Floating phase versus chiral transition in 2D incommensurate systems and 1D Rydberg cold atoms

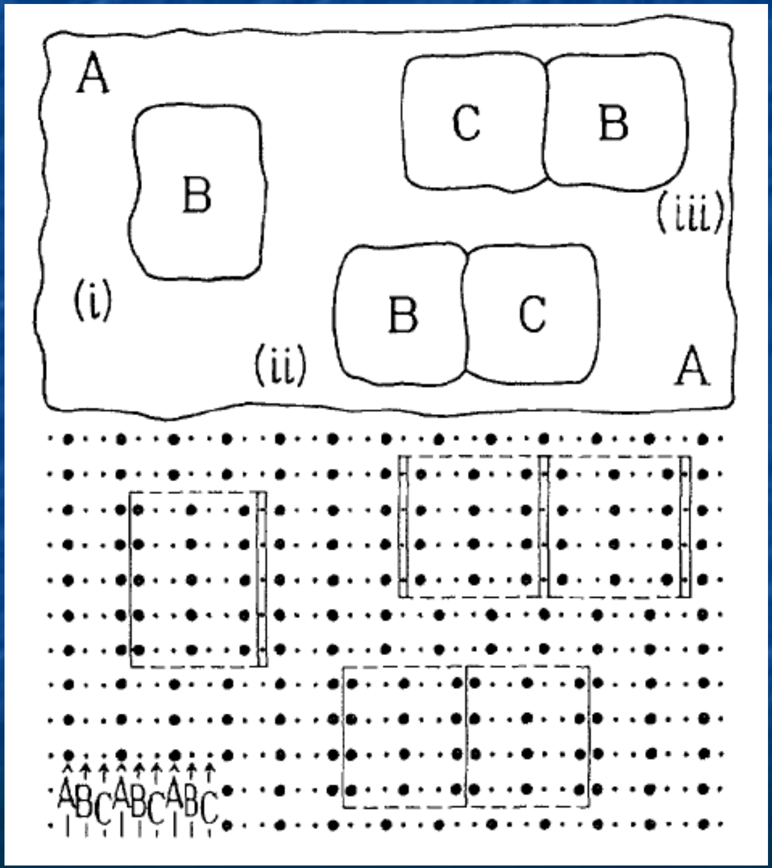
F. Mila
Ecole Polytechnique Fédérale de Lausanne
Switzerland

Natalia Chepiga
Amsterdam

Scope

- A classical unsolved problem of the eighties
  → C-IC transition in 2D period-3 systems
- Rydberg atoms (2017): quantum 1D version
- Quantum Dimer Model on a ladder (2019)
- DMRG phase diagram of quantum problem
  → Potts transition
  → Intermediate critical floating phase
  → Chiral transition (Huse-Fisher 1982)
- Conclusions

C-IC transition in adsorbed layers



3 types of domains
→ 3-state Potts?

Not so simple!

A B C ≠ A C B
Chiral perturbation
Huse-Fisher, 1982

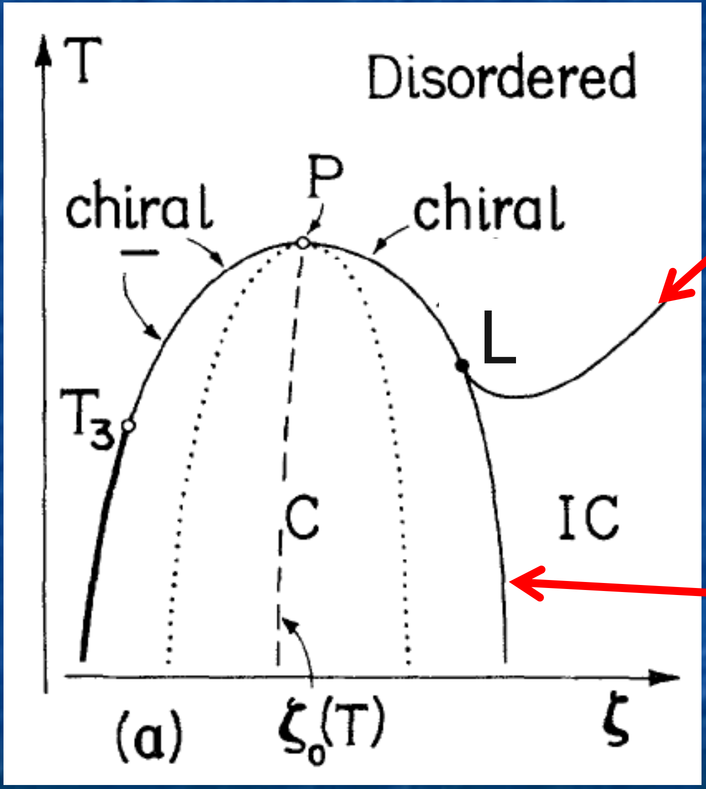
Asymmetric 3-state Potts in 2D

$$\mathcal{H} = -J_x \sum_{\langle ij \rangle}^{x} \cos\left[\frac{2\pi}{3}(n_i - n_j + \Delta)\right] - J_y \sum_{\langle ij \rangle}^{y} \cos\left[\frac{2\pi}{3}(n_i - n_j)\right]$$

n_i=0,1 or 2

- Huse-Fisher: possibility of a chiral transition between a Potts point and a Lifshitz point
- Intermediate (floating) critical phase beyond Lifshitz point

Huse-Fisher phase diagram



Kosterlitz-Thouless

Pokrovsky-Talapov

Huse and Fisher, 1982

Difference between the 3 cases

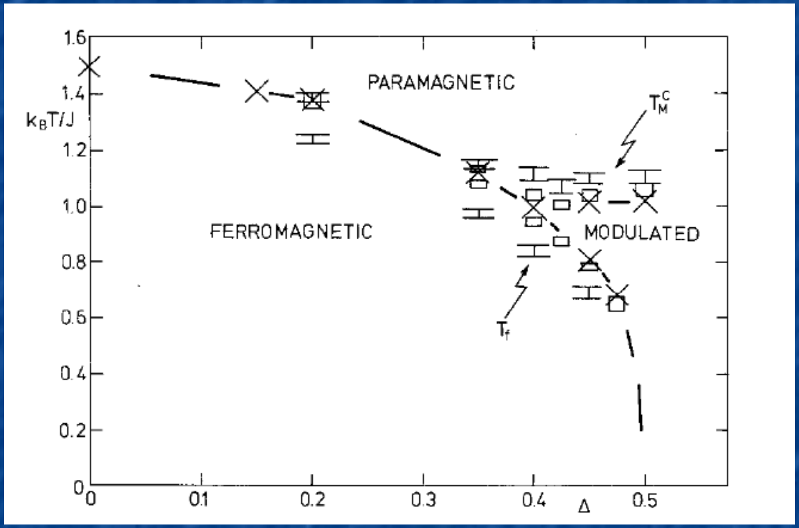
- Δq: distance to $2\pi/3$; ξ : correlation length
- $\Delta q\ \xi \rightarrow 0$ for Potts

