

Classification theory of topological crystalline gapless superconductivity

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SS & Y. Yanase, Phys. Rev. B **97**, 134512 (2018).

S. Kobayashi, SS, Y. Yanase, & M. Sato, Phys. Rev. B **97**, 180504(R) (2018).

SS, T. Nomoto, K. Shiozaki, & Y. Yanase, Phys. Rev. B **99**, 134513 (2019).



Topology in gapped systems

- ▶ Topological invariant: defined for **gapped** Hamiltonian
 - e.g.) insulators, fully gapped SCs
- ▶ Classification by **symmetry** and **dimensionality**
 - Onsite symmetry: 10 **Altland-Zirnbauer (AZ) classes**

A. Altland & M. R. Zirnbauer, PRB (1997)

 - TRS, PHS, & CS
 - "*Topological periodic table*"

A. P. Schnyder et al. (2008)
A. Kitaev (2009) / S. Ryu et al. (2010)

TRS	PHS	CS	AZ class
0	0	0	A
0	0	1	AIII
+1	0	0	AI
+1	+1	1	BDI
0	+1	0	D
-1	+1	1	DIII
-1	0	0	AII
-1	-1	1	CII
0	-1	0	C
+1	-1	1	CI

Cartan	d												
	0	1	2	3	4	5	6	7	8	9	10	11	...
<i>Complex case:</i>													
A	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	...
AIII	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	...
<i>Real case:</i>													
AI	\mathbb{Z}	0	0	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	...
BDI	\mathbb{Z}_2	\mathbb{Z}	0	0	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	...
D	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	...
DIII	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	...
AII	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	...
CII	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	...
C	0	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	$2\mathbb{Z}$	0	...
CI	0	0	0	$2\mathbb{Z}$	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	$2\mathbb{Z}$...

Topology in gapped systems

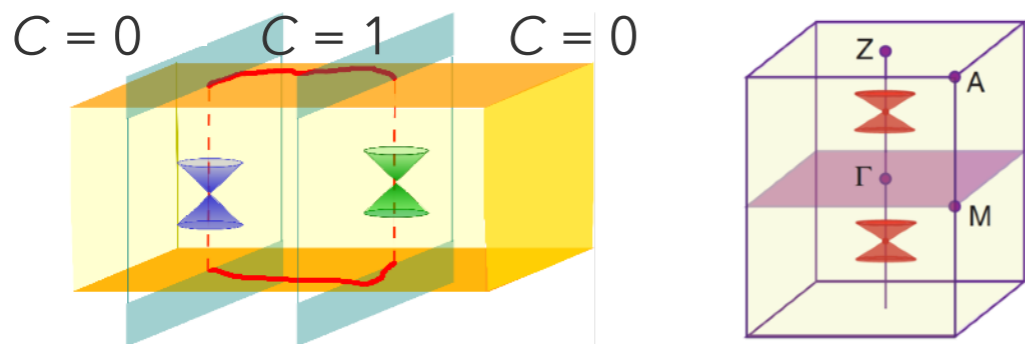
- ▶ Topological invariant: defined for **gapped** Hamiltonian
 - e.g.) insulators, fully gapped SCs
- ▶ Classification by **symmetry** and **dimensionality**
 - Onsite symmetry: 10 **Altland-Zirnbauer (AZ) classes**
A. Altland & M. R. Zirnbauer, PRB (1997)
 - TRS, PHS, & CS
 - "*Topological periodic table*" A. P. Schnyder *et al.* (2008)
A. Kitaev (2009) / S. Ryu *et al.* (2010)
 - Crystal symmetry: **point groups, space groups**
 - "*Topological crystalline insulators*" L. Fu (2011)
 - Various methods
 - Symmetry-based indicator: H. C. Po *et al.* (2017), H. Watanabe *et al.* (2018)
 - Topological quantum chemistry: B. Bradlyn *et al.* (2017)
 - Atiyah-Hirzebruch spectral sequence: K. Shiozaki *et al.* (2018)

Topology in gapless systems

- ▶ Topology is useful even for **gapless** Hamiltonian

Semimetals

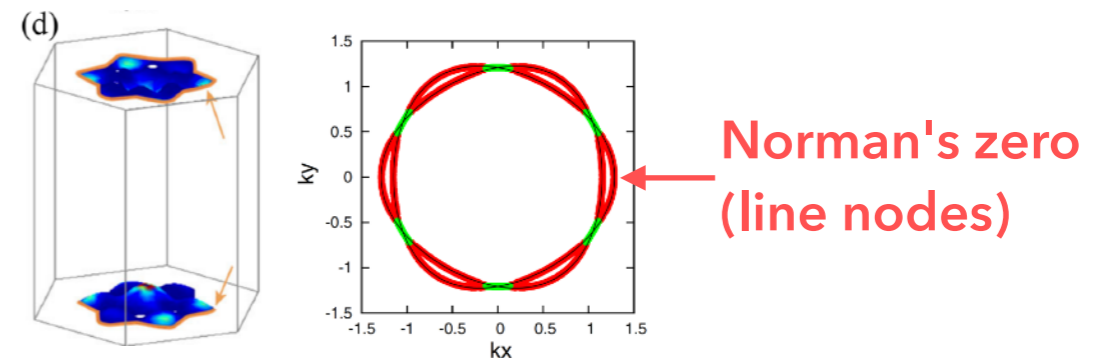
- ▶ Jumps of topological # in BZ = **Topological gapless points**
 - Weyl points (Chern #)
 - Dirac points on a C_n -axis



Yang-Nagaosa (2014)

Nodal superconductors

- ▶ Gapless points = **SC nodes**
 - **Unusual nodes** due to crystal symmetry Norman (1995) Nomoto-Ikeda (2017)
 - Topological protection ?



