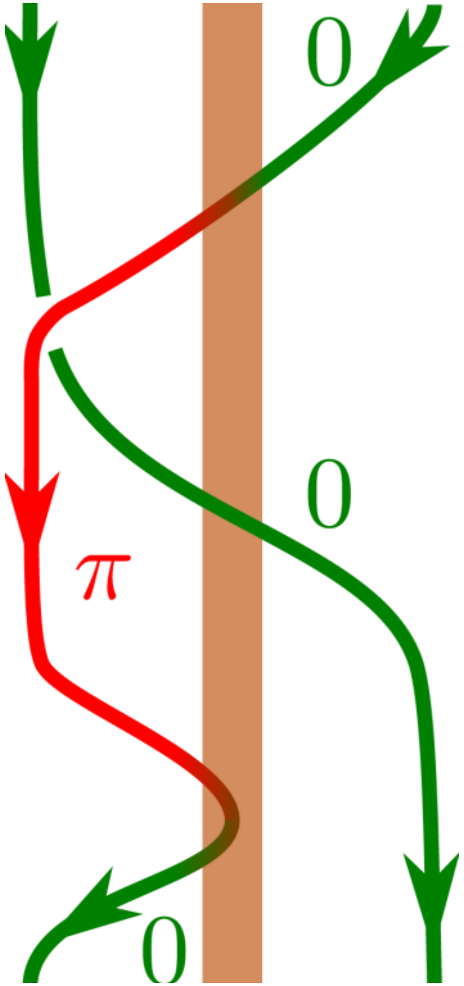

Topologically protected braiding in a single wire using Floquet Majorana modes

Bela Bauer
Microsoft Station Q, Santa Barbara

Tami Pereg-Barnea (McGill/Weizmann)
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Maria-Theresa Rieder (Weizmann)
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Phys. Rev. B 100, 041102 (2019)

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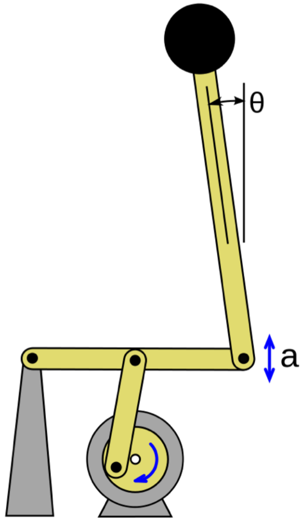
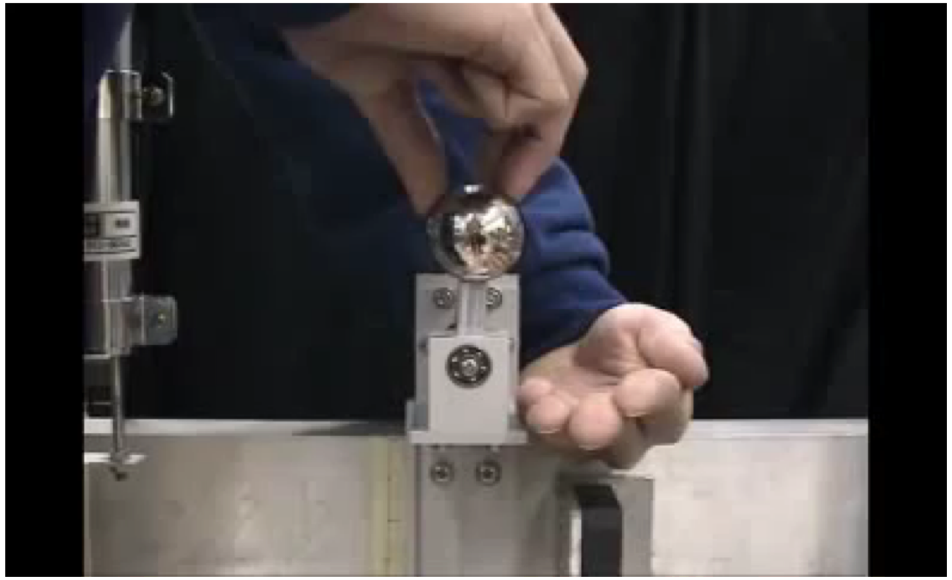




