Particle-hole symmetry without particle-hole symmetry in the quantum Hall effect

Dima Feldman
Brown University

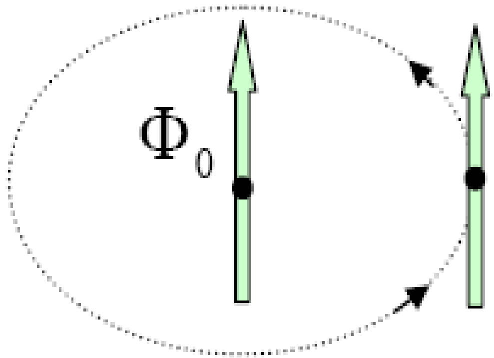
P. T. Zucker and D. E. Feldman, PRL **117**, 096802 (2016)
G. Yang and D. E. Feldman, PRB **90**, 161306 (2014)

Laughlin State

$$\nu = \frac{1}{2n+1}; \quad \Psi = \prod (z_i - z_j)^{2n+1} \exp\left(-\sum |z_i|^2/4l^2\right); \; z_k = x_k + i y_k$$

Quasiparticle charge and statistics

$$q = \nu e$$



$$\theta = 2\pi\nu$$

Edge theory

$$L = -\frac{1}{4\pi\nu} \int dx\,dt \left[\partial_t \varphi \partial_x \varphi + v(\partial_x \varphi)^2\right]$$

Composite fermion interpretation:

IQHE of (electrons+2n flux quanta)

$\nu = 5/2$ and the Halperin 331 state

Even denominator = Cooper pairs of composite fermions

$$\Psi = \hat{A}\exp(-\frac{1}{4l^2}\sum[|z_{l\downarrow}|^2 + |z_{a\uparrow}|^2]) \prod_{k<l}(z_{k\downarrow} - z_{l\downarrow})^3 \prod_{a<b}(z_{a\uparrow} - z_{b\uparrow})^3 \prod_{k,a}(z_{k\downarrow} - z_{a\uparrow})$$

K-matrix: $K = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}$ Charge vector: $\boldsymbol{t} = (1,1)$

$$\nu = \boldsymbol{t}K^{-1}\boldsymbol{t}^T = 1/2$$

2 types of anyons of charge and statistics

$$e[(K^{-1})_{11} + (K^{-1})_{12}] = \frac{e}{4}$$

$$\theta_{12} = 2\pi(K^{-1})_{12} = \frac{3\pi}{4}$$

$$\theta_{11} = 2\pi(K^{-1})_{11} = -\frac{\pi}{4}$$

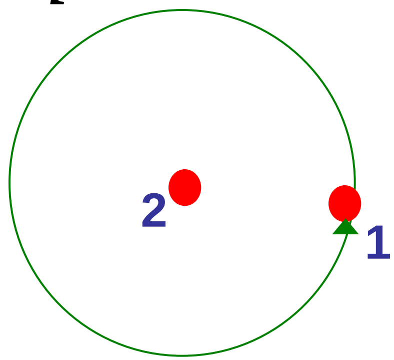
Abelian state: anyon+anyon = uniquely defined anyon

statistical phases of particles add for a composite object

Moore-Read (Pfaffian) state

$$\nu = 5/2$$

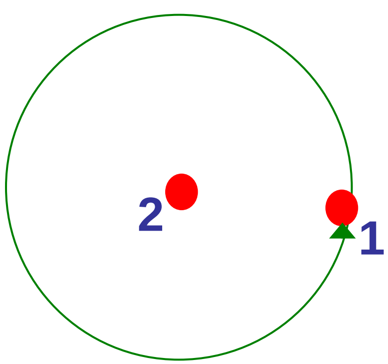
$$q = e/4; \text{non-Abelian statistics}$$



$$|\psi_f\rangle \neq \exp(i\theta)|\psi_i\rangle$$

Several states at given quasiparticle positions

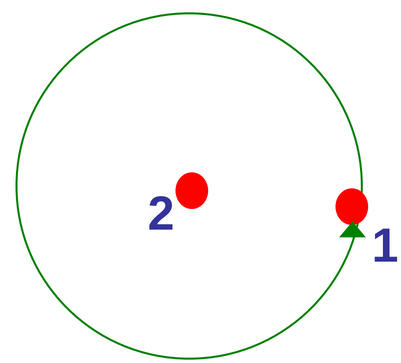
Vacuum superselection sector $|1\rangle$