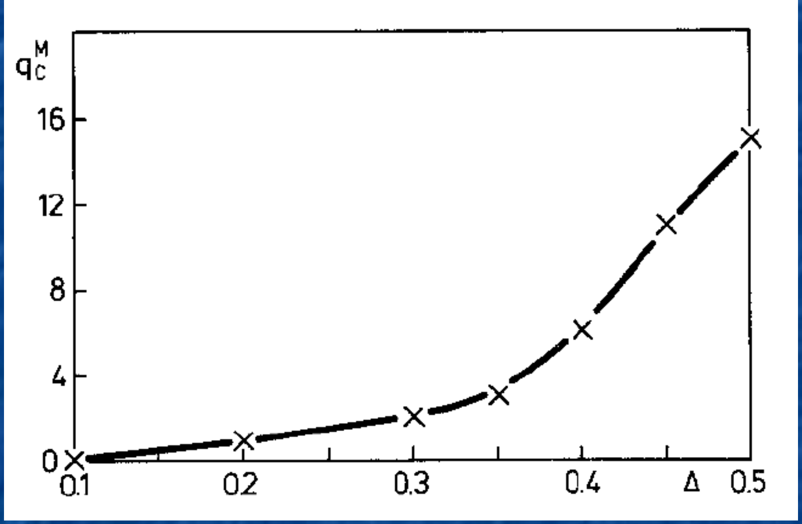
  $\rightarrow$ cst > 0 for chiral

  $\rightarrow + \infty$ for KT transition
- Monte Carlo simulations in the eighties

  $\rightarrow$ systems too small to extract Δq with sufficient precision Selke and Yeomans, 1982

