Quarkonium spectra and decays in a QCD-based potential model.

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So, what is a quarkonium?

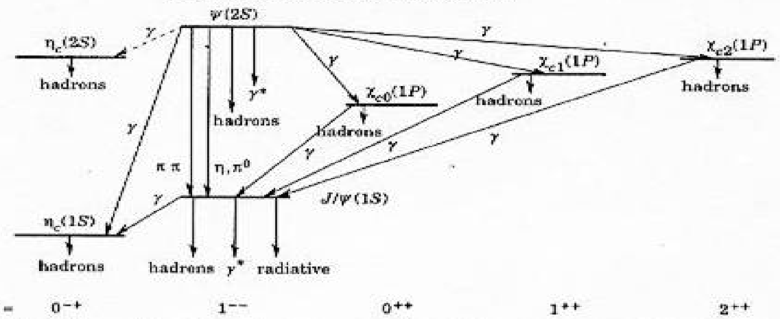
- A quarkonium is a bound state of one quark and one anti-quark ($q\ \bar{q}$)
  - Quarkonia built from light quarks have been known for a long time – these are the "mesons"
  - The positronium system is an EM paradigm

- Heavy quarkonia – those built with c, b, or t quarks and anti-quarks provide us with experimental and modeling opportunities
  - Their experimental signatures are very narrow and can be distinguished from the background in experiments
  - The heavy quarks move slowly in these systems
    - Confinement effects are (sort of) calculable
    - Relativistic effects are to some extent ignorable
  - We can do perturbative field theory and make predictions

Some facts about heavy quarkonia

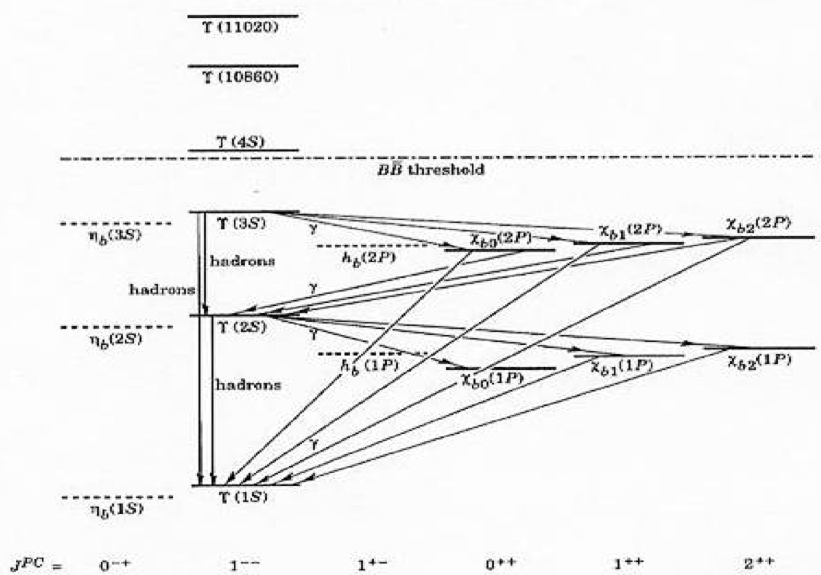
- The first heavy quarkonium discovered was the J/y ($c\bar{c}$) discovered in 1974
  - Additional mass and spin states soon followed
- Members of the system of $b\bar{b}$ quarkonia were first measured in the early 1980's
- Systems of $t\bar{t}$ have not been seen due to the huge mass of the top quark
- A variety of mixed heavy-heavy and heavy-light states have been observed

THE CHARMONIUM SYSTEM



The current state of knowledge of the charmonium system and transitions, as interpreted by the charmonium model. Uncertain states and transitions are indicated by dashed lines. The notation γ^* refers to decay processes involving intermediate virtual photons, including decays to e^+e^- and $\mu^+\mu^-$.

THE BOTTOMONIUM SYSTEM



The level scheme of the $b\bar{b}$ states showing experimentally established states with solid lines. Singlet states are called η_b and h_b, triplet states Υ and χ_{bJ}. In parentheses it is sufficient to give the radial quantum number and the orbital angular momentum to specify the states with all their quantum numbers. *E.g.*, $h_b(2P)$ means 2^1P_1 with $n = 2$, $L = 1$, $S = 0$, $J = 1$, $PC = +-$. If found, D-wave states would be called $\eta_b(nD)$ and $\Upsilon_J(nD)$, with $J = 1,2,3$ and $n = 1,2,3,4,\cdots$. For the χ_b states, the spins of only the $\chi_{b2}(1P)$ and $\chi_{b1}(1P)$ have been experimentally established. The spins of the other χ_b are given as the preferred values, based on the quarkonium models. The figure also shows the observed hadronic and radiative transitions.

Understanding heavy quarkonia

• First principles – lattice gauge theories. This approach continues to make progress, but hasn't yet attacked the spin splittings.

• 'Heavy quark effective theories' – useful for heavy-light systems where one can expand in the quark mass ratio.

• Potential models – able to treat both relativistic and quantum corrections with a track record of success in $c\bar{c}$ and $b\bar{b}$ systems.

So, what's the point?

