Mimicking the edge of 2d topological superconductors and insulators in 1d lattice models

Max Metlitski

“Topological quantum matter: concepts and realizations”.
KITP, October 10, 2019

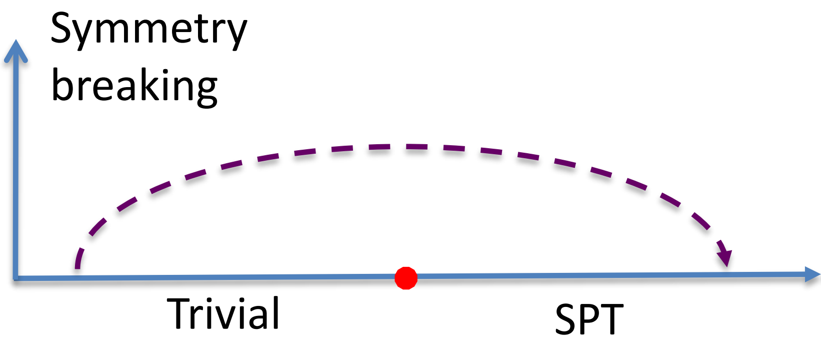
Robert Jones,
MIT

R. A. Jones and MM, arXiv:1902.05957
MM, arXiv:1908.08958

Symmetry protected topological phases (SPTs)

- Generalization of non-interacting TIs
- Protected by symmetry G
- Gapped bulk

  - no anyon exciations



- Boundary must be non-trivial

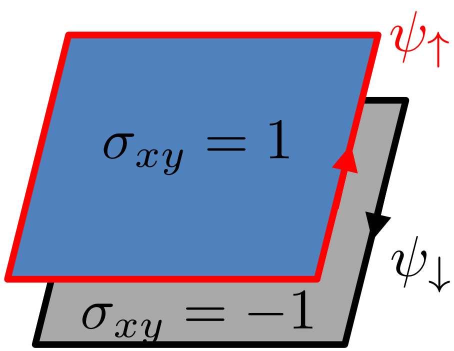
  - cannot be (trivially) gapped without breaking the symmetry

Boundary anomaly

- Boundary cannot exist* without the bulk

  - ``doubling theorems"

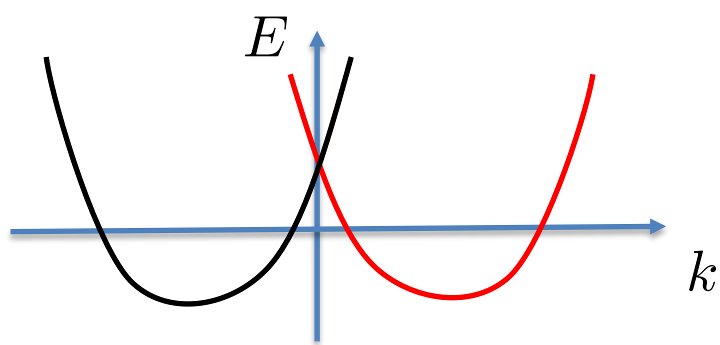
  - e.g. 2d quantum spin-Hall insulator



$$\mathcal{T}: \; \psi_\uparrow \to \psi_\downarrow \qquad i \to -i \qquad \mathcal{T}^2 = (-1)^F$$

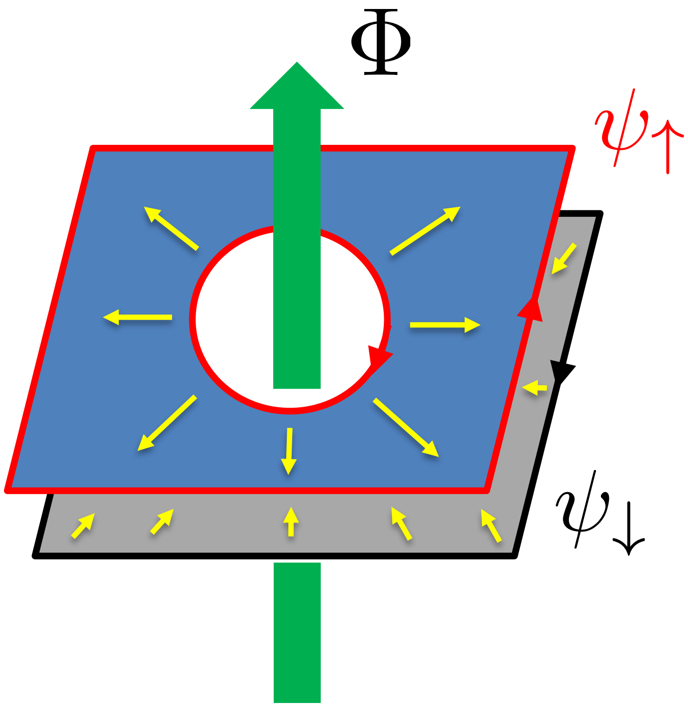
$$\psi_\downarrow \to -\psi_\uparrow$$

- 1d:



Anomaly inflow

- Non-perturbative argument:



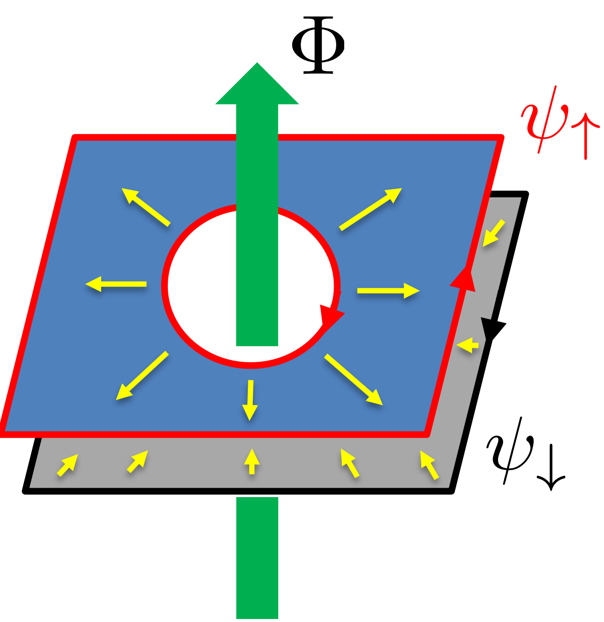
$$\Delta S^z = \frac{\Phi}{2\pi} \qquad \Delta N = 0$$

- spin on the boundary is not conserved when flux is threaded

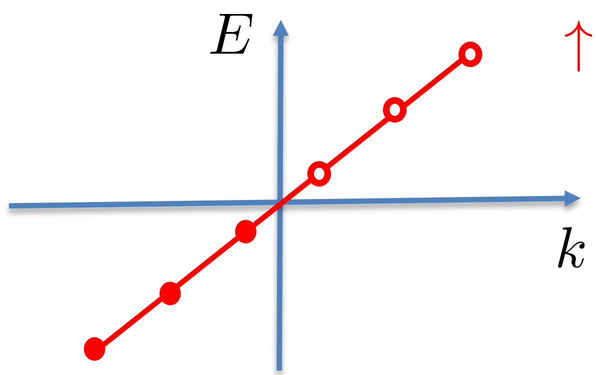
- Anomaly inflow

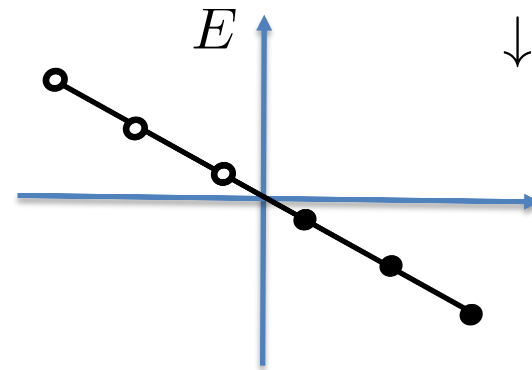
$$L_{bulk} = \frac{1}{2\pi}\epsilon_{\mu\nu\lambda}A^s_\mu \partial_\nu A^c_\lambda$$