MC simulations of asymmetric Potts model



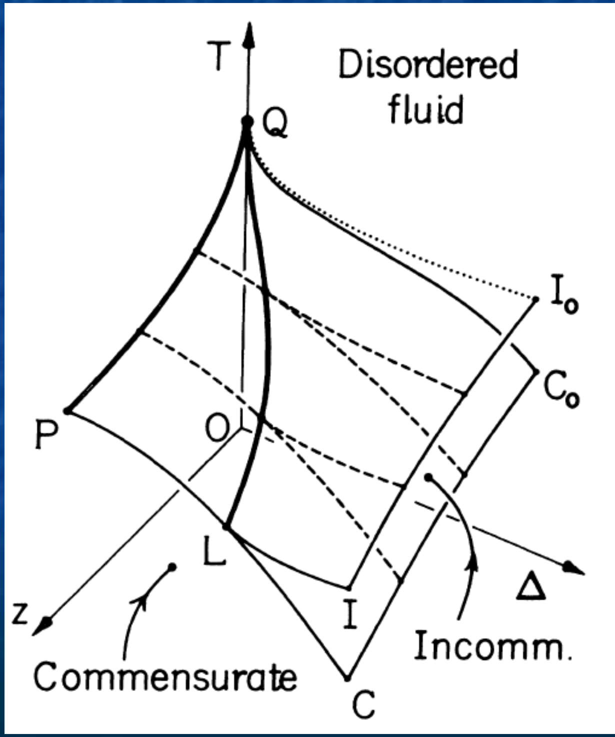


Selke and Yeomans, 1982

Field theory argument

Intermediate phase away from Potts

Haldane, Bak, Bohr, 1983; Schulz, 1983

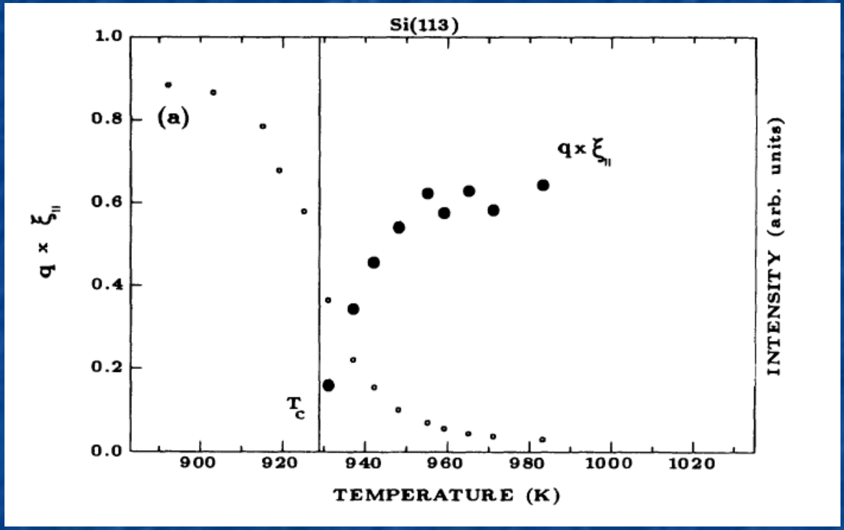


Not necessarily true
if dislocations are allowed

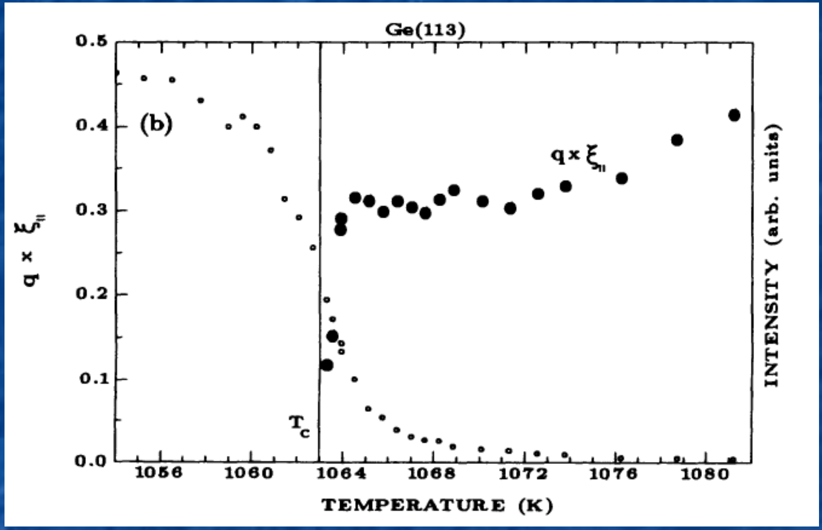
Huse-Fisher, 1984

Si (113) 3 x 1 and Ge(113) 3 x 1

Potts

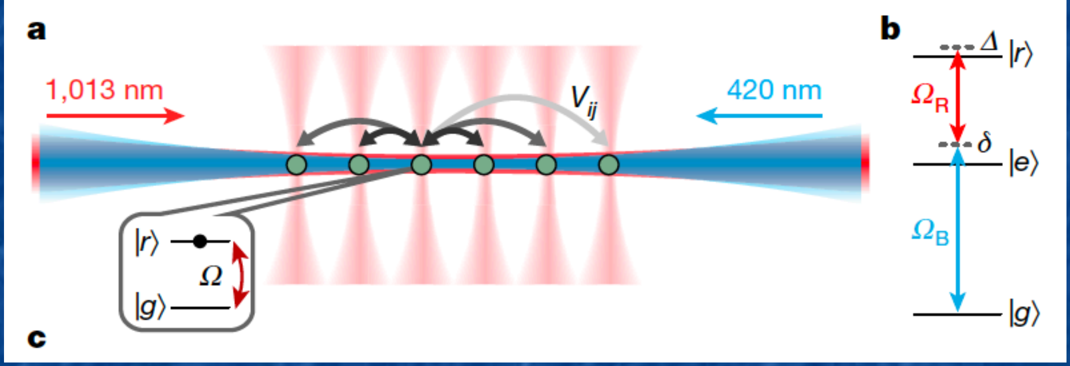


Chiral



J. Schreiner, K. Jacobi, W. Selke, PRB 1994

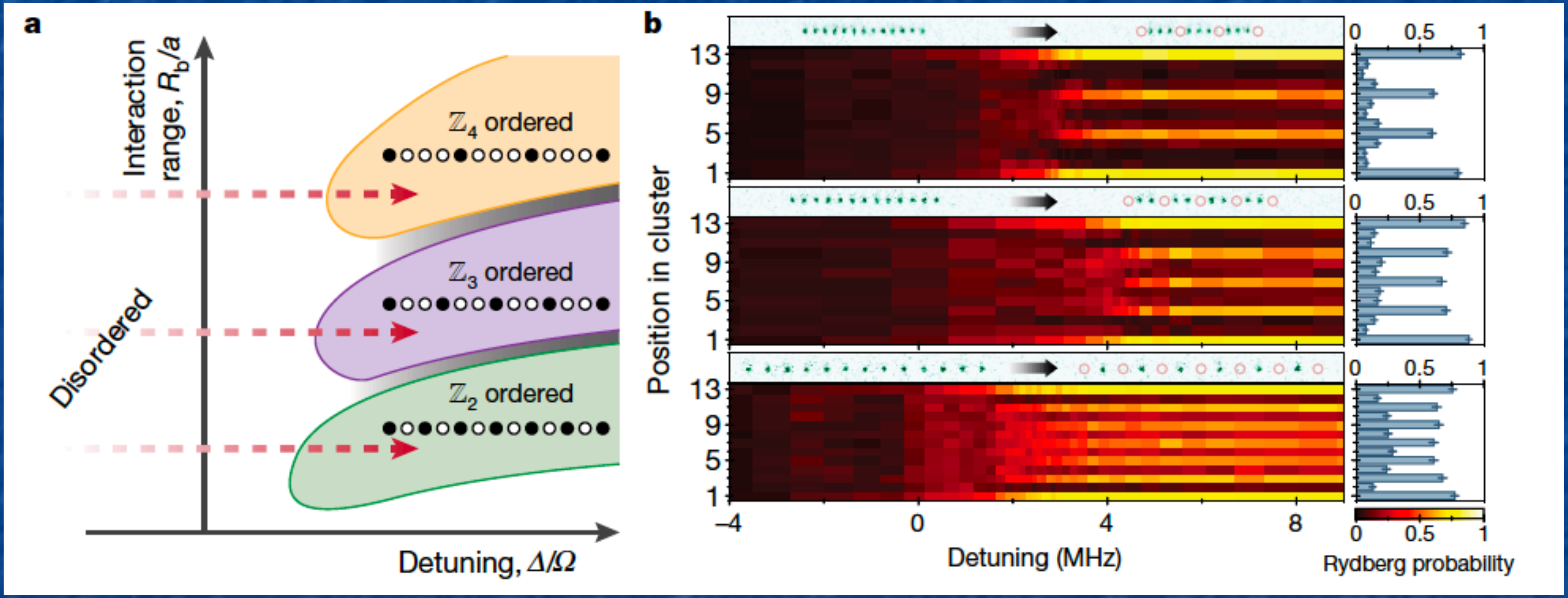
Rydberg atoms



$$\frac{\mathcal{H}}{\hbar} = \sum_i \frac{\Omega_i}{2}\sigma_x^i - \sum_i \Delta_i n_i + \sum_{i<j} V_{ij} n_i n_j$$

