3rd EPiQS-TMS alliance workshop on
Topological Phenomena in Quantum Materials

Dielectric breakdown of strongly correlated insulators in one dimension

Universal formula from non-Hermitian sine-Gordon theory

arXiv:1908.06107

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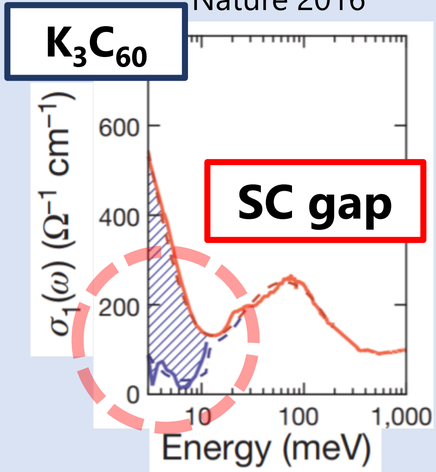
Collaborators: **Masaya Nakagawa** (Tokyo), **Norio Kawakami** (Kyoto)

Nonequilibrium phases of matter

Light-induced “ordered” phase (transient)

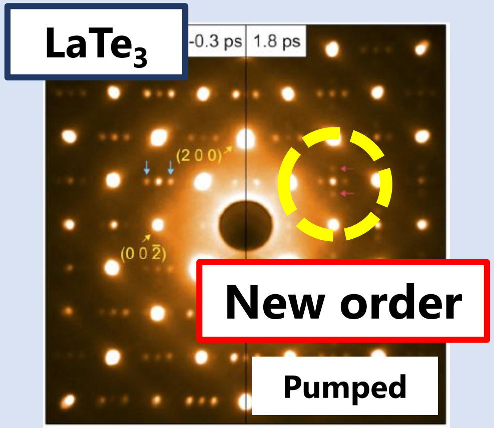
Superconductivity

Mitrano *et al.* (MPI Hamburg)
Nature 2016



Charge density wave

Kogar *et al.* (MIT) arXiv 2019

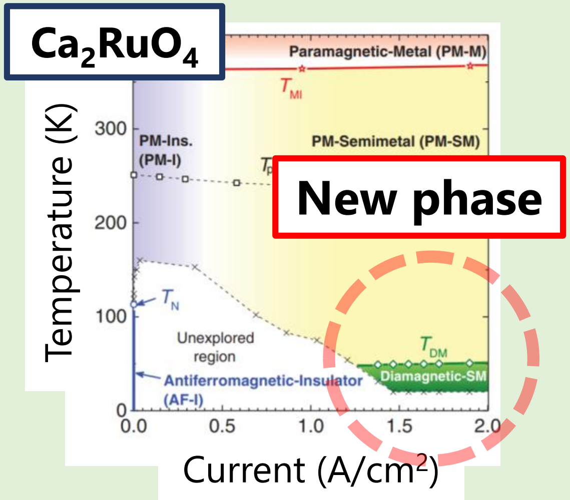


(Diffraction pattern)

DC-current-induced phase (steady)

Diamagnetic phase

Chanchal *et al.* (Kyoto) Science 2017



- New and exciting frontier in condensed matter physics
- Theoretical understanding is still lacked, particularly in interacting systems

c.f. In equilibrium, well-established concepts and theories

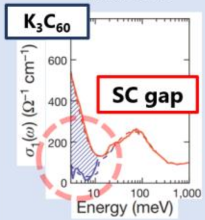
Universality class, Renormalization group, Landau theory, CFT, etc.

How can we develop the theoretical understanding?