- We can use experiments and these models to investigate the consequences of QCD as a theory of nature
  - Model fits and predictions can be validated and tested
  - Quark mass values can be estimated
  - Begin to get a phenomenological (at least) handle on how confinement works
- Valuable since these are significant aspects of the Standard Model
- Several new(!) charmonium(?) and upsilon(?) states

Modeling heavy quarkonia

- Several theoretical (motivated principally by QCD considerations) and phenomenological (motivated principally by data fit considerations) models were used to fit the J/y and U data

- The best "early" model was reported in 1982 (Gupta, Radford, Repko, Phys Rev. D26, 3305)
  - Linear (phenomenological) confining potential with relativistic corrections
  - QCD interaction potential terms to fourth order in the QCD coupling constant
  - Excellent fit to then-known J/y and U spectra
  - Outrageously successful predictions for later-measured U states

Modeling heavy quarkonia

The discovery of the J/ψ led to the introduction of simple potential models for the $c\bar{c}$ bound states. To keep the quarks confined, a potential of the form[1]

$$V(r) = -\frac{a}{r} + Ar + C$$

was suggested. This model ignores the rather considerable effects of spin, but can account for the average features of the $c\bar{c}$ spectrum with

$$a = 0.597 \quad A = 0.179\,GeV^2 \quad C = 2.896\,GeV \quad m_c = 1.92\,GeV.$$

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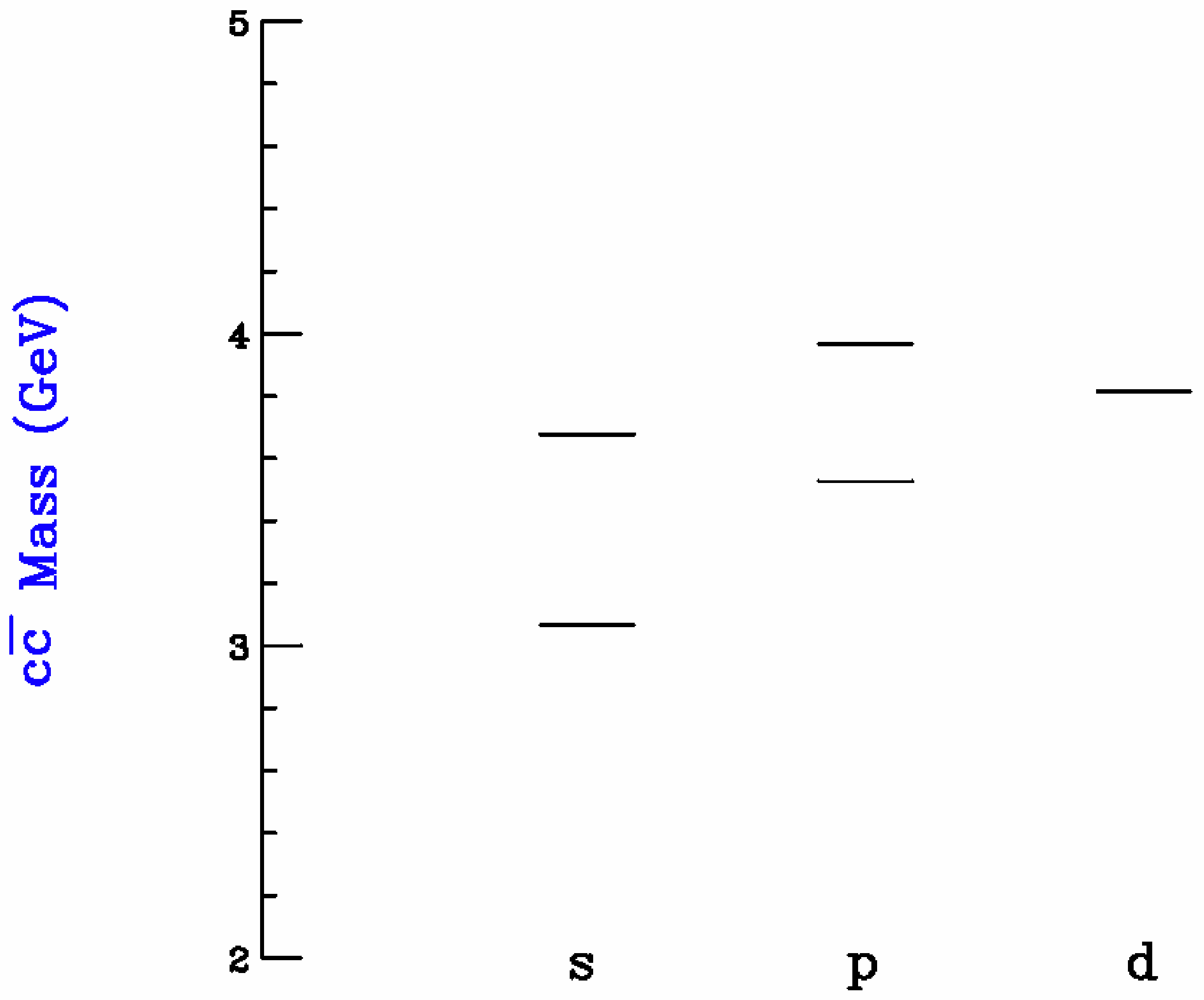
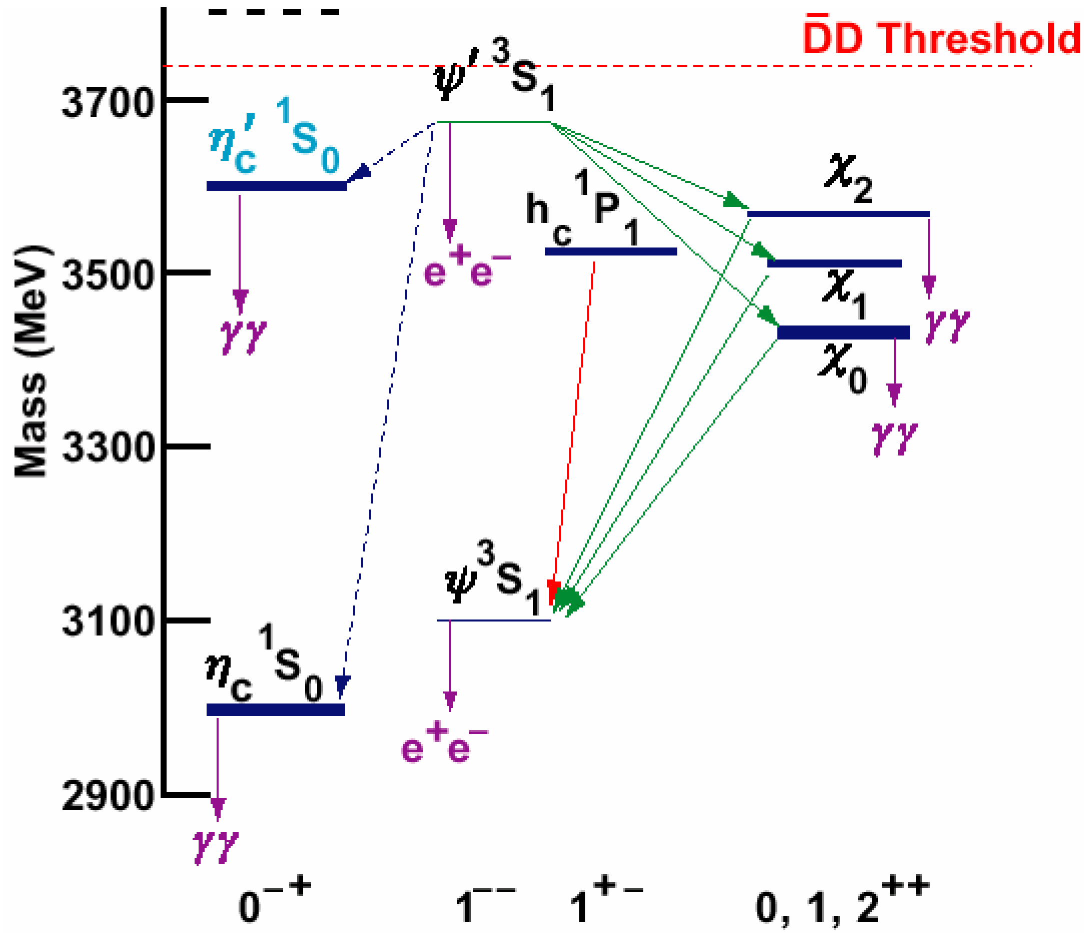
1. E.Eichten, et al., Phys. Rev. Lett., 34, 369 (1975).