H. Bernien ... M. Lukin, Nature 2017

Commensurate phases



H. Bernien … M. Lukin, Nature 2017

Hard boson model

- Hard boson model of Z_3 ordered phase

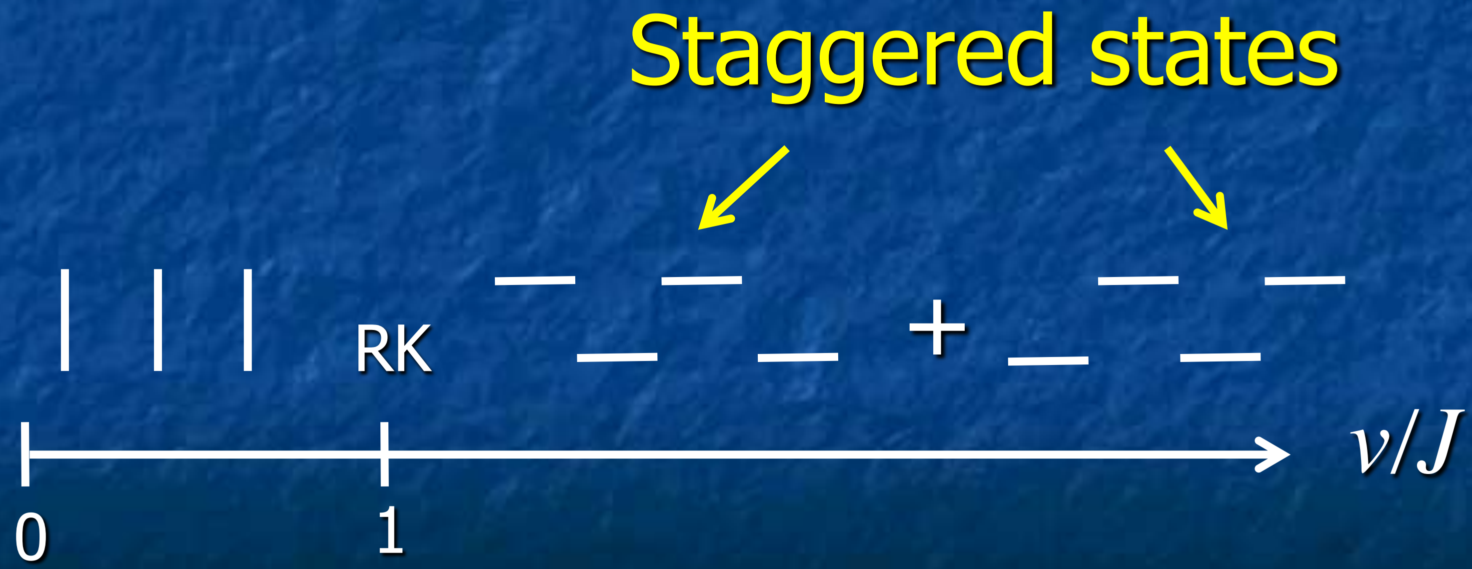
$$H_{\mathrm{HB}} = \sum_j \left[-w(d_j^\dagger + d_j) + U n_j + V n_{j-1} n_{j+1} \right]$$

- Two constraints $n_j(n_j - 1) = 0$ $n_j n_{j+1} = 0$
- Hilbert space: grows as Fibonacci number

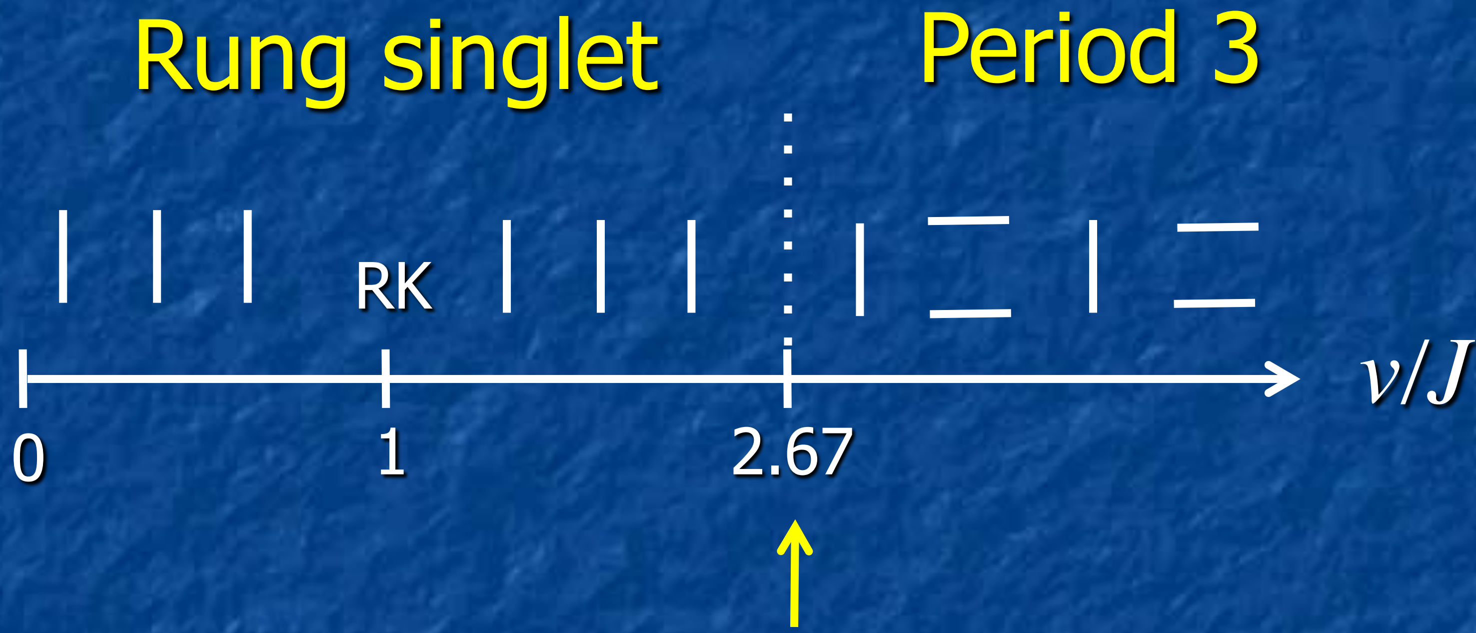
Fendley, Sengupta, Sachdev 2004

Quantum Dimer Model on ladder

$$H_{QDM} = \sum_{\text{Plaquettes}} -J\left(|\,\|\;\|\,\rangle\langle\,=\,| + \text{h.c.}\right) + v\left(|\,=\,\rangle\langle\,=\,| + |\,\|\;\|\,\rangle\langle\,\|\;\|\,|\right)$$