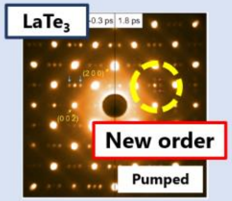
Beautiful experimental results

Photo-induced "ordered" phase (transient)

Superconductivity
Mitrano *et al.* (MPI Hamburg) Nature 2016



Charge density wave
Kogar *et al.* (MIT) arXiv 2019



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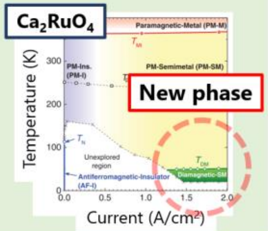
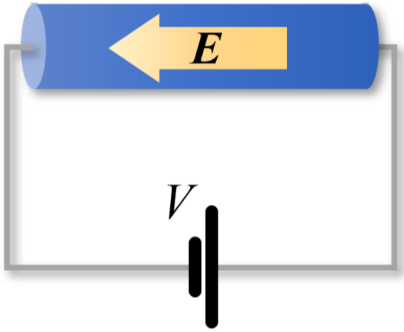


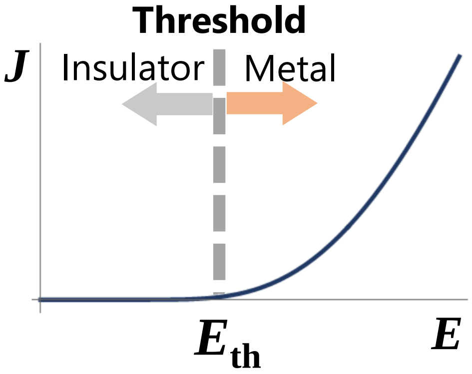
Lack of theoretical understanding

Universality Class,
Renormalization Group,
Landau Theory, CFT, etc.

for ***nonequilibrium phases of matter***

Strat from very simple problem:
dielectric breakdown in 1D interacting insulators

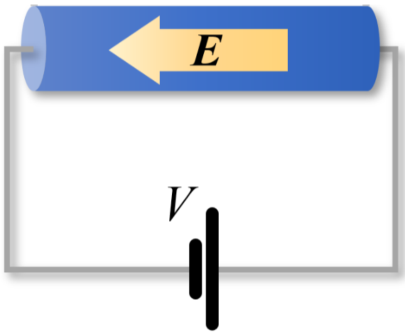


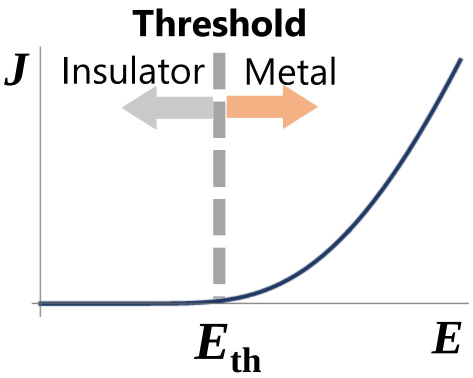


Why dielectric breakdown? : Simple but fundamental nonequilibrium phase transition

Why 1D? : We have many reliable theoretical tools (even for interacting systems)

Dielectric breakdown





- **Band insulator** (non-interacting): well-understood (Zener breakdown)

C. Zener 1934

- **Interacting systems** **(Fermionic) Mott insulators**

Fukui-Kawakami PRB 1998, ..., H. Yamakawa et al Nat. Mat. 2017, ...

→ **Theory for other insulators (Bose Mott, CDW, Kondo, etc.)?**

Experimentally relevant!

- **Universal property** common in all the above insulators?
  (e.g. Universality class in equilibrium → Out of equilibrium?)

Dielectric breakdown in generic insulators in 1D

Summary of this study

KT, M. Nakagawa, N. Kawakami, arXiv:1908.06107

Motivation

Theoretical understanding of nonequilibrium phases of matter (e.g. universality)

Goal of this study

Construct a theory for dielectric breakdown in **generic 1D insulators**

Find the **universal properties** common in the insulators

Results

1. Construct **the effective-field-theory description** for dielectric breakdown

Non-Hermitian sine-Gordon theory

2. Derive a **formula of the threshold field** **universally applicable** to 1D insulators

$$F_{\mathrm{th}} = \frac{e}{e^*} \cdot \frac{(\Delta_0/2)^2}{v}$$

Many-body generalization
of Landau-Zener formula

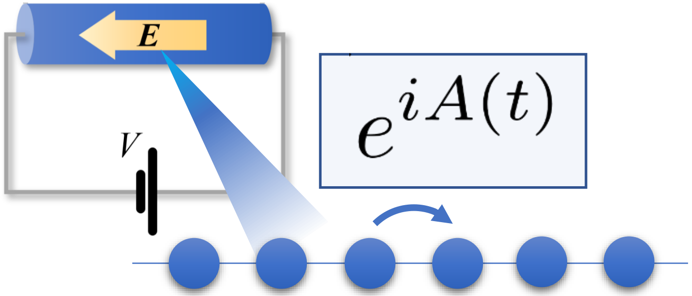
3. Apply the formula to **lattice models** and find nice agreement

