Tidal Disruption Rates

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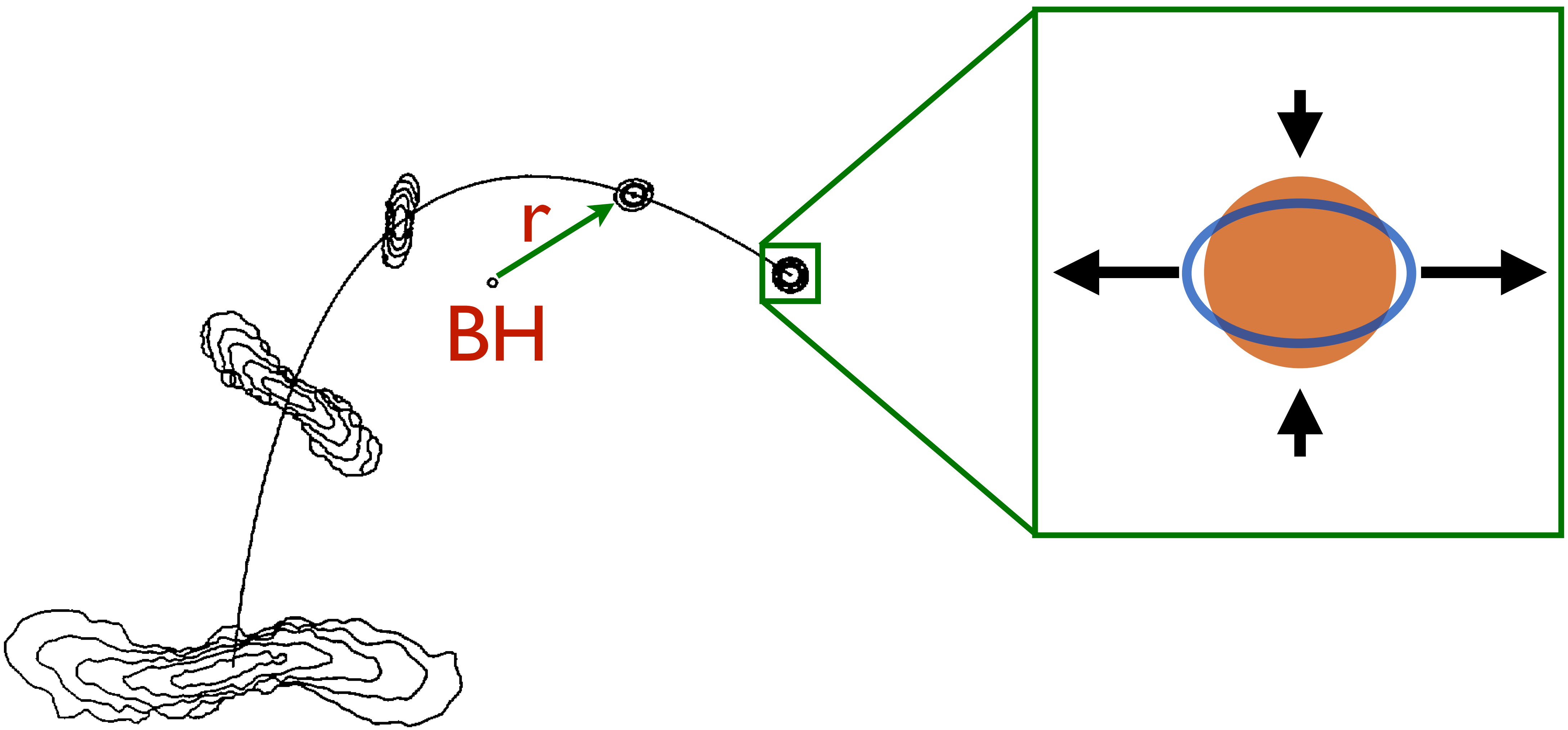
Galactic Nuclei