Exclude staggered states



3-state Potts transition? Not clear

→ Gap closing, but complicated spectrum

Chepiga and FM, SciPost 2019

From quantum dimer ladder to hard bosons

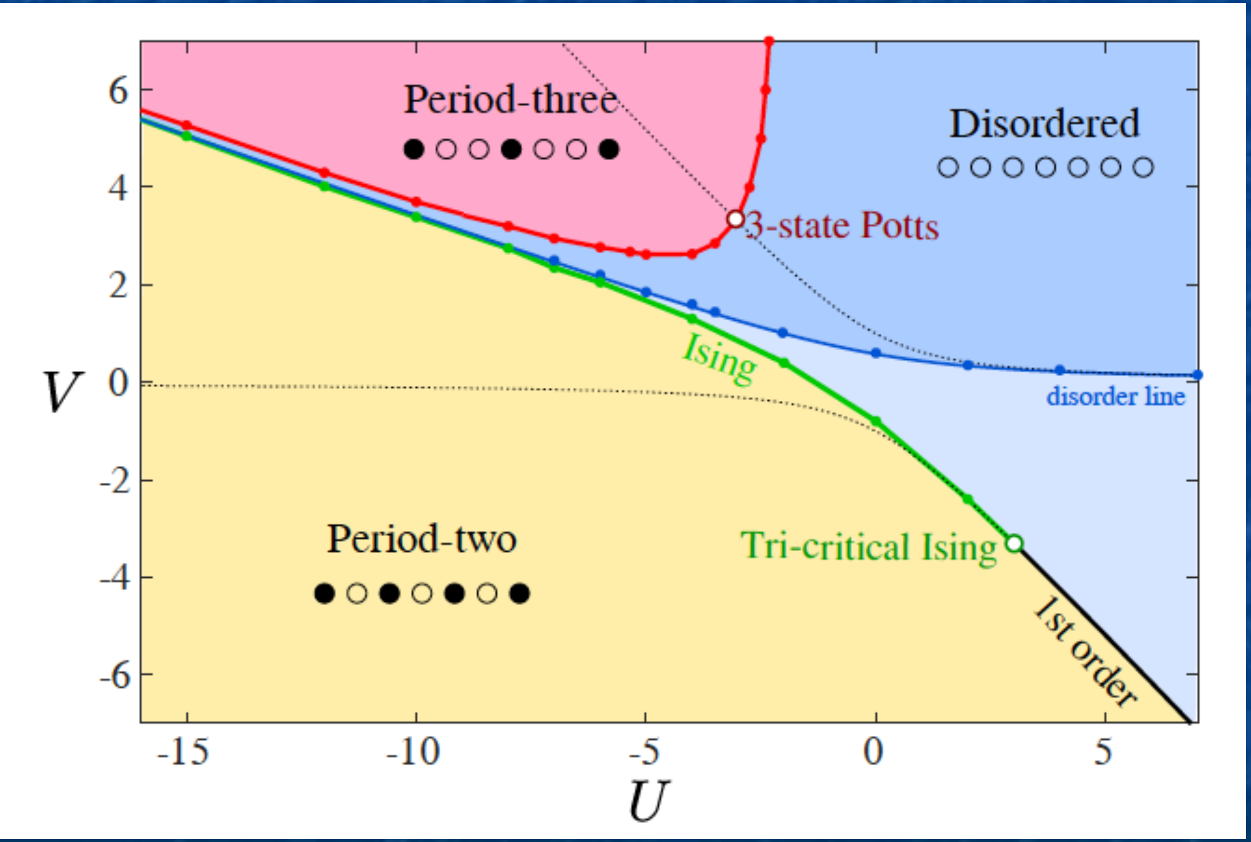
$$H_{\mathrm{QDM}} = \sum_{\mathrm{Plaquettes}} \left[-J \left(|\mathrm{I}\;\mathrm{I}\rangle\langle = | + \mathrm{h.c.} \right) + v_{\mathrm{rung}} |\mathrm{I}\;\mathrm{I}\rangle\langle \mathrm{I}\;\mathrm{I}| + v_{\mathrm{leg}} |=\rangle\langle =| \right]$$

$$\Downarrow$$

$$H_{\mathrm{QDM}}^{\mathrm{HB}} = \sum_j \left[-J(d_j^\dagger + d_j) + (v_{\mathrm{leg}} - 3v_{\mathrm{rung}})\, n_j + v_{\mathrm{rung}}\, n_j n_{j+2} \right]$$

Chepiga and FM, SciPost 2019

Hard-bosons: phase diagram



Fendley et al, PRB 2004; Chepiga and FM, PRL 2019

Phase transitions

- Transition out of period-2 phase:
  → Ising
  → Tricritical Ising point
  → First order
- Disorder line: entirely inside the disordered phase
- Transition out of period-3 phase: Commensurate-Incommensurate transition

Transition out of period 3

- Fendley-Sengupta-Sachdev (2004)

  → Intermediate phase for $U \to -\infty$

  → Probable intermediate phase up to Potts
- Samajdar, Choi, Pichler, Lukin, Sachdev (2018)

  → Evidence of non-integer dynamical exponent between Potts and $V \to +\infty$

  → Chiral transition between Potts and $V \to +\infty$

DMRG

- Algorithm that takes the constraint into account when building the MPS

  → Huge reduction of Hilbert space

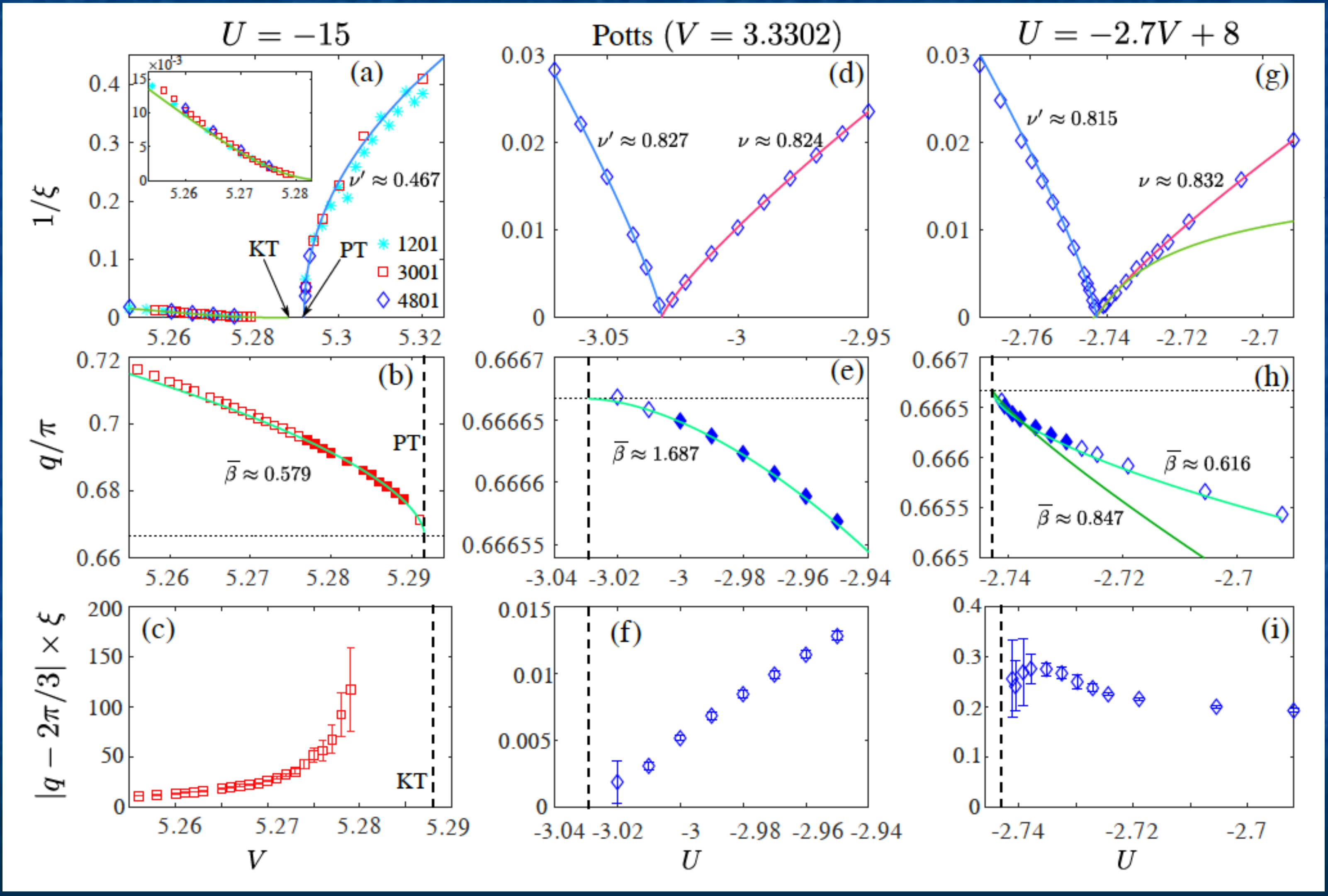
$$\Omega_0(N_r) \approx \varphi^{N_r} \approx 1.6^{N_r} \text{ instead of } \Omega(N_r) \approx 8^{N_r}$$