Kramers Switching



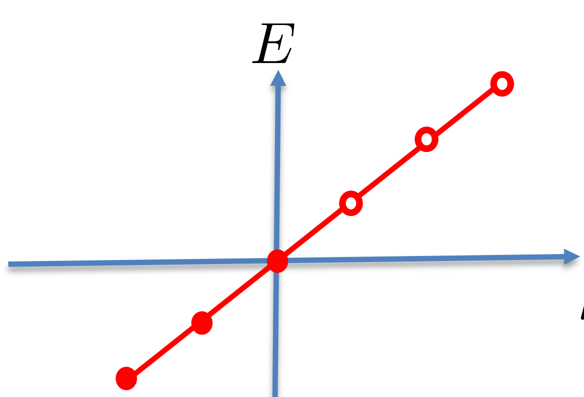
- $\Phi = 0$ $\qquad \mathcal{T}^2 = 1$

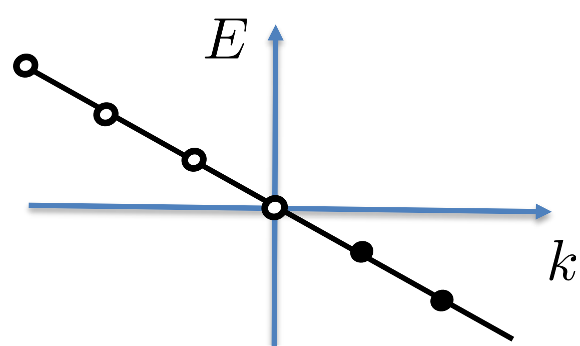




- $\Phi = \pi$ $\qquad \mathcal{T}^2 = -1$ $\qquad \Delta N = 0$

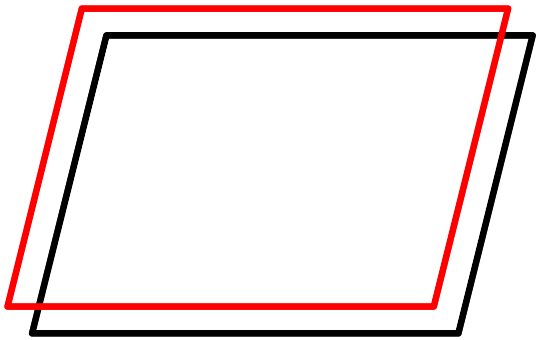
$$\mathcal{T}^2 = (-1)^N$$



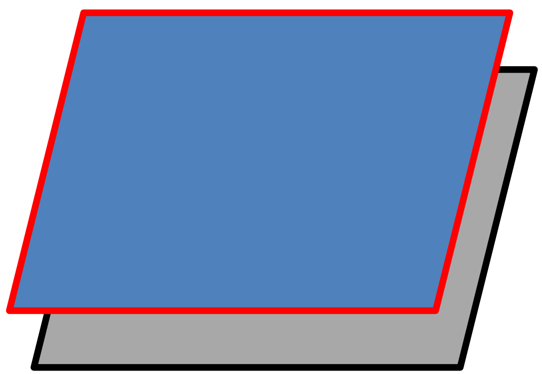


Surface anomaly

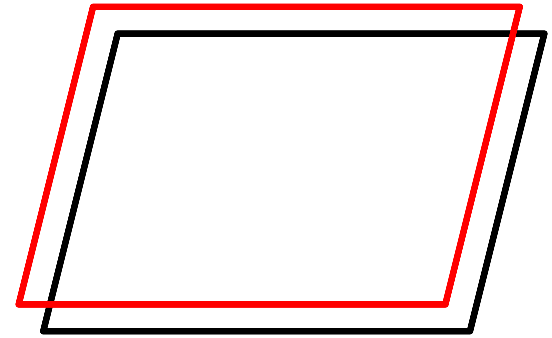
- Cannot consistently couple the boundary to a background G gauge field (``generalized” flux insertion fails)



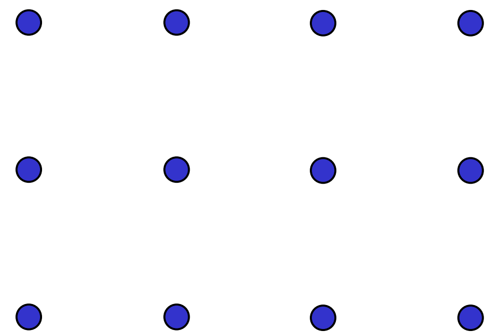
- Can consistently couple the entire system (bulk + boundary)



Q: Can we realize/mimick the boundary in a lattice model without the bulk?



A: No, subject to



i) $V = \otimes_i V_i$

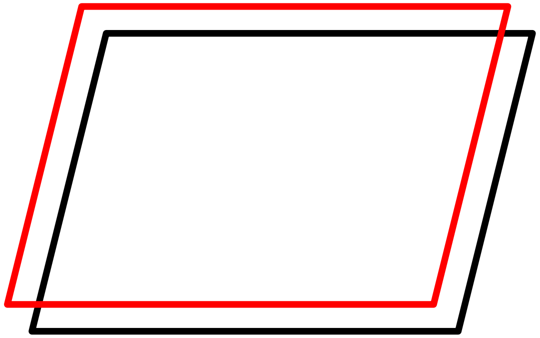
ii) $U(g) = \prod_i U_i(g)$

$$U_i(g)U_i(h) = U_i(gh)$$

(On-site symmetry)

- Can always be coupled to a gauge field!

Q: Can we realize/mimick the boundary in a lattice model without the bulk?



A: Relax?

i) $V = \otimes_i V_i$

ii) $U(g) = \prod_i U_i(g)$

Motivation:

- Simulating edge of SPT without the bulk overhead

- A more lattice based understanding of anomalies

Cohomology phases of bosons

- Class of boson SPT phases X. Chen, Z.-C. Gu, Z.-X, Liu, X.G.-Wen (2012)

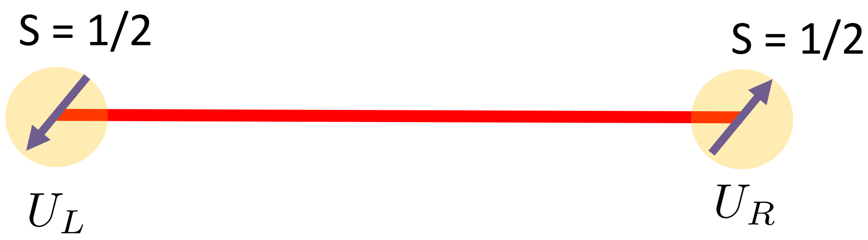
- Classified by $w \in H^{d+1}(G, U(1))$

- Edge can be mimicked if ii) is dropped

i) $V = \otimes_i V_i$

ii) ~~$U(g) = \prod_i U_i(g)$~~

Example: Haldane S = 1 chain (1d)



$$G = SO(3)$$

$$U(g) \sim U_L(g)U_R(g), \quad g \in SO(3)$$

$$U_L(g)U_L(h) = e^{i\phi(g,h)}U_L(gh)$$ - Projective (S=1/2) rep of SO(3)

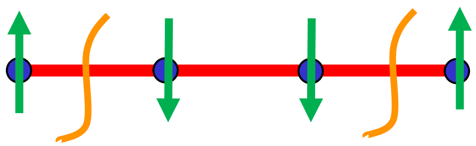
($H^2(G, U(1))$ - projective representations of G)

Example: 2d bulk

- $G = Z_2$

- $H^3(G, U(1)) = Z_2$

- 1d boundary



S = 1/2

$$U = (-1)^{N_{dw}/2} \prod_i \sigma_i^x, \qquad U^2 = 1$$

$$U = \prod_i U_i, \quad U_i^2 = 1$$

X. Chen, Z. X. Liu, X.-G. Wen (2012)
M. Levin, Z. C. Gu (2012)

Extracting the anomaly co-cycle

1d "boundary" 

$$U(g) \longrightarrow w \in H^3(G, U(1))$$

-corresponding 2d SPT

Chen, Liu, Wen (2011); D. Else, C. Nayak (2014)