$$\psi \to \psi$$

$$\theta = 0$$

Fermion sector $|\varepsilon\rangle$



$$\psi \to -\psi$$

$$\theta = \pi$$

$$\alpha|1\rangle|\ \rangle + \beta|\varepsilon\rangle|\ \rangle \to \alpha|1\rangle|\ \rangle - \beta|\varepsilon\rangle|\ \rangle$$

Theoretical proposals at ν=5/2

Numerous ways to build Cooper pairs!

- Pfaffian state
- 331 state
- K=8 state
- $SU(2)_2$ state
- anti-Pfaffian state
- anti-331 state
- anti- $SU(2)_2$ state
- and so on

Numerics favors Pfaffian and anti-Pfaffian

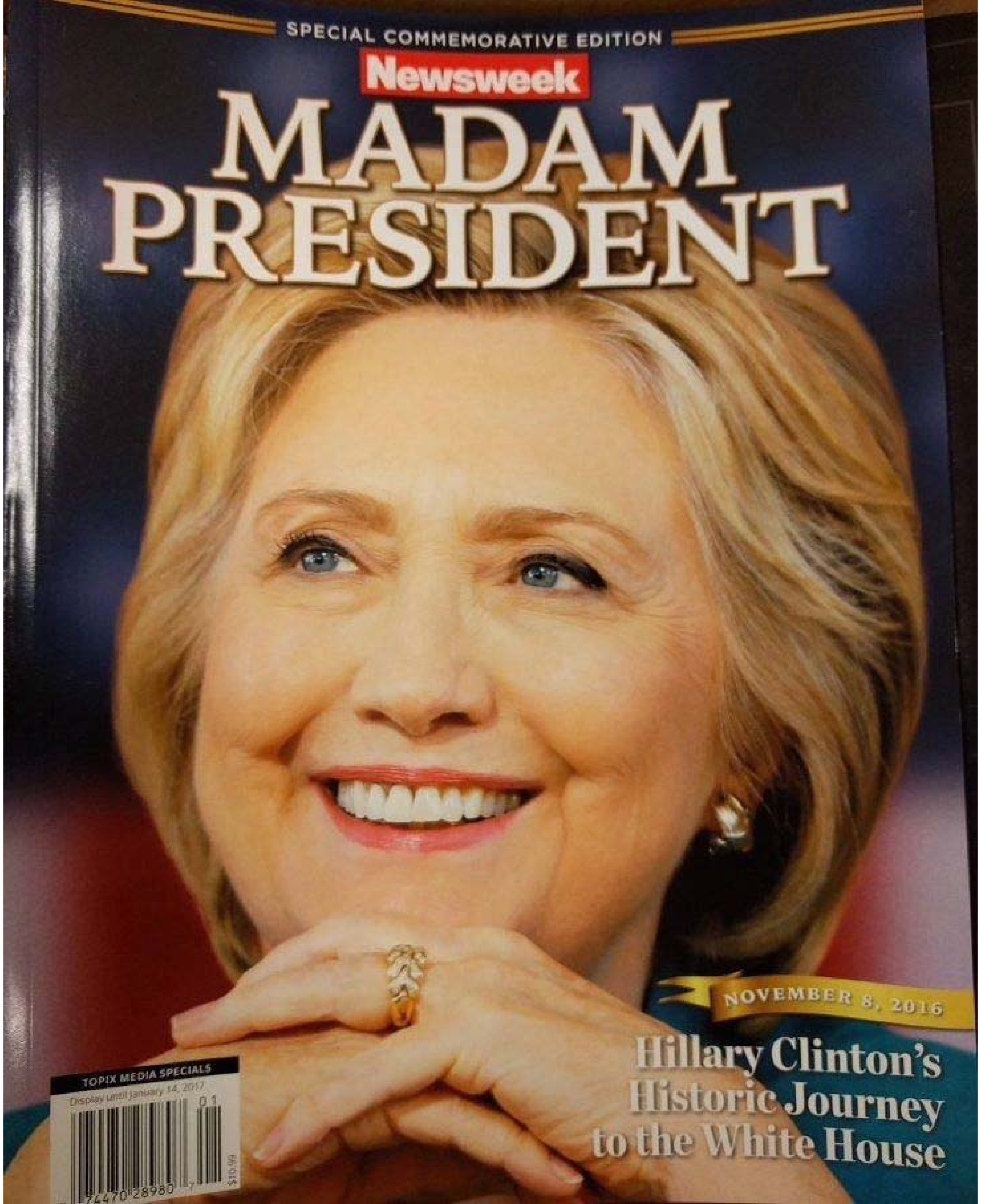
Pfaffian and anti-Pfaffian states

- numerics supports Pfaffain and anti-Pfaffian states in the absence of disorder and Landau level mixing
- poor results for the energy gap
  - strong disorder
  - LLM parameter ~ 1.3
- Small energy differences for proposed states

[J. Biddle *et al.*, *Phys. Rev. B* **87**, 235134 (2013)]

- no QHE at realistic LLM in numerics

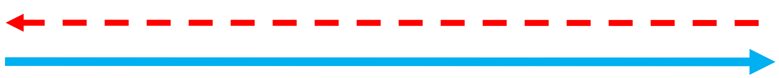
[K. Pakrouski *et al.*, *Phys. Rev. X* **5**, 021004 (2015)]


SPECIAL COMMEMORATIVE EDITION
Newsweek
MADAM PRESIDENT
NOVEMBER 8, 2016
Hillary Clinton's Historic Journey to the White House
TOPIX MEDIA SPECIALS

Experimental results

- Charge **e/4**
- Controversial results for spin polarization
- Fabry-Perot interferometry identical for all non-Abelian states and can be identical for Abelian states [**A. Stern, B. Rosenov, R. Ilan, and B.I. Halperin, PRB 82, 085321 (2010)**]

