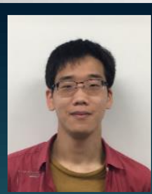
Half-integer thermal quantum Hall effect in a Kitaev quantum spin liquid

T. Yokoi, S.Ma, T. Ohnishi, Y. Kasahara, Y. Matsuda,
Department of Physics, Kyoto Univ., Japan




K. Sugii
ISSP, Univ. of Tokyo, Japan

Y. Mizukami, O. Tanaka, T. Shibauchi
Department of Adv. Materials Science, Univ. of Tokyo, Japan

N. Kurita, H. Tanaka — Single crystals
Department of Physics, Tokyo Institute of Technology, Japan

J. Nasu — Theory
Department of Physics, Yokohama National University, Japan

Y. Motome — Theory
Department of Applied Physics, Univ. of Tokyo, Japan

C. Hickey, S. Trebst — Theory
Department of Physics, Univ. of Cologne, Germany

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1. Kitaev quantum spin liquid
2. Half-integer thermal quantum Hall effect in α-$RuCl_3$

   Topologically protected chiral edge current of Majorana fermions

3. Additional test of the Kitaev physics in α-$RuCl_3$

   Influence of non-Kitaev interactions on the quantization

   Comparison of topological Chern number of α-$RuCl_3$ with that of Majorana band of the Kitaev model

Y. Kasahara *et al*., Phys. Rev. Lett. **120**, 217205 (2018).
Y. Kasahara *et al*., Nature **559**, 227 (2018).
T. Yokoi *et al*. a preprint.

Kitaev quantum spin liquid

S = 1/2 spin on honeycomb lattice with three inequivalent bonds

A. Kitaev, Ann. Phys. **321**, 2 (2006).

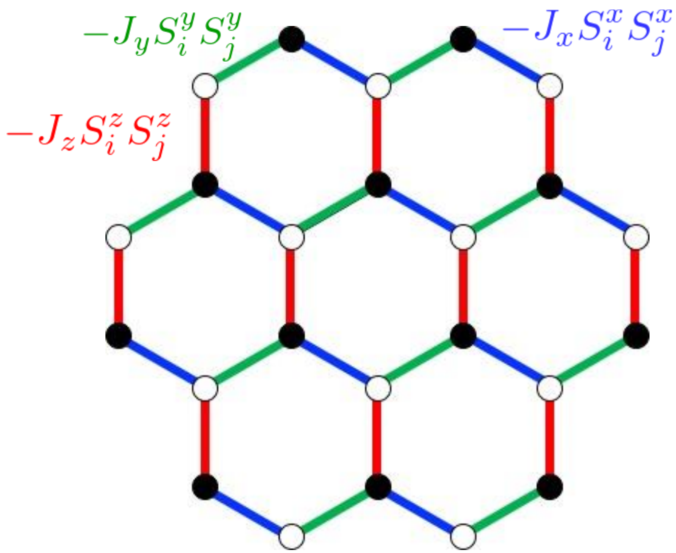
$$\mathcal{H} = -J_x \sum_{<ij>_x} S_i^x S_j^x - J_y \sum_{<ij>_y} S_i^y S_j^y - J_z \sum_{<ij>_z} S_i^z S_j^z$$

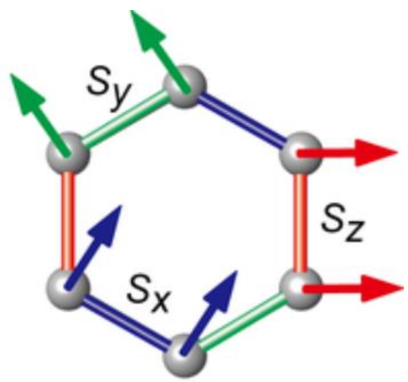
Kitaev interaction

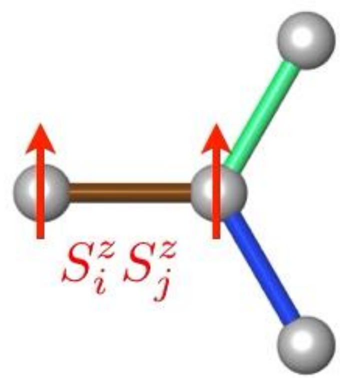
Bond-dependent Ising-like

Each bond favors different directions of *x*,*y* and *z*. → **Exchange frustration**

2D Honeycomb







Exactly solvable ground state of a quantum spin liquid

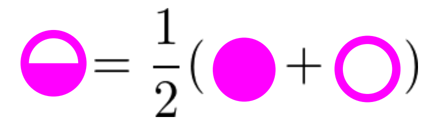
Why is the Kitaev quantum spin liquid interesting?

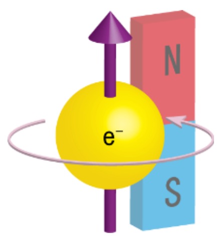
Topological order with emergent fractionalized particles

Majorana fermions

A particle is its own antiparticle $\gamma^\dagger = \gamma$

Half of conventional fermion $\gamma = \frac{1}{2}(f^\dagger + f)$





Fractionalization of electron ***spin***

Fractionalization of ***charge*** in fractional QHE

Non-Abelian anyons

Neither boson nor fermion

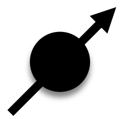
Identical to those of

p_x+ip_y superconductor N. Read and D. Green, Phys. Rev. B 61, 10267 (2000).

5/2 fractional quantum Hall state G. Moore and N. Read, Nucl. Phys. B 360, 362 (1991).

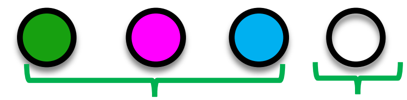
Fractionalization of spin into Majorana fermions

Spin-1/2 particle $\Rightarrow$ Fermions $\Rightarrow$ Majorana fermions



$f_1 \quad f_2$

$b^x \quad b^y \quad b^z \quad c$



localized itinerant

$$|\uparrow\rangle = |00\rangle$$

$$|\downarrow\rangle = |11\rangle$$

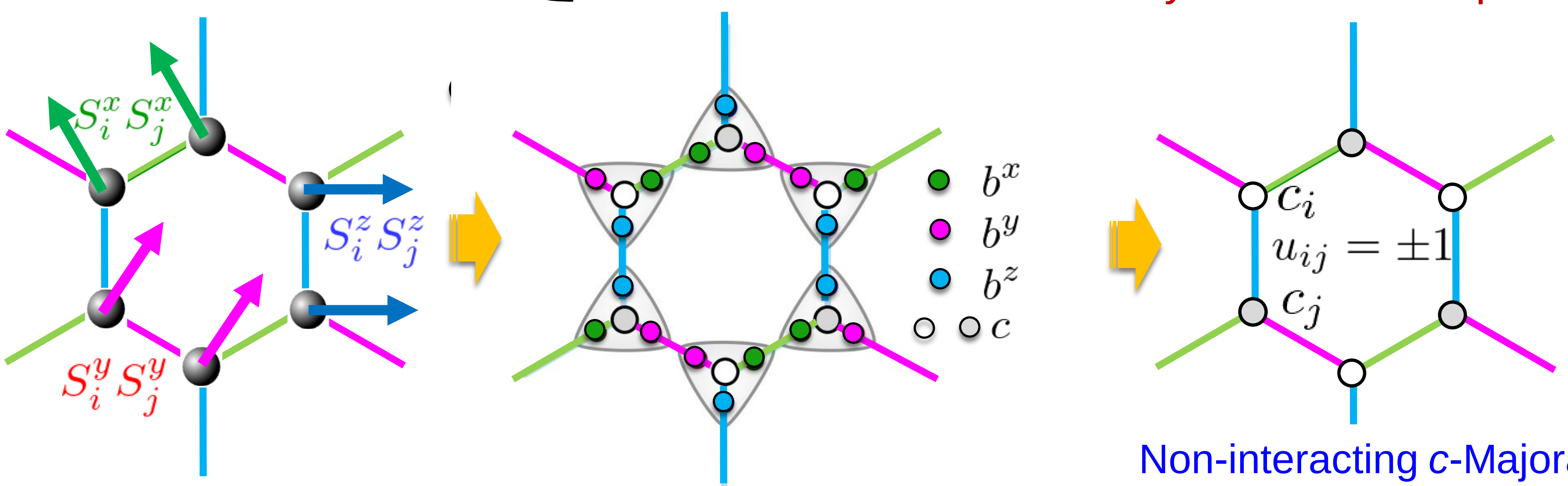
$$|11\rangle = f_1^\dagger f_2^\dagger |00\rangle$$

$$\begin{cases} c = f_1 + f_1^\dagger \\ b^x = i(f_1^\dagger - f_1) \\ b^y = f_2 + f_2^\dagger \\ b^z = i(f_2^\dagger - f_2) \end{cases}$$

$\{b^x, b^y, b^z, c\}$ anticommuting

$$b^{\alpha\dagger} = b^\alpha \quad c^\dagger = c$$

They are their own particles

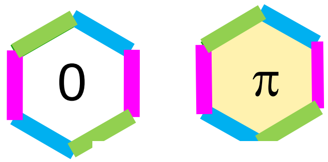


Non-interacting c-Majorana

Two types of Majorana fermions

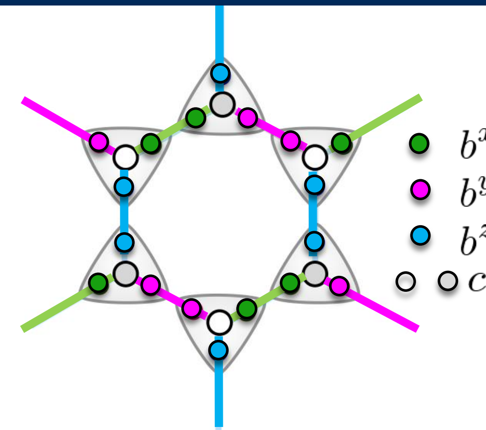
Spin 1/2 → Itinerant Majorana c

Spin 1/2 → Localized Majorana b^x, b^y, b^z

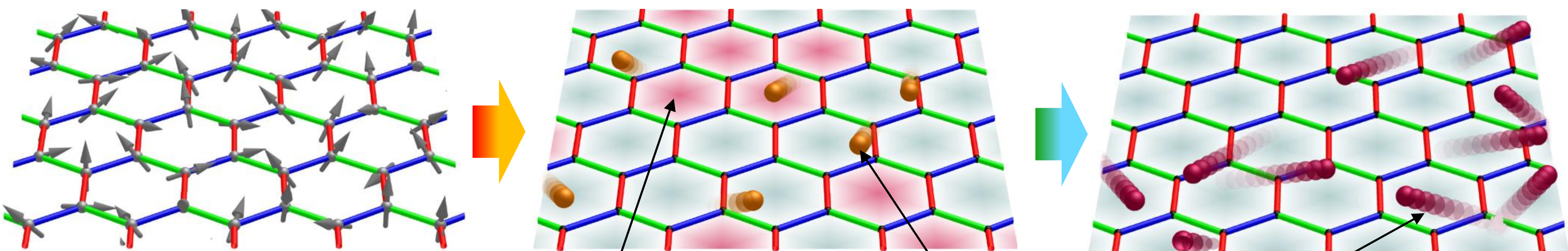


Z_2 vortex (Vison)

Energy gap: $\Delta_F \sim 0.1 J_K$



$T > J_K$	$T < J_K$	$T \ll J_K$
Paramagnetic	Spin fractionalization	Quantum spin liquid

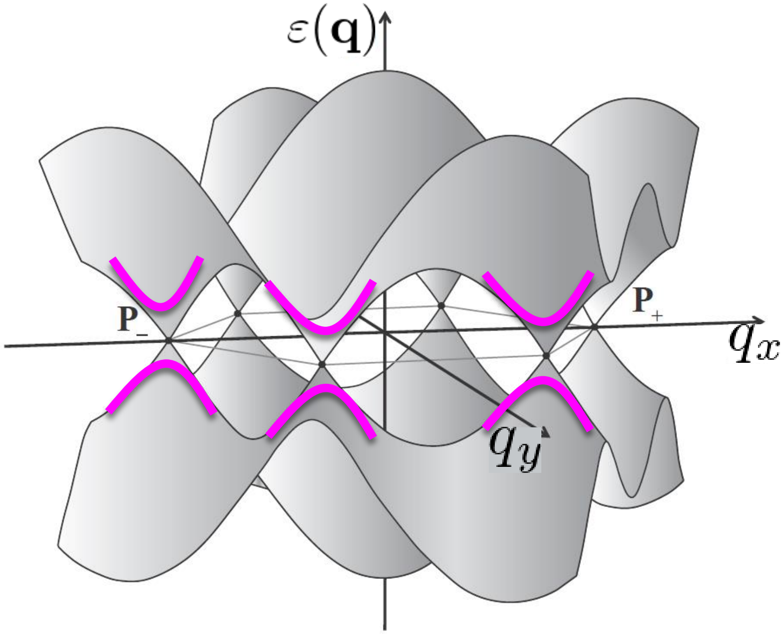


Z_2 vortex

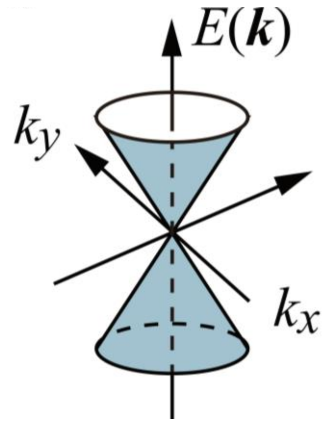
Itinerant Majorana fermions

J. Nasu and Y.Motome

Energy dispersion of itinerant Majorana

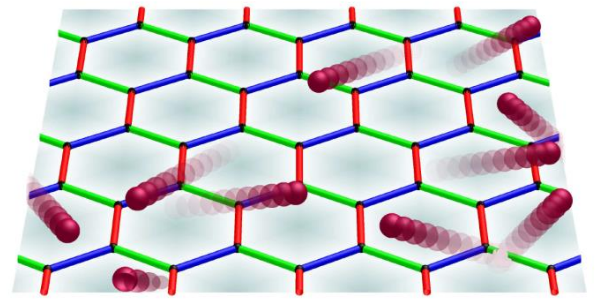


H = 0



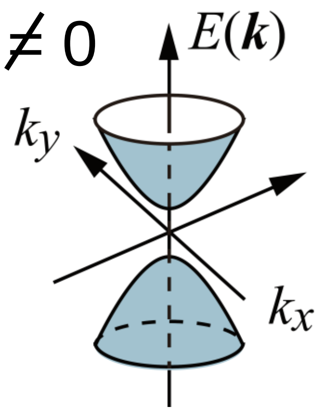
Dirac cone

Itinerant Majorana



Semimetal

H ≠ 0



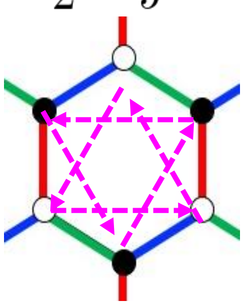
Third perturbation of Zeeman energy

$$\mathcal{H}_h^{3\mathrm{rd}} \sim -\frac{h_x h_y h_z}{J^2} \sum_{j,k,l} \sigma_j^x \sigma_k^y \sigma_l^z$$

Three body spin correlation

$$\sim -\frac{i}{2} \frac{h_x h_y h_z}{J^2} \sum_{j,k} c_j c_k$$

2nd nearest neighbor hopping of Majorana



QHE without Landau level

cf. F.D.M. Haldane, PRL 61, 2015 (1988).