5
4
3
2
$c\bar{c}$ Mass (GeV)
s
p
d

Charmonium Spin Splittings



Modeling heavy quarkonia

A complete description of the levels requires the inclusion of spin effects.

- Relativistic corrections: Pumplin, Repko and Sato; Schnitzer (1975).

- Quantum corrections: Gupta, Radford and Repko (1982).

Spin effects can be included in the long-range part to order v^2/c^2 in a straightforward way (Pumplin, WWR & Sato, Schnitzer, 1975) to obtain a Hamiltonian of the form

$$H = \frac{\vec{p}^{\,2}}{2\mu} + V(r) + V_{HF} + V_{LS} + V_{TEN} + V_{SI}$$

where V_{SI} consists of spin-independent terms including the kinetic energy correction. For scalar + vector confinement, the confining potential is

$$V_L = (1 - f_V) V_{SC} + f_V V_V$$

where f_V is the fraction of vector confinement and

$$V_{SC} = Ar - \frac{A}{2m^2 r} \vec{L} \cdot \vec{S}$$

$$V_V = Ar + \frac{4A}{3m^2 r} \vec{S}_1 \cdot \vec{S}_2 + \frac{3A}{2\,m^2 r} \vec{L} \cdot \vec{S} + \frac{A}{3m^2 r} (3\vec{S}_1 \cdot \hat{r}\, \vec{S}_2 \cdot \hat{r} - \vec{S}_1 \cdot \vec{S}_2) + \frac{A}{2\,m^2 r}$$

Modeling heavy quarkonia

The QCD Lagrangian is

$$L_{\mathrm{QCD}} = -\frac{1}{4} F_{\mu\nu}^{(a)} F^{(a)\mu\nu} + i \sum_q \overline{\psi}_q^i \gamma^\mu (D_\mu)_{ij} \psi_q^j - \sum_q m_q \overline{\psi}_q^i \psi_{qi} ,$$

$$F_{\mu\nu}^{(a)} = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g_s f_{abc} A_\mu^b A_\nu^c ,$$