Super-cohomology phases of fermions

i) $V = \otimes_i V_i$

ii) $U(g) = \prod_i U_i(g)$

- Extends to "super-cohomology" phases of fermions

  (explicit construction for 1d boundary and discrete G in MM, arXiv:1908.08958)

This talk

- Beyond (super)cohomology phases:

  - 2d topological superconductors

$$V = \otimes_i V_i$$

$$U(g) = \prod_i U_i(g)$$

R. A. Jones and MM, arXiv:1902.05957

This talk

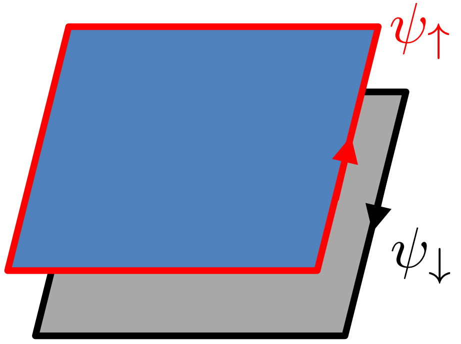
- (Super)cohomology phases with a continuous symmetry group G

  - infinite dimensional site Hilbert space?

- 2d quantum spin Hall: $G \supset U(1)$

$$V = \otimes_i V_i$$

$$\cancel{U(g) = \prod_i U_i(g)}$$

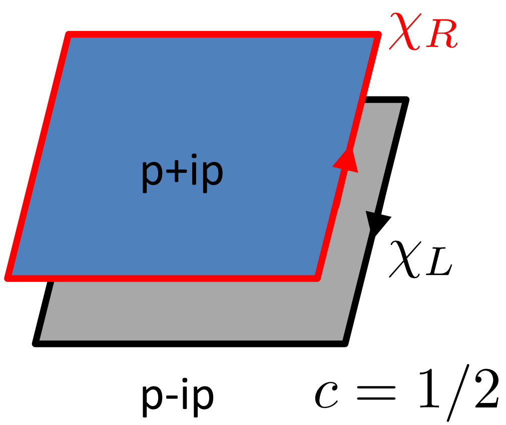


MM, arXiv:1908.08958

See also J. H. Son and J. Alicea, 1906.11846

2d topological superconductor

- 2d, fermions, time-revesal $\mathcal{T}$, with $\mathcal{T}^2 = (-1)^F$ (class DIII).



$$\mathcal{T}: \ \chi_R \to \chi_L, \qquad i \to -i$$
$$\chi_L \to -\chi_R$$

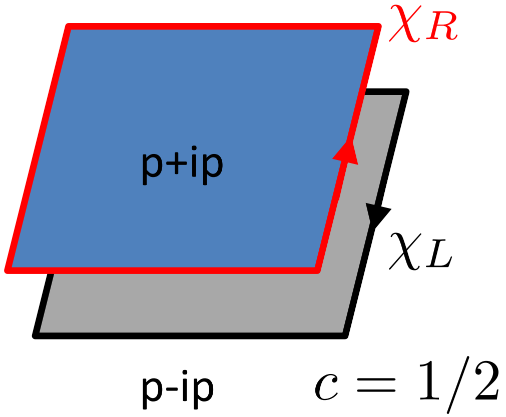
$$\delta L = im\chi_R\chi_L$$

$$\mathcal{T}: \ m \to -m$$

Unitary Z_2

$$G = Z_2 \times Z_2^f$$

Gu, Levin (14)



$$Z_2 : \chi_R \to -\chi_R, \quad \chi_L \to \chi_L$$

$$\delta L = im\chi_R\chi_L$$

$$m \to -m$$

Kramers-Wannier duality:

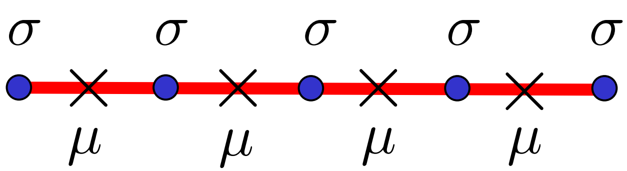
High T ⟷ low T

- Mimick in 1d?

Kramers-Wannier

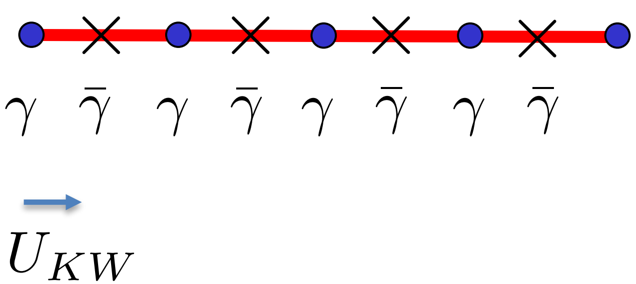
$$H_{TFIM} = -J \sum_i \sigma_i^x \sigma_{i+1}^x + h \sum_i \sigma_i^z$$

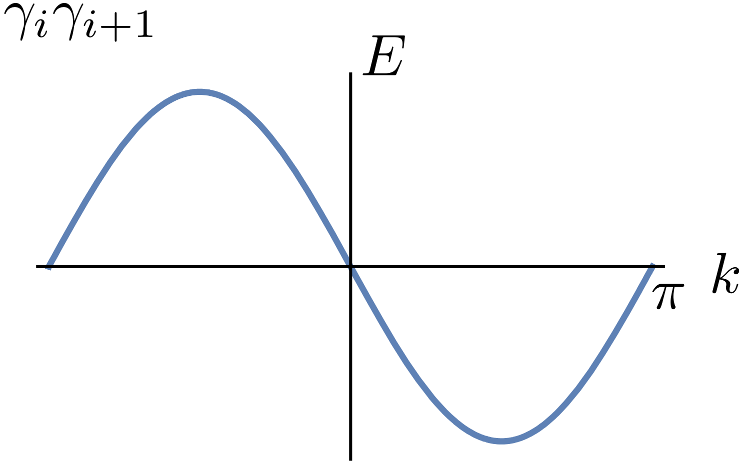
$$\mu_{i+1/2}^z = -\sigma_i^x \sigma_{i+1}^x, \quad \mu_{i-1/2}^x \mu_{i+1/2}^x = -\sigma_i^z \qquad J \leftrightarrow h$$



$$U_{KW}^2 = T_x$$

- Fermionize: $H = ih \sum_i \gamma_i \bar{\gamma}_i + iJ \sum_i \bar{\gamma}_i \gamma_{i+1}$





i) $V = \bigotimes_i V_i$ $\qquad V_{phys} \subset V$

ii) $U(g) = \prod_i U_i(g)$

- not enough to drop this if $U^2 = 1$

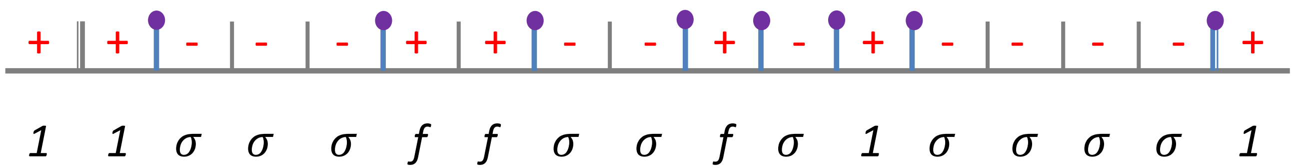
- Starting point:
  Exactly solvable bulk model of Tarantino and Fidkowski (2016)

Intuition

$$\delta L = i m \chi_R \chi_L$$

$$m > 0$$

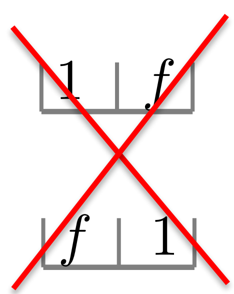
- Restore symmetry – Majorana liquid $\qquad \dim V_{Maj} = 2^{N_{dw}/2}$