https://www.youtube.com/watch?v=is_ejYsvAjY

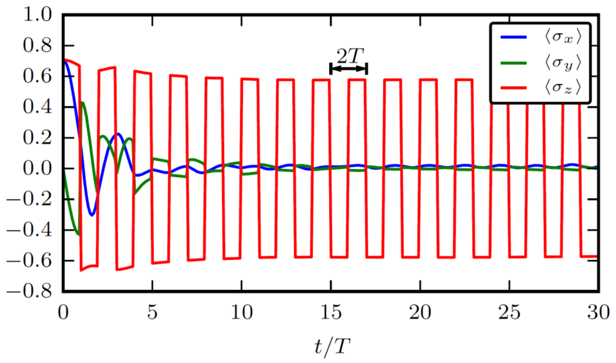
Discrete time-translation symmetry

- Time-translation symmetry broken to discrete symmetry:

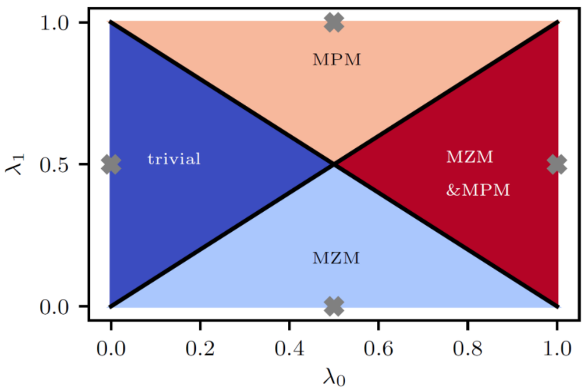
$$\frac{\partial}{\partial t} H(t) \neq 0 \qquad\qquad H(t+T) = H(t)$$

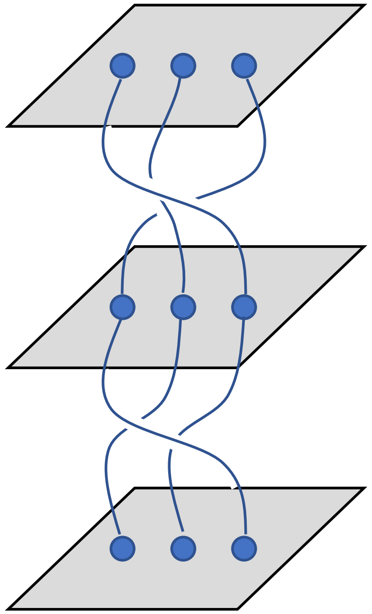
- New physics arising from *discrete time translation symmetry?*

Spontaneous symmetry breaking



Symmetry-protected phases



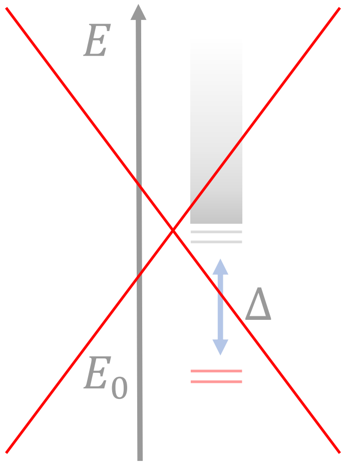


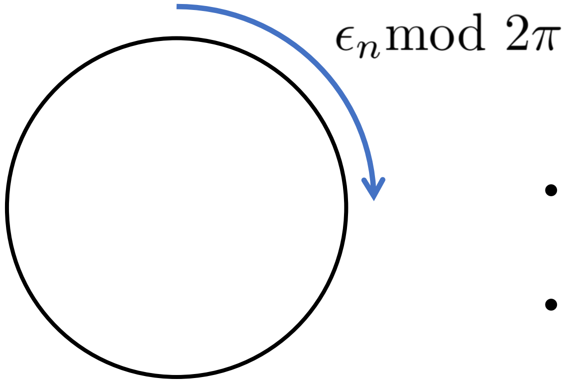
Quantum phases in driven systems

- Quantum phases ↔ *low-energy* phase of matter.