Theoretical setup

Model: 1D lattice model + DC electric field

$$H(t) = -\sum_{i\alpha} \left(e^{iA(t)} c^\dagger_{i\alpha} c_{i\alpha} + \text{h.c.} \right) + V_{\text{int}}$$

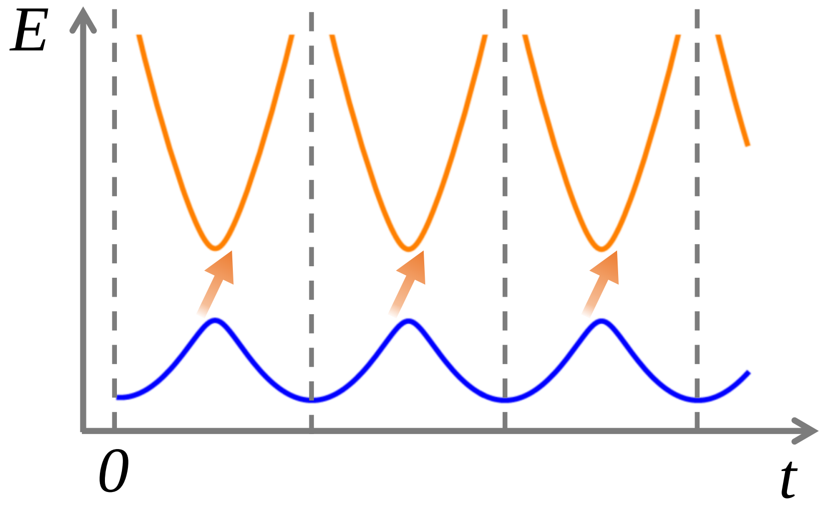
$$A(t) = -Ft,\ F = eE$$



How to treat dielectric breakdown? → Non-adiabatic transition

(Theory) T. Oka et al PRL 2003, T. Oka and H. Aoki, PRB 2010, etc.
(Exp.) Y. Taguchi et al PRB 1998, H. Yamakawa et al Nat. Mat. 2017, etc.

Many-body energy spectrum (schematic)



Ground state shows no current

Non-adiabatic transition to excited states → Finite current

Dielectric breakdown
= **Rapid increase of the transition rate**

How to calculate the transition rate?

Theory of quantum tunneling

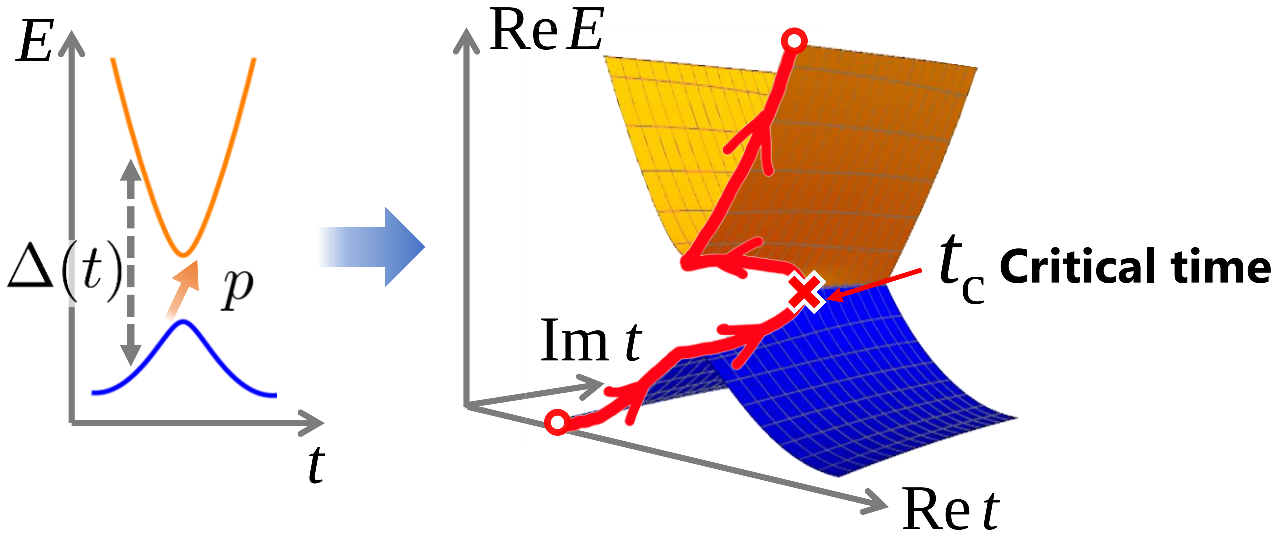
How to (approximately) calculate the non-adiabatic transition rate?