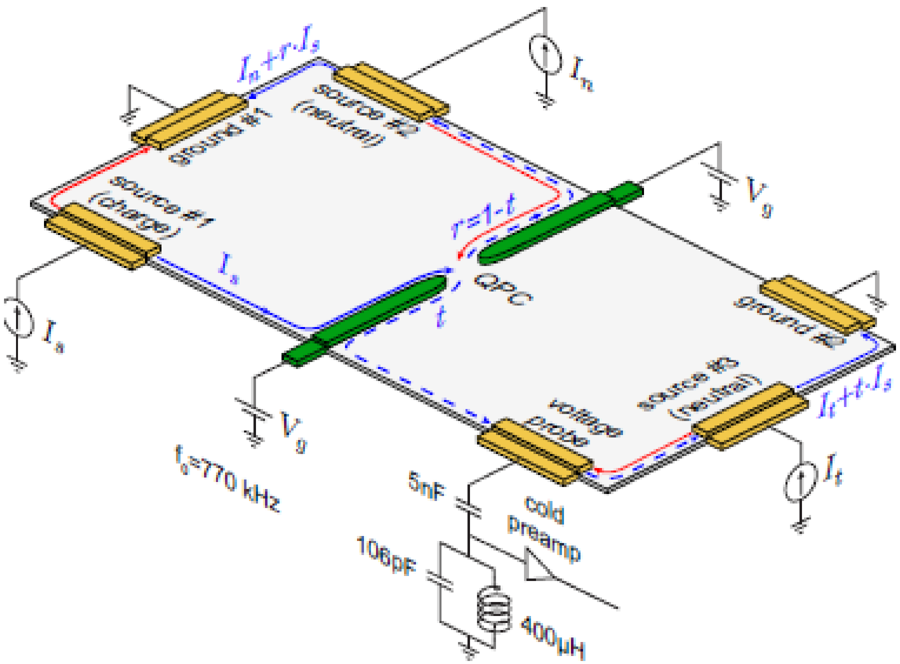
Upstream modes



Topologically protected

Edge reconstruction

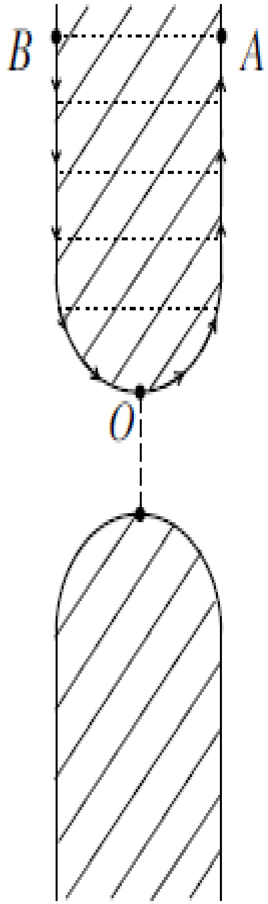
Upstream neutral modes



Observation of a topologically protected upstream neutral mode [A. Bid *et al.,* Nature **466**, 585 (2010); M. Dolev *et al*., PRL **107**, 036805 (2011)]. Compatible with anti-Pfaffian. Incompatible with Pfaffian.

The observed physics at ν=5/2 is similar to ν=8/3. It differs from the filling factors such as 7/3 where no topologically protected upstream modes are present but edge reconstruction is possible [H. Inoue *et al*., Nature Comm. **5**, 4067 (2014).]

Tunneling



Theory: $G \sim T^{2g-2}$

- Pfaffian: $g = \frac{1}{4}$
- Anti-Pfaffian: $g = \frac{1}{2}$

Experiment: $g_{\text{exp}} > g_{\text{theor}}$

Experiment gives an upper bound on g
The upper bound of 0.4 is consistent with Pfaffian and excludes anti-Pfaffian