Nomoto-Ikeda(2016) / Yanase (2016)

Bulk property	Normal	Superconducting
Gapped	Topological insulators	(Fullgap) topological SCs
Gapless	Topological semimetals	????

Background: gapless physics

- ▶ Semimetal case:
 - **Gapless points** characterized by a **topological invariant**
 - **Crystal sym.** → an **additional invariant** of gapless points
- ▶ Nodal SC case: examples of **symmetry-protected nodes**

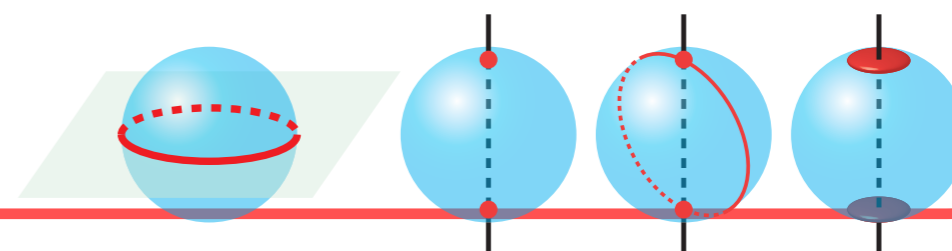
Aim

- ▶ *Do SC nodes meet topology ?*
- ▶ *Do crystal symmetries give an invariant of nodes ?*

Topologically classify symmetry-protected SC nodes for centrosymmetric superconductors

(a) (Line) nodes on high-symmetry plane

(b) Nodes on high-symmetry line



Collaborators

(a) SS-Yanase, PRB (2018). / Kobayashi-SS-Yanase-Sato, PRB (2018).
(b) SS-Nomoto-Shiozaki-Yanase, PRB (2019).

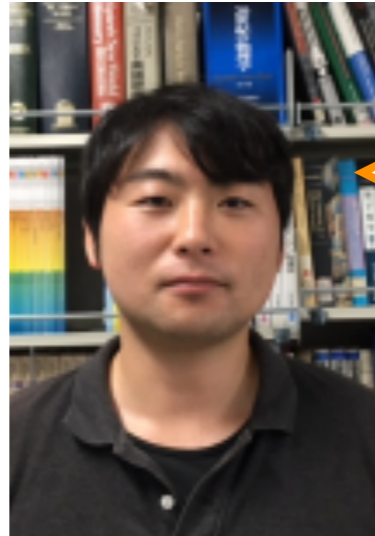
Dept of Phys., Kyoto Univ.
Youichi Yanase