- Black hole masses: $M_\bullet \sim 10^6\text{–}10^9\,M_\odot$
- Stellar densities: $n \propto r^{-\alpha}$ $(\gtrsim 10^4\text{–}10^5\,\mathrm{pc}^{-3};\quad \alpha \simeq 1\text{–}2)$
- Stellar velocities: $\sigma \propto r^{-\beta}$ $(\gtrsim 100\,\mathrm{km\,s}^{-1};\quad \beta \simeq 0\text{–}1/2)$
- Black hole sphere of influence: $r_h \sim \dfrac{GM_\bullet}{\sigma_\infty^2}$ (few pc)
- Dynamical time: $\tau_d \sim \dfrac{r}{\sigma(r)}$ $(\sim 10^4\,\mathrm{yr}$ near $r_h)$
- Relaxation time: $\tau_r \sim \dfrac{\sigma(r)^3}{(Gm)^2 n(r)}$ $(\gtrsim 10^{10}\,\mathrm{yr}$ near $r_h)$

Disruption Criteria (I)



$$\frac{GM_*}{R_*^2} \sim \frac{GM_\bullet R_*}{r_t^3} \Rightarrow \boxed{r_t \sim R_* \left(\frac{M_\bullet}{M_*}\right)^{1/3}}$$

Tidal Radius

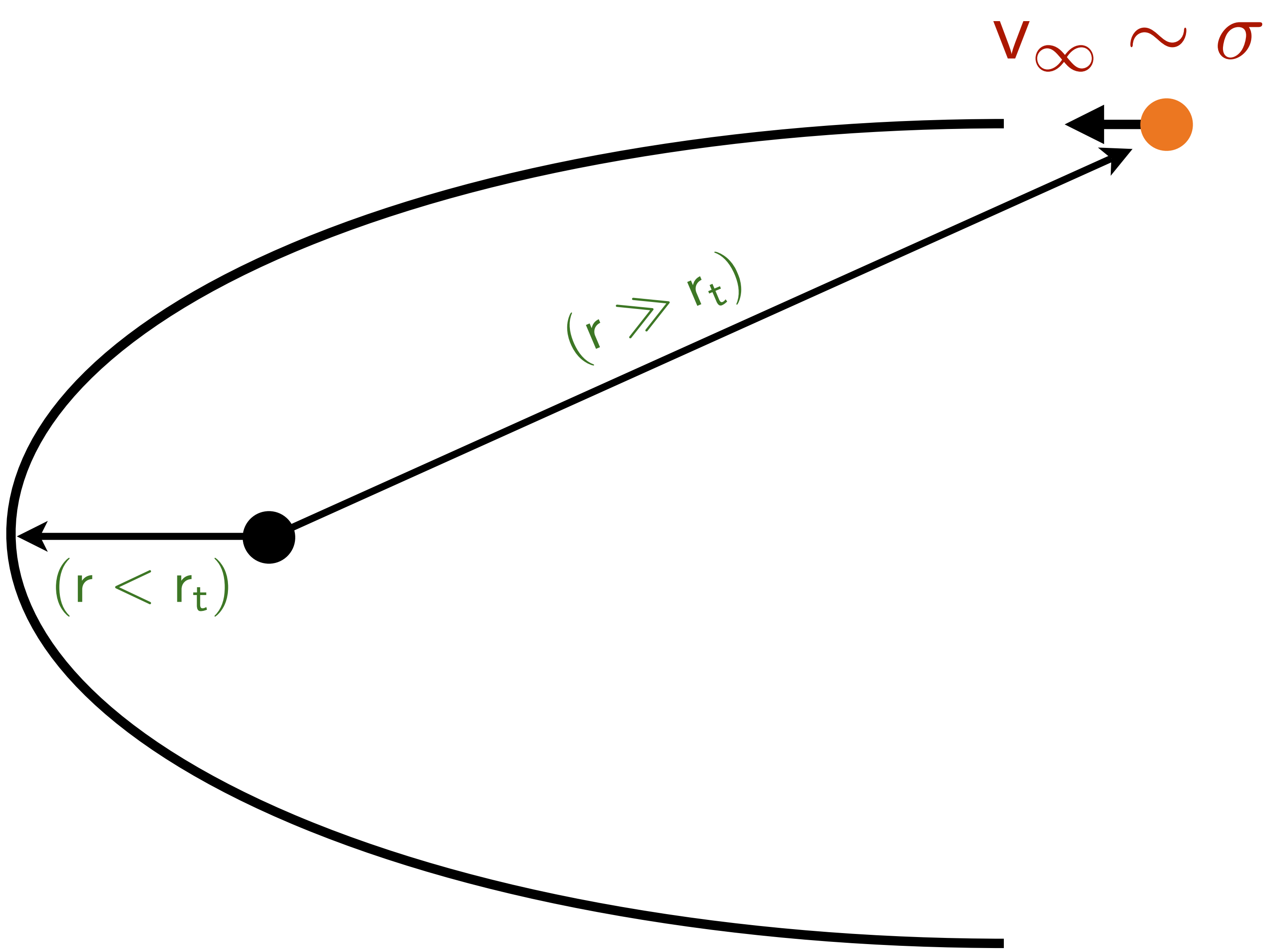
$$r_t \sim R_* \left(\frac{M_\bullet}{M_*}\right)^{1/3} = 100 R_* \left[\frac{(M_\bullet/10^6\,M_\odot)}{(M_*/M_\odot)}\right]^{1/3}$$