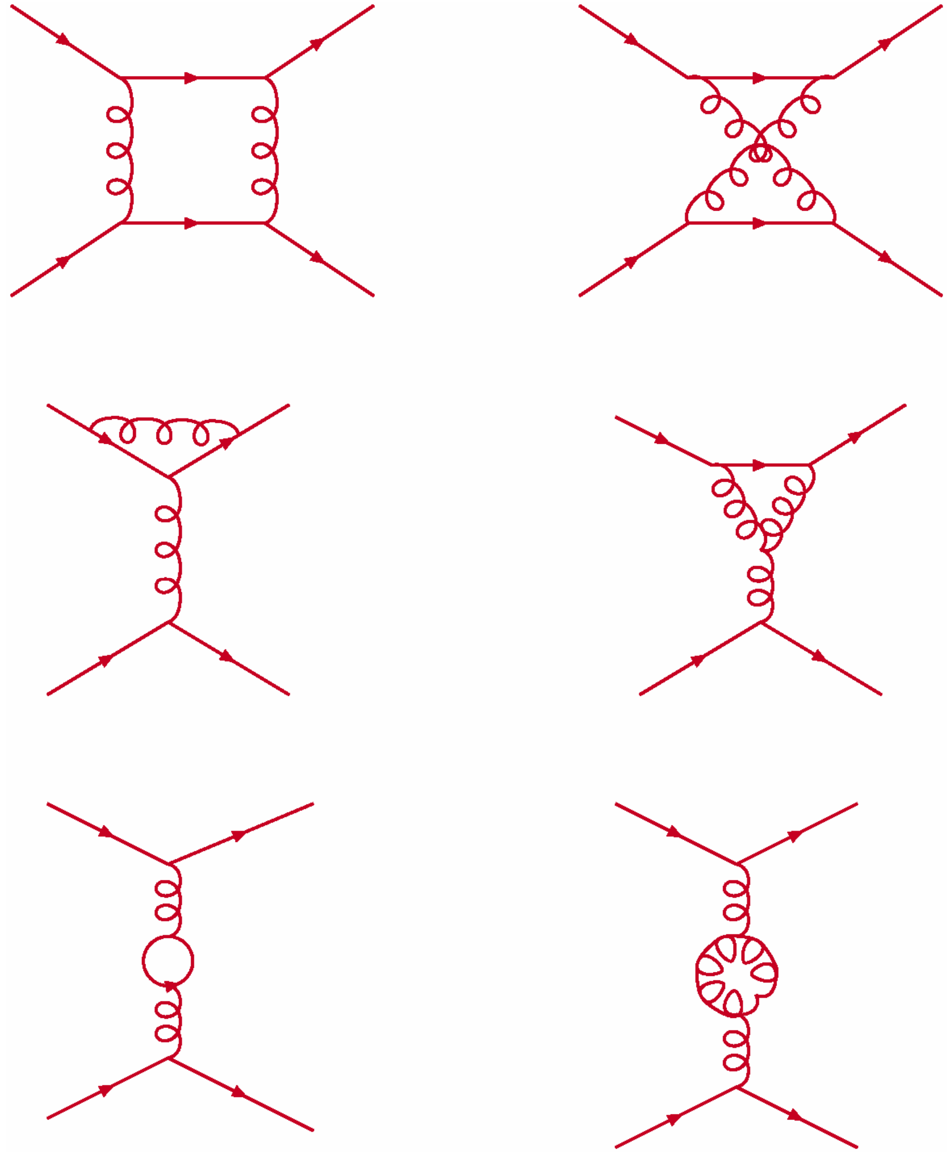
$$(D_\mu)_{ij} = \delta_{ij} \partial_\mu + i g_s \sum_a \frac{\lambda_{i,j}^a}{2} A_\mu^a ,$$

where g_s is the QCD coupling constant, f_{abc} are the SU(3) structure constants, $\psi^i_q(x)$ are the quark field spinors, and the $A^a_\mu(x)$ represent the gluon fields.

Modeling heavy quarkonia

Notice that even the simplest relativistic corrections produce short distance terms in the potential of the form $\delta(\vec{r})$. Because of asymptotic freedom, it is possible make a reliable perturbative calculation of the correction to the short distance part of the potential using QCD.

This calculation was performed by S. N. Gupta and S. F. Radford at the one loop level in 1981. It involves computing a set of one-loop scattering diagrams and then extracting the potential.



Inclusion of the one loop QCD corrections to the short distance potential (Gupta & SFR, 1981; Gupta, SFR & WWR, 1982)

$$V_{HF} = \frac{32\pi\alpha_S}{9m^2}\vec{S}_1\cdot\vec{S}_2\left[\quad - \frac{\alpha_S}{12\pi}\left(26+9\ln 2\right)\right]\delta(\vec{r})$$

$$+\frac{32\pi\alpha_S}{9m^2}\vec{S}_1\cdot\vec{S}_2\left\{-\frac{\alpha_S}{24\pi^2}(33-2n_f)\nabla^2\left[\frac{\ln(\mu r)+\gamma_E}{r}\right]+\frac{21\alpha_S}{16\pi^2}\nabla^2\left[\frac{\ln(mr)+\gamma_E}{r}\right]\right\}$$

$$V_{LS} = \frac{2\alpha_S}{m^2}\frac{\vec{L}\cdot\vec{S}}{r^3}\left\{\quad - \frac{11\alpha_S}{18\pi}+\frac{\alpha_S}{6\pi}(33-2n_f)(\ln\mu r+\gamma_E-1)-\frac{2\alpha_S}{\pi}(\ln mr+\gamma_E-1)\right\}$$