Topologically non-trivial state

Chern number $C_h = \frac{\Delta}{|\Delta|} = \pm 1$

Majorana Chern insulator

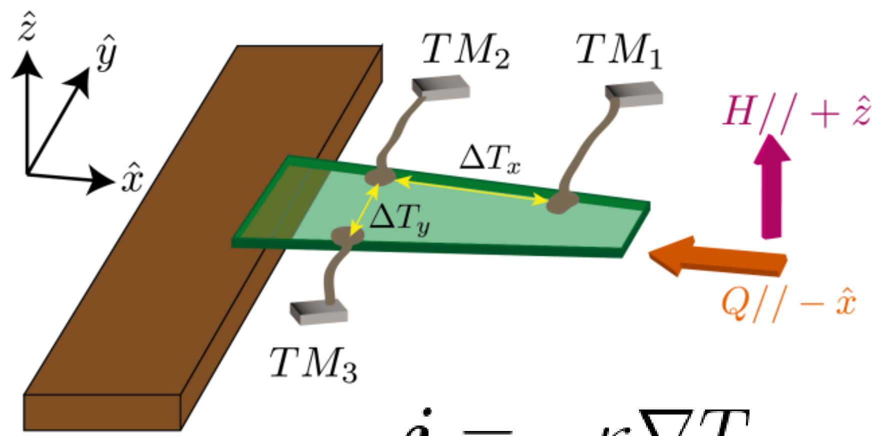
How to detect Majorana

Majorana fermions: **Charge neutral**.
do not respond to electric field but can carry the heat (entropy).

Thermal transport measurements

Thermal Hall conductivity

Thermal analog of the electronic Hall effect



$$\boldsymbol{j} = -\kappa \nabla T$$

$$\begin{pmatrix} j_x \\ 0 \end{pmatrix} = \begin{pmatrix} \kappa_{xx} & \kappa_{xy} \\ -\kappa_{xy} & \kappa_{xx} \end{pmatrix} \begin{pmatrix} -\nabla_x T \\ -\nabla_y T \end{pmatrix}$$

Phonon usually does not contribute
to the thermal Hall effect

Thermal Hall effect is absent in
conventional insulating magnets.

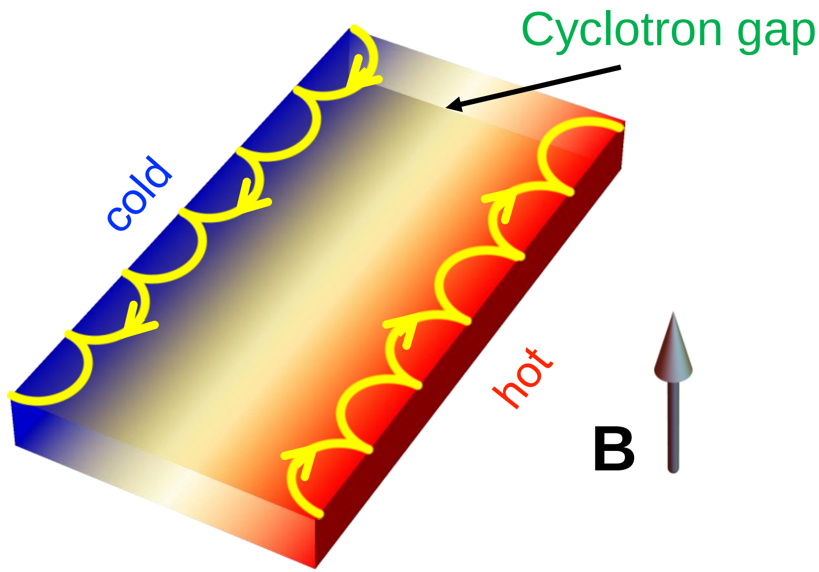
J.H. Han and H. Lee, J.Phys. Soc.Jpn **86**, 011007 (2017).

**Thermal Hall conductance is quantized
in Kitaev spin liquid.**

A. Kitaev, Ann. Phys. **321**, 2 (2006).

2D electron gas and Kitaev spin liquid

Integer quantum Hall state



$$\sigma_{xy}^{2D} = \nu \frac{G_0}{2} \qquad G_o = \frac{2e^2}{h} \qquad \nu = 1, 2, 3 \cdots$$

Quantum conductance

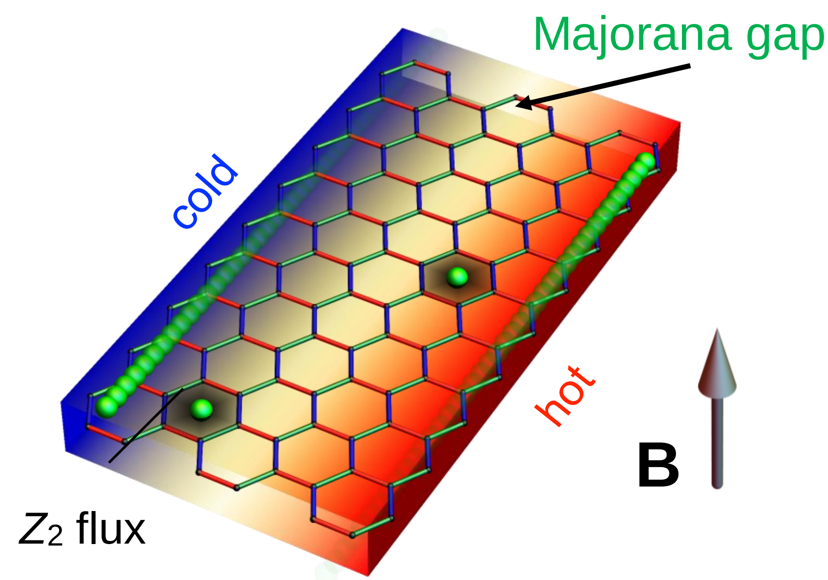
$$\kappa_{xy}^{2D} = \nu K_0 \qquad K_0 = \frac{\pi^2 k_B^2}{3h} T$$

Quantum thermal conductance

M. Banerjee *et al*. Nature **545**, 75 (17).
C.L. Kane and M.P.A. Fisher, PRB **55**, 15832 (97).

Chiral edge currents of electrons

Kitaev quantum spin liquid state



Chiral edge currents of Majorana fermions

$$\kappa_{xy}^{2D} = \frac{1}{2} K_0$$

Half integer thermal QHE

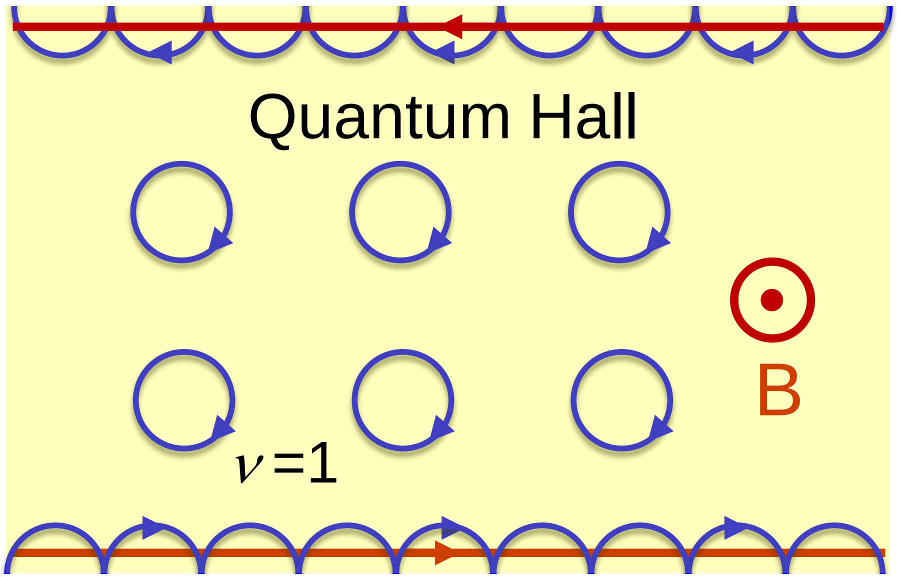
One Majorana = "*half*" of a fermion

$$\gamma = \frac{1}{2}(f^\dagger + f)$$

$$◓ = \frac{1}{2}(● + ○)$$

Chiral edge currents

2D electron gas



Skipping motion of electron

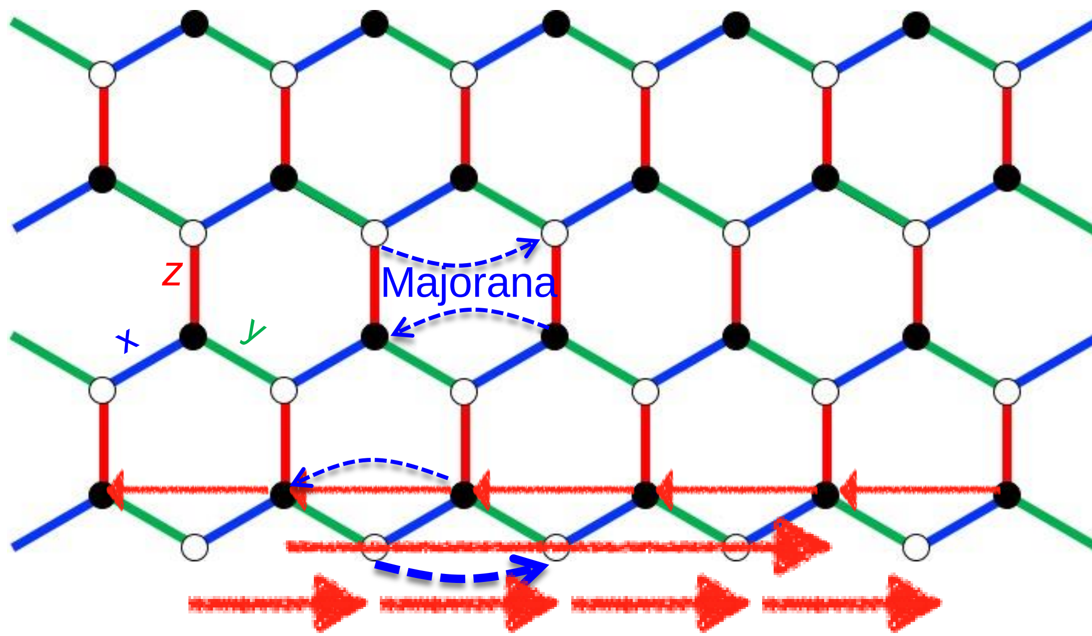
Lorentz force

Kitaev quantum spin liquid

$$\mathcal{H}_h^{3\mathrm{rd}} \sim -\frac{h_x h_y h_z}{J^2} \sum_{j,k,l} \sigma_j^x \sigma_k^y \sigma_l^z = -\frac{i}{2} \frac{h_x h_y h_z}{J^2} \sum_{j,k} c_j c_k$$

Three body spin correlation

2nd NN hopping of Majorana



Two contributions do not cancel out.

Zeeman

1. Kitaev quantum spin liquid

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Topologically protected chiral edge current of Majorana fermions

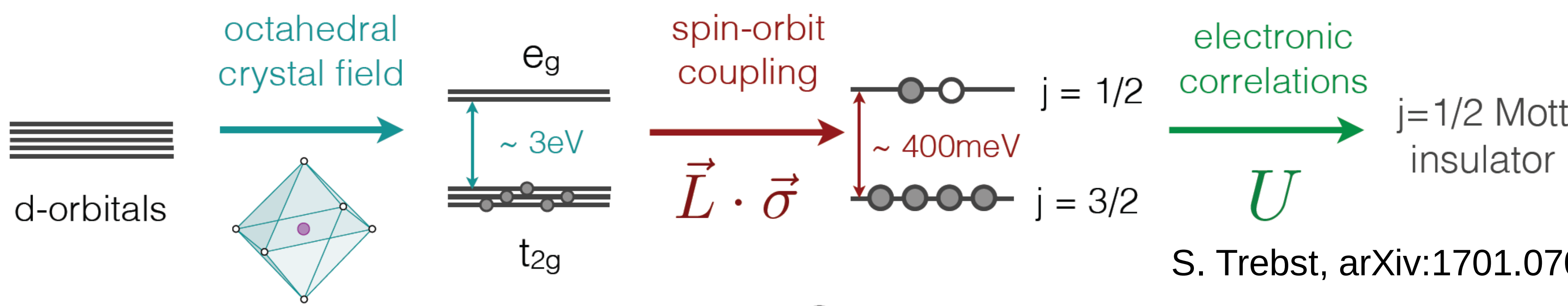
3. Additional test of the Kitaev physics in α-$RuCl_3$

Influence of non-Kitaev interactions on the quantization

Comparison of topological Chern number of α-$RuCl_3$ with that of Majorana band of the Kitaev model

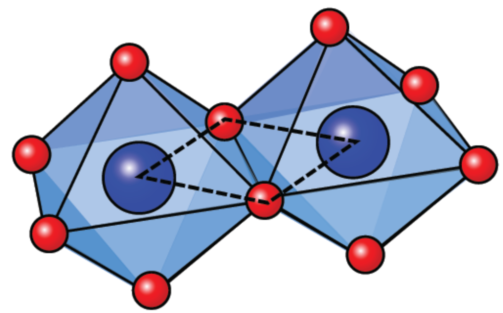
Kitaev-type interactions in materials

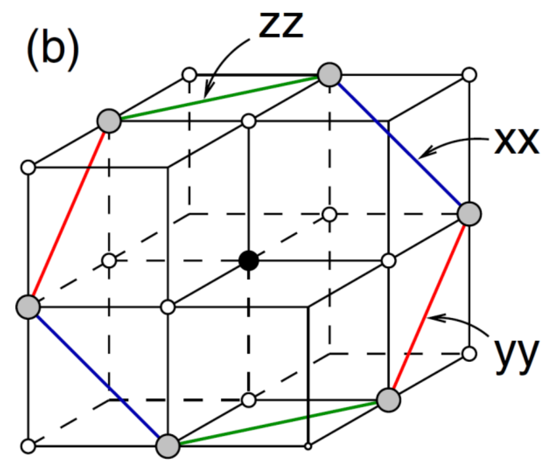
Half filled d^5 orbital + t_{2g} orbitals + Strong spin-orbit coupling + Strong el-el interaction

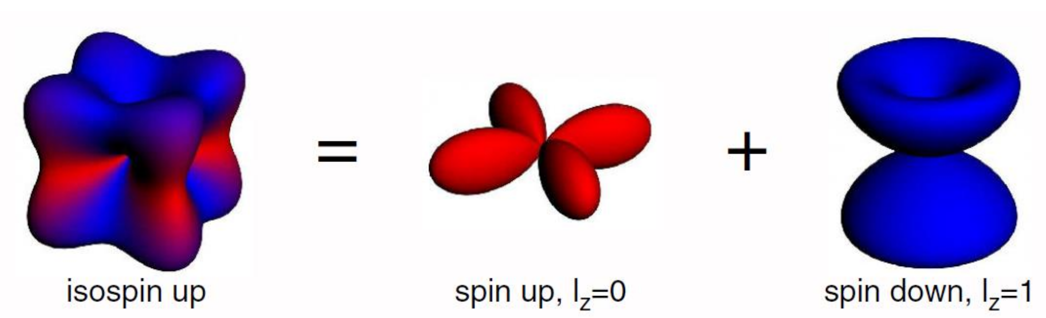


S. Trebst, arXiv:1701.07056

+ 90 deg bond by edge-sharing octahedra







G.Jackeli & G.Khaliullin, PRL**102**, 017205 (2006).