I. P. Radu *et al.*, *Science* **320**, 899 (2008); X. Lin *et al.*, *Phys. Rev. B* **85**, 165321 (2012); S. Baer *et al.*, *Phys. Rev. B* **90**, 075403 (2014); H. Fu *et al.*, *PNAS* **113**, 12386 (2016).

Filling factor 1/2

[D. T. Son, *Phys. Rev. X* **5**, 031027 (2015)]

Imagine exact particle-hole symmetry between filling factors f and 1-f.

Geometric resonance experiments are compatible with such symmetry.

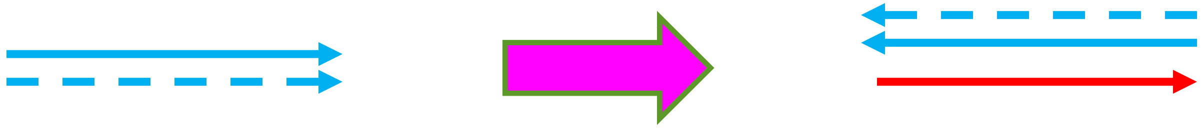
The theory can be made explicitly symmetric by assuming that composite fermions are Dirac particles.

What about Cooper pairing of Dirac particles in the *s*-channel?

PH-Pfaffian state

s-pairing of Dirac fermions

Particle-hole symmetry:



$$G = \frac{e^2}{2h}; \qquad k = \pi^2 T/6h$$

Edge theory:

$$-\frac{2}{4\pi}[\partial_t\varphi\partial_x\varphi + v_c\partial_x\varphi\partial_x\varphi] + i\psi(\partial_t - v_n\partial_x)\psi$$

$$\psi = \psi^+$$

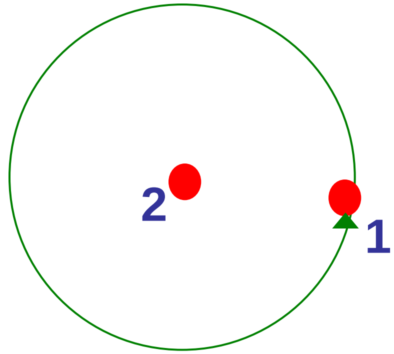
Wave function:

$$\int\{d^2 s_i\}\mathrm{Pf}\left\{\frac{1}{\bar{s}_i - \bar{s}_j}\right\}\prod(s_i - s_j)^2\exp[-2|s_i|^2 + 2\bar{s}_i z_i - |z_i|^2]$$