For normal stars $r_h \gg r_t$

$$r_{Sch} \simeq 4.3\,(M_\bullet/10^6\,M_\odot)\,R_\odot$$

Star swallowed whole if $M_\bullet \gtrsim 10^8\,M_\odot$

Dynamics of Disrupted Stars



$$r \gg r_t \Rightarrow$$

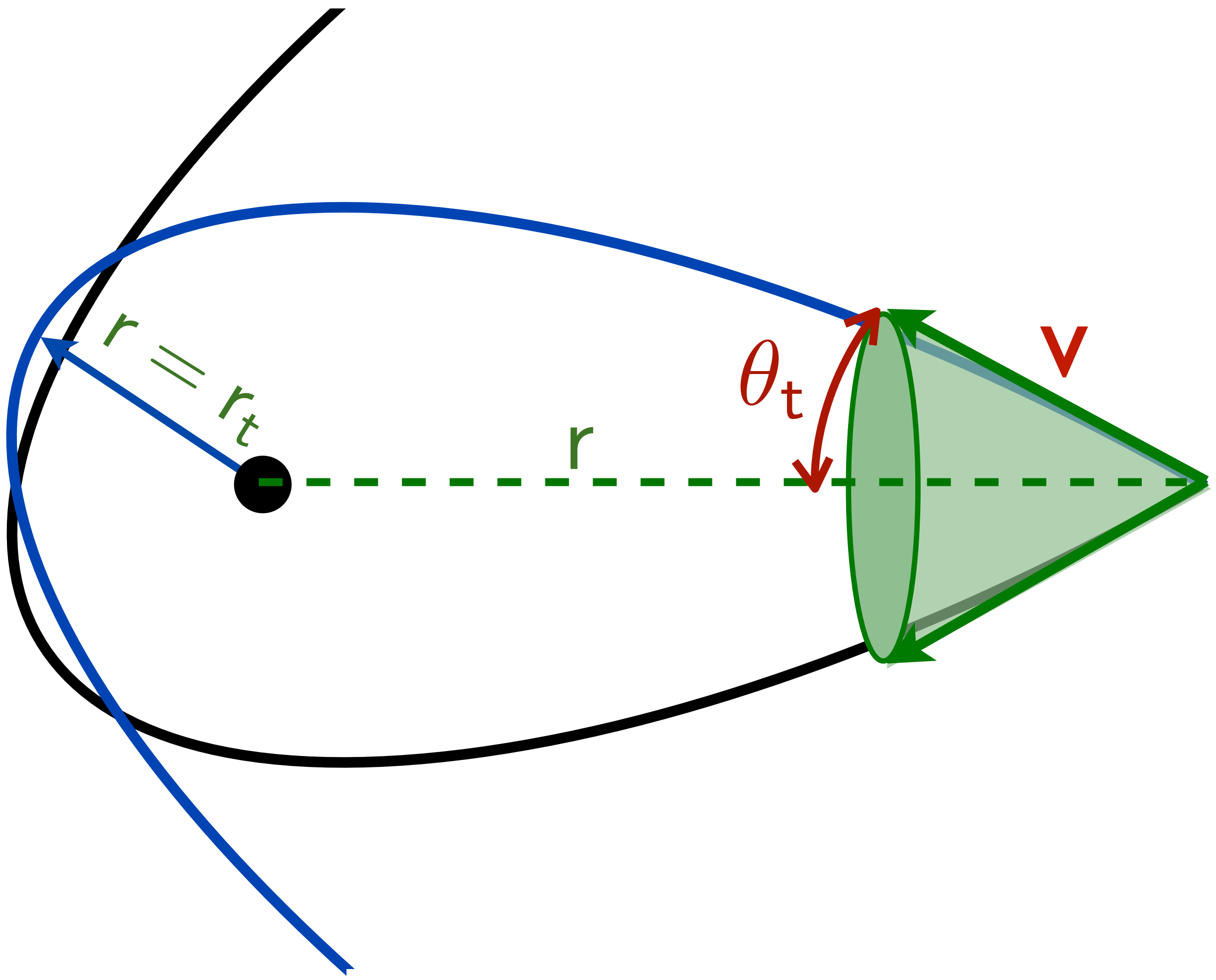
$$E \simeq 0$$

&

$$J < J_t \simeq (2GM_\bullet r_t)^{1/2}$$

$$\left(E = \frac{1}{2} v_\infty^2 = \frac{1}{2} v^2(r_{min}) - \frac{GM_\bullet}{r_{min}} \right)$$

'Loss Cone' Angle



Angular momentum:

$$J_t = rv\sin\theta_t \simeq rv\theta_t$$

$$\Rightarrow$$

$$\theta_t^2 \sim \frac{GM_\bullet r_t}{r^2\sigma^2}$$

Disrupted stars have: $J < J_t\,,\ \theta < \theta_t$

Loss cone emptied on a dynamical time $\tau_d \sim \dfrac{r}{\sigma}$

Orbital Diffusion

- Stellar motion deflected by weak scattering.
- Angle θ undergoes a random walk.
- In a dynamical time: $\theta_{\mathrm{d}} \sim \left(\frac{\tau_{\mathrm{d}}}{\tau_{\theta}}\right)^{1/2}$
- We expect $\tau_{\theta} \sim \tau_{\mathrm{r}}$