→ **Dykhne-Davis-Pechukas (DDP) formula**

(applicable to wide range of 2-level systems)

Dykhne, Sov. Phys. JETP 1962
Davis-Pechukas, J. Chem. Phys. 1976
(c.f. Fukushima-Shimazaki, arXiv:1907.12224)

Time-dep. Energy gap

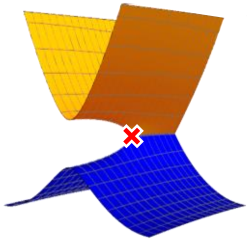


$$p = \exp\left(-2\mathrm{Im}\int_0^{t_c}\Delta(t)dt\right), \qquad \Delta(t_c) = 0$$

- Why complex time? → Integral path connecting g.s. and 1st excited state
- Works very well in (1) various 2-level models, e.g. Kitamura-Morimoto-Nagaosa arXiv:1908.00819
  (2) 1D Hubbard model (checked with t-DMRG)

Oka-Aoki, PRB 2010; Oka PRB 2012

Non-Hermitian theory

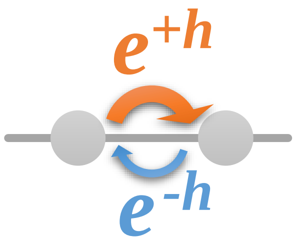


Spectrum on **complex time** has the information of non-adiabatic transition and **dielectric breakdown**

$A(t) = -Et$ ⇒ Complex time ~ **Complex gauge field**

Introducing *A=ih*, **Asymmetric hopping** (non-Hermitian)

$$H = -\sum_{i\alpha}\left(e^{-h}c^{\dagger}_{i\alpha}c_{i+1\alpha} + e^{+h}c^{\dagger}_{i+1\alpha}c_{i\alpha}\right) + V_{\mathrm{int}}.$$



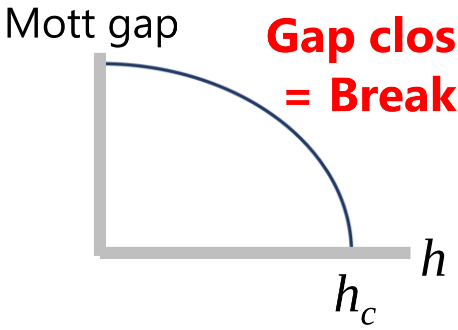
1. Asymmetric hopping “delocalizes” the particles

V_{int} = Random pot. → Hatano-Nelson model

V_{int} = Hubbard interaction

→ **Fukui-Kawakami**, PRB 1998

First effective model of Mott breakdown



Gap closing
= Breakdown

2. Experimentally realizable in ultracold atomic systems

Z. Gong, ..., **KT**, ..., M. Ueda, PRX 2018

Non-Hermitian sine-Gordon theory

$$H = -\sum_{i\alpha}\left(e^{-h}c^{\dagger}_{i\alpha}c_{i+1\alpha} + e^{+h}c^{\dagger}_{i+1\alpha}c_{i\alpha}\right) + V_{\text{int}}.$$