PH-Pfaffian state

$$\nu = 5/2$$

$$q = e/4;\ \text{non - Abelian statistics}$$

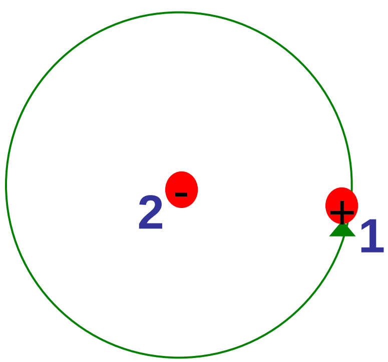


$$\left|\psi_f\right\rangle \neq \exp(i\theta)\left|\psi_i\right\rangle$$

Several states at given quasiparticle positions

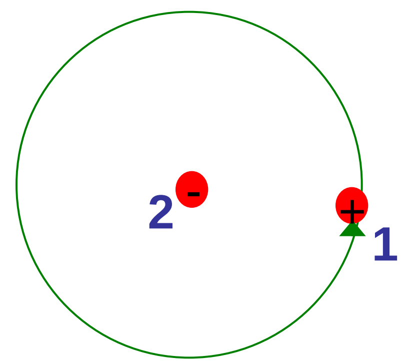
Vacuum superselection sector $\left|1\right\rangle$

Fermion sector $\left|\varepsilon\right\rangle$



$$\psi \rightarrow \psi$$

$$\theta = \pi/2$$



$$\psi \rightarrow -\psi$$

$$\theta = -\pi/2$$

$$\alpha\left|1\right\rangle\left|\ \right\rangle + \beta\left|\varepsilon\right\rangle\left|\ \right\rangle \rightarrow i\alpha\left|1\right\rangle\left|\ \right\rangle - i\beta\left|\varepsilon\right\rangle\left|\ \right\rangle$$

Fusion and braiding rules

- 6 quasiparticle types:

  topological charge σ and electric charges $\pm e/4$;

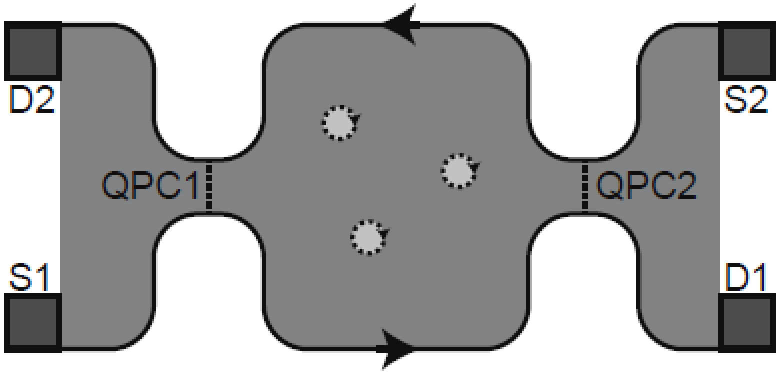
  topological charges 1 and ψ and electric charges

  0 and $e/2$
- Fusion rules $\psi \times \psi = 1; \psi \times \sigma = \sigma; \sigma \times \sigma = 1 + \psi$
- Braiding rules determine the phase accumulated by a quasiparticle moving around another quasiparticle. The phase depends on the fusion channel.

See L. Fidkowski et al., PRX 3, 041016 (2013);
P. Bonderson et al., J. Stat. Mech. P09016 (2013)