Heisenberg→ Ising

A prime candidate for hosting an approximate Kitaev QSL

Strongly spin-orbit coupled j=1/2 Mott insulator α-$RuCl_3$

$$\mathcal{H} = \sum[J\vec{S}_i \cdot \vec{S}_j + J_K S_i^\gamma S_j^\gamma + \Gamma(S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha)]$$

Heisenberg | Kitaev | off-diagonal

$J_K = -6.7\,\mathrm{meV}$ Ferromagnetic

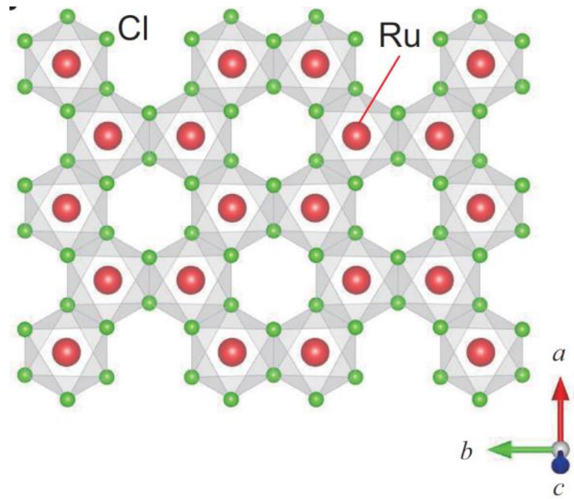
$J = -1.4\,\mathrm{meV}$

$\Gamma = 3.5\,\mathrm{meV}$

Significant Kitaev term

Seung-Hwan Do, Sungdae Ji *et al.*

S. M. Winter *et al.*, PRB **93**, 214431 (2016).



K.W. Plumb et al. PRB **90**, 041112 (2014).

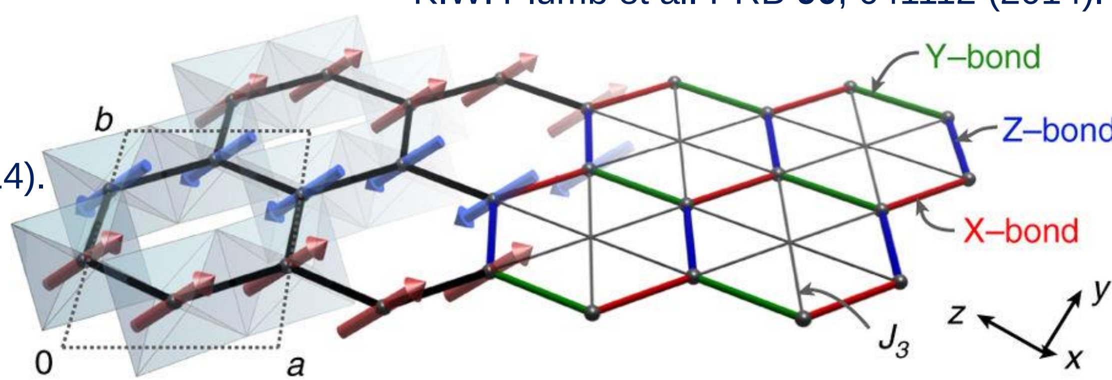
Non-Kitaev interaction

AFM order with zigzag spin structure at T_N ~ 7.5 K

J.G. Rau, E. K-H Lee and H-Y Kee, PRL 112, 077204 (2014).

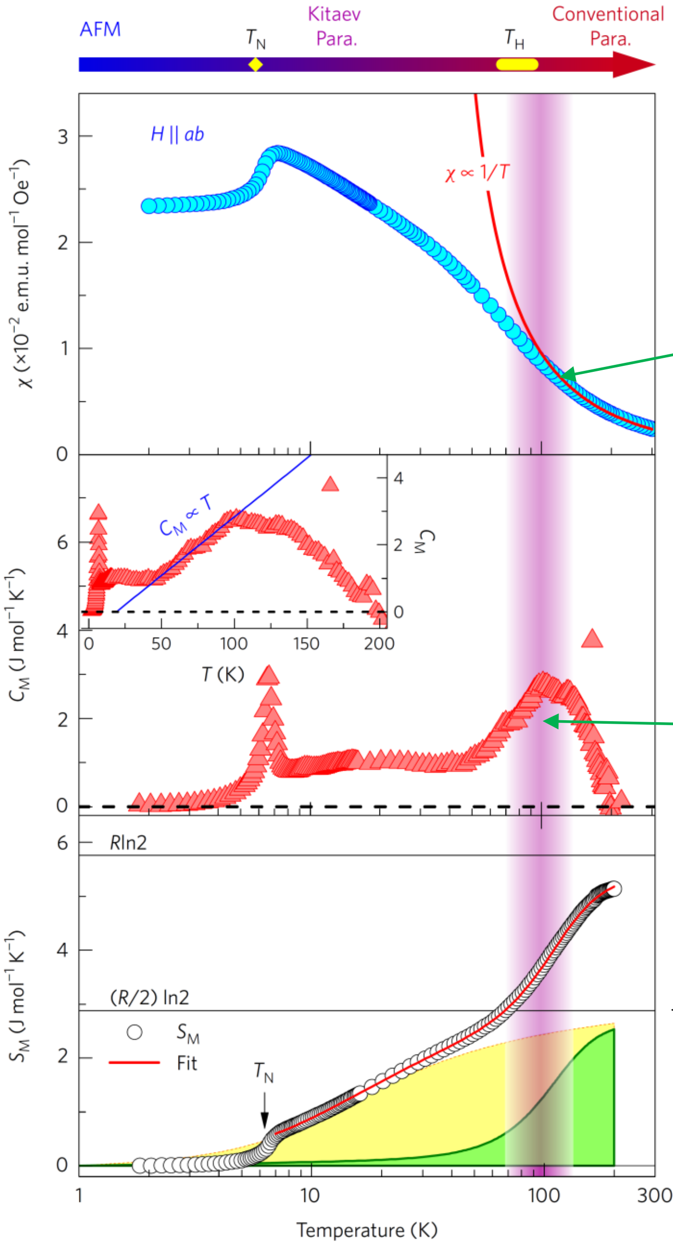
A. Catuneanu *et al*. npj Quantum Materials **3**,23 (2018)

Z-X. Liu and B. Normand, PRL **120**, 187201 (2018).

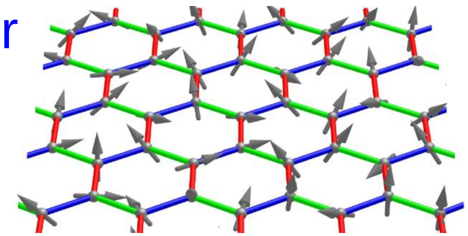


S. M. Winter *et al*., Nature Commun. 8, 1152 (2017).

Signature of fractionalization in α-$RuCl_3$

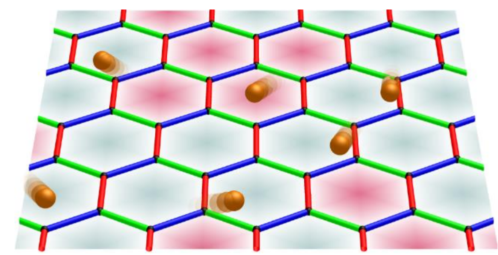


Curie-like behavior



Phonon reference $ScCl_3$ $C_M \propto T$

Fermi degeneracy



Half of *R*ln2 in S=1/2 spin system

$\leftarrow \frac{R}{2} \ln 2$

Entropy release due to itinerant Majorana fermions

Fingerprint of itinerant Majorana

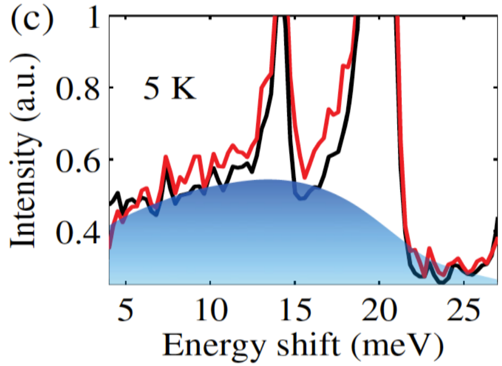
S-H.Do *et al*. Nature Phys. **13**, 1079 (2017).

Phonon reference $RhCl_3$+*ab initio* calculation of phonon

S. Widmann *et al*. Phys. Rev. B **99**, 094415 (2019).

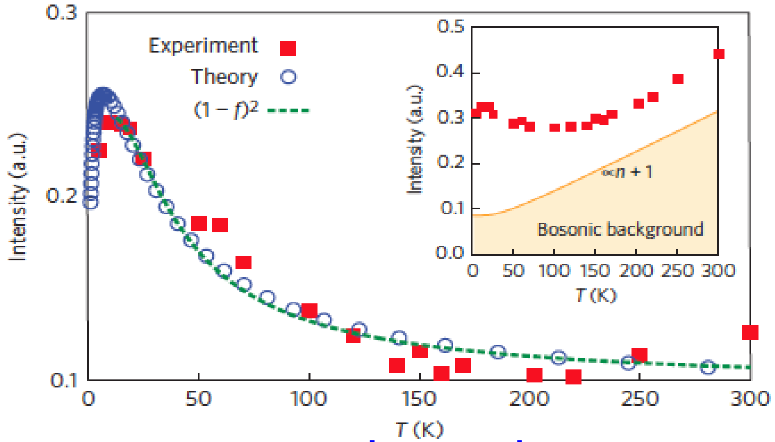
Signature of fractionalization in α-$RuCl_3$

Raman scattering



Broad magnetic continuum

L. J. Sandilands *et al.*, PRL **114**, 147201 (2015).



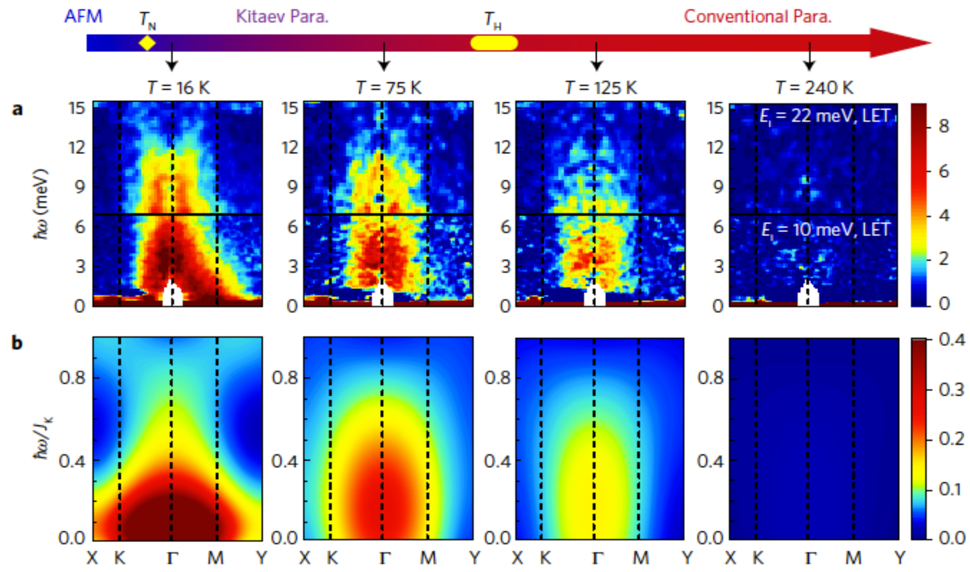
Temperature dependence

J. Nasu *et al.*, Nat. Phys. **12**, 912 (2016).

Energy dependence

Y, Wang, *et al*. arXiv:1809.07782

Inelastic neutron scattering



Magnetic continuum below ~ J_K/k_B

Seung-Hwan Do, Sungdae Ji *et al.*, Nat. Phys. (2017).
A. Banerjee *et al*., Nat. Mater. **15**, 733 (2016).
A. Banerjee *et al*., Science **356**, 1055 (2017).

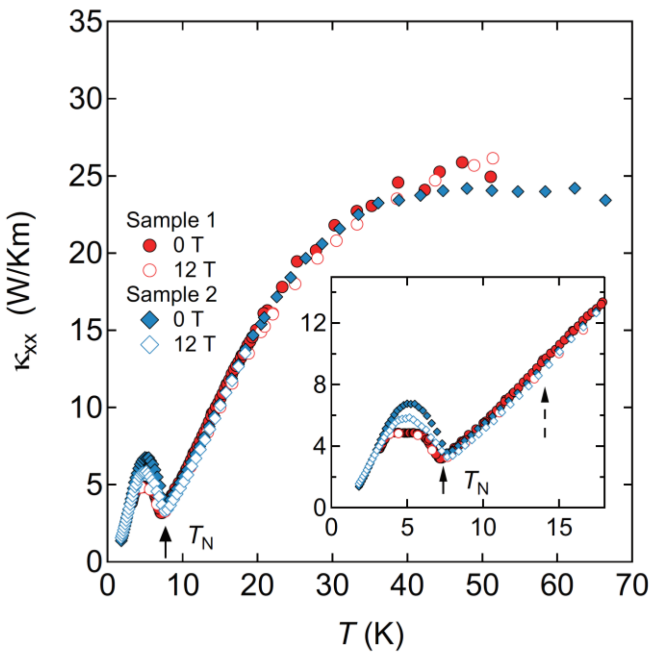
Inconsistent with bosonic magnons

Fermionic excitations → reflect the proximity to the Kitaev model

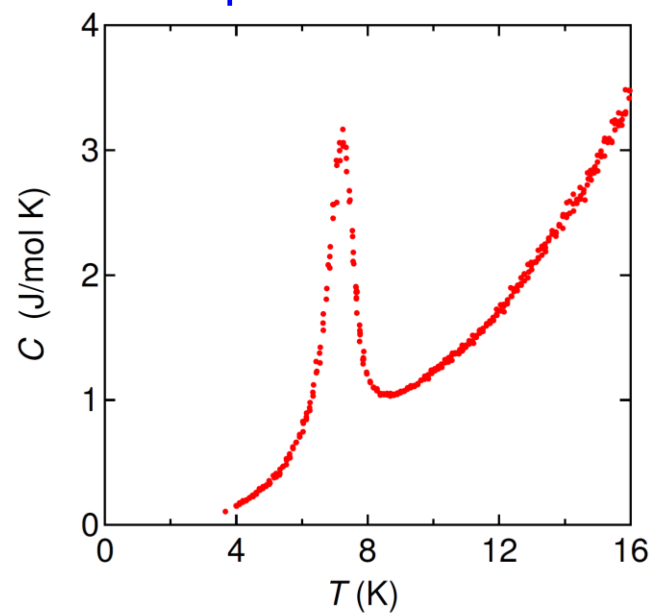
Measurements

Thermal conductivity

I.A. Lehy *et al*. PRL **118**, 187203 (2017).
D. Hirobe *et al.* PRB **95**, 241112(R) (2017).
Y. J. Yu *et al*. PRL **120**, 067202 (2018).