  → Simulations up to 9'000 sites

- Δq: extremely precise results (third digit)
- ξ: access to values of several hundreds

    → meaningful evaluation of Δq ξ

Three cases



$U = -15$
Potts ($V = 3.3302$)
$U = -2.7V + 8$
(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(i)
$1/\xi$
q/π
$|q - 2\pi/3| \times \xi$
V
U
U
$\nu' \approx 0.467$
KT
PT
1201
3001
4801
$\times 10^{-3}$
$\bar{\beta} \approx 0.579$
PT
KT
$\nu' \approx 0.827$
$\nu \approx 0.824$
$\bar{\beta} \approx 1.687$
$\nu' \approx 0.815$
$\nu \approx 0.832$
$\bar{\beta} \approx 0.616$
$\bar{\beta} \approx 0.847$

Three cases

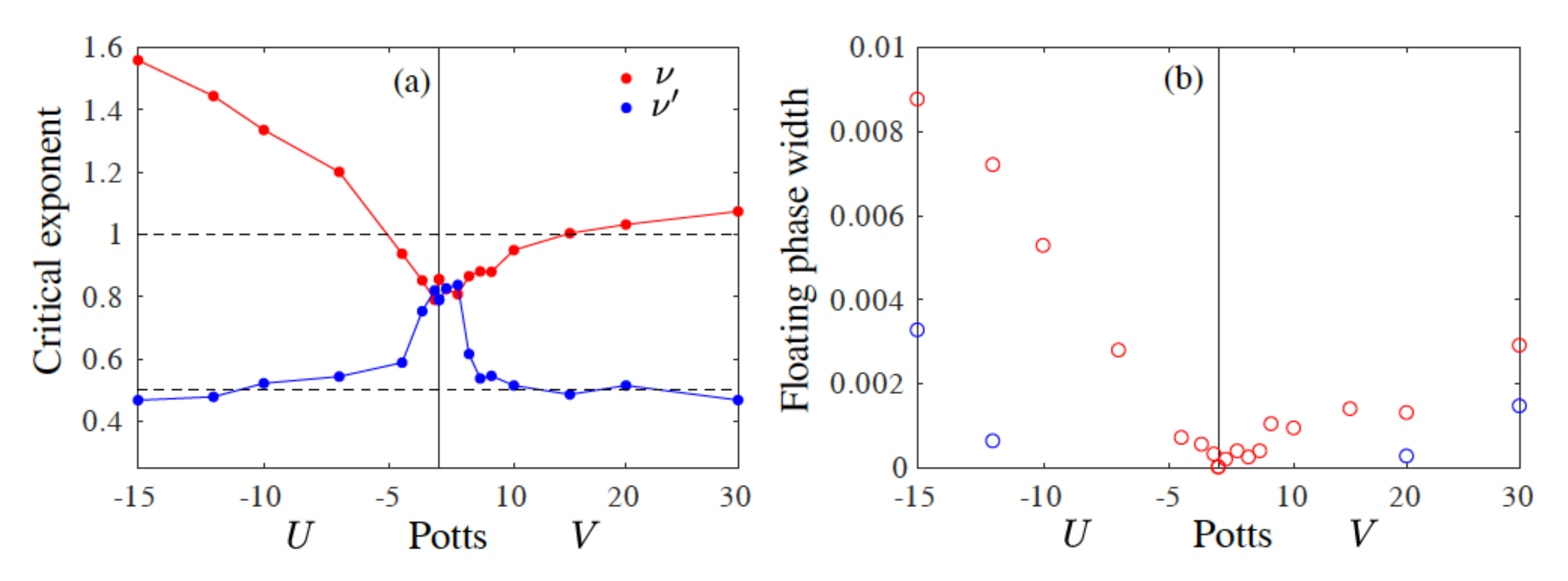
- Potts: Δq goes to zero with exponent 5/3, ξ diverges with exponent 5/6

  → $\Delta q\ \xi \to 0$
- Far from Potts: Two transitions : KT and PT, and intermediate critical phase in both directions
- Vicinity of Potts: consistent with single transition