Supervisor

- ▶ Superconductivity
- ▶ SCES
- ▶ Multipole physics

(a) Dept. of Appl. Phys., Nagoya Univ.
Shingo Kobayashi



Clifford algebra

Quantum field theory

YITP, Kyoto Univ.
Masatoshi Sato



(b) Dept. of Appl. Phys., The Univ. of Tokyo
Takuya Nomoto



Group theory

K-theory, cohomology

YITP, Kyoto Univ.
Ken Shiozaki

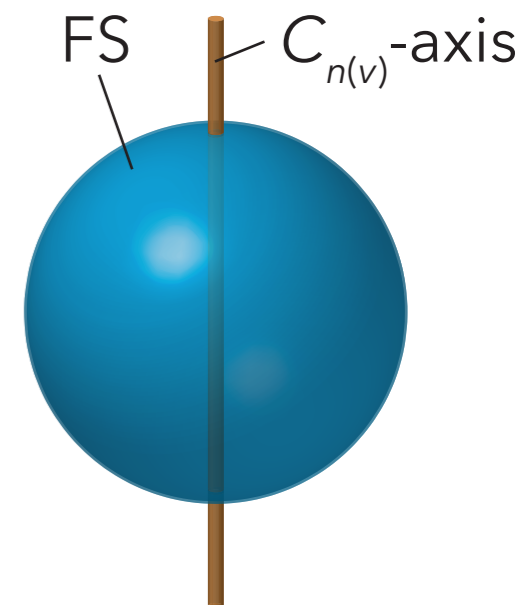


Methods

- ▶ High-sym. $\mathbf{k} \rightarrow$ normal Bloch state a_1, a_2, \dots

ex.) C_2 -axis

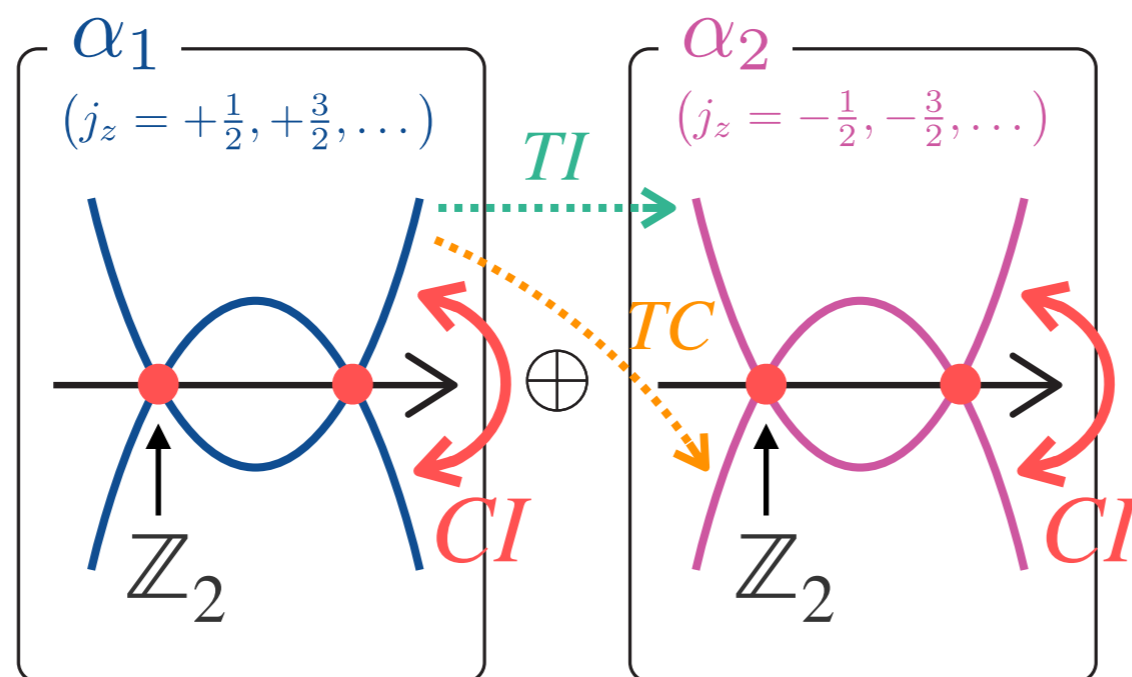
IRs	E	C_2	
α_1	1	$+i$	\sim spin up
α_2	1	$-i$	\sim spin down



- ▶ Symmetry of SC order parameter
 \rightarrow maps of (pseudo-) TRS, PHS, & CS among a_1, a_2, \dots

ex.) B_g sym.
 I : inversion,
 T : TRS, C : PHS

$$H_{\text{BdG}}(\mathbf{k}) \sim$$



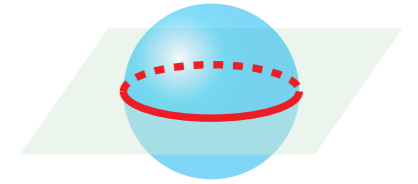
"Emergent" AZ class
 \rightarrow **Topological invariant!**

PHS w/ $(CI)^2 = +E$: class D \rightarrow Classification = \mathbf{Z}_2

Main results

(a) SS-Yanase, PRB (2018). / Kobayashi-SS-Yanase-Sato, PRB (2018).
(b) SS-Nomoto-Shiozaki-Yanase, PRB (2019).

(a) For mirror- or glide-invariant SCs:

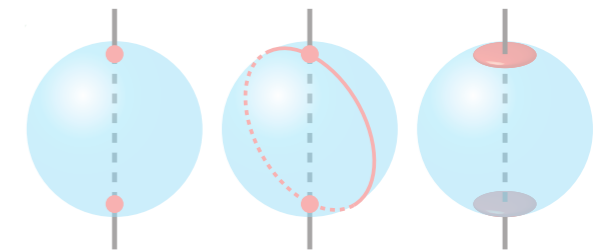


- **Complete classification** on **high-symmetry planes** for all symmorphic & nonsymmorphic symmetries
- No unusual node beyond previous examples

UPt₃: Norman (1995), Micklitz-Norman (2009), Kobayashi-Yanase-Sato (2016), Nomoto-Ikeda (2016)

CrAs: Micklitz-Norman (2017) / UCoGe, UPd₂Al₃: Nomoto-Ikeda (2017) / Sr₂IrO₄: SS-Nomoto-Yanase (2017)

(b) For rotation-invariant SCs:



- Classification on **high-symmetry axes** *only* for symmorphic symmetries
- **Novel type of gap structure** on C₃- and C₆-axes

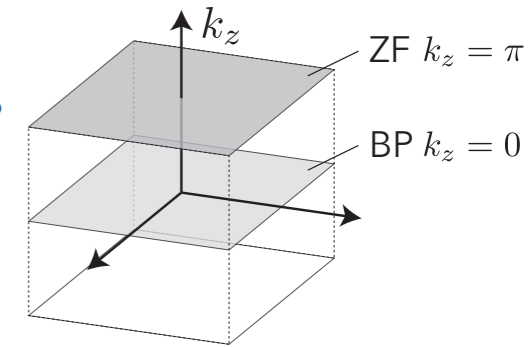
Complete classification on mirror- or glide-planes