It's still difficult to treat... (except for integrable cases)

Bosonization

e.g. T. Giamarchi's textbook

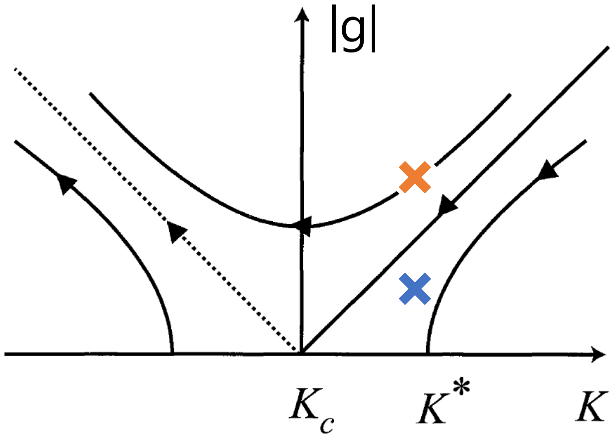
Low-energy effective field theory

Sine-Gordon model

$$H = \frac{v}{2\pi}\int dx\left\{K(\pi\Pi - ih)^2 + \frac{1}{K}(\nabla\phi)^2\right\} + g\int dx\cos(\beta\phi)$$

Free boson (gapless)

Interaction (gapful)



Small g → Gapless ("Metal")

Large g → Gapful ("Insulator")

Effective model of metal-insulator transition for generic insulators

(e.g. Bose Mott, CDW, Kondo, etc.)

Time-dependent lattice model

$$H(t) = -\sum_{i\alpha}\left(e^{iA(t)}c^{\dagger}_{i\alpha}c_{i\alpha} + \mathrm{h.c.}\right) + V_{\mathrm{int}}$$

DDP formula, Complex time → Complex gauge field, A=ih

Non-Hermitian lattice model

$$H = -\sum_{i\alpha}\left(e^{-h}c^{\dagger}_{i\alpha}c_{i+1\alpha} + e^{+h}c^{\dagger}_{i+1\alpha}c_{i\alpha}\right) + V_{\mathrm{int}}$$

Bosonization, Low-energy effective theory

Non-Hermitian field theory

$$\boxed{H = \frac{v}{2\pi}\int dx\left\{K(\pi\Pi - ih)^2 + \frac{1}{K}(\nabla\phi)^2\right\} + g\int dx\cos(\beta\phi)}$$

Effective field theory description of “dielectric breakdown phase transition”

Non-Hermitian theory works as an effective theory for nonequilibrium phenomena(?)

A first step towards “universality class in nonequilibrium”

Summary of this study

KT, M. Nakagawa, N. Kawakami, arXiv:1908.06107

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$$F_{\mathrm{th}} = \frac{e}{e^*} \cdot \frac{(\Delta_0/2)^2}{v}$$