Link labels: $\{1, \sigma, f\}$, $\qquad \sigma \times \sigma = 1 + f$

- σ

+ 1 or f



$$\dim(V_{phys}) \sim (\sqrt{2} + 1)^N$$

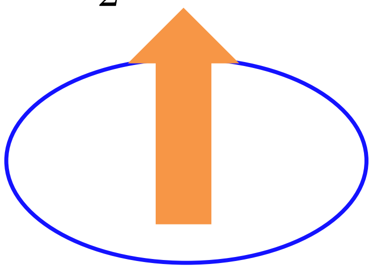
Fermion parity symmetry

- $$H_{TFIM} = -J\sum_i \sigma_i^x \sigma_{i+1}^x + h\sum_i \sigma_i^z$$

$$Z_2^{Ising} = \prod_i \sigma_i^z$$

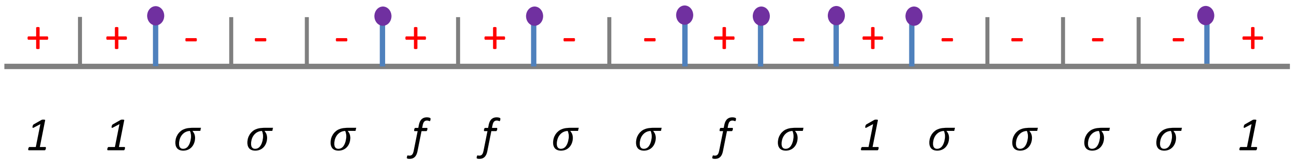
$$(-1)^F \leftrightarrow Z_2^{Ising}$$

Z_2^{Ising} flux



		Z_2^{Ising} flux	
		1	-1
Z_2^{Ising} charge	1	NS, $(-1)^F = 1$	R, $(-1)^F = -1$
	-1	R, $(-1)^F = 1$	NS, $(-1)^F = -1$

Fermion parity symmetry

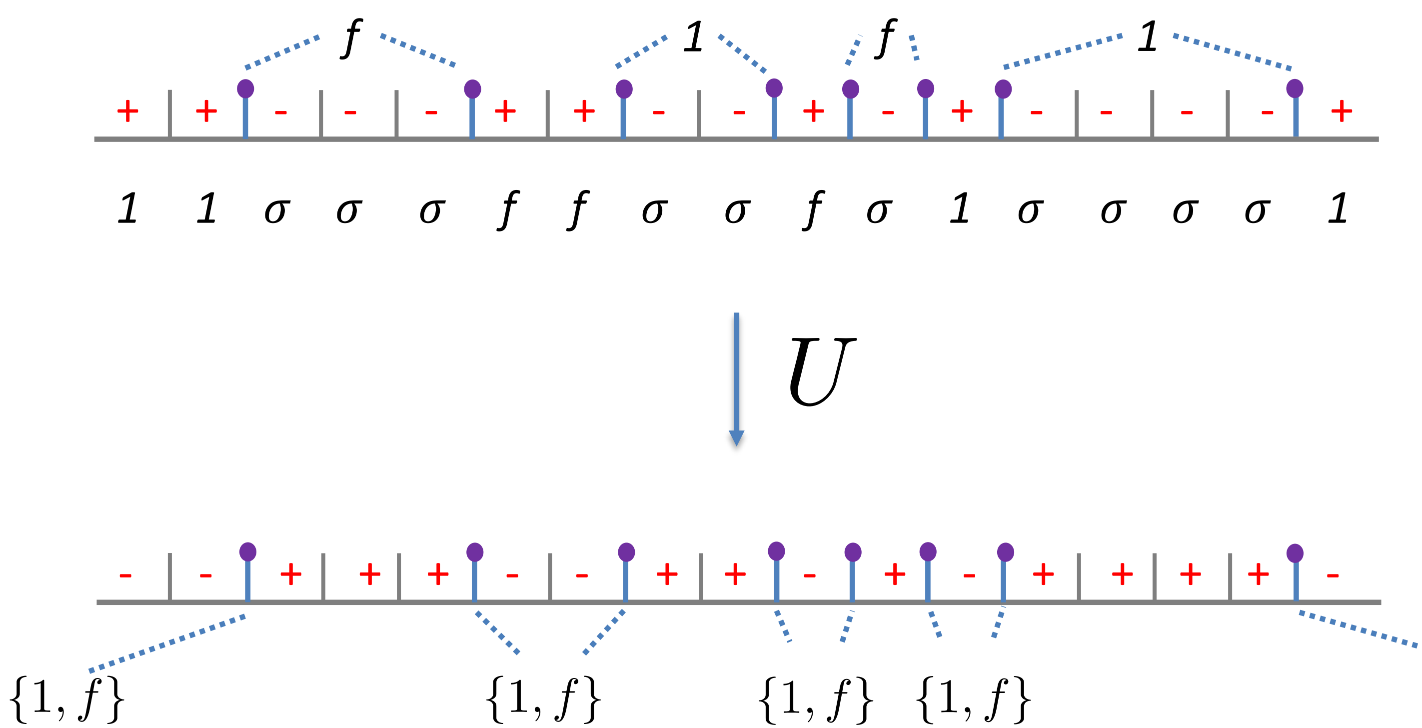


$$(-1)^F \leftrightarrow Z_2^{Ising}$$

$$Z_2^{Ising}: \qquad |1\rangle \leftrightarrow |f\rangle, \quad |\sigma\rangle \rightarrow |\sigma\rangle$$

		Z_2^{Ising} flux	
		1	-1
Z_2^{Ising} charge	1	NS, $(-1)^F = 1$	R, $(-1)^F = -1$
	-1	R, $(-1)^F = 1$	NS, $(-1)^F = -1$