Two regimes to consider: $\theta_{\mathrm{d}}/\theta_{\mathrm{t}} \gtrless 1$

Remarks on Scaling

$n(r) \propto r^{-\alpha} \quad (\alpha \simeq 1\text{–}2)$

$\sigma(r) \propto r^{-\beta} \quad (\beta \simeq 0\text{–}1/2)$

$$\theta_d^2 \propto \frac{rn}{\sigma^4} \propto r^{1+4\beta-\alpha}$$

Increasing for $r < r_h$ and decreasing for $r > r_h$

$$\theta_t^2 \propto (r\sigma)^{-2} \propto r^{2\beta-2}$$

Decreasing for all r

$$\frac{\theta_d^2}{\theta_t^2} \propto r^{3+2\beta-\alpha}$$

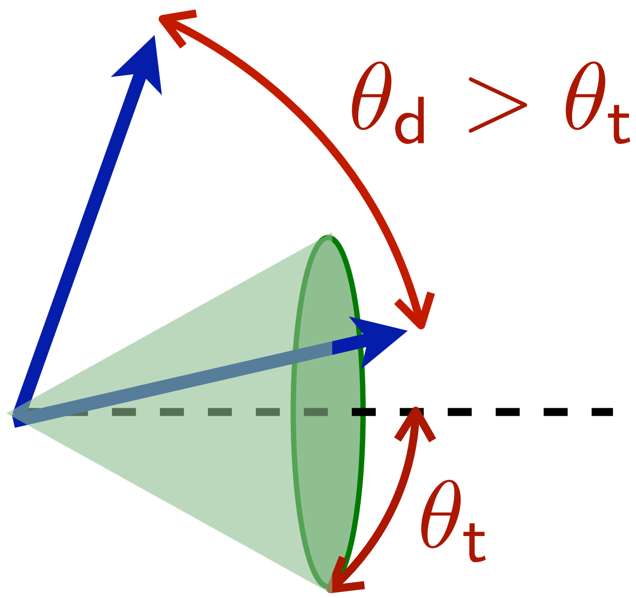
Increasing for all r

$$\frac{\theta_d^2}{\theta_t^2}(r_c) = 1 \Rightarrow \frac{r_c}{r_h} \sim 1$$

for solar-type stars

(decreases for smaller r_t)

Full Loss Cone



- Stars scatter in and out of LC on a dynamical time.
- The LC is full statistically.
- The disruption rate is proportional to the solid angle $\sim \theta_{\mathrm{t}}^2$.

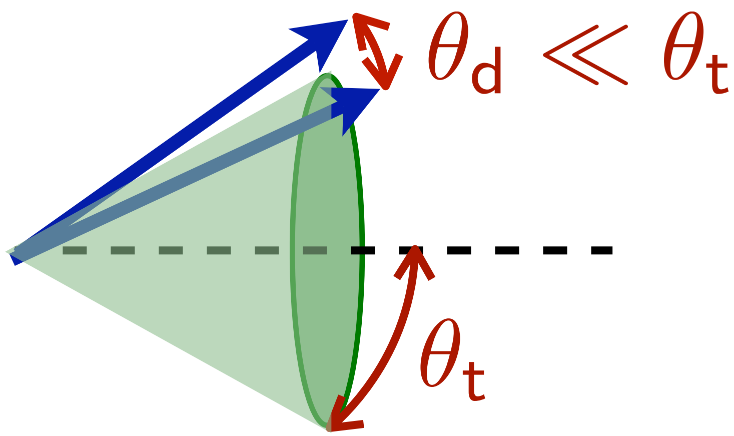
Local disruption rate per unit volume:

$$\frac{d\mathcal{R}_{\mathrm{full}}}{dV} \sim \frac{n\,\theta_{\mathrm{t}}^2}{\tau_{\mathrm{d}}}$$

Matters when $r > r_c$

Vanishes as $r \to \infty$

Empty Loss Cone



- Stars scatter in and out of LC on a dynamical time.
- The LC is mostly empty.
- LC refilling is driven by diffusion on a time $\sim \tau_r$.

Local disruption rate per unit volume:

$$\frac{d\mathcal{R}_{empty}}{dV} \sim \frac{n}{\tau_r} \sim \left(\frac{\theta_d}{\theta_t}\right)^2 \frac{d\mathcal{R}_{full}}{dV}$$

Rates equal when $\dfrac{\theta_d}{\theta_t} \sim 1$

Total Rate

$$4\pi r^3 \frac{d\mathcal{R}_{\rm empty}}{dV} = \boxed{\frac{d\mathcal{R}_{\rm empty}}{d\ln r}} \sim \frac{4\pi r^3 n(r)}{\tau_r} \sim \boxed{\frac{N(r)}{\tau_r}}$$

Vanishes as $r \to 0$ and $r \to \infty$

Peaks near r_h

$$* \;\; \mathcal{R}_{\rm total} \sim \frac{N(r_h)}{\tau_r(r_h)} \sim 10^{-4} \left(\frac{M_\bullet}{10^6\, M_\odot}\right)^{-1/4} {\rm yr}^{-1} \;\; *$$

(Scaling with $M_\bullet$ only a crude approximation.)