  → Δq goes to zero with exponent smaller than 1

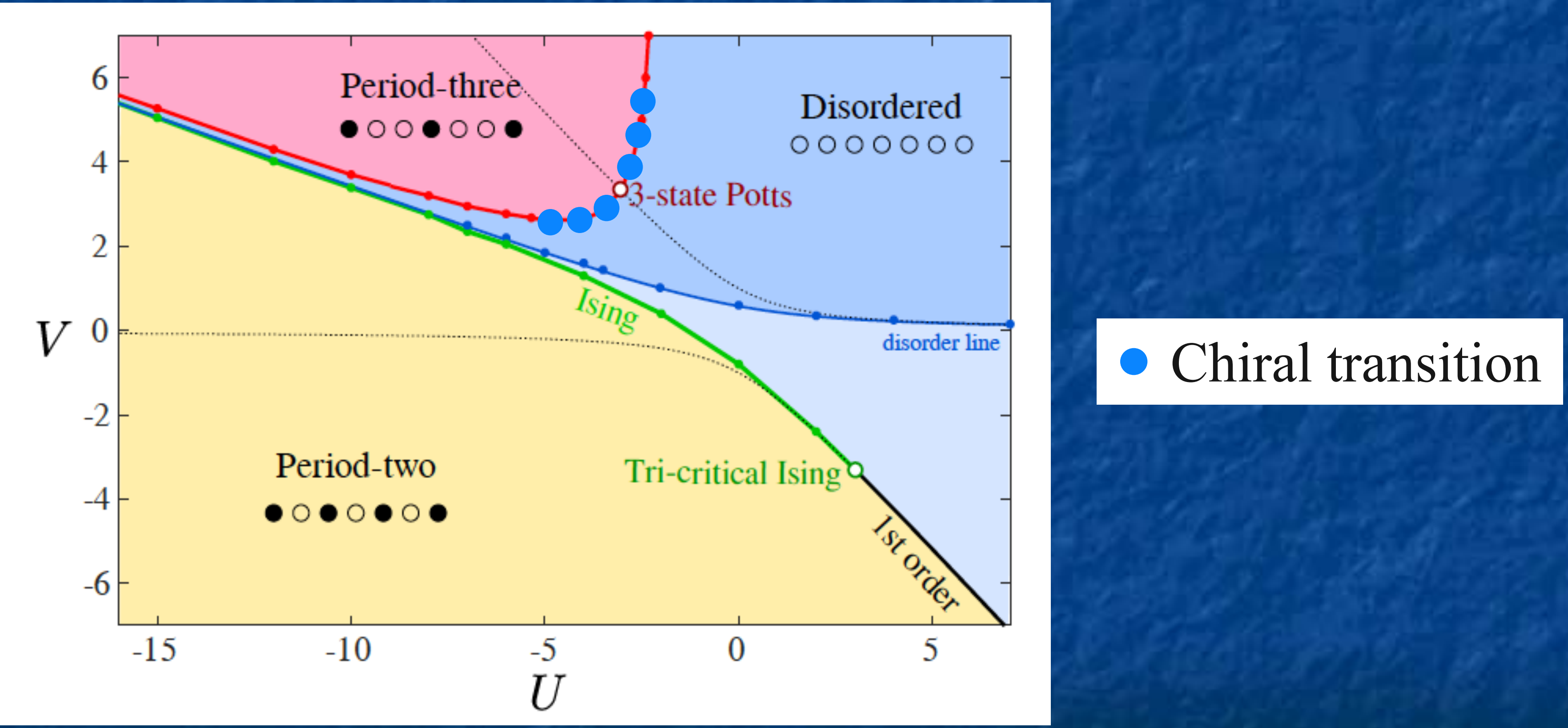
  → $\Delta q\ \xi \to$ constant

Nature of phase transition



- U<-4.5 or V>6: Intermediate phase
- U>-4.5 and V>6: chiral phase

Hard-bosons: phase diagram

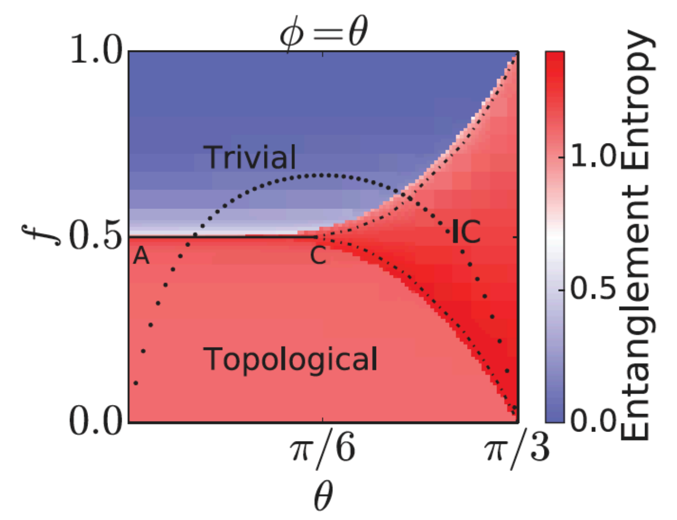


Chepiga and FM, PRL 2019

Chiral clock model I

$$H_3 = -f\sum_{j=1}^{L}\tau_j^{\dagger}e^{-i\phi} - J\sum_{j=1}^{L-1}\sigma_j^{\dagger}\sigma_{j+1}e^{-i\theta} + \text{H.c.}$$

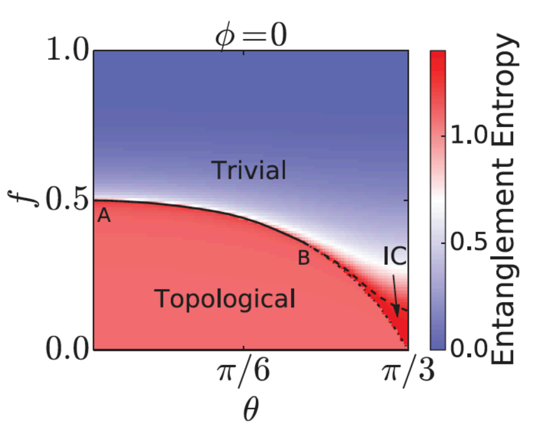
Impossible to decide if IC phase sets in at $\theta > 0$



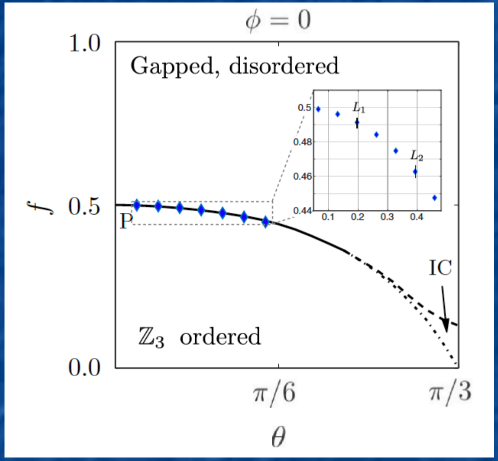
Zhuang, Changlani, Tubnan, Hughes, PRB 2015

Chiral clock model II

Zhuang et al, PRB 2015



(a)



Samajdar et al, PRA 2018

θ	f_c	z	z_ξ	$1/\nu$
$\pi/48$	0.4990	1.003	1.00(7)	1.20(9)
$\pi/24$	0.4961	1.021	1.01(8)	1.21(8)
$\pi/16$	0.4913	1.022	1.02(1)	1.22(3)
$\pi/12$	0.4842	1.078	1.07(6)	1.25(1)
$5\pi/48$	0.4748	1.135	1.13(3)	1.27(7)
$\pi/8$	0.4627	1.229	1.22(7)	1.32(4)
$7\pi/48$	0.4475	1.368	1.36(6)	1.38(2)

Conclusions

- Phase diagram: very rich!