$$V_T = \frac{4\alpha_S}{3m^2}\frac{(3\vec{S}_1\cdot\hat{r}\,\vec{S}_2\cdot\hat{r}-\vec{S}_1\cdot\vec{S}_2)}{r^3}$$

$$\times\left\{\quad + \frac{4\alpha_S}{3\pi}+\frac{\alpha_S}{6\pi}(33-3n_f)(\ln\mu r+\gamma_E-\frac{4}{3})-\frac{3\alpha_S}{\pi}(\ln mr+\gamma_E-\frac{4}{3})\right\}$$

$$V_{SI} = \frac{4\pi\alpha_S}{3m^2}\left\{\left[\quad - \frac{\alpha_S}{2\pi}(1+\ln 2)\right]\delta(\vec{r})-\frac{\alpha_S}{24\pi^2}(33-2n_f)\nabla^2\left[\frac{\ln\mu r+\gamma_E}{r}\right]-\frac{7\alpha_S}{6\pi}\frac{m}{r^2}\right\}$$

Modeling heavy quarkonia

The complete treatment of both the $c\bar{c}$ and $b\bar{b}$ systems in the non-relativistic case,

$$H = \frac{\vec{p}^{\,2}}{2\mu} + Ar - \frac{4\alpha_S}{3r}\Lambda(r) + V_L + V_P$$

$$\Lambda(r) = 1 - \frac{3\alpha_S}{2\pi} + (33 - 2n_f)[\ln(\mu r) + \gamma_E]$$

was published in 1982 (SNG, SFR and WWR). The results were embarrassingly good for the experimental situation of the time. This version of the potential approach has held up well over the intervening 20 years, but the experimental situation has recently become quite active leading us to reexamine what potential models can do with the new data.

TABLE II. $b\bar{b}$ spectrum with $m_b = 4.78$ GeV, $\mu = 3.75$ GeV, $\alpha_s(\mu) = 0.288$, and $A = 0.177$ GeV^2.

	State	Mass (GeV)	State	Mass (GeV)
	$1\,^3S_1(\Upsilon)$	9.462	$1\,^3D_3$	10.167
	$1\,^1S_0(\eta_b)$	9.427	[redacted]	
			$1\,^3D_1$	10.155
	$2\,^3S_1(\Upsilon')$	10.013	$1\,^1D_2$	10.163
	$2\,^1S_0(\eta_b')$	9.994		
			$2\,^3D_3$	10.459
	$3\,^3S_1(\Upsilon'')$	10.355	$2\,^3D_2$	10.454
	$3\,^1S_0(\eta_b'')$	10.339	$2\,^3D_1$	10.447
			$2\,^1D_2$	10.455
CUSB 1983/4	$1\,^3P_2$	9.910		
	$1\,^3P_1$	9.893	$1\,^3F_4$	10.365
	$1\,^3P_0$	9.868	$1\,^3F_3$	10.364
	$1\,^1P_1$	9.900	$1\,^3F_2$	10.361
			$1\,^1F_3$	10.364
CLEO 1991, CUSB 1992	$2\,^3P_2$	10.266		
	$2\,^3P_1$	10.252		
	$2\,^3P_0$	10.232		
	$2\,^1P_1$	10.258		

$M(\Upsilon(^3D_2)) = 10161.1 \pm 0.6 \pm 1.6$ MeV (CLEO 2003)

Modeling heavy quarkonia

In reexamining the earlier potential model treatment, we use a modified Hamiltonian, which gives a nod to the fact that v^2/c^2 is not all that small for charmonium. Specifically, the starting point is

$$H = 2\sqrt{\vec{p}^{\,2} + m^2} + Ar - \frac{4\alpha_S}{3r}\Lambda(r) + V_L + V_P.$$

V_L contains the scalar and vector order v^2/c^2 corrections to Ar and V_P includes all v^2/c^2 and one-loop QCD corrections to the short distance potential. Two versions of the model are examined

Modeling heavy quarkonia

- $V_L + V_P$ treated as a perturbation
- All terms treated nonperturbatively

Because of the complexity of the one loop corrections, we use a variational technique to determine the various levels and, importantly, the wave functions. The method itself simply seeks to minimize the functional

$$E = \frac{\langle \psi | H | \psi \rangle}{\langle \psi | \psi \rangle}.$$

Starting with a trial function of the form

$$\psi_\ell^m(\vec{r}) = \sum_{n=1}^{N} C_n \left(r/R\right)^{n+\ell-1} e^{-(r/R)^\beta} Y_\ell^m(\Omega),$$

or its counterpart in momentum space to evaluate the kinetic term, minimization with respect to the C_n's leads to an eigenvalue equation of the form

$$\sum_{n'=1}^{N} H_{nn'} C_{n'} = \lambda \sum_{n'=1}^{N} N_{nn'} C_{n'}.$$

For a fixed λ, the resulting radial wave functions are orthogonal and the N eigenvalues λ_n are upper bounds on the true energies E_n.

Modeling heavy quarkonia: status

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Potential model calculations and predictions for heavy quarkonium

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We investigate the spectroscopy and decays of the charmonium and upsilon systems in a potential model consisting of a relativistic kinetic energy term, a linear confining term including its scalar and vector relativistic corrections and the complete perturbative one-loop quantum chromodynamic short distance potential. The masses and wave functions of the various states are obtained using a variational technique, which allows us to compare the results for both perturbative and nonperturbative treatments of the potential. As well as comparing the mass spectra, radiative widths and leptonic widths with the available data, we include a discussion of the errors on the parameters contained in the potential, the effect of mixing on the leptonic widths, the Lorentz nature of the confining potential, and the possible $c\bar{c}$ interpretation of recently discovered charmoniumlike states.