Many-body generalization
of Landau-Zener formula

3. Apply the formula to **lattice models** and find nice agreement

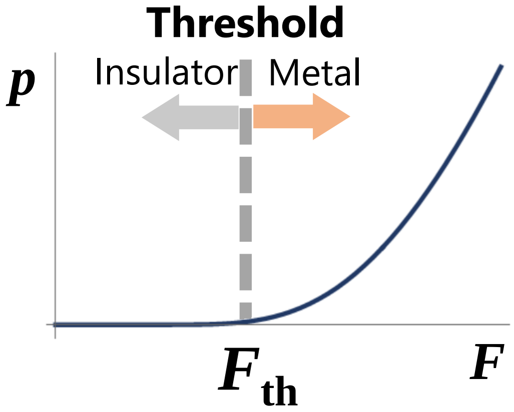
Threshold field

Dielectric breakdown = Rapid increase of *p*

DDP formula

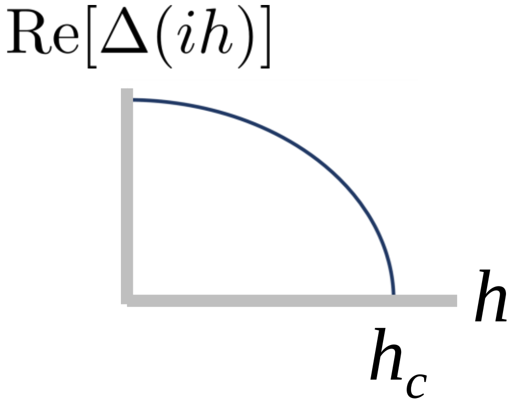
$$p = \exp\left(-2\mathrm{Im}\int_0^{t_c}\Delta(t)dt\right)$$

$$= \exp\left(-\pi\frac{F_{\mathrm{th}}}{F}\right)$$



Threshold field

$$\boxed{F_{\mathrm{th}} = \int_0^{h_c}\mathrm{Re}[\Delta(A=ih)]dh}$$



To obtain F_{th}, calculate

(1) **Critical value**

(2) **Change of energy gap** (“dispersion”)

Derivation

Skip all the details of the derivation. Please see our preprint arXiv:1908.06107.

(1) **Critical value** h_c

Space-(imaginary)time transposition to the action $(\tilde{x}, \tilde{\tau}) = (v\tau, x/v)$

→ **Mapping to a Hermitian model (doped insulator)!!**

From the Hermitian model,

$$\boxed{h_c = \frac{e}{e^*} \cdot \frac{\Delta_0}{2v}}$$

Δ_0	Original many-body energy gap
v	Velocity of elementary excitations
$e^*/e = 2/\beta$	Charge of elementary excitations

(2) **Change of energy gap** ("dispersion") $\Delta(ih)$

Bethe ansatz approach to sine-Gordon model

(Key point: Elementary excitation is *soliton* which has a relativistic dispersion)

$$\boxed{\Delta(ih) = \Delta_0\sqrt{1 - \left(\frac{e^* 2vh}{e\Delta_0}\right)^2} = \Delta_0\sqrt{1 - \left(\frac{h}{h_c}\right)^2}}$$

Main result: Universal formula

$$\boxed{F_{\mathrm{th}} = \frac{e}{e^*} \cdot \frac{(\Delta_0/2)^2}{v}}$$

$$F_{\mathrm{th}} = \int_0^{h_c} \mathrm{Re}[\Delta(A = ih)]dh$$

1. Many-body generalization of Landau-Zener formula

Δ_0 Original many-body energy gap

v Velocity of elementary excitations

Charge of elementary excitations

$$e^*/e = 2/\beta$$

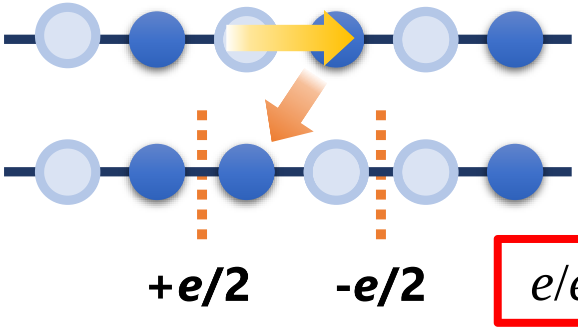
2. Appearance of a fractional charge

Simplest non-trivial example:

Spinless fermions

with n.n. repulsive interaction

"Smaller elementary charge needs a stronger field"



$\boxed{e/e^*=2}$

3. Applicable to various insulators beyond integrable systems

(Bose Mott, CDW, Kondo, etc.)