$$U_F = \mathcal{T} \exp i \int_0^T H(t)dt$$

$$U_F|n\rangle = e^{i\epsilon_n}|n\rangle$$

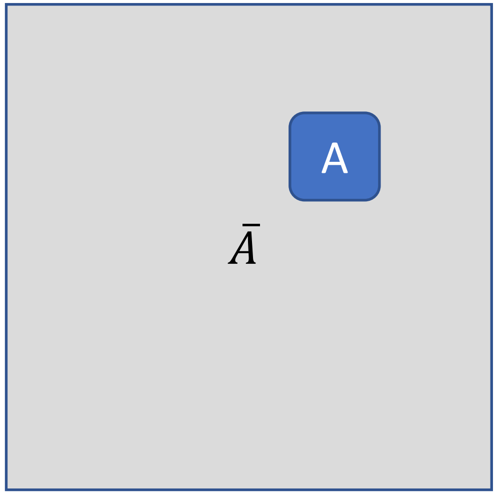




- No notion of high or low energy
- No ground state

Looking for driven quantum phases

$$\text{Energy}/\text{Volume} \;\rightarrow\; \text{Entropy}/\text{Volume}$$



$$\rho_A = \mathrm{Tr}_{\bar{A}}\, |\psi\rangle\langle\psi|$$
$$S_A = -\,\mathrm{Tr}\, \rho_A \log \rho_A$$

Area law ~ low-energy state

Looking for driven quantum phases

$$\text{Energy}/_{\text{Volume}} \quad \rightarrow \quad \text{Entropy}/_{\text{Volume}}$$

- Drive supplies energy → system heats up
- Single universal long-time fixed point:

$$U_f^n|\psi_0\rangle = |\psi_n\rangle$$
$$\lim_{n\to\infty} \mathrm{Tr}_B\left(|\psi_n\rangle\langle\psi_n|\right) \approx 1$$

- Conventional wisdom: *no distinct phases in closed quantum Floquet systems!*
  - Expected for all systems that obey "eigenstate thermalization hypothesis".

Exceptions

Free fermions/ integrable

Restricted dynamics
prevent thermalization

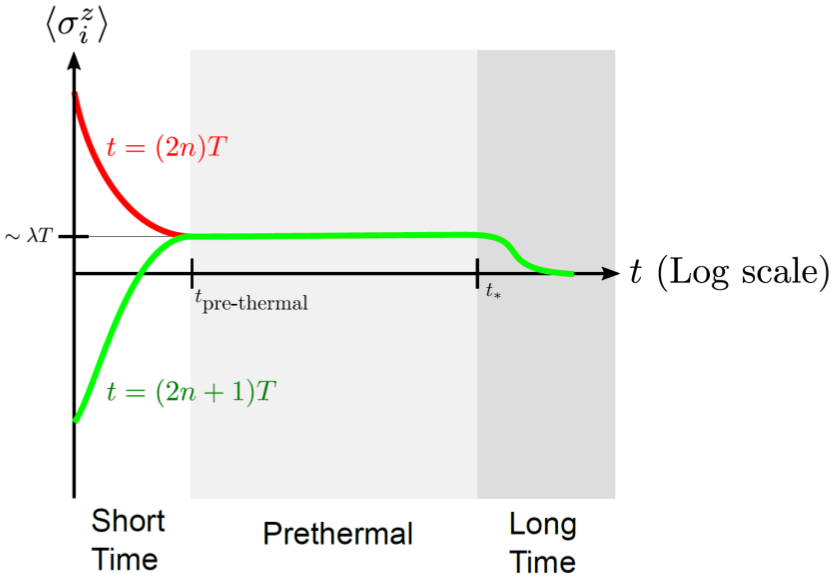
Many-body localization

Interactions
+ disorder
= robust emergent integrability

Ponte, Papic, Huveneers & Abanin, PRL 2015

Interactions
+ disorder
+ driving
= emergent integrability

Prethermalization



Abanin, De Roeck, Huveneers PRL 2015
Else, BB, Nayak PRX 2017

Vedika Khemani, Achilleas Lazarides, Roderich Moessner, S. L. Sondhi, PRL 2015

The Kitaev chain

$$H = \sum_i \left[\boxed{-\mu c_i^\dagger c_i} - \boxed{\frac{w}{2}\left(c_i^\dagger c_{i+1} + \mathrm{h.c.}\right)} + \boxed{\frac{\Delta}{2}\left(c_i c_{i+1} + \mathrm{h.c.}\right)} \right]$$