Modeling heavy quarkonia: status

TABLE I. Fitted parameters for the $c\bar{c}$ and $b\bar{b}$ systems.

	$c\bar{c}$ pert	$c\bar{c}$ nonpert	$b\bar{b}$ pert	$b\bar{b}$ nonpert
A (GeV^2)	$0.166^{+0.002}_{-0.002}$	$0.186^{+0.003}_{-0.001}$	$0.177^{+0.006}_{-0.002}$	$0.193^{+0.004}_{-0.001}$
α_S	$0.334^{+0.009}_{-0.009}$	$0.332^{+0.003}_{-0.004}$	$0.296^{+0.004}_{-0.007}$	$0.295^{+0.002}_{-0.006}$
m_q (GeV)	$1.51^{+0.07}_{-0.08}$	$1.80^{+0.03}_{-0.05}$	$5.36^{+0.87}_{-0.42}$	$6.61^{+0.35}_{-0.18}$
μ (GeV)	2.60	1.32	4.74	3.73
f_V	0.00	0.24	0.00	0.21

TABLE IV. The leptonic widths of the $J = 1^{--}$ states are shown.

$\Gamma_{e\bar{e}}$ (keV)	Pert	Nonpert	Expt
$\psi(1S)$	4.28	1.89	5.55 ± 0.14
$\psi(2S)$	2.25	1.04	2.48 ± 0.06
$\psi(3S)$	1.66	0.77	0.86 ± 0.07
$\psi(4S)$	1.33	0.65	0.58 ± 0.07
$\psi(1D)$	0.09	0.23	0.242 ± 0.030
$\psi(2D)$	0.16	0.45	0.83 ± 0.07

TABLE VII. The leptonic widths of the $\Upsilon(nS)$ states are shown.

$\Gamma_{e\bar{e}}$ (keV)	Pert	Nonpert	Expt
$\Upsilon(1S)$	1.33	1.33	1.340 ± 0.018
$\Upsilon(2S)$	0.61	0.62	0.612 ± 0.011
$\Upsilon(3S)$	0.46	0.45	0.443 ± 0.008
$\Upsilon(4S)$	0.35	0.30	0.272 ± 0.029

TABLE II. Perturbative and nonperturbative results for the $c\bar{c}$ spectrum are shown. The perturbative fit uses the indicated states and the leptonic widths of the $\psi(1S)$ and $\psi(2S)$. In the non-perturbative fit the $\eta_c(2S)$ and $\psi(1D)$ are included and no leptonic widths are used.

$m_{c\bar{c}}$ (MeV)	Pert	Nonpert	Expt
$\eta_c(1S)^*$	2980.3	2981.7	2980.4 ± 1.2
$\psi(1S)^*$	3097.36	3096.92	3096.916 ± 0.011
$\chi_{c0}(1P)^*$	3415.7	3415.2	3414.76 ± 0.35
$\chi_{c1}(1P)^*$	3508.2	3510.6	3510.66 ± 0.07
$\chi_{c2}(1P)^*$	3557.7	3556.2	3556.20 ± 0.09
$h_c(1P)$	3526.9	3523.7	3525.93 ± 0.27
$\eta_c(2S)$	3597.1	3619.2	3638.0 ± 4.0
$\psi(2S)^*$	3685.5	3686.1	3686.093 ± 0.034
$\psi(1D)$	3803.8	3789.4	3771.1 ± 2.4
1^3D_2	3823.8	3822.1	
1^3D_3	3831.1	3844.8	
1^1D_2	3823.6	3822.2	
$\chi_{c0}(2P)$	3843.7	3864.3	
$\chi_{c1}(2P)$	3939.7	3950.0	
$\chi_{c2}(2P)$	3993.7	3992.3	3929. ± 5.4
$h_c(2P)$	3960.5	3963.2	
1^3F_2	4068.5	4049.9	
1^3F_3	4069.6	4069.0	
1^3F_4	4061.8	4084.3	
1^1F_3	4066.2	4066.9	
$\eta_c(3S)$	4014.0	4052.5	
$\psi(3S)$	4094.9	4102.0	4039. ± 1
$\psi(2D)^*$	4164.2	4159.2	4153. ± 3
2^3D_2	4189.1	4195.8	
2^3D_3	4202.3	4218.9	
2^1D_2	4190.7	4196.9	
$\psi(4S)$	4433.3	4446.8	4421. ± 4
$\psi(3D)$	4477.3	4478.9	

TABLE III. The radiative decays of the charmonium system are shown. The $\psi(1D) \to \chi_J(1P)$ widths marked with a * are from [48]; see also [43].