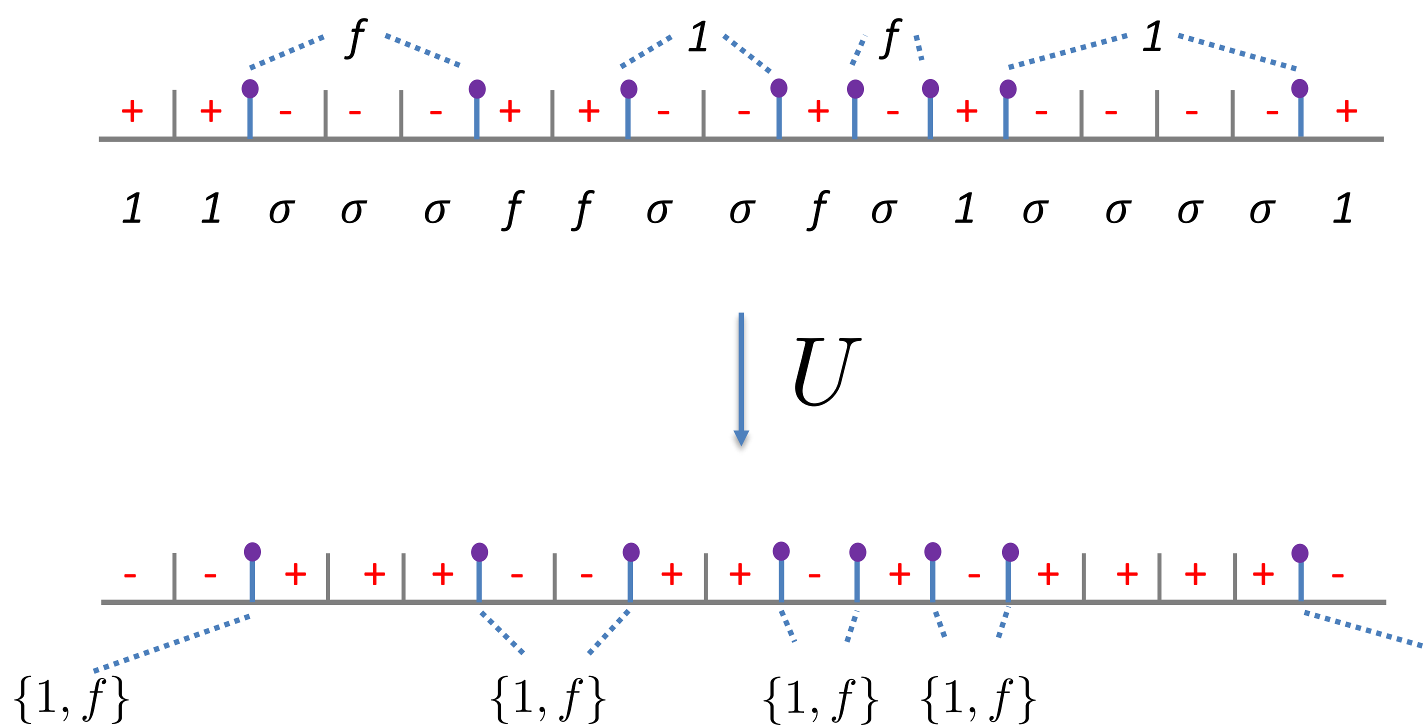
“Kramers-Wannier” symmetry action



- Strings: $\{1, f\} \;\rightarrow\; \sigma$

  $\sigma \;\rightarrow\; \{1, f\}$ -superposition

Kramers time-reversal



$$\mathcal{T}: \ \chi_R \to \chi_L, \qquad i \to -i$$

$$\chi_L \to -\chi_R$$

Just add complex conjugation!

Hamiltonian

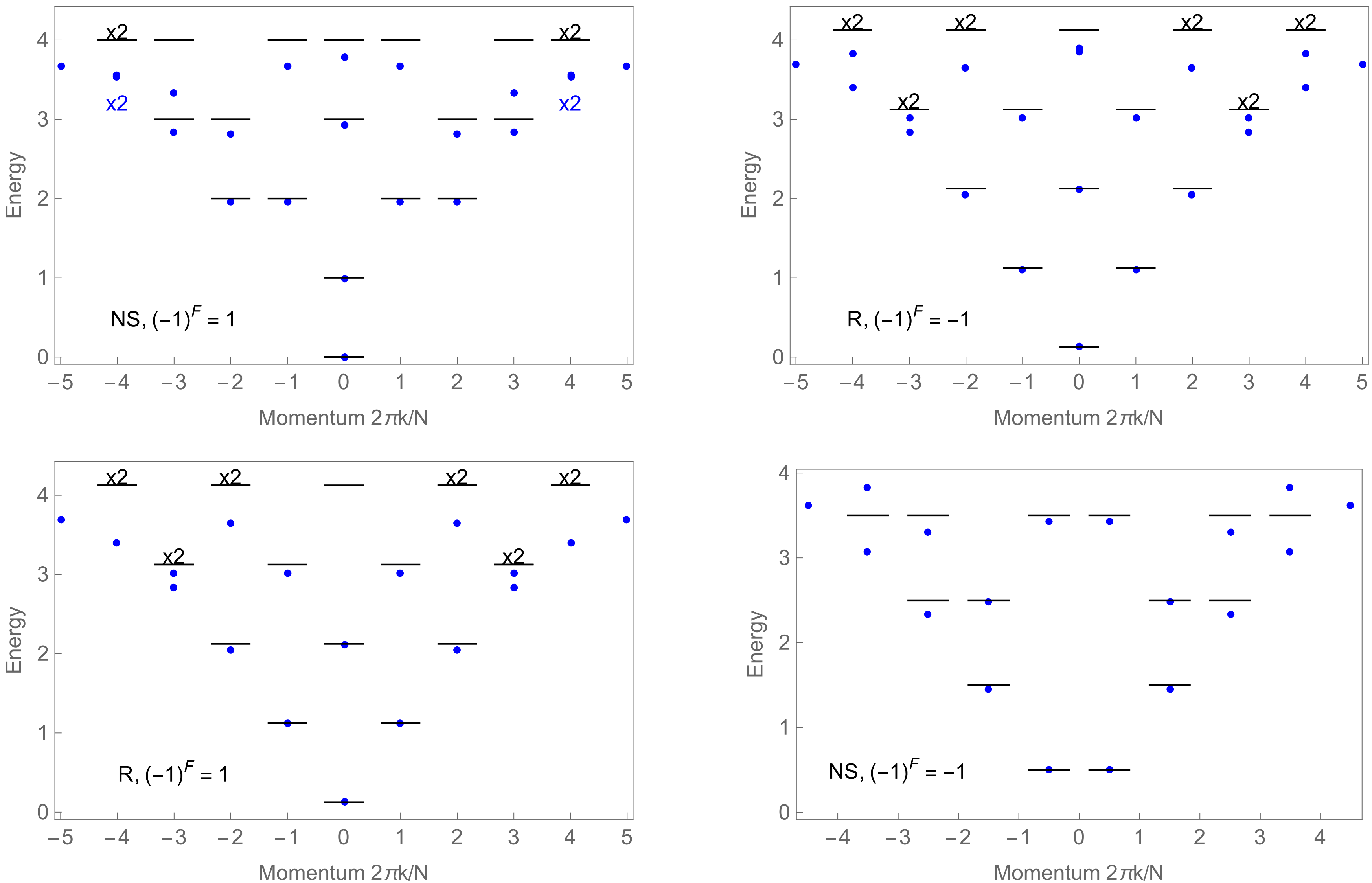
c_1

c_2

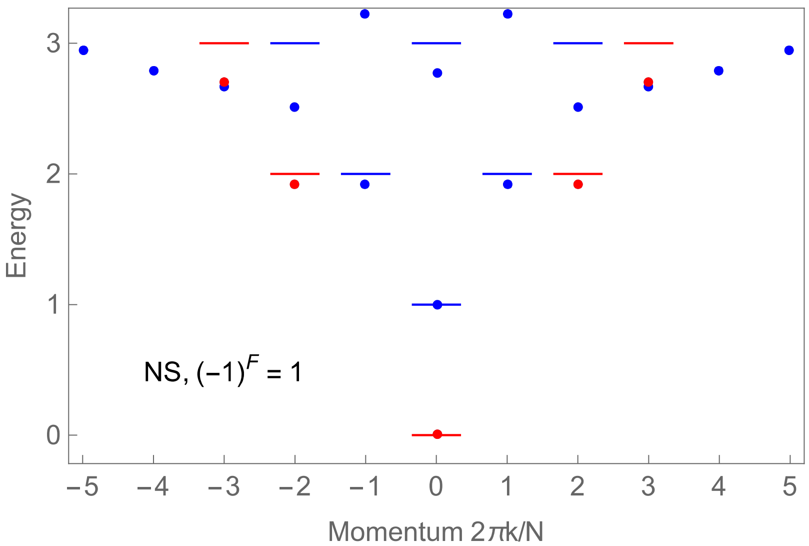
- $c_1 \sim c_2$ - solid numerical evidence (ED & DMRG) for Ising CFT

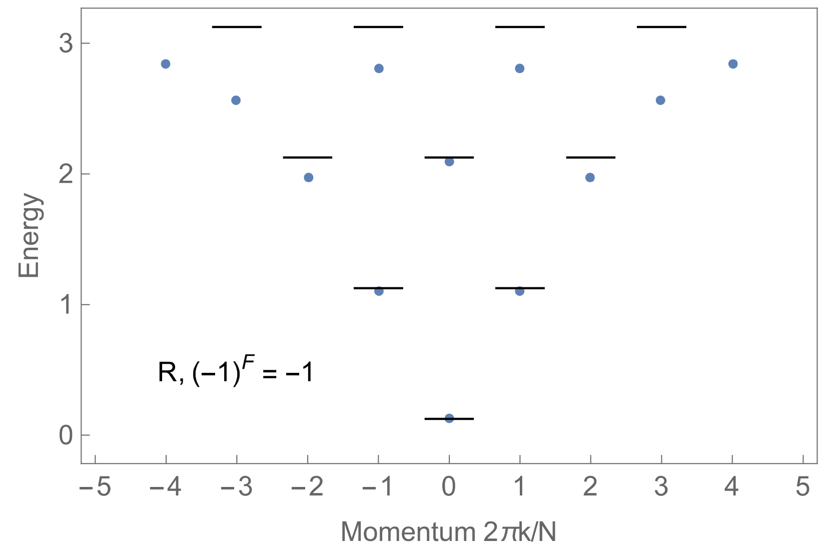
$$Z_2 : \chi_R \to -\chi_R, \quad \chi_L \to \chi_L$$