Chemical potential

Kinetic energy

p-wave pairing

Kitaev 2001

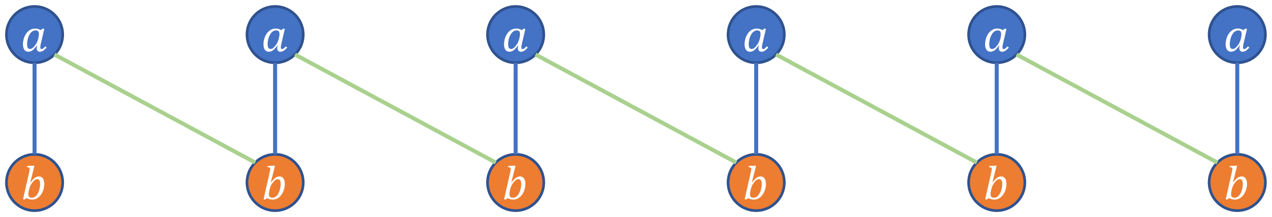
The Kitaev chain

$$\Delta = -w: \quad H = \sum_i \left[-\mu c_i^\dagger c_i - \frac{w}{2} \left(c_i^\dagger c_{i+1} + c_i c_{i+1} + \text{h.c.} \right) \right]$$

Change to basis of Majorana operators:

$$2c_n = a_n + ib_n$$
$$\{a_n, a_m\} = \{b_n, b_m\} = 2\delta_{nm}, \; \{a_n, b_m\} = 0$$

$$H = -\sum_k \left[\frac{\mu}{2} i a_k b_k + \frac{w}{2} i a_k b_{k+1} \right]$$

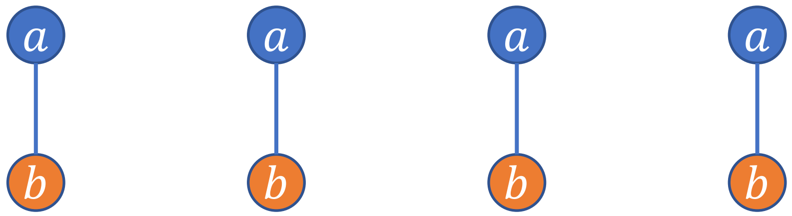


Kitaev 2001

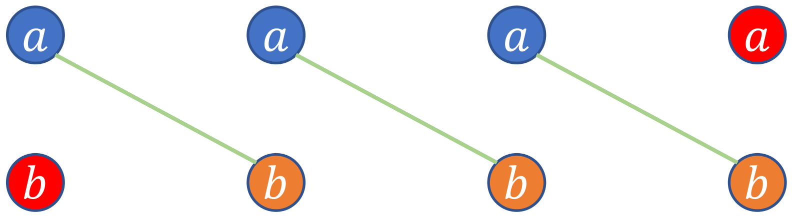
The Kitaev chain

$$H = -\sum_k \left[\frac{\mu}{2} i a_k b_k + \frac{w}{2} i a_k b_{k+1}\right]$$

Trivial phase ($\mu > 0, w = 0$)



Topological phase ($\mu = 0, w > 0$)



$$[H, b_1] = [H, a_L] = 0$$

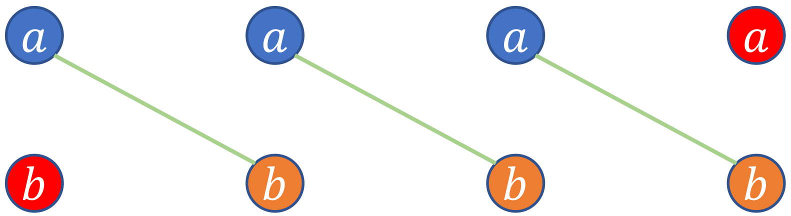
- “Majorana zero modes”
- Two degenerate ground states
- No operator in the bulk can distinguish