  → Ising

  → Tricritical Ising

  → 3-state Potts

  → Intermediate floating phase with KT and PT transitions

  → Possibly Huse-Fisher chiral transition

Related models

- Four rigorously equivalent models

  → Quantum Dimer Model

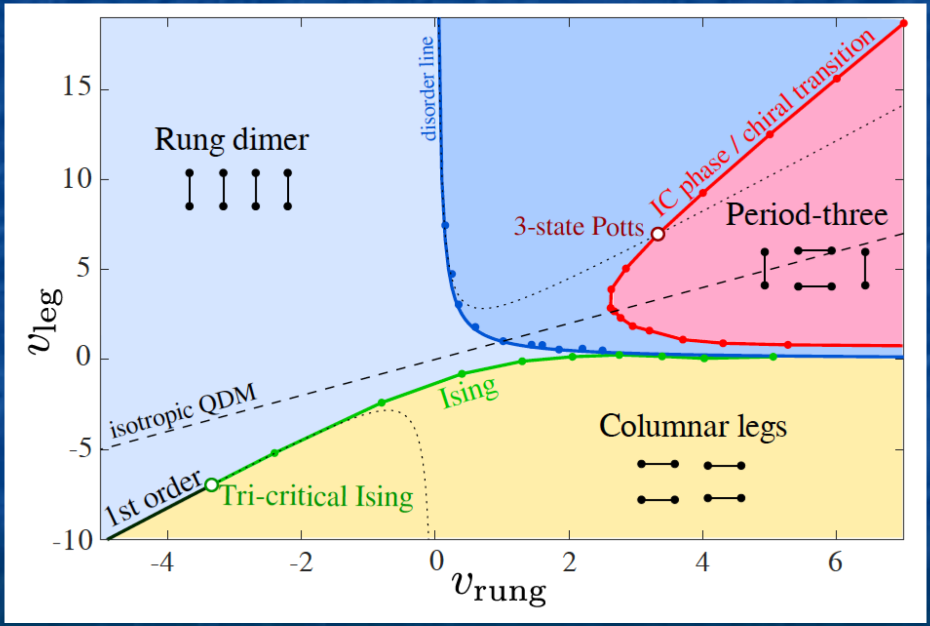
  → Quantum Loop Model

  → Hard boson model

  → Fibonacci anyon chain: tricritical Ising or Potts point depending on the sign of the density terms matrix elements

Quantum Dimer Model

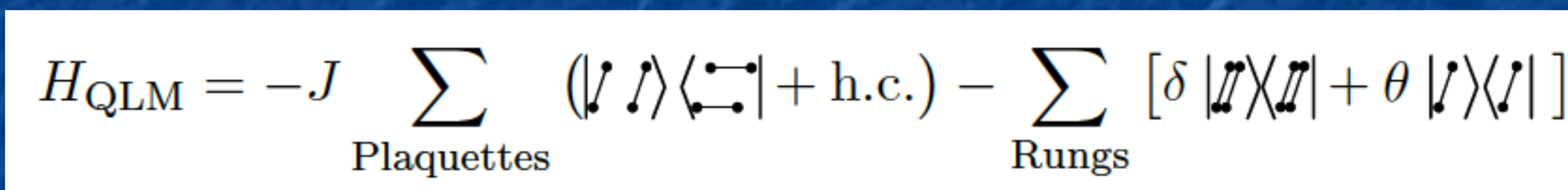
$$H_{\rm QDM}^{\rm HB} = \sum_j \left[-J(d_j^\dagger + d_j) + (v_{\rm leg} - 3v_{\rm rung})\, n_j + v_{\rm rung}\, n_j n_{j+2} \right]$$



Chepiga and FM, SciPost 2019

Quantum loop model

Counts double bonds

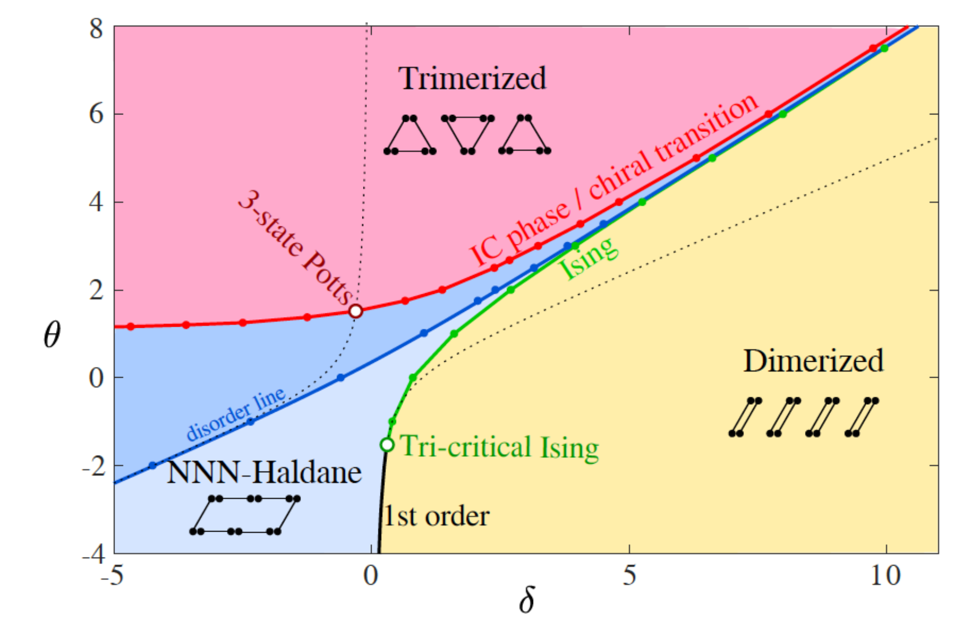


Plaquette flipping

Counts single bonds

Quantum Loop Model

$$H_{\text{QLM}}^{\text{HB}} = \sum_j \left[-J(d_j^\dagger + d_j) - 2\theta n_j + (2\theta - \delta) n_{j-1} n_{j+1} \right]$$



Chepiga and FM, SciPost 2019