Comparison with the experiment

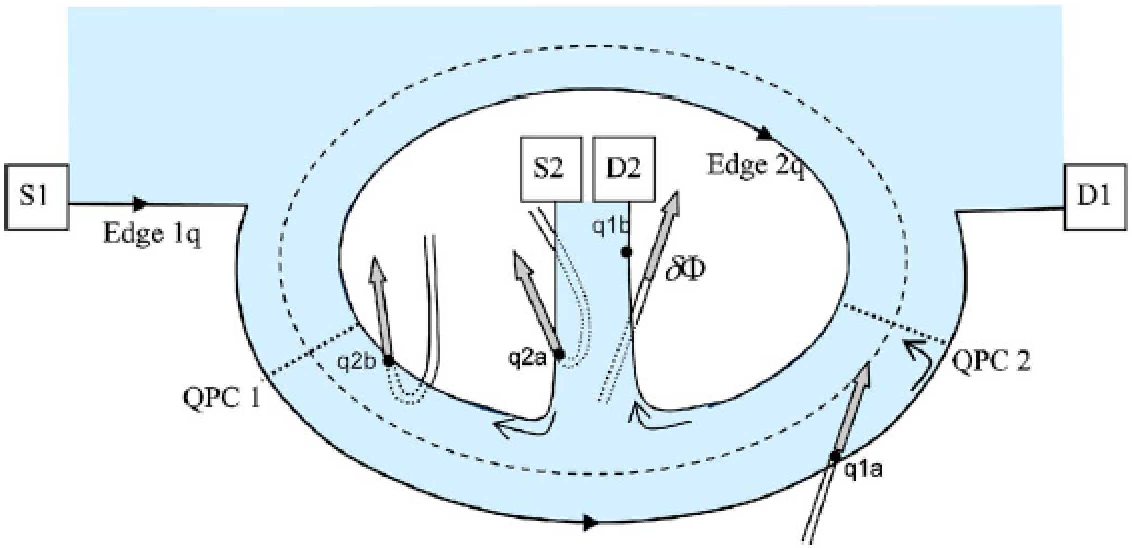
- An upstream neutral mode
- Tunneling exponent $g = \frac{1}{4}$
- Topological even-odd effect



New experimental signatures

Thermal Hall conductance $\dfrac{\pi^2 k^2 T^2}{6h}$

Mach-Zehnder interferometry

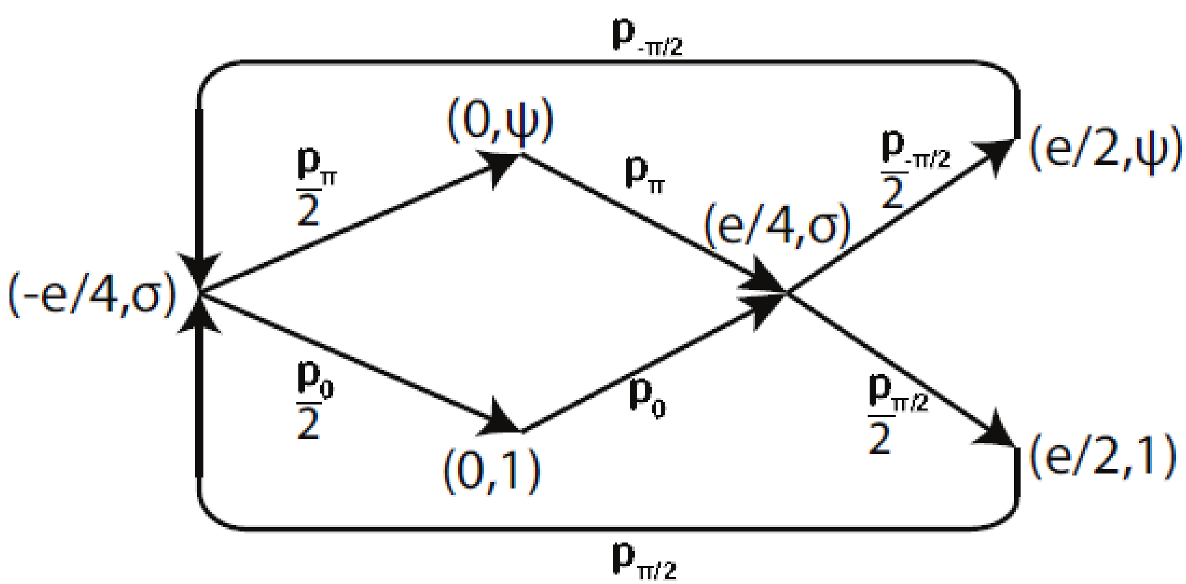


No magnetic field dependence of the current.
Shot noise diverges at some magnetic fields.