S. Kobayashi, SS, Y. Yanase, & M. Sato, Phys. Rev. B **97**, 180504(R) (2018).

▶ Nontrivial results: differences between BP & ZF

• **Nonsymmorphic (screw and/or AFM) sym.**

→ **unusual node structures on ZF**



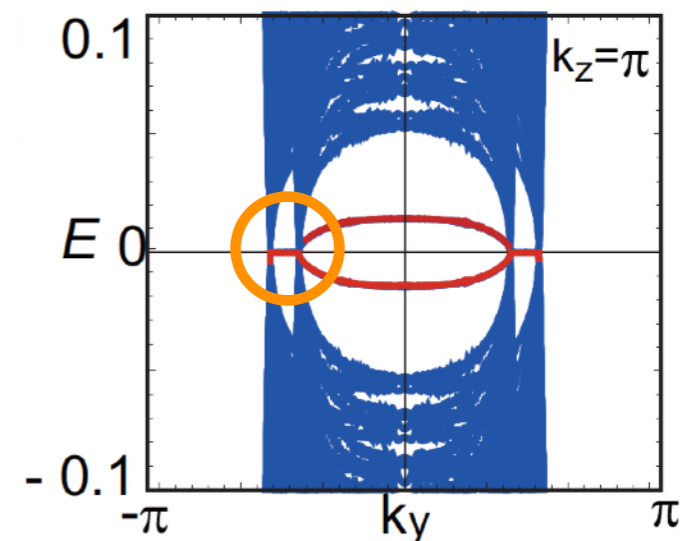
Symmetry of SC order parameter

		A_g		B_g		A_u		B_u	
Cases		BP	ZF	BP	ZF	BP	ZF	BP	ZF
Rotation / PM	(a)	0	→ 0	(ii) \mathbb{Z}_2	→ \mathbb{Z}_2	0	→ 0	0	→ 0
Screw / PM	(b)	0	→ 0	(ii) \mathbb{Z}_2	→ $2\mathbb{Z}$	(i) 0	→ $2\mathbb{Z}$	0	→ 0
Rotation / AFM	(c)	(iii) 0	→ $2\mathbb{Z}$	(iii) \mathbb{Z}_2	→ 0	0	→ 0	(i) 0	→ $2\mathbb{Z}$
Screw / AFM	(d)	(iii) 0	→ \mathbb{Z}_2	(iii) \mathbb{Z}_2	→ 0	0	→ 0	0	→ 0

UPt₃ (Norman's zero)
UCoGe, CrAs?

Sr₂IrO₄ **UPd₂Al₃**

▶ Edge mode: **Majorana flat band**



Classification of 59 space groups

SS & Y. Yanase, Phys. Rev. B **97**, 134512 (2018).

UPd₂Al₃

UCoGe

Sr₂IrO₄

UCoGe

CrAs

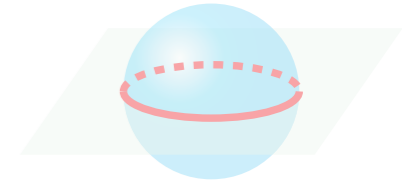
UPt₃

(a)			(b)				(c)				
No.	Short	$\perp = y$	No.	Short	$\perp = x$	$\perp = y$	$\perp = z$	No.	Short	$\perp = z$	$\perp = x, y$
10	<i>P2/m</i>	(RM)	47	<i>Pmmm</i>	(RM)	(RM)	(RM)	83	<i>P4/m</i>	(RM)	N/A
11	<i>P2₁/m</i>	(SM)	48	<i>Pnnn</i>	(RG)	(RG)	(RG)	84	<i>P4₂/m</i>	(RM)	N/A
13	<i>P2/c</i>	(RG)	49	<i>Pccm</i>	(RG)	(RG)	(RM)	85	<i>P4/n</i>	(RG)	N/A
14	<i>P2₁/c</i>	(SG)	50	<i>Pban</i>	(RG)	(RG)	(RG)	86	<i>P4₂/n</i>	(RG)	N/A
			51	<i>Pmma</i>	(SM)	(RM)	(RG)	123	<i>P4/mmm</i>	(RM)	(RM)
			52	<i>Pnna</i>	(RG)	(SG)	(RG)	124	<i>P4/mcc</i>	(RM)	(RG)
			53	<i>Pmna</i>	(RM)	(RG)	(SG)	125	<i>P4/nbm</i>	(RG)	(RG)
			54	<i>Pcca</i>	(SG)	(RG)	(RG)	126	<i>P4/nnc</i>	(RG)	(RG)
			55	<i>Pbam</i>	(SG)	(SG)	(RM)	127	<i>P4/mbm</i>	(RM)	(SG)
			56	<i>Pccn</i>	(SG)	(SG)	(RG)	128	<i>P4/mnc</i>	(RM)	(SG)
			57	<i>Pbcm</i>	(RG)	(SG)	(SM)	129	<i>P4/nmm</i>	(RG)	(SM)
			58	<i>Pnnm</i>	(SG)	(SG)	(RM)	130	<i>P4/ncc</i>	(RG)	(SG)
			59	<i>Pmmn</i>	(SM)	(SM)	(RG)	131	<i>P4₂/mmc</i>	(RM)	(RM)
			60	<i>Pbcn</i>	(SG)	(RG)	(SG)	132	<i>P4₂/mcm</i>	(RM)	(RG)
			61	<i>Pbca</i>	(SG)	(SG)	(SG)	133	<i>P4₂/nbc</i>	(RG)	(RG)
			62	<i>Pnma</i>	(SG)	(SM)	(SG)	134	<i>P4₂/nnm</i>	(RG)	(RG)
			63	<i>Cmcm</i>	N/A	N/A	(SM)	135	<i>P4₂/mbc</i>	(RM)	(SG)
			64	<i>Cmca</i>	N/A	N/A	(SG)	136	<i>P4₂/mnm</i>	(RM)	(SG)
			65	<i>Cmmm</i>	N/A	N/A	(RM)	137	<i>P4₂/nmc</i>	(RG)	(SM)
			66	<i>Cccm</i>	N/A	N/A	(RM)	138	<i>P4₂/ncm</i>	(RG)	(SG)
			67	<i>Cmma</i>	N/A	N/A	(RG)				
			68	<i>Ccca</i>	N/A	N/A	(RG)				
(d)				(e)							
No.	Short	$\perp = z$	$\perp = [1-10], [120], [210]$	No.	Short	$\perp = x, y, z$					
175	<i>P6/m</i>	(RM)	N/A	200	<i>Pm$\bar{3}$</i>	(RM)					
176	<i>P6₃/m</i>	(SM)	N/A	201	<i>Pn$\bar{3}$</i>	(RG)					
191	<i>P6/mmm</i>	(RM)	(RM)	205	<i>Pa$\bar{3}$</i>	(SG)					
192	<i>P6/mcc</i>	(RM)	(RG)	221	<i>Pm$\bar{3}m$</i>	(RM)					
193	<i>P6₃/mcm</i>	(SM)	(RM)	222	<i>Pn$\bar{3}n$</i>	(RG)					
194	<i>P6₃/mmc</i>	(SM)	(RG)	223	<i>Pm$\bar{3}n$</i>	(RM)					
				224	<i>Pn$\bar{3}m$</i>	(RG)					