Γ_γ (keV)	Pert	Nonpert	Expt
$\psi(1S) \to \eta_c(1S)$	2.7	1.8	1.21 ± 0.37
$\psi(2S) \to \eta_c(2S)$	1.2	0.4	<0.7
$\psi(2S) \to \eta_c(1S)$	0.0	0.45	0.88 ± 0.14
$\psi(2S) \to \chi_{c0}(1P)$	45.0	25.2	31.0 ± 1.8
$\psi(2S) \to \chi_{c1}(1P)$	40.9	29.1	29.3 ± 1.8
$\psi(2S) \to \chi_{c2}(1P)$	26.5	25.2	27.3 ± 1.7
$\eta_c(2S) \to h_c(1S)$	8.3	17.4	
$\psi(3S) \to \chi_{c0}(2P)$	87.3	30.1	
$\psi(3S) \to \chi_{c1}(2P)$	65.7	45.0	
$\psi(3S) \to \chi_{c2}(2P)$	31.6	36.0	
$\psi(3S) \to \chi_{c0}(1P)$	1.2	2.1	
$\psi(3S) \to \chi_{c1}(1P)$	2.5	0.3	<880
$\psi(3S) \to \chi_{c2}(1P)$	3.3	2.4	<1360
$\chi_{c0}(1P) \to \psi(1S)$	142.2	139.3	135 ± 15
$\chi_{c1}(1P) \to \psi(1S)$	287.0	293.7	317 ± 25
$\chi_{c2}(1P) \to \psi(1S)$	390.6	384.1	417 ± 32
$h_c(1P) \to \eta_c(1S)$	610.0	546.4	
$\chi_{c0}(2P) \to \psi(2S)$	53.6	89.7	
$\chi_{c1}(2P) \to \psi(2S)$	208.3	235.8	
$\chi_{c2}(2P) \to \psi(2S)$	358.6	319.4	
$\chi_{c0}(2P) \to \psi(1S)$	20.8	24.0	
$\chi_{c1}(2P) \to \psi(1S)$	28.4	5.1	
$\chi_{c2}(2P) \to \psi(1S)$	33.2	36.7	
$\chi_{c0}(2P) \to \psi(1D)$	1.2	7.4	
$\chi_{c1}(2P) \to \psi(1D)$	11.1	12.3	
$\chi_{c2}(2P) \to \psi(1D)$	1.2	0.8	
$\chi_{c1}(2P) \to 1^3D_2$	20.9	23.5	
$\chi_{c2}(2P) \to 1^3D_2$	12.7	9.1	
$\psi(1D) \to \chi_{c0}(1P)$	415.4	243.9	172 ± 30*
$\psi(1D) \to \chi_{c1}(1P)$	146.7	104.9	70 ± 17*
$\psi(1D) \to \chi_{c2}(1P)$	5.8	1.9	<21*
$1^3D_2 \to \chi_{c1}(1P)$	317.3	256.7	
$1^3D_2 \to \chi_{c2}(1P)$	65.7	61.8	
$1^3D_3 \to \chi_{c2}(1P)$	62.7	39.5	
$\psi(2D) \to \chi_{c0}(1P)$	8.9	23.3	
$\psi(2D) \to \chi_{c1}(1P)$	4.7	0.02	<721
$\psi(2D) \to \chi_{c2}(1P)$	0.26	0.23	<1340

TABLE V. Perturbative and nonperturbative results for the $b\bar{b}$ spectrum are shown. The perturbative fit uses the indicated states.

$m_{b\bar{b}}$ (MeV)	Pert	Nonpert	Expt
$\eta_b(1S)$	9413.70	9421.02	
$\Upsilon(1S)^*$	9460.69	9460.28	9460.30 ± 0.26
$\chi_{b0}(1P)^*$	9861.12	9860.43	9859.44 ± 0.52
$\chi_{b1}(1P)^*$	9891.33	9892.83	9892.78 ± 0.40
$\chi_{b2}(1P)^*$	9911.79	9910.13	9912.21 ± 0.40
$h_b(1P)$	9899.99	9899.94	
$\eta_b(2S)$	9998.69	10 003.6	
$\Upsilon(2S)^*$	10 022.5	10 023.5	$10\,023.26 \pm 0.31$
$\Upsilon(1D)$	10 149.5	10 148.8	
1^3D_2	10 157.1	10 157.0	$10\,161.1 \pm 1.7$
1^3D_3	10 162.9	10 164.1	
1^1D_2	10 158.4	10 158.3	
$\chi_{b0}(2P)^*$	10 230.5	10 231.4	$10\,232.5 \pm 0.6$
$\chi_{b1}(2P)^*$	10 255.0	10 257.6	$10\,255.46 \pm 0.55$
$\chi_{b2}(2P)^*$	10 271.5	10 271.1	$10\,268.65 \pm 0.55$
$h_b(2P)$	10 262.0	10 263.1	
1^3F_2	10 353.0	10 351.0	
1^3F_3	10 355.8	10 355.6	
1^3F_4	10 357.5	10 359.7	
1^1F_3	10 355.9	10 355.9	
$\eta_b(3S)$	10 344.8	10 350.4	
$\Upsilon(3S)$	10 363.6	10 365.6	$10\,355.2 \pm 0.5$
$\Upsilon(2D)$	10 443.1	10 443.7	
2^3D_2	10 450.3	10 451.2	
2^3D_3	10 455.9	10 457.5	
2^1D_2	10 451.6	10 452.4	
2^3F_2	10 610.0	10 609.0	
2^3F_3	10 613.0	10 613.4	
2^3F_4	10 615.0	10 617.3	
2^1F_3	10 613.2	10 613.7	
$\eta_b(4S)$	10 622.8	10 631.5	
$\Upsilon(4S)$	10 643.0	10 643.4	$10\,579.4 \pm 1.2$

TABLE VI. The radiative decays of the upsilon system are shown.