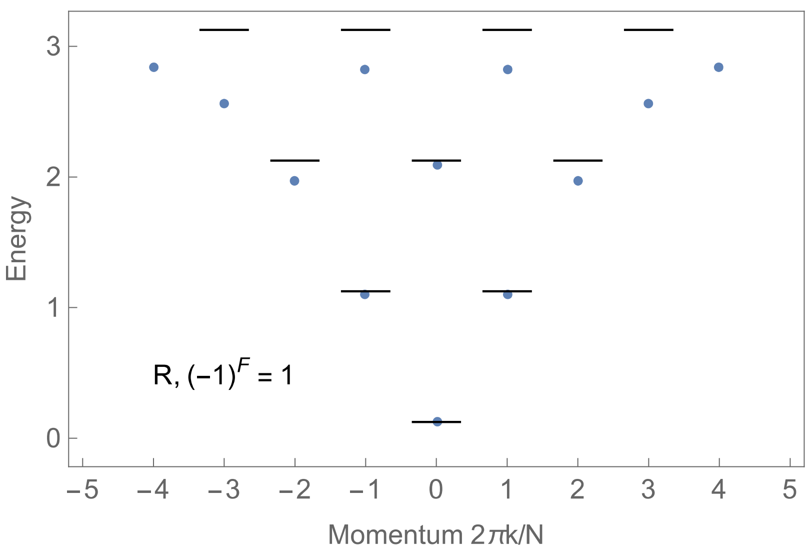
Exact diagonalization

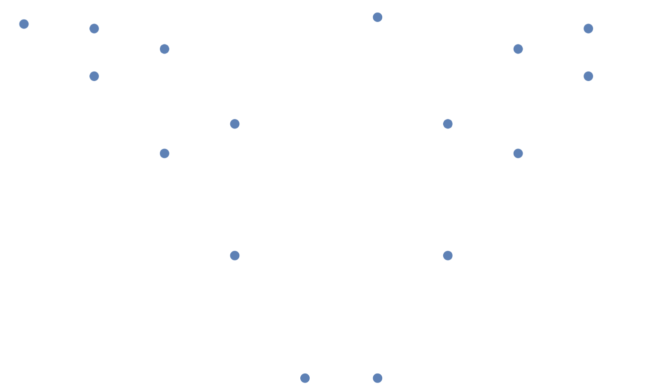


Exact diagonalization

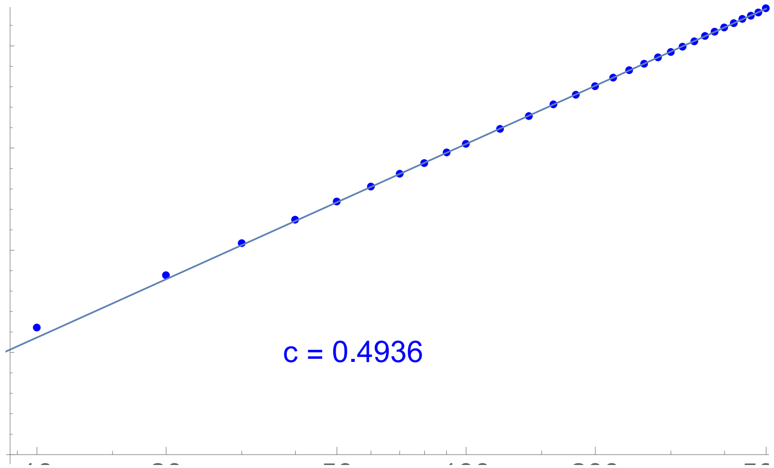


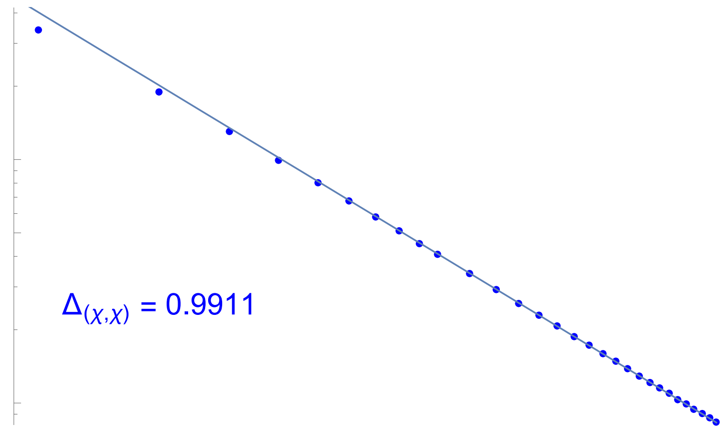


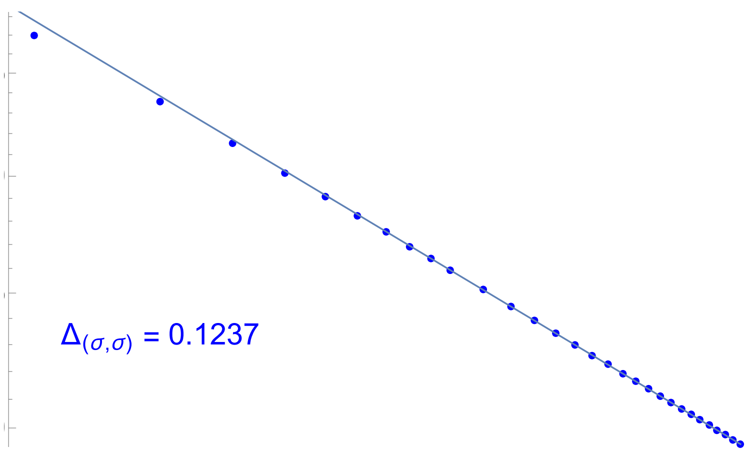


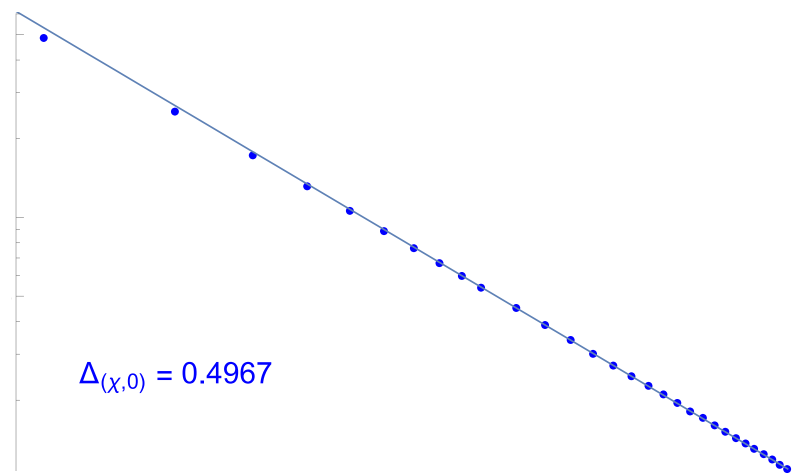


DMRG


c = 0.4936


$\Delta_{(\chi,\chi)} = 0.9911$


$\Delta_{(\sigma,\sigma)} = 0.1237$


$\Delta_{(\chi,0)} = 0.4967$

Lessons & open questions

- Mimick edge of 2d beyond super-cohomology fermion SPT

  - constrained Hilbert space

  - reminiscent of 3d class AIII

$\nu = 1/2$

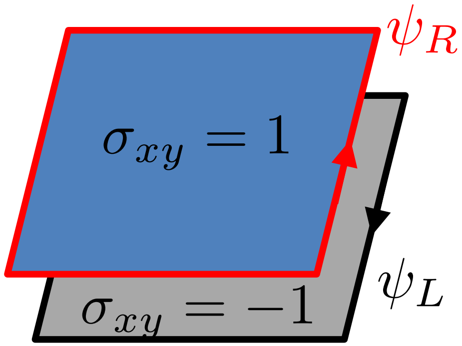
  - generalizes to 2d fSPTs with $G \times Z_2^f$

  - obstruction?