Main results

(a) SS-Yanase, PRB (2018). / Kobayashi-SS-Yanase-Sato, PRB (2018).
(b) SS-Nomoto-Shiozaki-Yanase, PRB (2019).

(a) For mirror- or glide-invariant SCs:

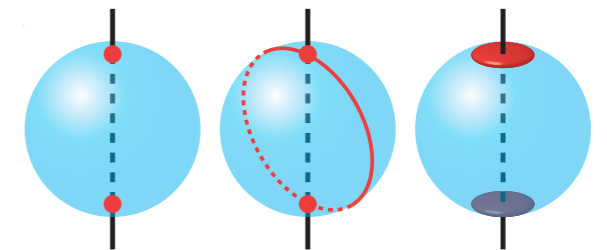


- **Complete classification** on **high-symmetry planes** for all symmorphic & nonsymmorphic symmetries
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UPt₃: Norman (1995), Micklitz-Norman (2009), Kobayashi-Yanase-Sato (2016), Nomoto-Ikeda (2016)

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(b) For rotation-invariant SCs:



- Classification on **high-symmetry axes**

only for symmorphic symmetries

- **Novel type of gap structure** on C₃- and C₆-axes

Background of SrPtAs

- ▶ SrPtAs: a pnictide SC w/ a hexagonal lattice (D_{6h})
- ▶ Pairing symmetry is still under debate
- ▶ **Even-parity chiral d -wave (E_{2g})** order parameter

M. H. Fischer *et al.* (2014, 2015)

$$C_3 \hat{\Delta}_{\pm}(\mathbf{k}) C_3^T = e^{\pm i 2\pi/3} \hat{\Delta}_{\pm}(\mathbf{k})$$

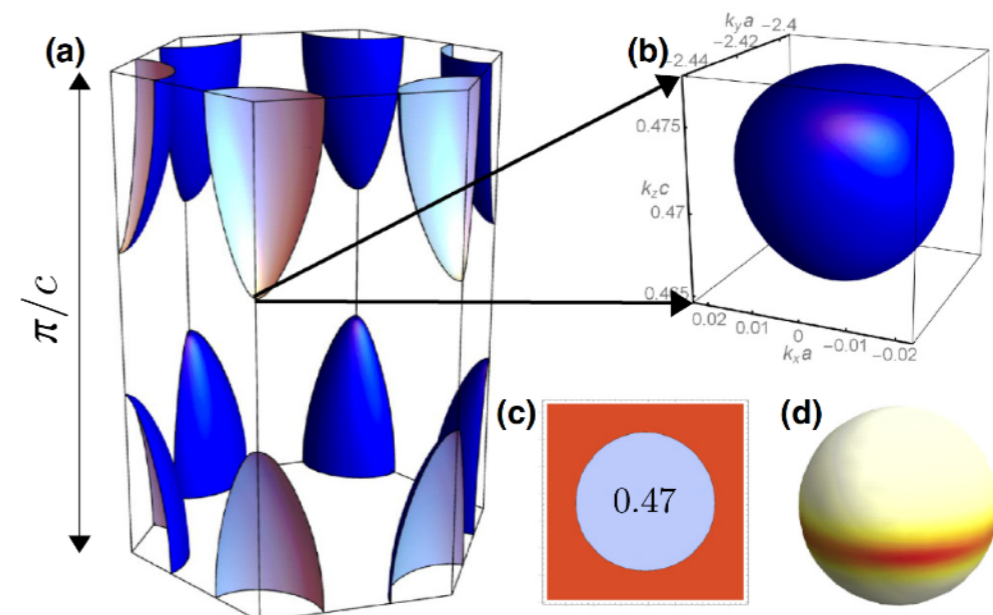
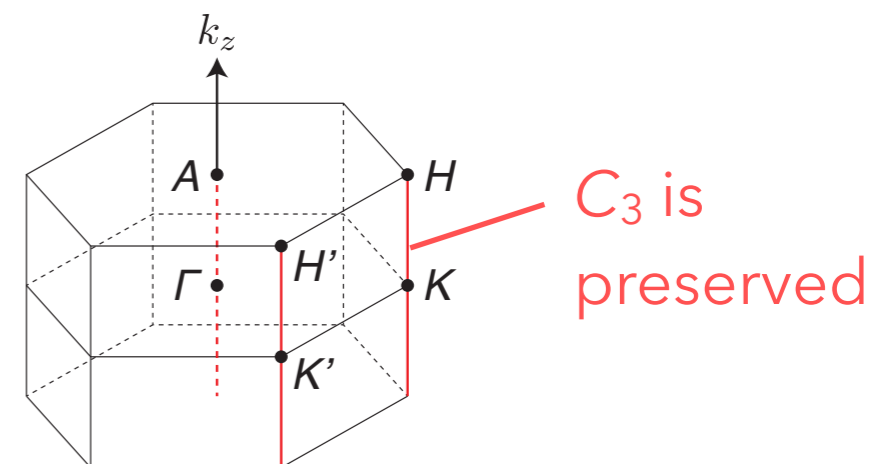
- **TRS broken & C_3 preserved**

