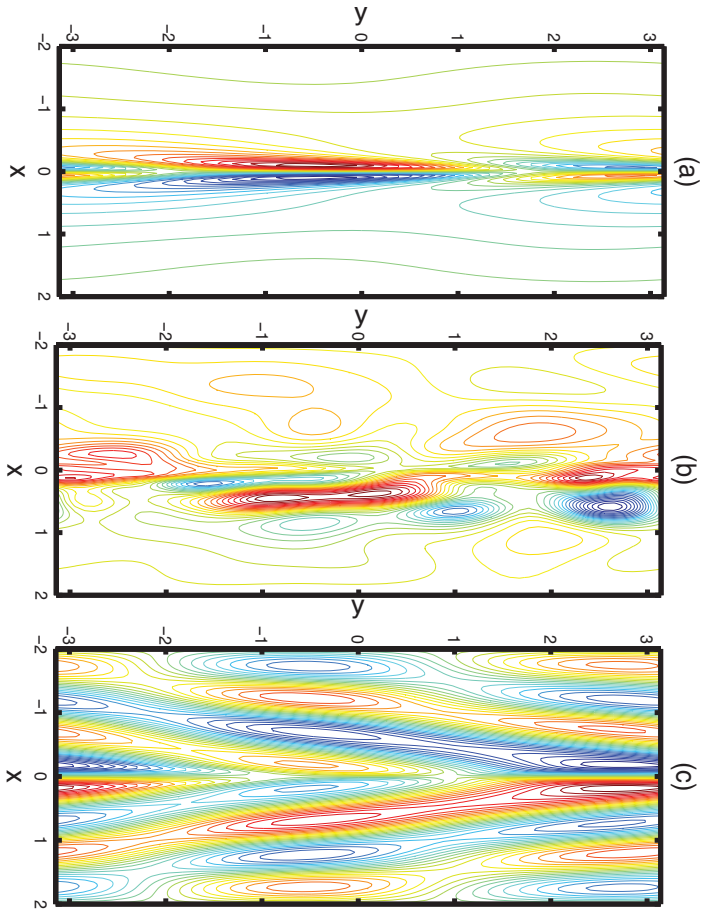
- The transition between states is easily observable: it is the surest way to determine the state.



R. Buttery & COMPASS-D group, Nucl. Fusion 1999

What is the role of diamagnetic flows?

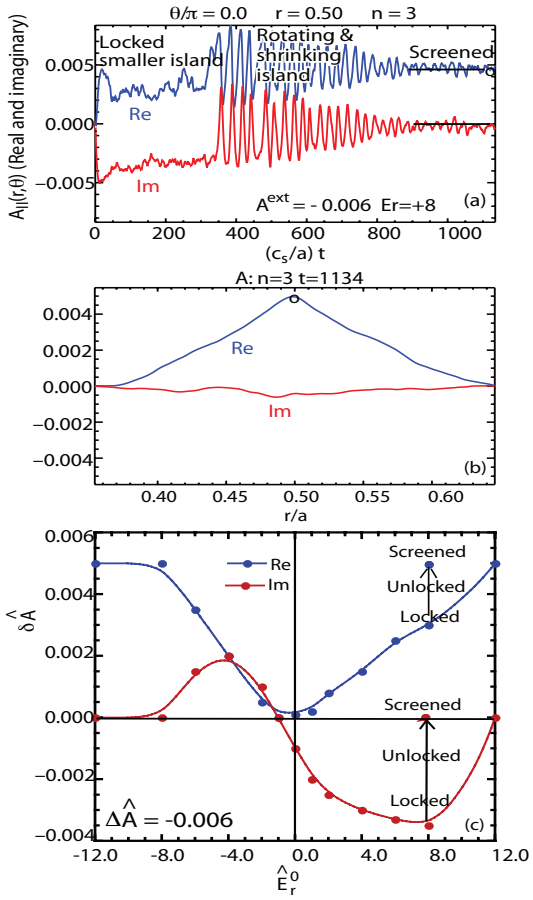
Near the locking threshold, the perturbation excites drift-acoustic waves



- Outside the band of velocity where drift waves (analogous to Rossby waves) propagate,
  $k^2 = \omega_{*e}/\omega - 1 > 0$
  the response is coherent and localized
- At the locked-mode threshold there appears irregular convection cells (turbulence?)
- When the rotation is further reduced, drift waves propagate away from the resonant surface and a locked mode grows.

F. Militello and F. L. Waelbroeck, NF 2009

EM Gyrokinetic simulations also show unlocking and suppression



Waltz and Waelbroeck, PoP 2012.

- Island initially shifts its phase to achieve force balance.
- Further phase slippage leads to unlocking and healing.
- Final state is a screened island with the response field canceling the vacuum field at the resonant surface.