  - other dimensions?

Quantum spin-Hall

- 2d, fermions



$c = 1$

$$U: \ \psi_R \to -\psi_R, \quad \psi_L \to \psi_L$$

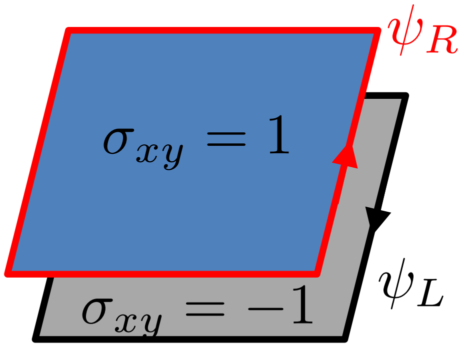
$$U(1): \ N = N_R + N_L$$

$$\mathcal{T}_{NK}: \ \psi_R \leftrightarrow \psi_L, \quad i \to -i. \quad \mathcal{T}_{NK}^2 = 1$$

$$\mathcal{T} = U\mathcal{T}_{NK}, \qquad \mathcal{T}^2 = (-1)^F$$

Domain wall

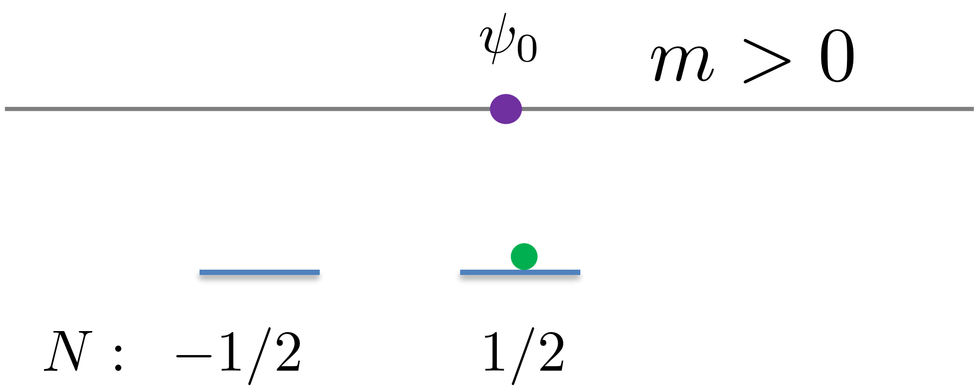
- 2d, fermions



$$U: \; \psi_R \to -\psi_R, \quad \psi_L \to \psi_L$$

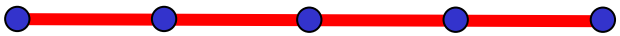
$$\delta L = m(\psi_R^\dagger \psi_L + h.c.)$$

$$U: \; m \to -m$$ (also $\mathcal{T}$)



QSH edge model

$$c_i = \frac{1}{2}(\gamma_i + i\bar{\gamma}_i)$$



$$g_{i,i+1} \in \{0,1\}$$

$$\tau^z_{i,i+1} = (-1)^{g_{i,i+1}}$$

$$U = \left(\prod_{j=1}^{L} \tau^x_{j,j+1}\right)\left(\prod_{j=1}^{L} \gamma_j^{g_{j-1,j}+g_{j,j+1}}\right)(-i)^{N_{dw}/2}$$

- inspired by Tantivasadakarn and Vishwanath (18)

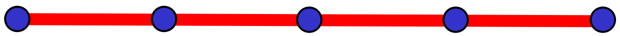
$$N = \sum_i n_i, \quad n_i = (-1)^{g_i(g_{i+1}+1)} c_i^\dagger c_i$$

$$\mathcal{T}_{NK} = \left(\prod_{j=1}^{L} (-1)^{g_{j,j+1}\, c_j^\dagger c_j}\right)\mathcal{T}_0$$

MM, arXiv:1908.08958

Charge density

$$c_i = \frac{1}{2}(\gamma_i + i\bar{\gamma}_i)$$



$$g_{i,i+1} \in \{0, 1\}$$

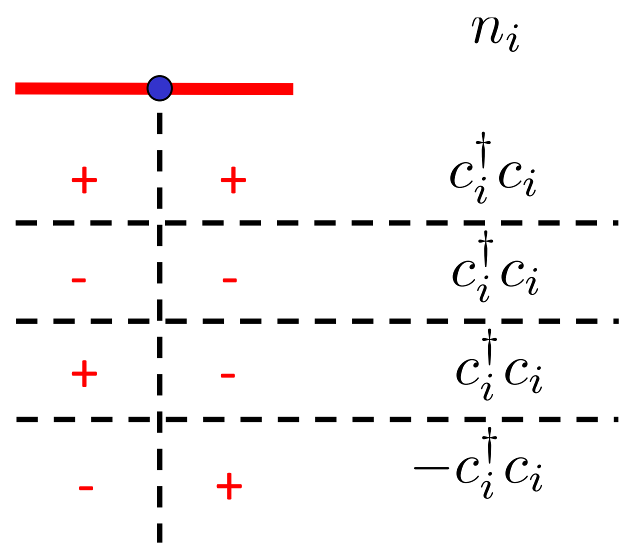
$$\tau^z_{i,i+1} = (-1)^{g_{i,i+1}}$$

$$N = \sum_i n_i, \quad n_i = (-1)^{g_i(g_{i+1}+1)} c_i^\dagger c_i$$

$$n_i = c_i^\dagger c_i \ (mod\ 2) \qquad (-1)^N = (-1)^F$$

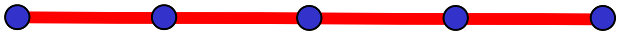
$$\mathcal{T} n_i \mathcal{T}^\dagger = n_i + \frac{1}{2}(\tau^z_{i,i+1} - \tau^z_{i-1,i})$$

$$\mathcal{T} N \mathcal{T}^\dagger = N$$



Charge on domain walls

$$c_i = \frac{1}{2}(\gamma_i + i\bar{\gamma}_i)$$



$$g_{i,i+1} \in \{0, 1\}$$

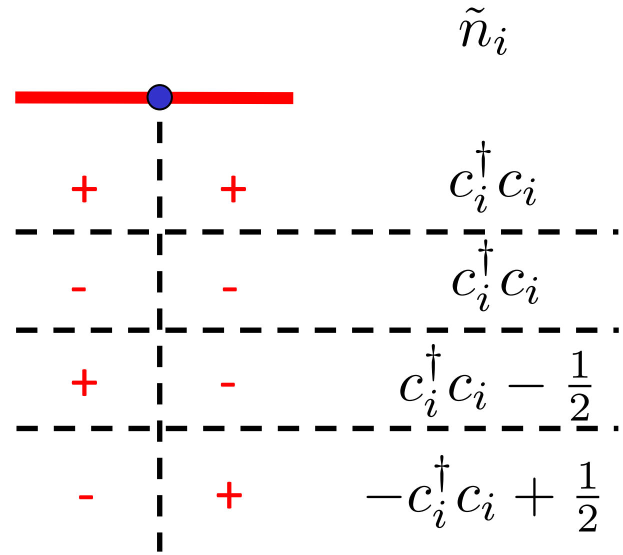
$$\tau^z_{i,i+1} = (-1)^{g_{i,i+1}}$$

$$\mathcal{T} n_i \mathcal{T}^\dagger = n_i + \frac{1}{2}(\tau^z_{i,i+1} - \tau^z_{i-1,i})$$