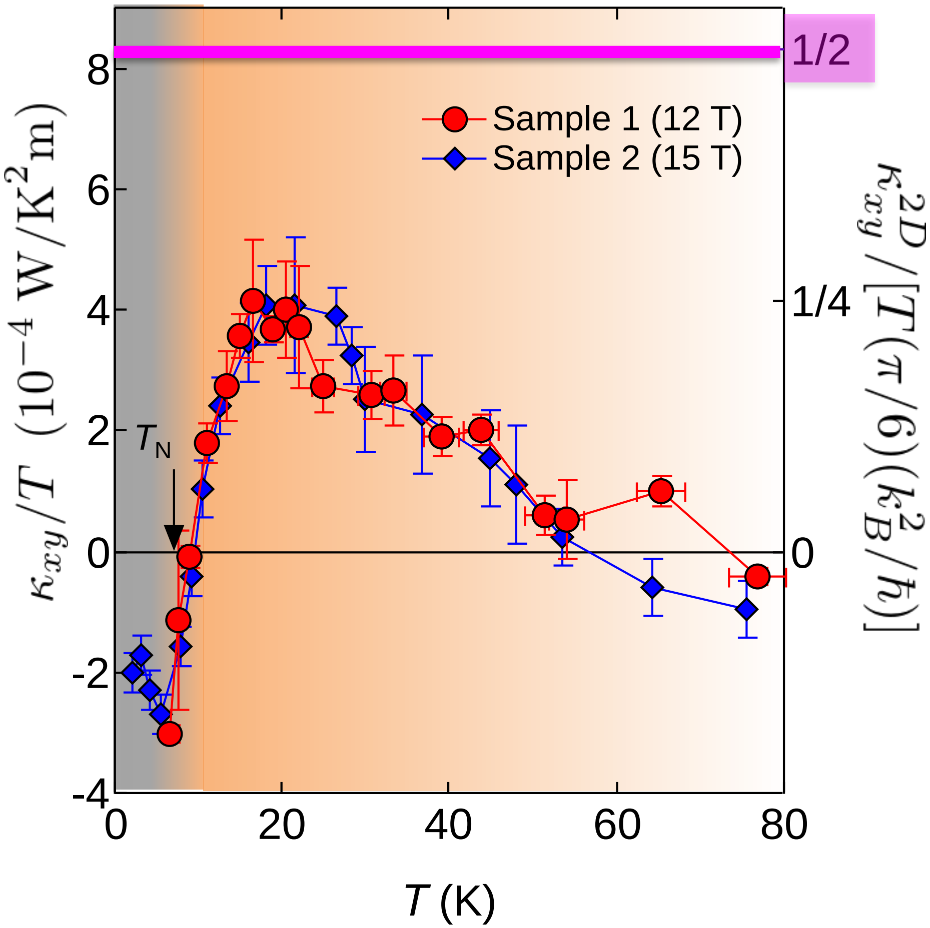
Specific heat

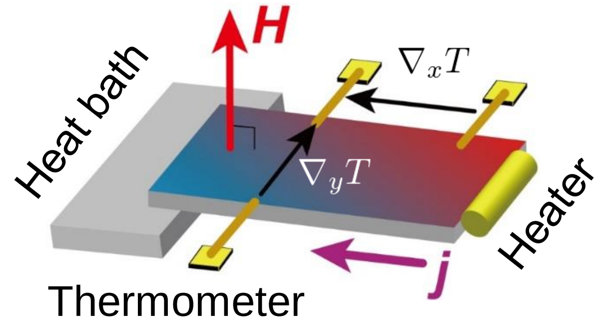


- Clear anomaly at T_N ~ 7.5 K
- No discernible anomaly at T_{N2}~ 14 K

k=(1/2,0.3/2) order
Staking every 2 layers

Thermal Hall conductivity κ_{xy} of α-$RuCl_3$





Non zero κ_{xy} below T~J_K !

Thermal Hall effect is usually absent in disordered insulating magnets.

➡ Non-trivial excitations

κ_{xy}/T appears to approach

half of the quantized value!

But…..

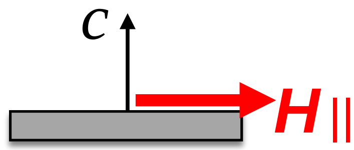
quantization is not attained.

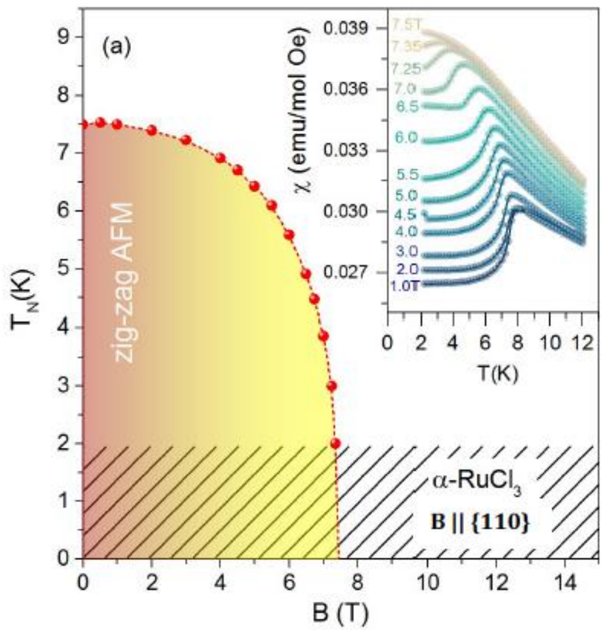
Low-temperature properties are masked by the magnetic order.

Y. Kasahara, Y.M. *et al*., PRL **120**, 217205 (2018).

See also R. Hentrich *et al*., Phys. Rev. B **99**, 085136 (2018).

Suppression of AFM order by in-plane field



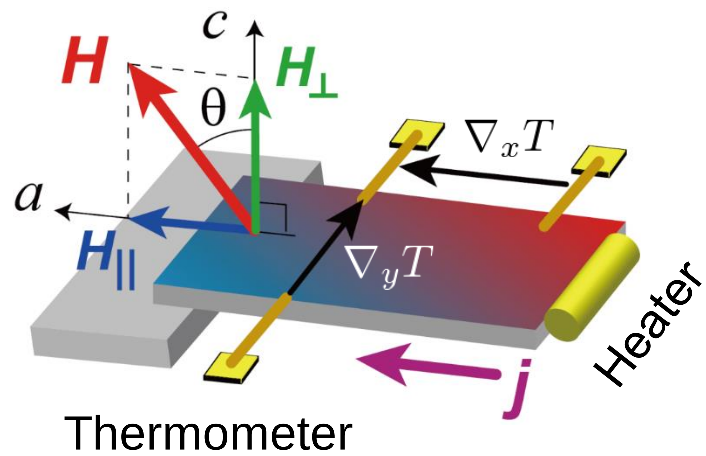


$H^*_{||}$ ~ 7-8 T

A. Banerjee *et al*., npj Quantum Materials **3**, 8 (2018)

M. Majumder *et al*., PRB **91**, 180401(R) (2015).

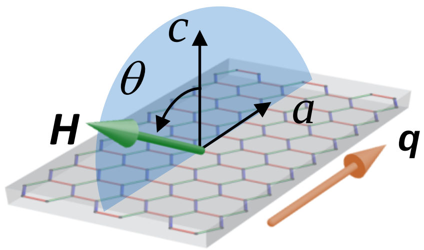
$\boldsymbol{H}_{||}$ suppresses the AFM ordering



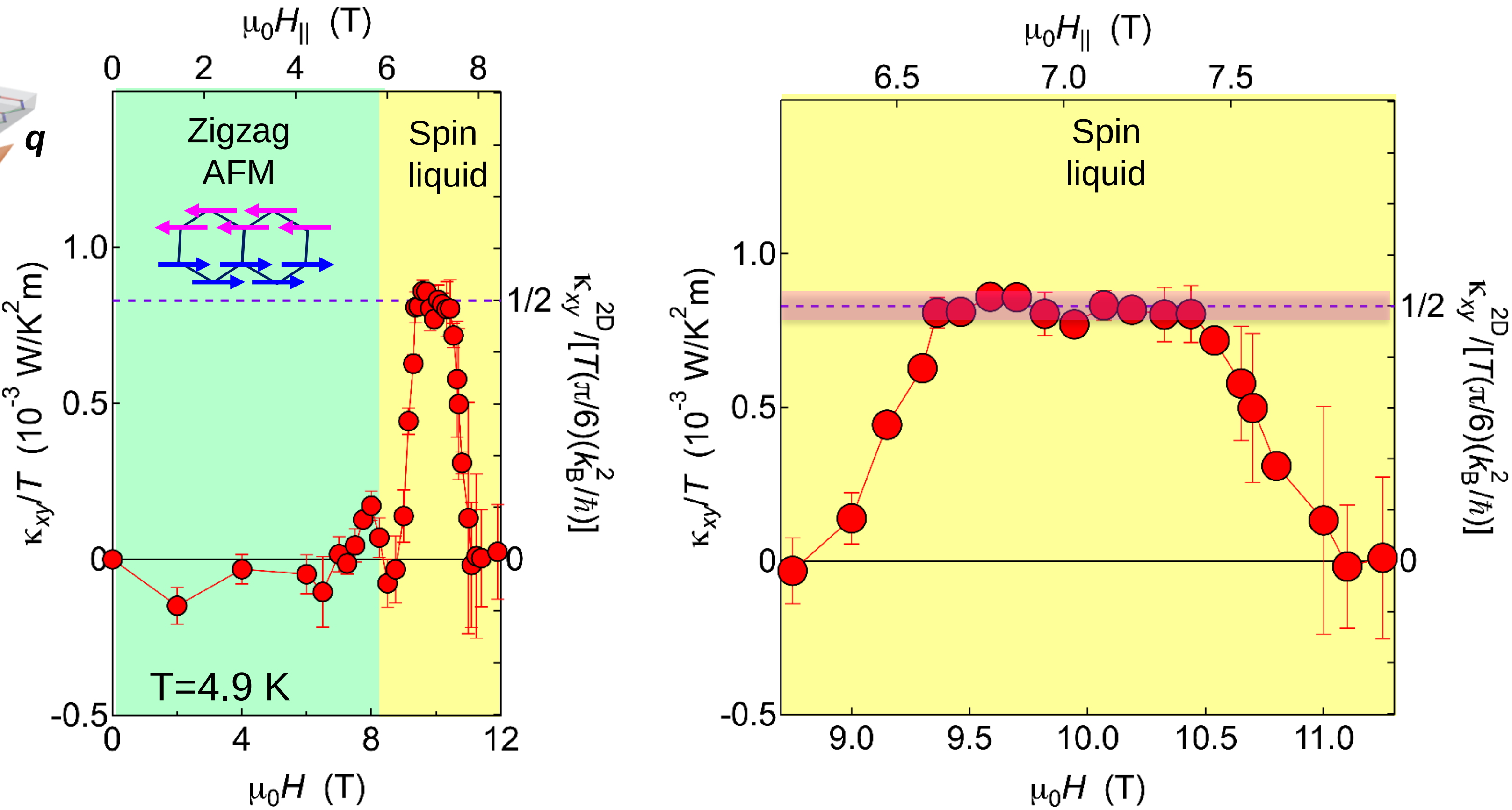
Thermal Hall effect in tilted fields

AFM order is suppressed by $H_{||}$

Half-integer quantized thermal Hall conductance plateau



$\theta = -45^\circ$



Thermal Hall plateau in the spin liquid state

$$\kappa_{xy}^{2D} = \frac{1}{2} K_0$$

$$K_0 = \frac{\pi^2 k_B^2}{3h} T$$

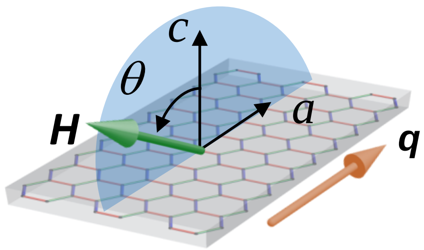
Quantum Thermal Conductance

Half of the quantized value for ν =1 of QHE

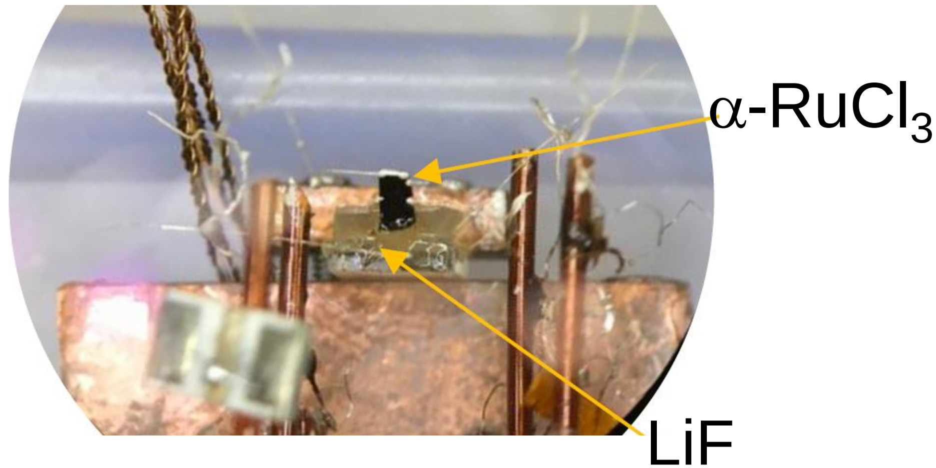
$$\kappa_{xy}^{2D} = K_0$$

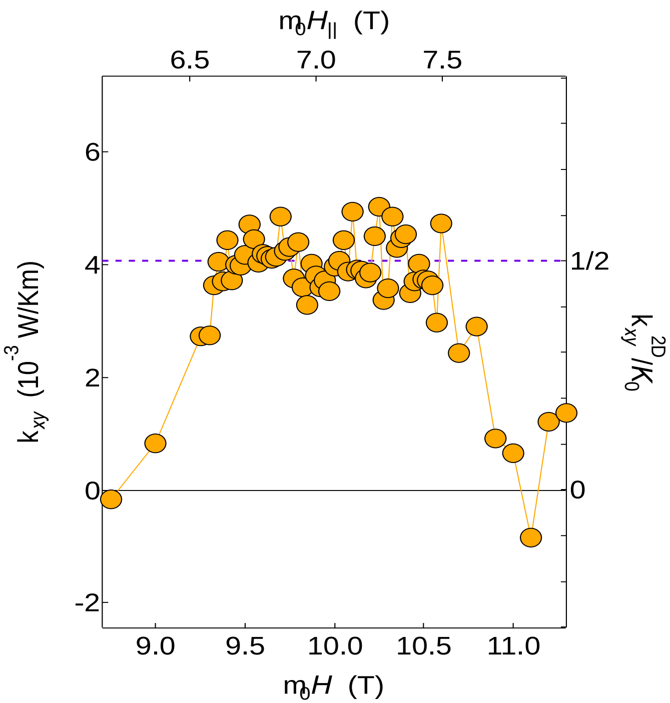
Y. Kasahara *et al*., Nature **559**, 227 (2018).