by choosing one of " \pm "

- **Surface nodes** on K - H line

D. F. Agterberg *et al.*, PRL (2017)

T. Bzdušek & M. Sigrist, PRB (2017)

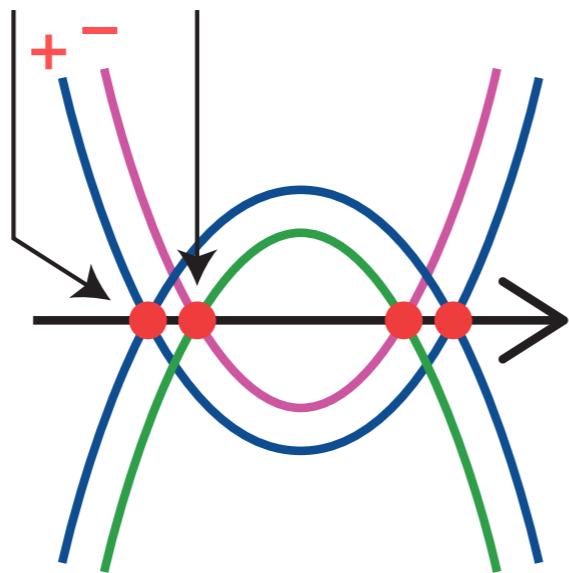


Results for even-parity SrPtAs

- ▶ j_z -dependent topological protection on K - H line

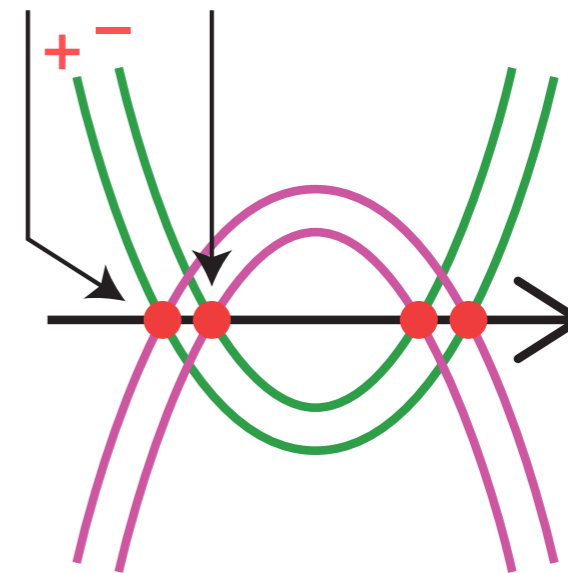
$$j_z = \pm 1/2$$

$$\mathbb{Z}_2 \quad \mathbb{Z}$$



$$j_z = \pm 3/2$$

$$\mathbb{Z} \quad \mathbb{Z}$$

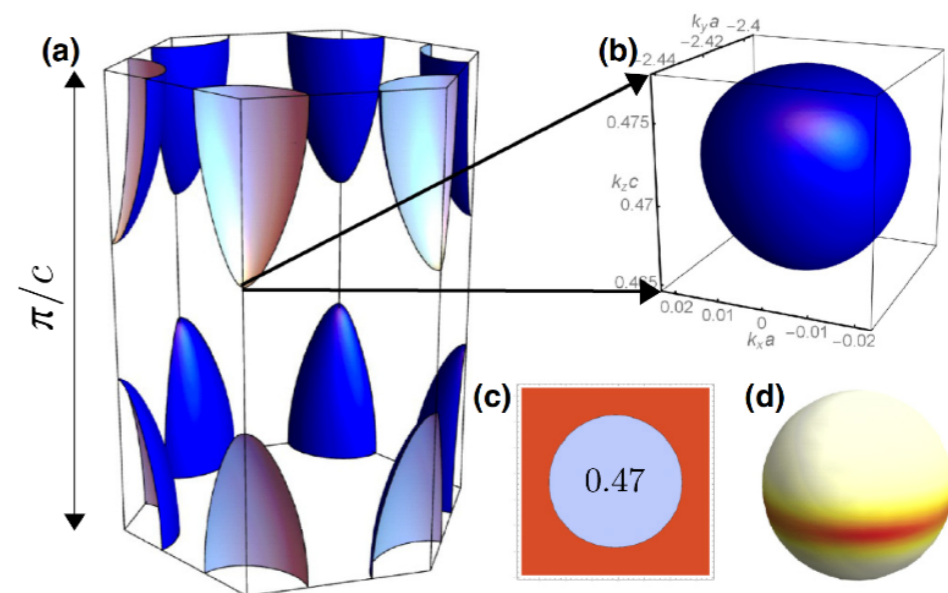


- ▶ Nodes on K - H line
= Parts of **Bogoliubov FSs**

irrespective of j_z