$$\tilde{n}_i = n_i + \frac{1}{4}(\tau^z_{i,i+1} - \tau^z_{i-1,i})$$

$$N = \sum_i \tilde{n}_i$$

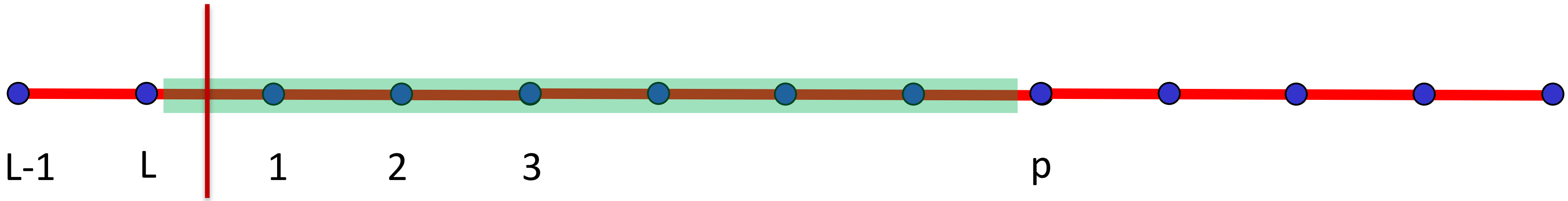
$$\mathcal{T} \tilde{n}_i \mathcal{T}^\dagger = \tilde{n}_i$$



- Half integer charge on domain walls!

Kramers parity switching

$$N = \sum_i \tilde{n}_i \qquad \mathcal{T} \tilde{n}_i \mathcal{T}^\dagger = \tilde{n}_i$$



$$S(\Phi) = e^{i\Phi \sum_{j=1}^{p} \tilde{n}_j} \qquad H_{cut} \to S(\Phi) H_{cut} S^\dagger(\Phi)$$

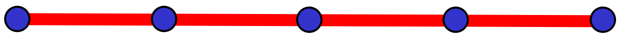
$$S(\Phi + 2\pi) = \tau^z_{L,1} \tau^z_{p,p+1} S(\Phi) \qquad \mathcal{T} S(\Phi) \mathcal{T}^\dagger = S(-\Phi)$$

$$\mathcal{T}^{\Phi=\pi} = \tau^z_{L,1} \mathcal{T}$$

$$\mathcal{T}^2 = (-1)^N, \qquad (\mathcal{T}^{\Phi=\pi})^2 = -(-1)^N$$

Bosonization

$$c_i = \frac{1}{2}(\gamma_i + i\bar{\gamma}_i)$$



$$g_{i,i+1} \in \{0, 1\}$$

$$\tau^z_{i,i+1} = (-1)^{g_{i,i+1}}$$

$$\tilde{n}_i$$

+	+	$c_i^\dagger c_i = 0$
-	-	$c_i^\dagger c_i = 0$
+	-	$c_i^\dagger c_i - \frac{1}{2} = \frac{1}{2}$
-	+	$-c_i^\dagger c_i + \frac{1}{2} = \frac{1}{2}$

$$[a_i, a_j^\dagger] = \delta_{ij}$$

$$\tilde{n}_i = \frac{1}{2} a_i^\dagger a_i$$

$$(-1)^{a_i^\dagger a_i} = \tau^z_{i-1,i}\tau^z_{i,i+1}$$

Z_2 Gauss law

$$N_b = \sum_i a_i^\dagger a_i \in 2\mathbb{Z}$$

$$N = \frac{1}{2} N_b$$

Bosonization

$$(-1)^{a_i^\dagger a_i} = \tau^z_{i-1,i}\tau^z_{i,i+1}$$

$\tau^z_{i,i+1}$ - Z_2 electric field

$$N_b = \sum_i a_i^\dagger a_i \in 2\mathbb{Z}$$

$$N = \frac{1}{2}N_b$$

$$U = \exp\left(\frac{\pi i}{4}N_b\right)\underbrace{\prod_i \tau^x_{i,i+1}}_{Z_2 \text{ flux}} \times \begin{cases} 1, & N - even \\ -i, & N - odd, \end{cases}$$

Z_2 flux

$$\mathcal{T}_{NK} = \begin{cases} K, & N - even \\ \tau^z_{L,1}K, & N - odd. \end{cases}$$

Hamiltonian

$$H = \sum_i H_{i,i+1}$$

$$H_{i,i+1} = -J(a_i^\dagger a_{i+1} + a_{i+1}^\dagger a_i)\tau_{i,i+1}^x, \quad N - \text{even}$$

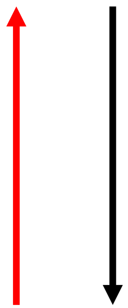
$$H_{L,1} = -J(ia_L^\dagger a_1 - ia_L a_1^\dagger)\tau_{L,1}^x, \qquad N - \text{odd}$$

Hardcore boson coupled to Z_2 gauge field

Exactly solvable – QSH edge

Continuum bosonization

ψ_R ψ_L



$$\psi_{R/L} = e^{-i\phi_{R/L}}$$

$$L = \frac{-1}{4\pi}\partial_x\phi_R\partial_t\phi_R + \frac{1}{4\pi}\partial_x\phi_L\partial_t\phi_L - \frac{1}{4\pi}((\partial_x\phi_L)^2 + (\partial_x\phi_R)^2)$$

$$U: \ \phi_R \to \phi_R + \pi, \quad \phi_L \to \phi_L$$

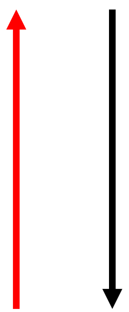
Standard Jordan-Wigner: $\phi = \frac{\phi_R + \phi_L}{2}, \quad \theta = \phi_L - \phi_R$

$$L = \frac{1}{2\pi}\partial_x\phi\partial_t\theta - \frac{1}{2\pi}\left(K(\partial_x\phi)^2 + \frac{1}{4K}(\partial_x\theta)^2\right),$$ $e^{i\phi}$ - JW boson

Fractionalize: $\phi = 2\tilde{\phi}, \quad \theta = \tilde{\theta}/2$

$$L = \frac{1}{2\pi}\partial_x\tilde{\phi}\partial_t\tilde{\theta} - \frac{1}{2\pi}\left(\tilde{K}(\partial_x\tilde{\phi})^2 + \frac{1}{4\tilde{K}}(\partial_x\tilde{\theta})^2\right)$$

$\psi_R \quad \psi_L$



$$L = \frac{1}{2\pi}\partial_x\tilde{\phi}\partial_t\tilde{\theta} - \frac{1}{2\pi}\left(\tilde{K}(\partial_x\tilde{\phi})^2 + \frac{1}{4\tilde{K}}(\partial_x\tilde{\theta})^2\right)$$

$$\psi_R\psi_L \sim e^{-4i\tilde{\phi}} \qquad \psi_L^\dagger\psi_R \sim e^{i\tilde{\theta}/2}$$

$$(-1)^{a_i^\dagger a_i} = \tau^z_{i-1,i}\tau^z_{i,i+1} \qquad H_{i,i+1} = -J(a_i^\dagger a_{i+1} + a_{i+1}^\dagger a_i)\tau^x_{i,i+1}$$

$$a_i \sim e^{-i\tilde{\phi}}$$

$\tau^z_{i,i+1} \sim \cos(\tilde{\theta}/2)$ - flips Z_2 flux $\prod_i \tau^x_{i,i+1}$

$$U = \exp\left(\frac{\pi i}{4}N_b\right)\prod_i \tau^x_{i,i+1} \times \begin{cases} 1, & N - even \\ -i, & N - odd, \end{cases} \qquad \psi_R \to -\psi_R, \quad \psi_L \to \psi_L$$

Open questions

- Mimick edge of 2d quantum spin-Hall

  - simulating QSH edge with disorder and interactions?

  Wu, Bernevig, Zhang (06); Xu, Moore (06); Chou, Nandkishore, Radzihovsky (18)

- mimick other SPTs with continuous symmetry groups?

  - continous vs discrete anomalies?

Thank you!