Γ_γ (keV)	Pert	Nonpert	Expt
$\Upsilon(1S) \to \eta_b(1S)$	0.004	0.001	
$\Upsilon(2S) \to \eta_b(2S)$	0.0005	0.0002	
$\Upsilon(2S) \to \eta_b(1S)$	0.0	0.005	<0.02
$\Upsilon(2S) \to \chi_{b0}(1P)$	1.15	0.74	1.22 ± 0.16
$\Upsilon(2S) \to \chi_{b1}(1P)$	1.87	1.40	2.21 ± 0.22
$\Upsilon(2S) \to \chi_{b2}(1P)$	1.88	1.67	2.29 ± 0.22
$\eta_b(2S) \to h_b(1P)$	4.17	20.4	
$\Upsilon(3S) \to \chi_{b0}(2P)$	1.67	1.07	1.20 ± 0.16
$\Upsilon(3S) \to \chi_{b1}(2P)$	2.74	2.05	2.56 ± 0.34
$\Upsilon(3S) \to \chi_{b2}(2P)$	2.80	2.51	2.66 ± 0.41
$\Upsilon(3S) \to \chi_{b0}(1P)$	0.03	0.03	0.061 ± 0.023
$\Upsilon(3S) \to \chi_{b1}(1P)$	0.09	0.003	
$\Upsilon(3S) \to \chi_{b2}(1P)$	0.13	0.11	
$\chi_{b0}(1P) \to \Upsilon(1S)$	22.1	19.6	
$\chi_{b1}(1P) \to \Upsilon(1S)$	27.3	23.9	
$\chi_{b2}(1P) \to \Upsilon(1S)$	31.2	26.3	
$h_b(1P) \to \eta_b(1S)$	37.9	4.61	
$\chi_{b0}(2P) \to \Upsilon(2S)$	9.90	9.91	
$\chi_{b1}(2P) \to \Upsilon(2S)$	13.7	12.4	
$\chi_{b2}(2P) \to \Upsilon(2S)$	16.8	13.5	
$\chi_{b0}(2P) \to \Upsilon(1S)$	6.69	1.83	
$\chi_{b1}(2P) \to \Upsilon(1S)$	7.31	4.81	
$\chi_{b2}(2P) \to \Upsilon(1S)$	7.74	6.86	
$\chi_{b0}(2P) \to \Upsilon(1D)$	1.13	1.05	
$\chi_{b1}(2P) \to \Upsilon(1D)$	0.62	0.52	
$\chi_{b2}(2P) \to \Upsilon(1D)$	0.04	0.03	
$\chi_{b1}(2P) \to 1^3D_2$	1.48	1.31	
$\chi_{b2}(2P) \to 1^3D_2$	0.47	0.35	
$\Upsilon(1D) \to \chi_{b0}(1P)$	18.1	12.5	
$\Upsilon(1D) \to \chi_{b1}(1P)$	9.82	7.59	
$\Upsilon(1D) \to \chi_{b2}(1P)$	0.51	0.44	
$1^3D_2 \to \chi_{b1}(1P)$	19.3	14.9	
$1^3D_2 \to \chi_{b2}(1P)$	5.07	4.35	
$1^3D_3 \to \chi_{b2}(1P)$	21.7	18.8	
Γ_1/Γ_2	**Pert**	**Nonpert**	**Expt**
$\Gamma_1(\chi_{b0})/\Gamma_2(\chi_{b0})$	1.48	5.42	5.11 ± 4.14
$\Gamma_1(\chi_{b1})/\Gamma_2(\chi_{b1})$	1.87	2.58	2.47 ± 0.60
$\Gamma_1(\chi_{b2})/\Gamma_2(\chi_{b2})$	2.17	1.97	2.28 ± 0.47

Heavy quarkonia: conclusions and outlook

- The semi-relativistic model consisting of the relativistic kinetic energy, a linear long-range confining potential with its v^2/c^2 corrections and the one-loop QCD potential provides a quantitatively good description of the $c\bar{c}$ and $b\bar{b}$ heavy quarkonium systems.

- The Lorentz structure of the confining potential is interesting. In both cases ($c\bar{c}$ and $b\bar{b}$) the perturbative treatment of the spin-dependent interactions always favors a pure scalar confining potential, while treating the spin terms non-perturbatively favors a scalar-vector mixture ~20% vector for both $c\bar{c}$ and $b\bar{b}$.

Heavy quarkonia: conclusions and outlook

- The calculated E_1 decays compare favorably with experiment. Transitions between $J/\psi, \chi$ and ψ' appear to be dominated by spin rather than open channel effects.

- Based on the model considered here, the X(3872) cannot be explained solely in terms of a charmonium state described by a potential, which suggests that its identification as a more complex bound state or "tetraquark' state.

- The X(3943) state is not compatible with a 2^3P_J charmonium level. It has been suggested that it may be the 3^1S_0 (η_c'') state.

Heavy quarkonia: conclusions and outlook

- The potential for unequal mass systems has also been calculated and can be used to investigate the D_S , B_S , and B_C mesons (Gupta, SR & WWR, 1981, 1985). We are continuing with that investigation.

- After a long period of inactivity, today there is a renewed interest in charmonium due to developments in the study of b quark states.

- At this time, there are several active experimental groups investigating heavy quarkonium states: CLEO II, Belle, BaBar, CDF.