Half-integer quantized thermal Hall conductance plateau

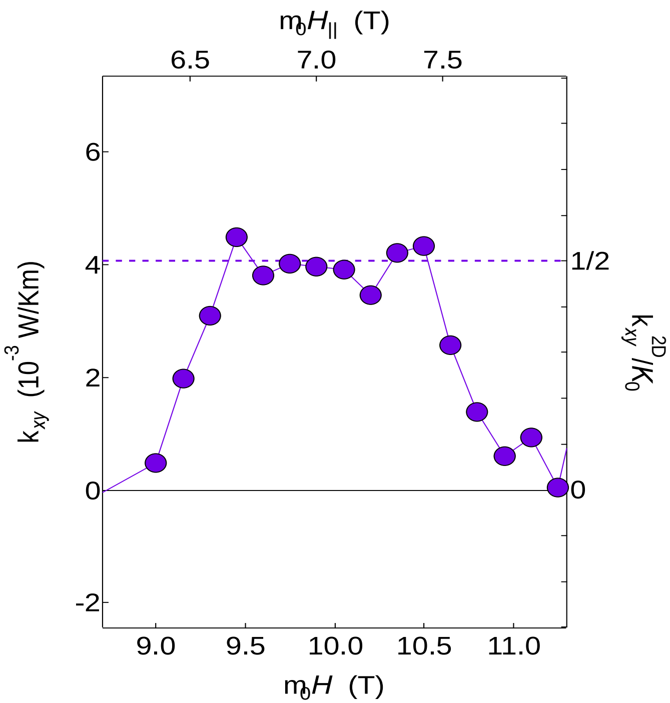


$\theta = -45°$

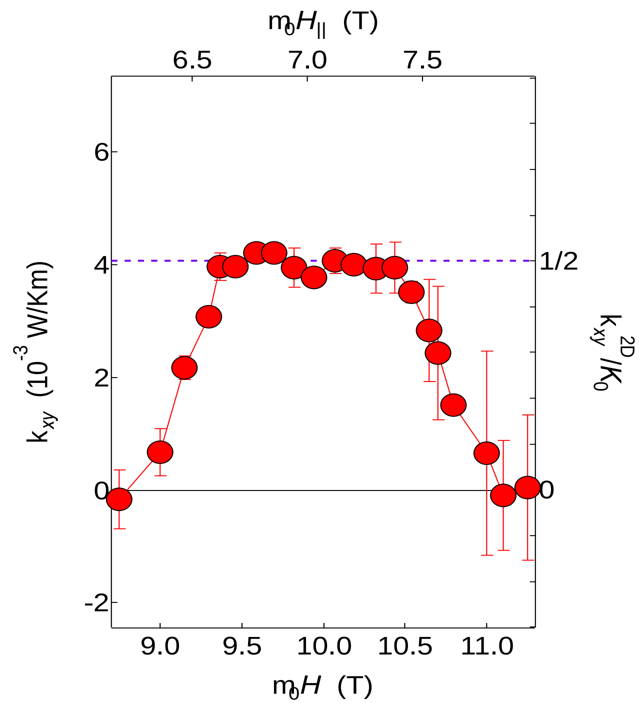




Raw data 1

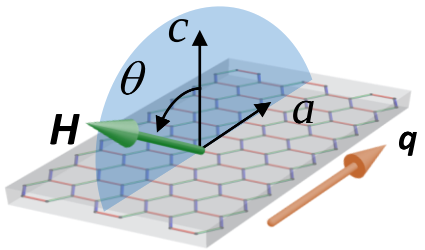


Raw data 2



Average for 5 measurements

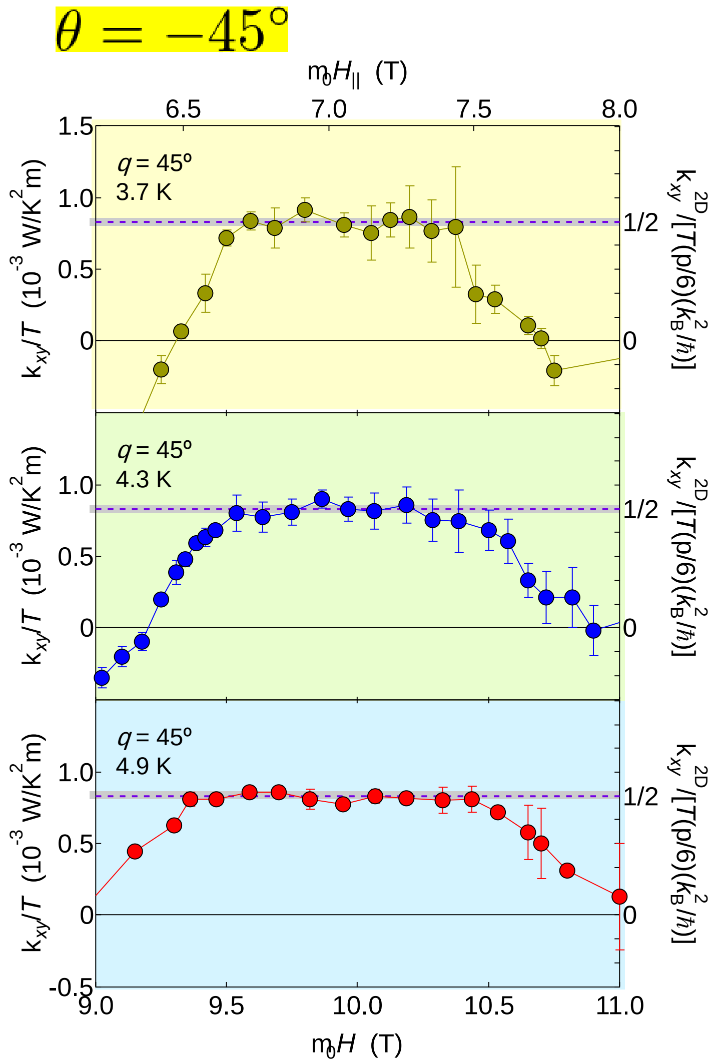
Half-integer quantized thermal Hall conductance plateau

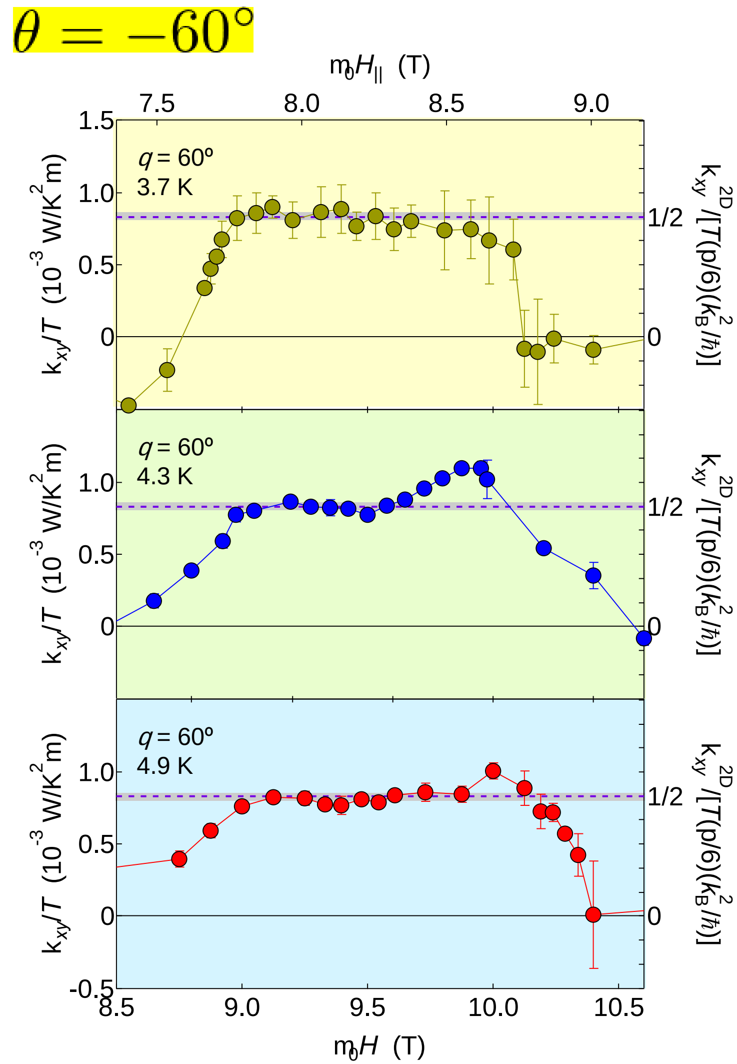


$$\kappa_{xy}^{2D} = \frac{1}{2} K_0$$

$$K_0 = \frac{\pi^2 k_B^2}{3h} T$$

Quantum Thermal Conductance





Insulator: Neutral quasiparticles

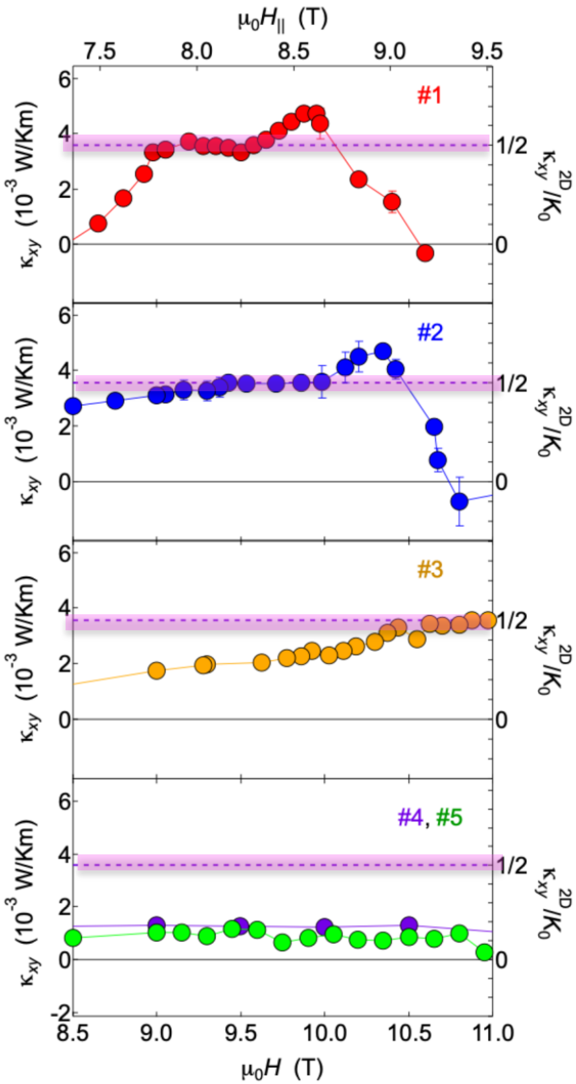
Quantized plateau: Topologically protected

Degrees of freedom: Half of conventional fermions

Majorana fermion

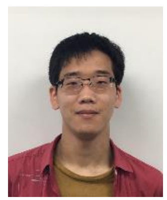
Reproducibility

Sample dependence $\theta = -60^\circ$



Crystal having large κ_{xx} is required to observe the ½ thermal QHE

Kyoto Univ.



Independent measurements by a different group

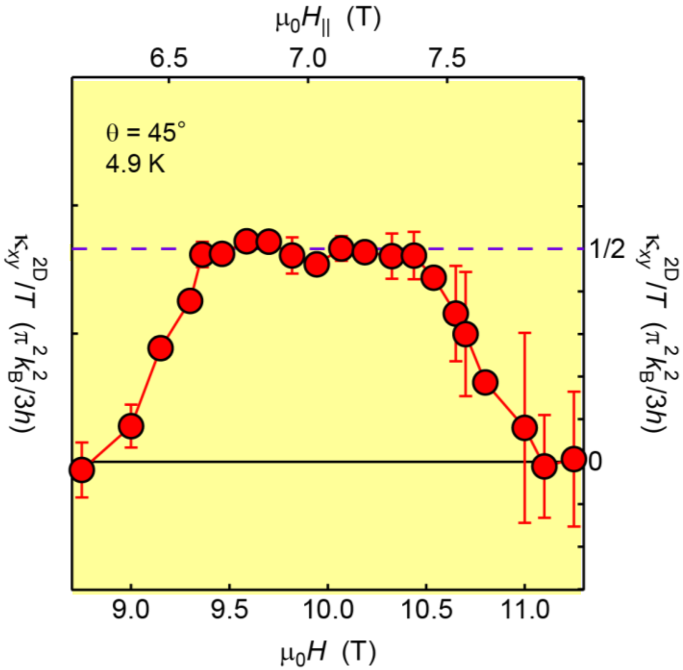
ISSP, Univ. of Tokyo

M. Yamashita *et al.* in preparation.

Crystal grown by N Kurita and H. Tanaka（Tokyo Institute of Technology)

The ½ quantization appears to be reproduced.