Role of Reynolds stress in fulfilled islands

With application to intrinsic tearing modes

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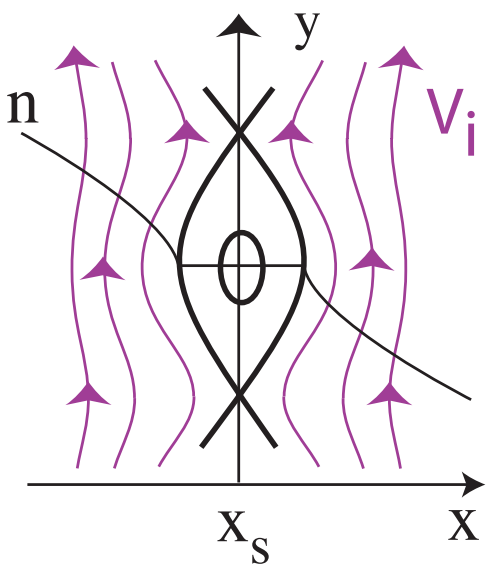
The Reynolds stress (ion inertia) gives rise to a polarization current



- Analytic calculations find

$$D_{pol}(W,\omega)=g\ V\ (V-V_{*i})/w^3$$

- Quadratic dependence on velocity explained by proportionality to product of flux (~V) and captured momentum (~V) as in Newton's theory for lift.

- The expression for the polarization "lift" may be viewed as plasma version of Kutta-Joukowsky theorem relating lift to circulation. Here the island width measures the nominally conserved flux, which replaces the circulation.
- There remains to determine V.

Ideal equations can be integrated to find generalized Bernoulli principles

- To lowest order, neglect resistivity, viscosity and particle diffusivity.
- The stream-functions for hot electrons and cold ions are φ+n and φ, respectively.
- Conservation of magnetic flux or the "frozen-in" property implies φ-n = H(ψ)
- Conservation of the potential vorticity

  $d\Pi/dt = v.\nabla\Pi = 0$ implies $\Pi = n+\nabla^2\varphi = L(\varphi)$
- Lastly, conservation of ion momentum yields

  $$J + \varphi\, dH/d\varphi = I(\psi)$$

  where lhs represents Maxwell and Reynolds stresses.

The profile functions are determined by transport

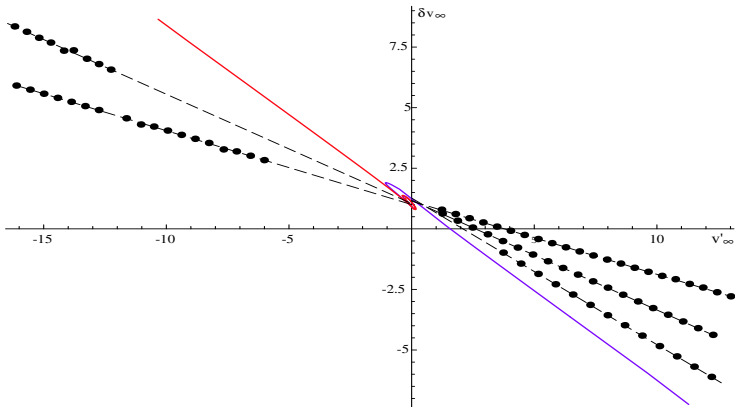
- Balancing the transport fluxes determines the profiles:

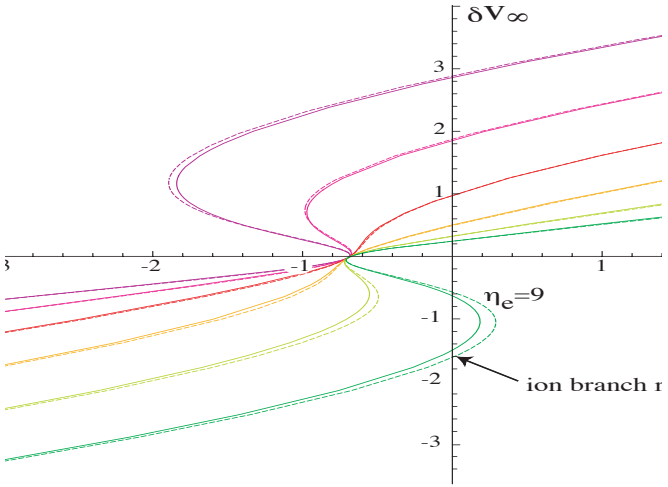
  $dL/d\varphi = -1/\langle \varphi_x^2 \rangle_\varphi$

  $dH/d\psi = 1 - 1/\langle \partial^\psi \varphi \rangle_\psi$

- We have solved the equilibrium & transport equations analytically in the limits of small ($W \ll \rho$) and large ($W \gg \rho$) islands and numerically for intermediate W.

The solutions reveal a complicated state space





- The result is best displayed in terms of the dependence of the slip velocity on the drag force
- For thin (unmagnetized) islands the island slips through the plasma, but for larger (magnetized) islands the flow is excluded from the island.
- This causes the slope of the slip velocity to reverse

Conclusion

- Magnetic islands are at the center of many interesting problems of plasma physics.
- Their mesoscopic nature challenges both macroscopic MHD theory and the microscopic theories of turbulent transport.
- The are often too small for direct observation, so their presence must be deduced from their theoretically-predicted effects.
- There remains many interesting problems such as
  - The role of the island wakefield;
  - The interaction of islands with turbulence;
  - The role of kinetic effects.