D. F. Agterberg *et al.*, PRL (2017)

T. Bzdušek & M. Sigrist, PRB (2017)



Background of UPt₃

▶ UPt₃: a heavy-fermion SC w/ a hexagonal lattice (D_{6h})

▶ Multiple SC phases: **odd-parity E_{2u} order parameter**

R. A. Fisher *et al.* (1989) / S. Adenwalla *et al.* (1990)

R. Joynt & L. Taillefer, RMP (2002)

$$\hat{\Delta}(\mathbf{k}) = \eta_1 \hat{\Gamma}_1^{E_{2u}} + \eta_2 \hat{\Gamma}_2^{E_{2u}}$$

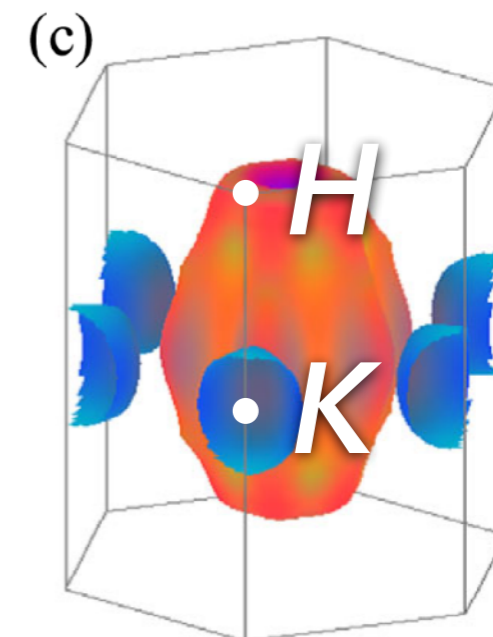
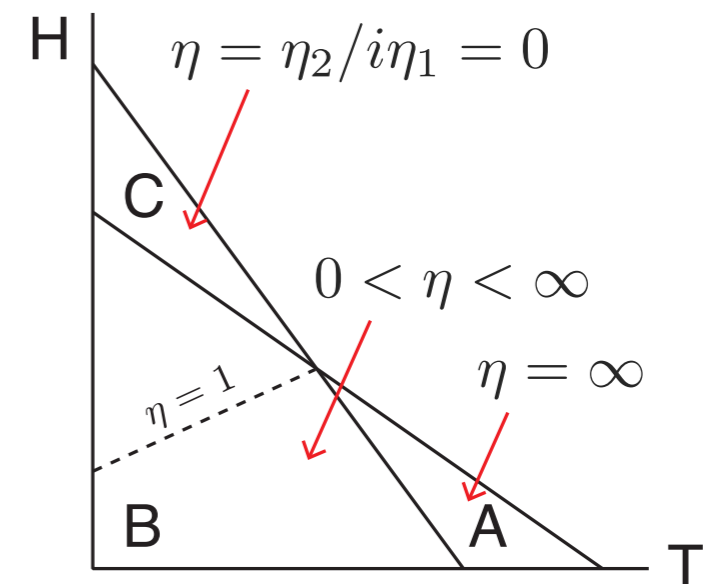
- **TRS broken** in B phase
- **C_3 preserved** on $\eta = 1$