Non-Abelian anyon



$$\kappa_{xy}^{2D} = cK_0 \qquad K_0 = \frac{\pi^2 k_B^2}{3h} T$$

Quantum Thermal Conductance

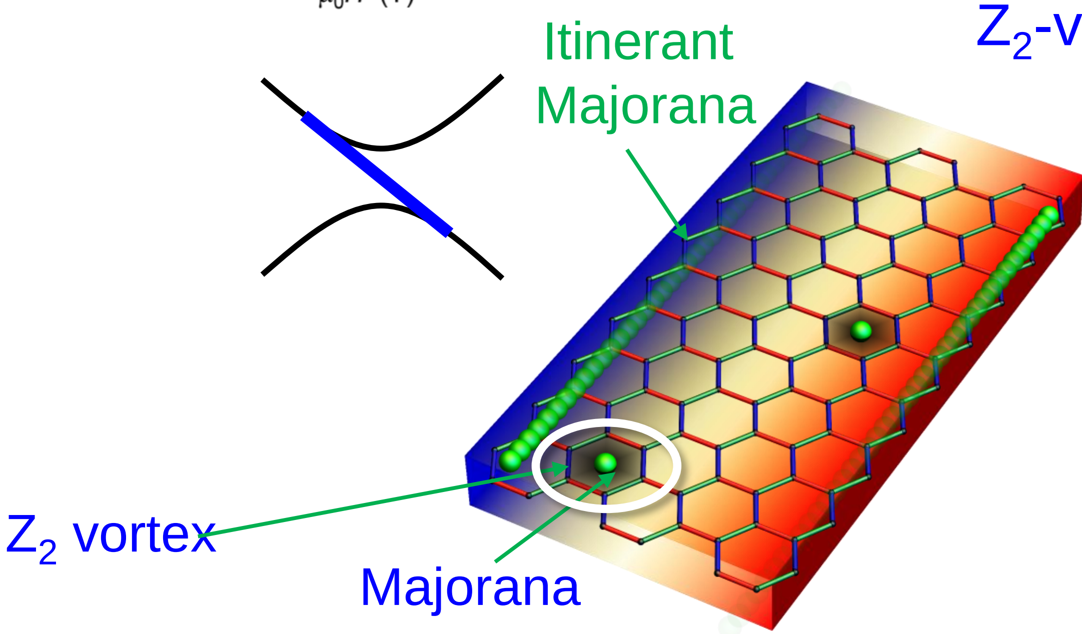
c: central charge

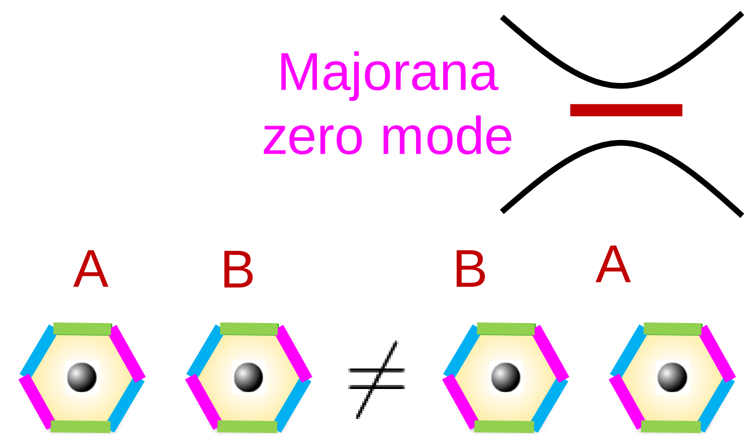
Quantization with $c < 1$

Non-Abelian anyon

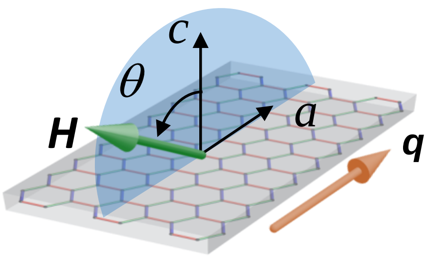
N. Read and D. Green, Phys. Rev. B **61**, 10267 (2000).

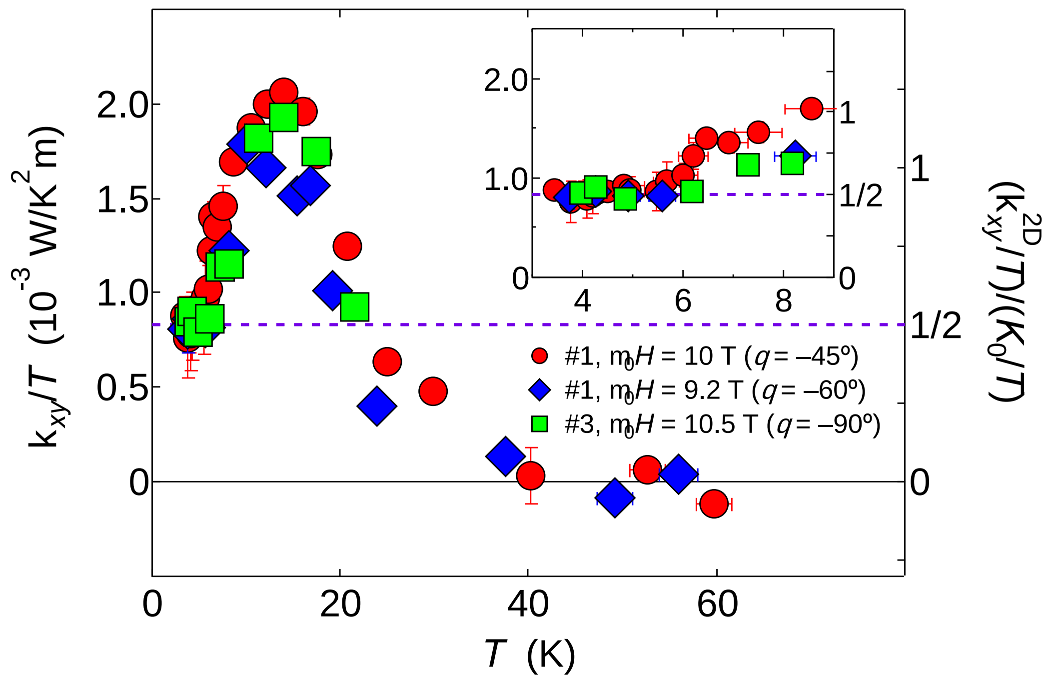
For non-trivial Chern numbers,
Z_2-vortices bind Majorana modes.





Half-integer quantized thermal Hall conductance





Quantization preserved up to ∼5 K

Large gap

NMR Δ ~30 K , Neutron Δ∼15 K at 9 T

N. Jansa *et al*., Nature Phys. (2018).
S.H. Baek *et al*. PRL **119**, 037201 (2017).

Quantization persists even in the presence of parallel phonon conduction.

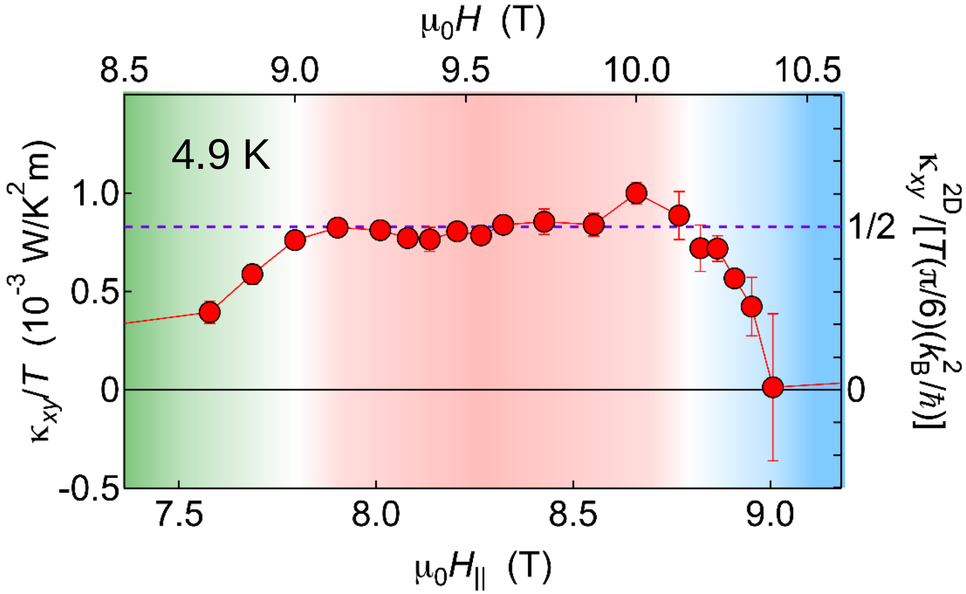
Y. Vinkler-Aviv and A. Rosch, Phys. Rev. X **8**, 021032 (2018).
M.Ye, G. B. Halász, L. Savary and L. Balents, Phys. Rev. Lett. **121**, 147201 (2018).

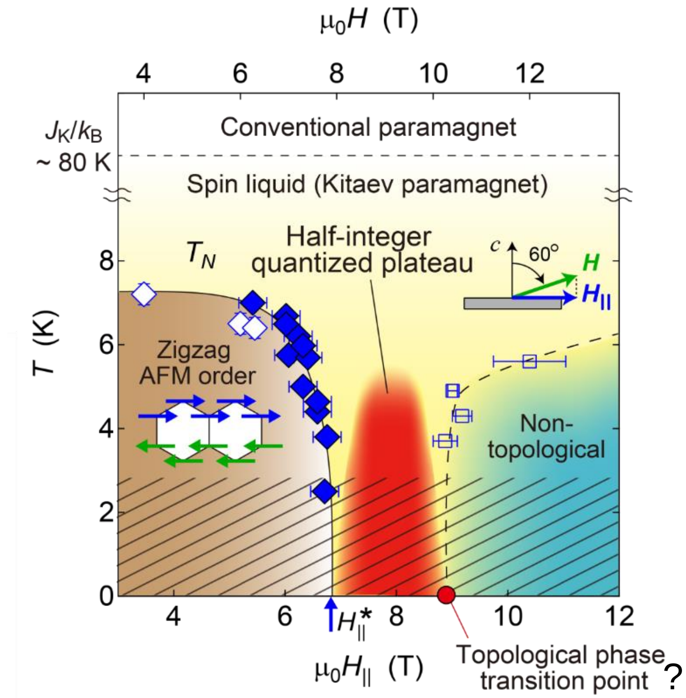
Enhancement from the quantized value above ~5 K

Signature of topological phase transition

A. Go.J. Jung and E-G. Moon, Phy. Rev. Lett. **122**, 147203 (2019).

Phase transition





Y. Kasahara et al. Nature 559, 227 (2018).

(I) $H_{\parallel}$ < 4.5 T (Magnetic order) :

κ_{xy} is small.

(II) Half-integer plateau

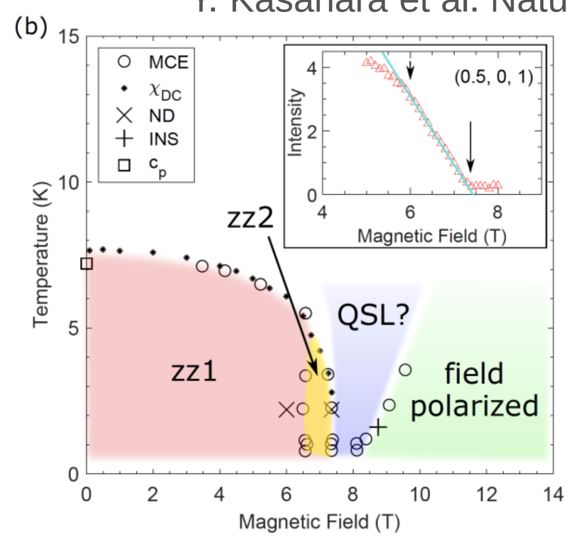
$$\kappa_{xy}^{2D} = \frac{1}{2} K_0 \quad K_0 = \frac{\pi^2 k_B^2}{3h} T$$

Topological phase transition?

(III) $H_{\parallel}$ > 9 T : $\kappa_{xy} = 0$

Forced ferromagnetic phase?

A new QSL state?



C. Balz *et al*. PRB **100** 060405(R) (2019).

1. Kitaev quantum spin liquid

2. Half-integer thermal quantum Hall effect in α-$RuCl_3$

Topologically protected chiral edge current of Majorana fermions

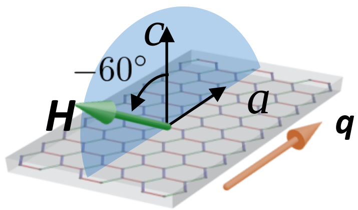
3. A big question; “Is the Kitaev model really relavent to α–$RuCl_3$?”

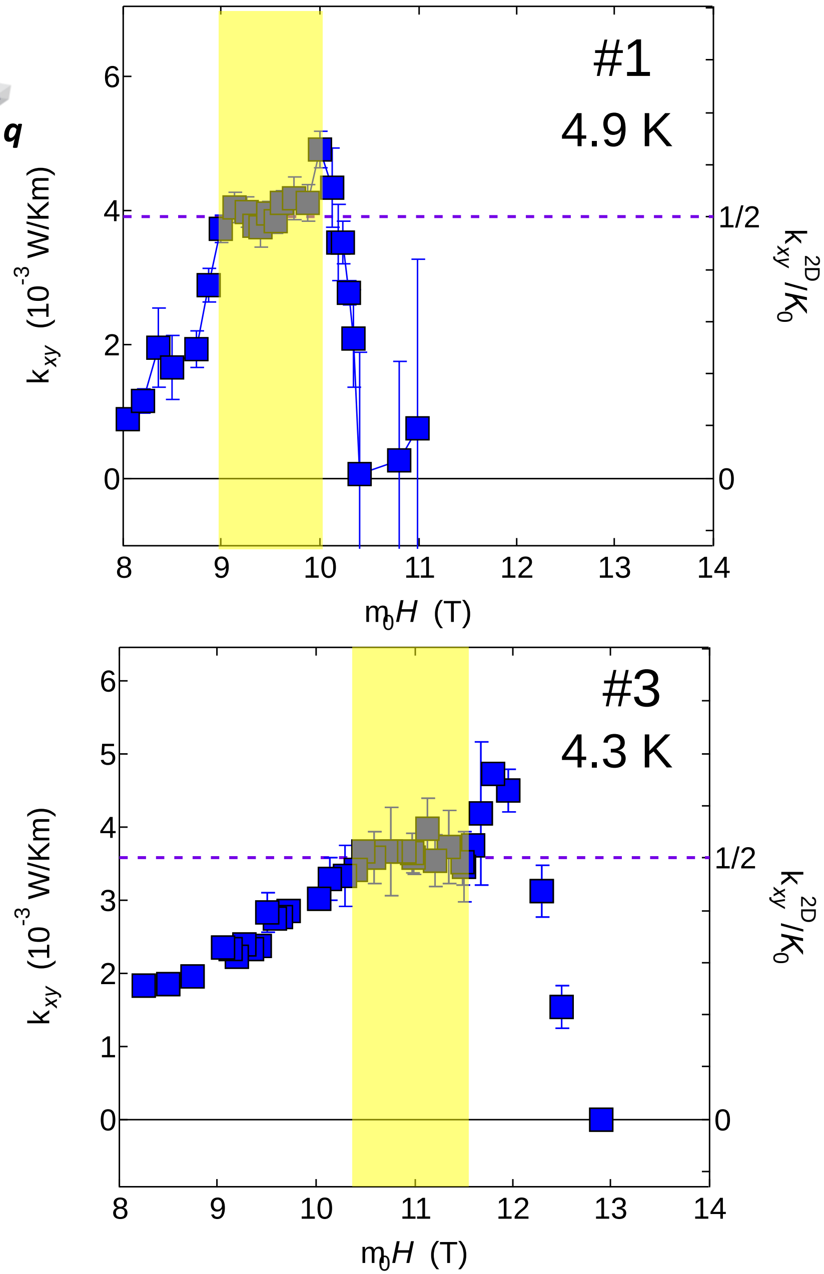
Additional test of the Kitaev physics in α-$RuCl_3$

Influence of non-Kitaev interactions on the quantization.