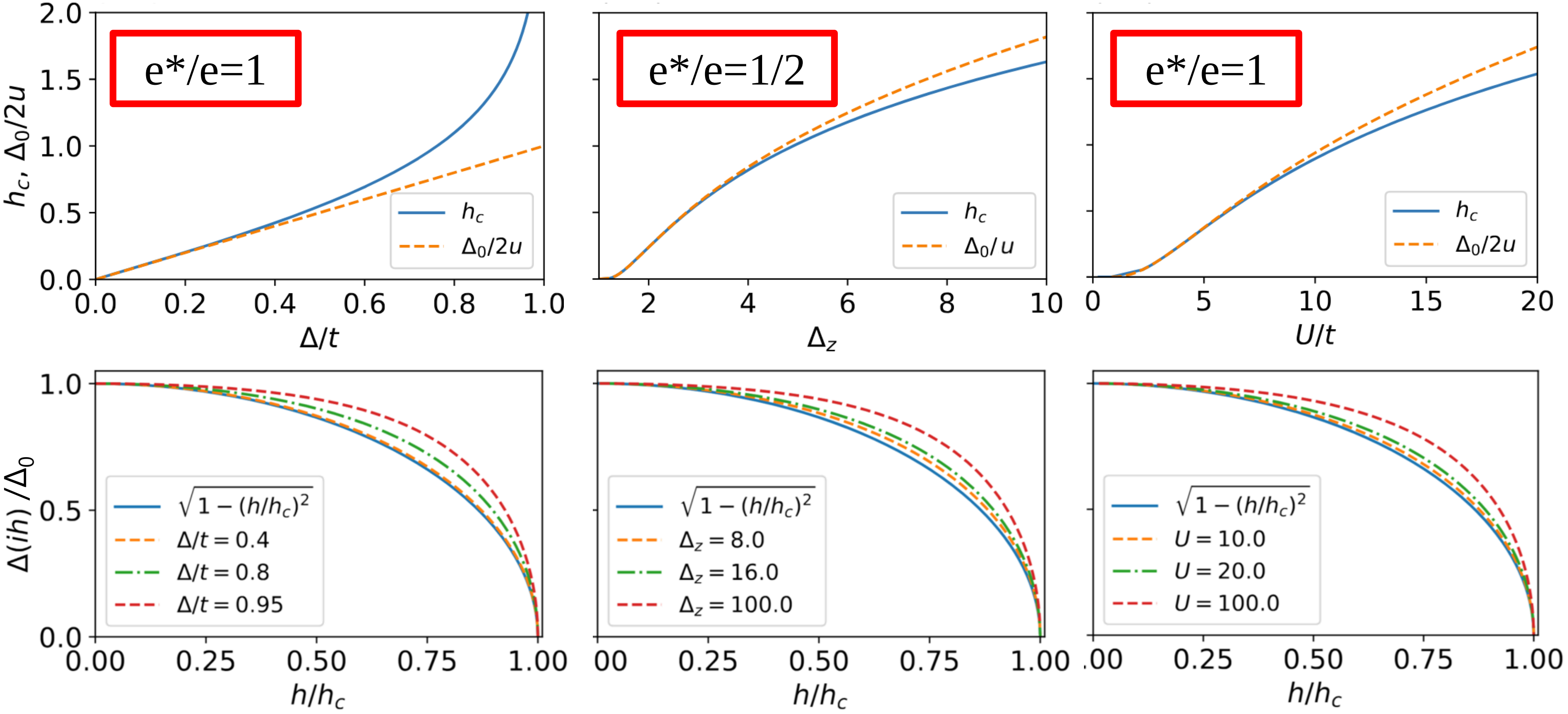
Field theoretical prediction → How good in lattice models?

Application to lattice models

Field theoretical prediction

$$\boxed{h_c = \frac{e}{e^*} \cdot \frac{\Delta_0}{2v}}$$

$$\boxed{\Delta(ih) = \Delta_0 \sqrt{1 - \left(\frac{h}{h_c}\right)^2}}$$



Band insulator (SSH)

CDW insulator (XXZ)

Fermionic Mott (Hubbard)

Nice agreement in a broad range including the weak coupling regime

Summary and Outlook

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$$F_{\rm th} = \frac{e}{e^*} \cdot \frac{(\Delta_0/2)^2}{v}$$

Many-body generalization of L-Z formula containing overlooked factor (**fractionalized** charge)

3. Apply the formula to **lattice models** and find nice agreement

Outlook

- Extension to the higher dimensions / the AC-driven cases
- Other universal properties as a nonequilibrium phase transition
- Study “field-induced metallic states” (nonequilibrium steady states, NESS)

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