▶ First-principles study of UPt₃

T. Nomoto & H. Ikeda, PRL (2016)

→ Γ -FSs, A-FSs, & K-FSs

- K-FSs: NOT sufficiently studied

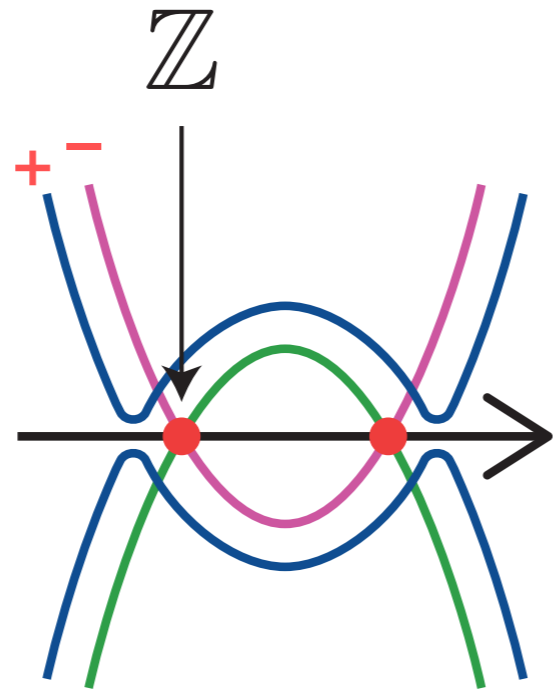


Results for odd-parity UPt_3

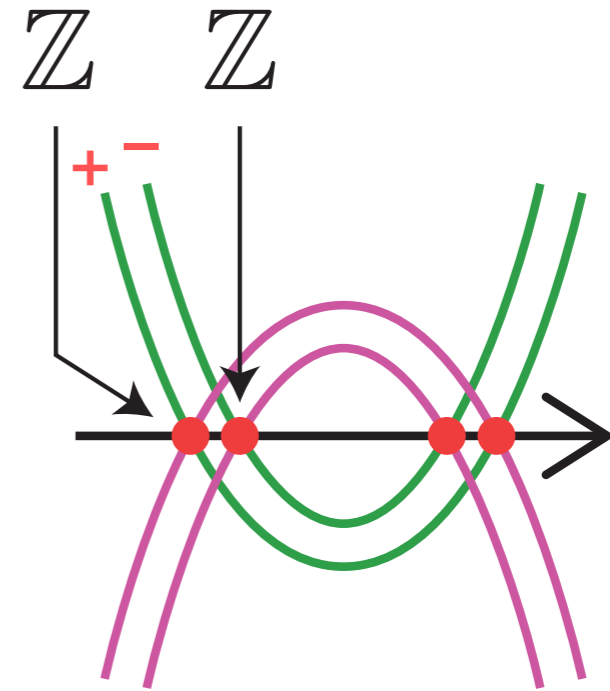
- ▶ j_z -dependent node structures on $K-H$ line

$$j_z = \pm 1/2$$

Perfectly
non-unitary
pairing



$$j_z = \pm 3/2$$

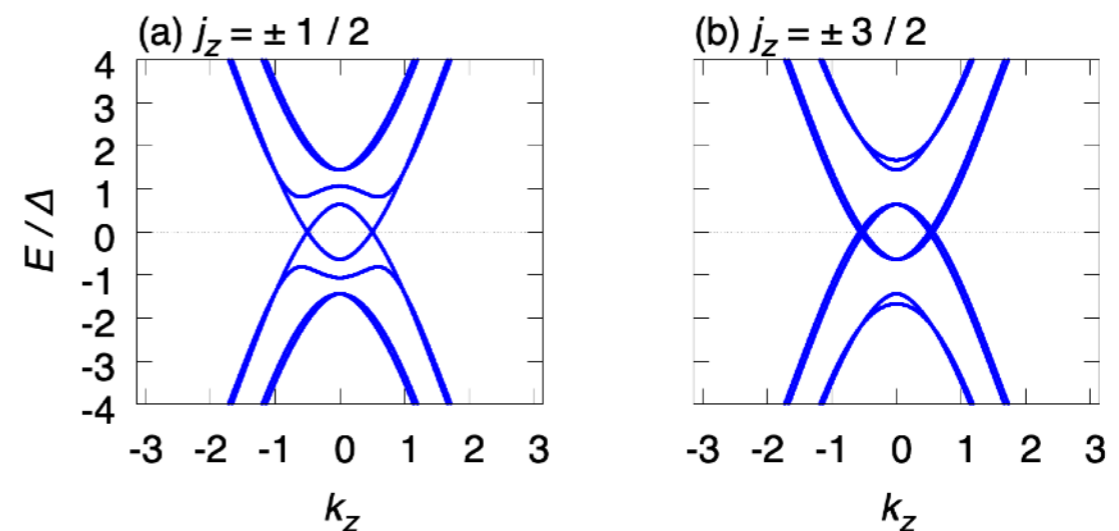


- ▶ Nodes on $K-H$ line = **Point nodes depending on j_z**

- ▶ **Novel type of nodes !**

- ▶ Numerical calc.

Model: Y. Yanase (2016)



Conclusion

SS & Y. Yanase, Phys. Rev. B **97**, 134512 (2018).

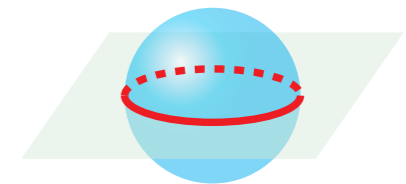
S. Kobayashi, SS, Y. Yanase, & M. Sato, Phys. Rev. B **97**, 180504(R) (2018).

SS, T. Nomoto, K. Shiozaki, & Y. Yanase, Phys. Rev. B **99**, 134513 (2019).

- ▶ *Do SC nodes meet topology ?* → Yes !
- ▶ *Do crystal symmetries give an invariant of nodes ?* → Yes !
- ▶ Classification of **topological crystalline SC nodes**

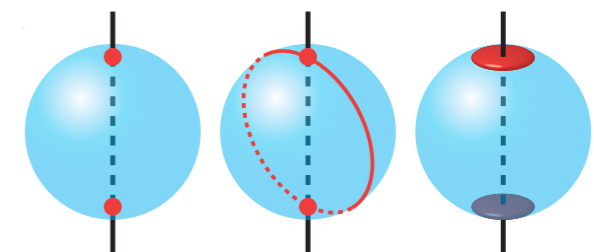
(a) For mirror or glide symmetry

- **Complete classification**
- Condition for **Majorana flat band**



(b) For rotation symmetry

- **Novel j_z -dep. node protections or structures on C_3 & C_6**
- Applications: SrPtAs & UPt₃



- ▶ **All nodes are topological ?**