Mach-Zehnder interferometry

Tunneling probability depends on the accumulated topological charge

$$P \sim |\Gamma_1|^2 + |\Gamma_2|^2 + 2u|\Gamma_1\Gamma_2|\cos(\varphi_{AB} + \varphi_{stat} + \alpha)$$

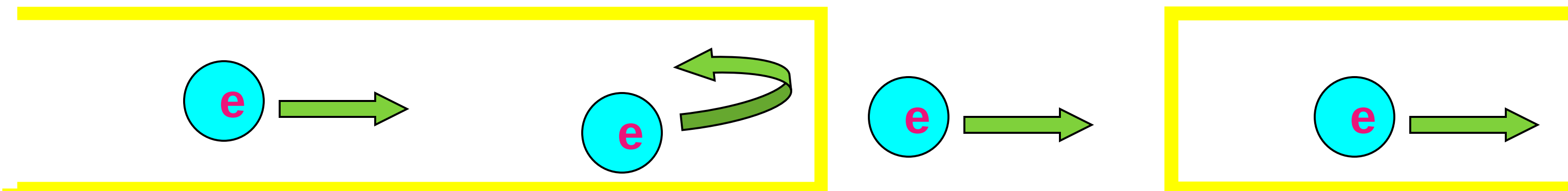


$$I = \mathrm{const}[|\Gamma_1|^2 + |\Gamma_2|^2]$$

Shot Noise

$$S=\int[\langle I(0)I(t)\rangle-\langle I\rangle^2]dt=q\langle I\rangle$$

Walter Shottky



L. Saminadayar et al., R. de Picciotto et al. (1997):q=e/3

Shot noise in MZ interferometer

$$S=q^*\langle I\rangle$$

$$q^*=\frac{e}{64}\sum p_i\sum\frac{1}{p_i}\ \text{diverges at some fields}$$

P. T. Zucker and D. E. Feldman, PRL **117**, 096802 (2016)

113 state

$$\Psi = \hat{A}\exp\left(-\frac{1}{4l^2}\sum[|z_{l\downarrow}|^2 + |z_{a\uparrow}|^2]\right)\prod_{k<l}(\partial_{z_{k\downarrow}} - \partial_{z_{l\downarrow}})^2 \prod_{a<b}(\partial_{z_{a\uparrow}} - \partial_{z_{b\uparrow}})^2 \times$$

$$\prod_{k<l}(z_{k\downarrow} - z_{l\downarrow})^3 \prod_{a<b}(z_{a\uparrow} - z_{b\uparrow})^3 \prod_{k,a}(z_{k\downarrow} - z_{a\uparrow})^3$$

Similar to the 112 state at $\nu = 2/3$: X. G. Wu *et al.*, PRL **71**, 153 (1993); I. A. McDonald and F. D. M. Haldane, PRB **53**, 15845 (1996).

K-matrix: $K = \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}$

2 types of anyons

$$\theta_{12} = 2\pi(K^{-1})_{12} = \frac{3\pi}{4}$$

$$\theta_{11} = 2\pi(K^{-1})_{11} = -\frac{\pi}{4}$$

G. Yang and D. E. Feldman, PRB **90**, 161306 (2014)

Transport

$$L = -\frac{1}{4\pi}\int dtdx\,[2\partial_t\varphi_c\partial_x\varphi_c - \partial_t\varphi_n\partial_x\varphi_n + 2v_c(\partial_x\varphi_c)^2 + v_n(\partial_x\varphi_n)^2 + 2v_{cn}\partial_x\varphi_c\partial_x\varphi_n]$$

φ_c and φ_n are charged and neutral modes;
v_c and v_n are their speeds, and v_{cn} is their interaction strength;
charge density $\rho = e\,\partial_x\varphi_c/2\pi$

√ The minus sign in the action reflects an upstream mode

?

$$g_\pm = \frac{1}{\sqrt{1-c^2}}\left(\frac{3}{8} \pm \frac{c}{2\sqrt{2}}\right);\; c = \frac{\sqrt{2}v_{cn}}{v_c+v_n}$$

$c = 0$ would agree with experiment. But $c \neq 0$!

√ Explanation: $v_c \gg v_n, v_{cn}$ because the charged mode participates in long-range Coulomb interaction and the neutral mode does not. Thus $c \ll 1$.

Closing Argument

- PH-Pfaffian topological order is consistent with all experiments
- Numerics with the particle-hole symmetric Hamiltonians supports states that break the particle-hole symmetry
- Realistic Hamiltonians have no symmetry
- The ground state is not symmetric, yet the topological order is compatible with the particle-hole symmetry