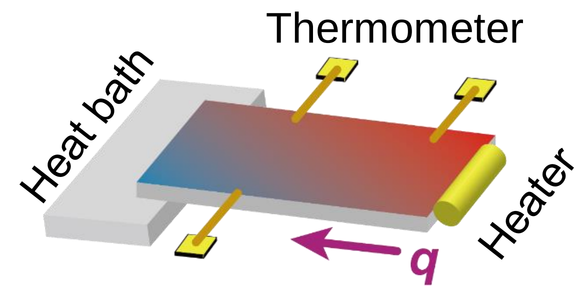
Topological Chern number of Majorana band
determined by field-angle variation of thermal QHE

Sample dependence of the quantized plateau field



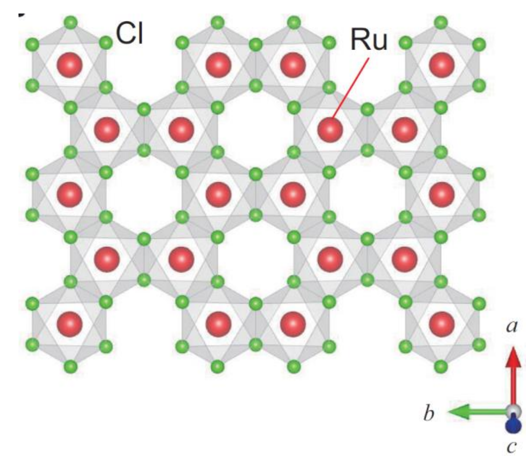


Sample variation of plateau location



Quantized plateau may be very sensitive to the strain.

Non-Kitaev terms sensitive to the distortion of octrahedra.



Topological Chern nuber of Kitaev spin liquid

$$\mathcal{H}_h^{\rm 3rd} \sim -\frac{h_x h_y h_z}{J^2} \sum_{j,k,l} \sigma_j^x \sigma_k^y \sigma_l^z = -\frac{i}{2}\frac{h_x h_y h_z}{J^2} \sum_{j,k} c_j c_k$$

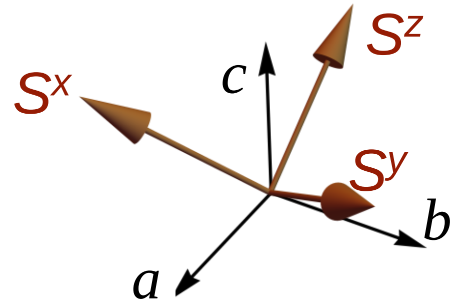
Three body spin correlation

$$\kappa_{xy}^{2D} = C_h \frac{1}{2} K_0 \qquad K_0 = \frac{\pi^2 k_B^2}{3h} T$$

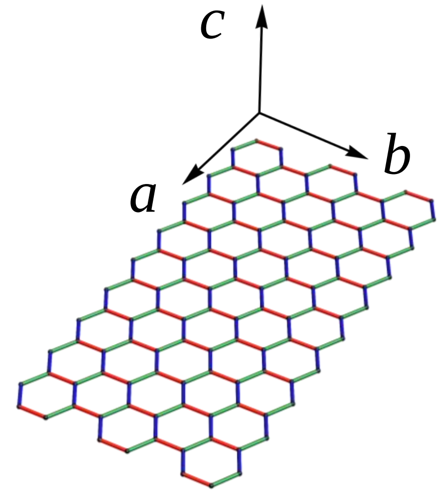
Chern number

$$C_h = \mathrm{sgn}(h_x h_y h_z) = \pm 1$$

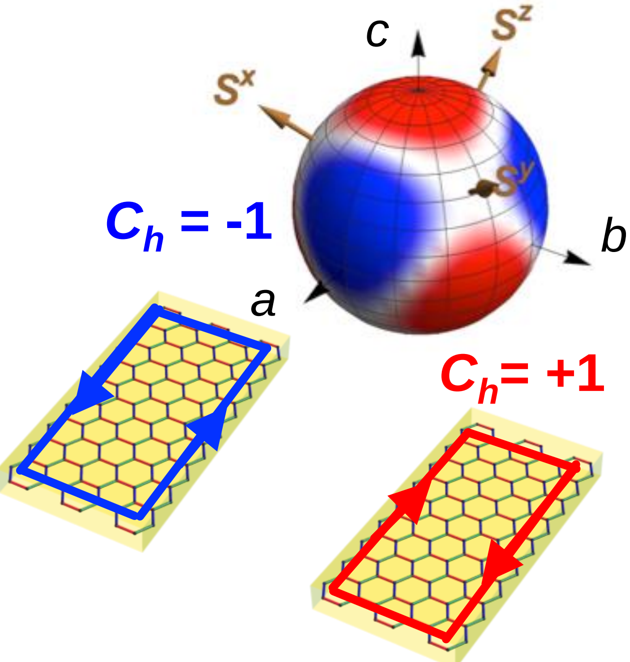
H-component with respect to the spin axis

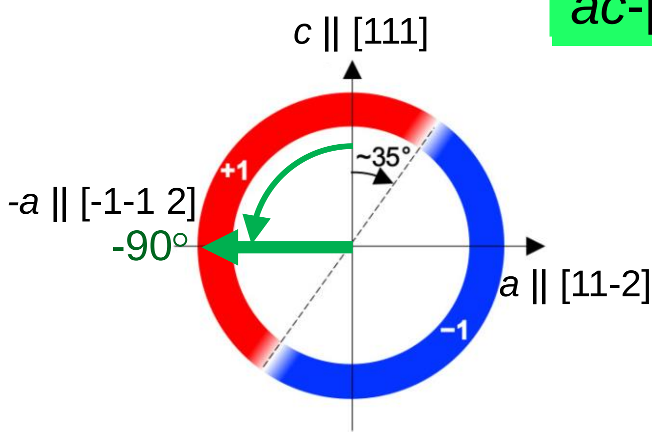


Spin axis is different from crystal axis

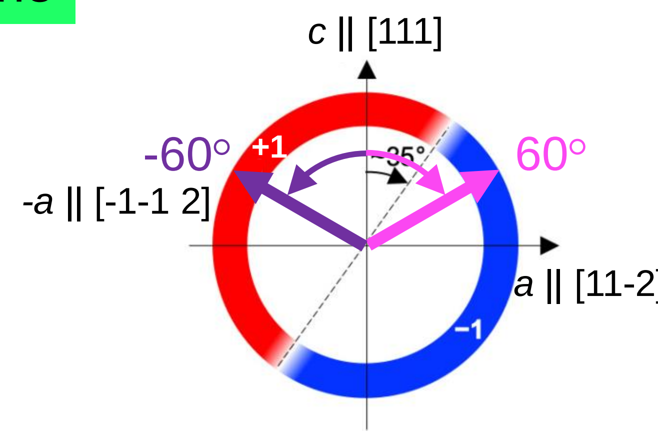


ac-plane





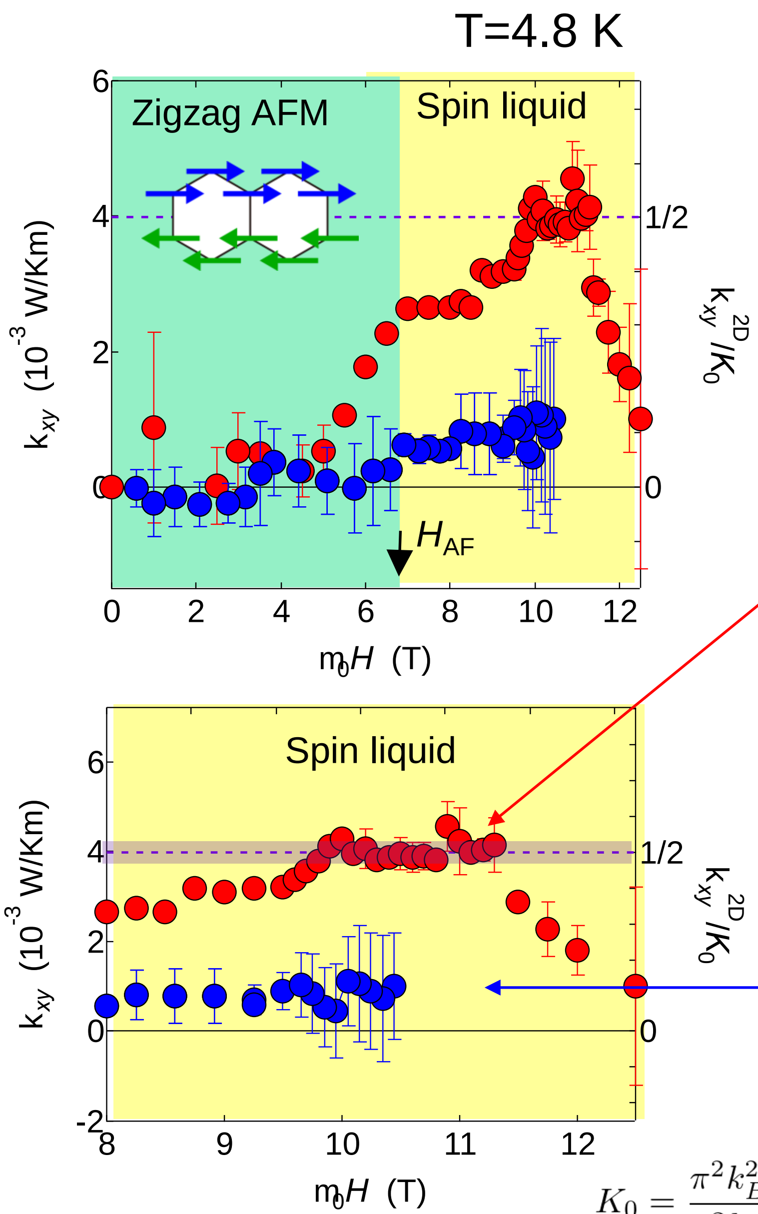
Half integer thermal QHE in parallel field

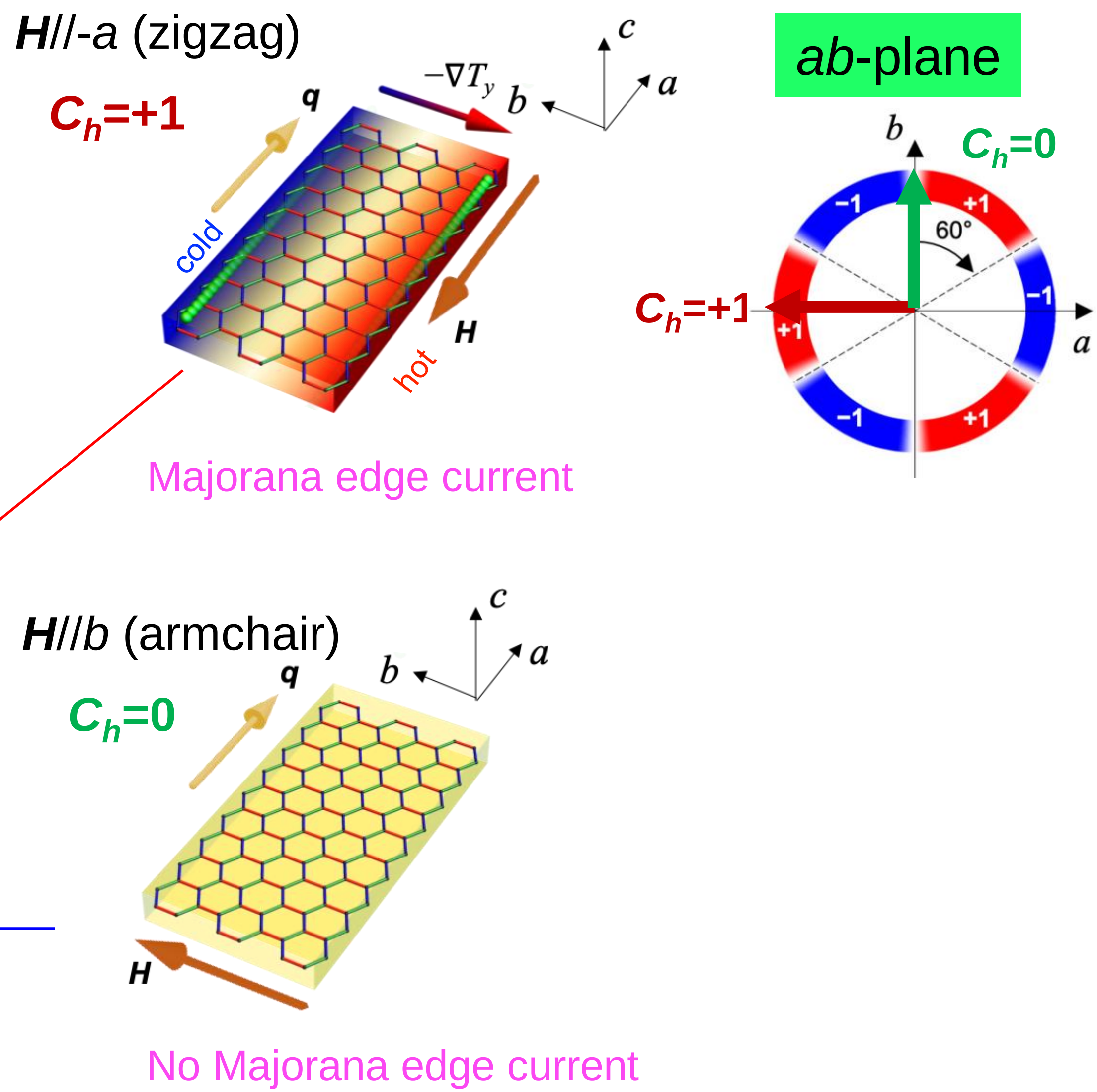


Hall sign change between $\theta = 60^\circ$ and -60°

Essentially different from the Hall effect of electron gas

Half-integer thermal QHE without out-of-plane magnetic field

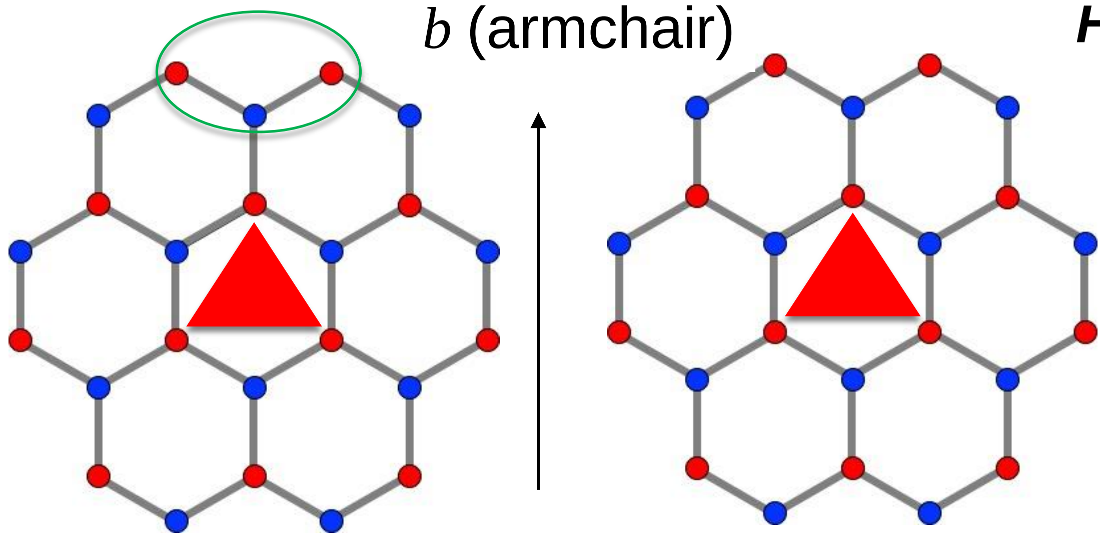




$$K_0 = \frac{\pi^2 k_B^2}{3h} T$$

Quantum thermal conductance

Chirality of the edge states

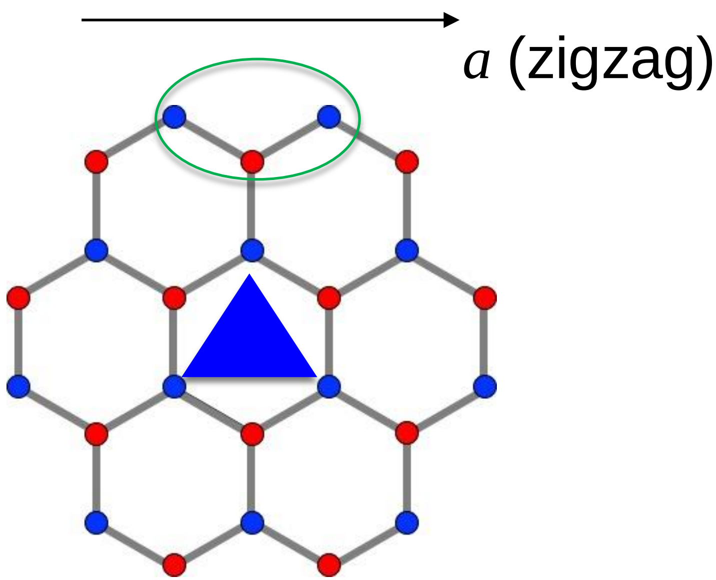


H//*b* (armchair) $\kappa_{xy} = 0$

Twofold rotation along *b*-axis

The same states

unable to distinguish



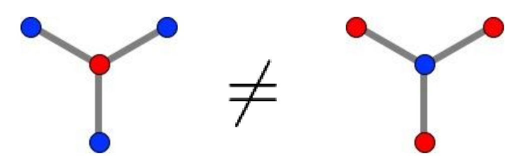
H//*a* (zigzag) $\kappa_{xy} \neq 0$

Twofold rotation along *a*-axis

swaps red- and blue- sublattices

In the Kataev spin liquid state,

Three spin correlation for
(NNN hopping of MF)

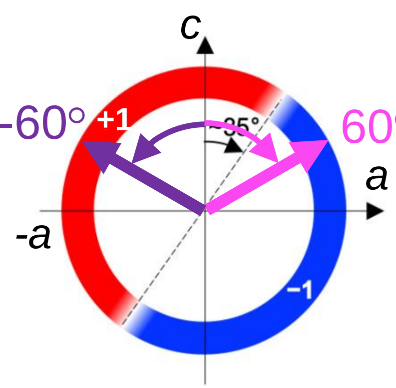


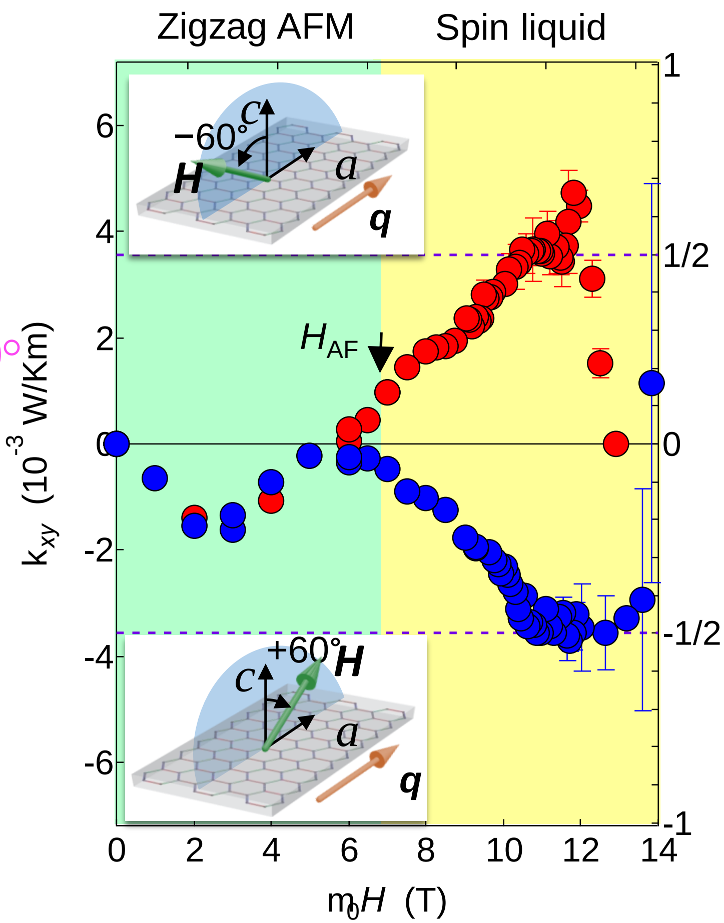
Chirality of the edge states is not reversed

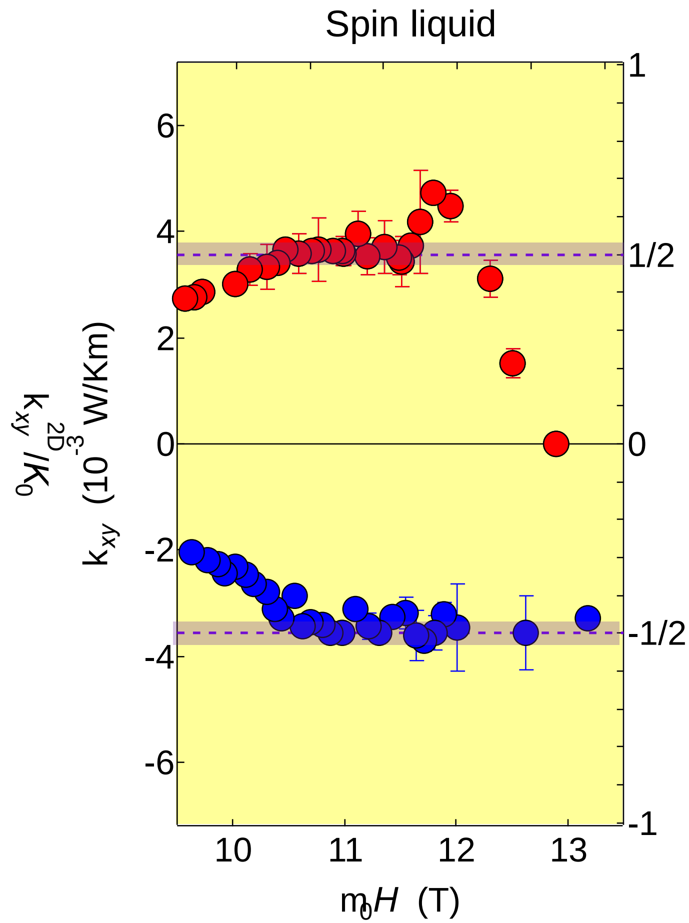
Net chirality $\neq 0$

Sign change of thermal Hall conductance

ac-plane







$$\kappa_{xy}^{2D} = C_h \frac{1}{2} K_0$$

$$K_0 = \frac{\pi^2 k_B^2}{3h} T$$

Quantum thermal conductance

Same thermal Hall sign well inside the AFM state (Magnon Hall effect)

J. Cookmeyer and J. Moore, Phys Rev. B **98**, 060412 (2018).

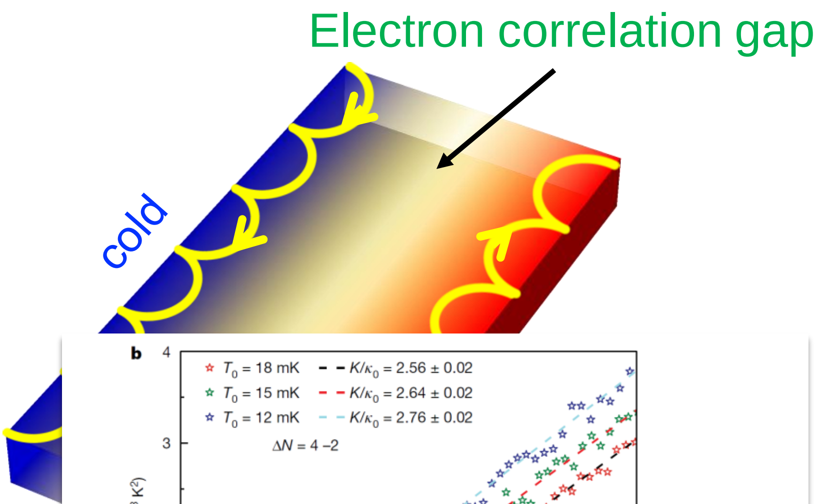
Sign of half-integer thermal QHE at $\theta = -60^\circ$ is positive while negative at -60°

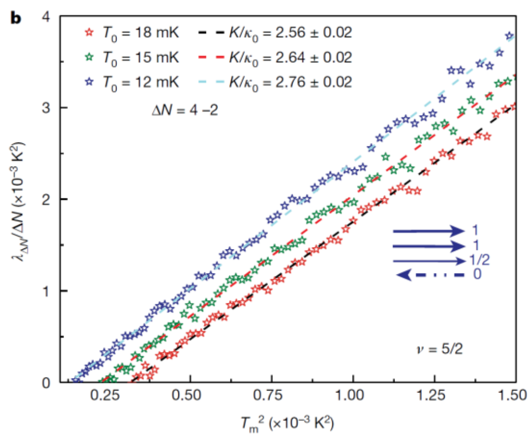
Hall sign is consistent with the Chern number of Majorana band.

T. Yokoi *et al.* a preprint.

2D electron gas and Kitaev spin liquid

Fractional quantum Hall state





$\sigma_{xy}^{2D} =$

$\kappa_{xy}^{2D} =$

$$\sigma_{xy}^{2\mathrm{D}} = \nu \frac{e^2}{h} \qquad \nu = \frac{5}{2}$$

$$\kappa_{xy}^{2D} = \frac{5}{2} K_0$$

Inter

Non-Abelian anyon (ν=5/2)

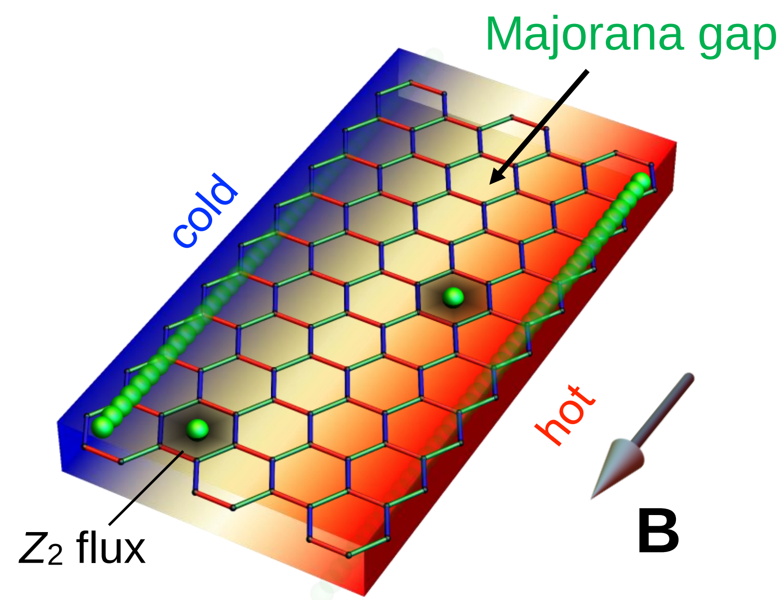
C.L)

M.

M. Banergee *et al*. Nature **559**, 205 (2018).

Abelian anyon

Kitaev quantum spin liquid state



Chiral edge currents of Majorana fermions

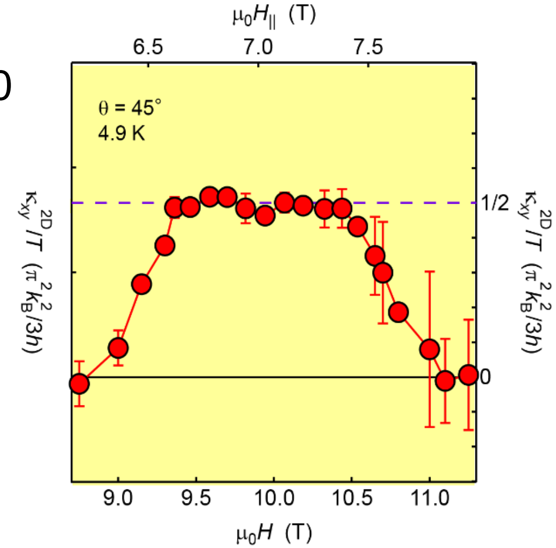
$$\kappa_{xy}^{2D} = \frac{1}{2} K_0$$

Half integer thermal QHE

NonAbelian anyon

Summary : Half-integer thermal QHE in α-$RuCl_3$

Thermal Hall conductance plateau at one-half of K_0



$$\kappa_{xy}^{2D} = C_h \frac{1}{2} K_0 \qquad C_h = \pm 1 \qquad K_0 = \frac{\pi^2 k_B^2}{3h} T$$

Chern number

Quantum thermal conductance

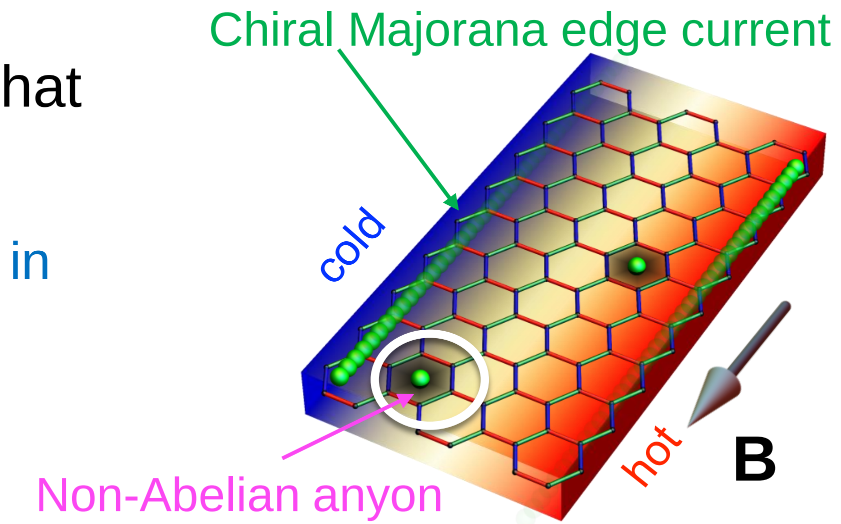
Majorana fermion and non-Abelian anyon
Fractionalization of quantum spin

Plateau field location may be sensitive to strain

Importance of the non-Kitaev interaction ?

Chern number of α-$RuCl_3$ in agreement with that of Majorana band of pure Kitaev model

Non-Abelian topological order persists even in the presence of non-Kitaev interactions



Y. Kasahara *et al*. PRL **120**, 217205 (2018).
Y. Kasahara *et al*. Nature **559**,227 (2018).
T. Yokoi *et al.* a preprint.