Kitaev 2001

The Kitaev chain

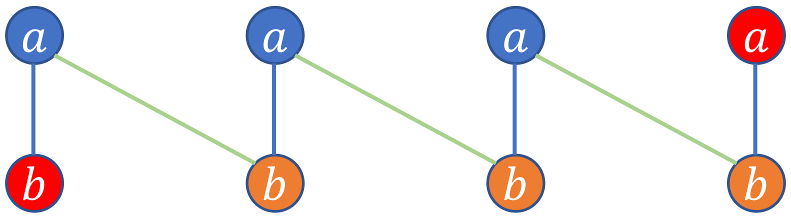
$$H = \sum_i \left[-\mu c_i^\dagger c_i - \frac{w}{2}\left(c_i^\dagger c_{i+1} + \text{h.c.}\right) + \frac{\Delta}{2}\left(c_i c_{i+1} + \text{h.c.}\right)\right]$$

Topological phase ($\mu = 0, w > 0$)



$$[H, ib_1 a_L] = 0$$

Topological phase ($\mu < w, t; t \neq w$)



$$[H, ib_1 a_L] = \mathcal{O}(e^{-L/\xi})$$

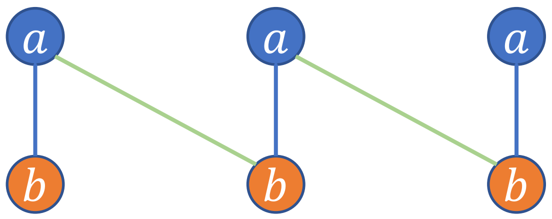
Kitaev 2001

Zero modes in the driven Kitaev chain

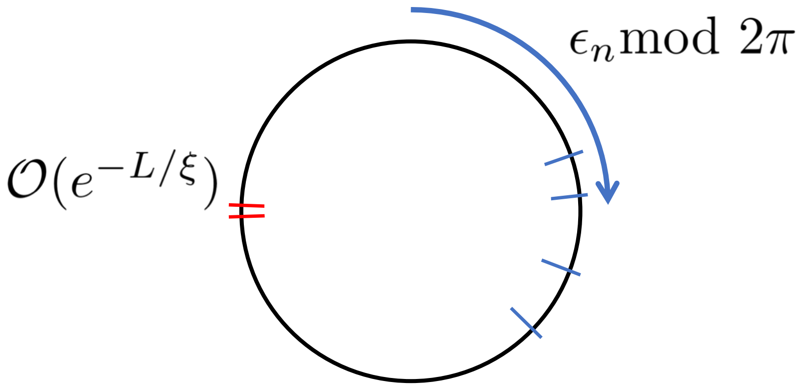
$$U_F = e^{i\frac{T}{2}H_0} e^{i\frac{T}{2}H_1}$$

$$H_0 = -i\frac{\pi}{T}\lambda_0 \sum_{n=1}^{N-1} a_n b_{n+1}$$

$$H_1 = -i\frac{\pi}{T}\lambda_1 \sum_{n=1}^{N} a_n b_n, .$$



$$\lambda_1 \ll \lambda_0$$



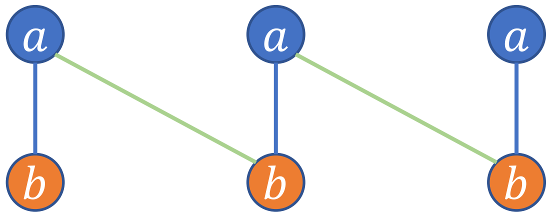
L. Jiang et al, 2011

π modes

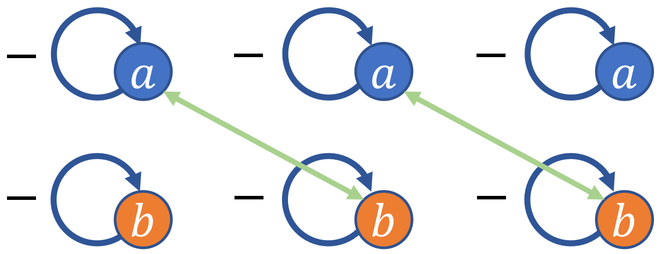
$$U_F = e^{i\frac{T}{2}H_0} e^{i\frac{T}{2}H_1}$$

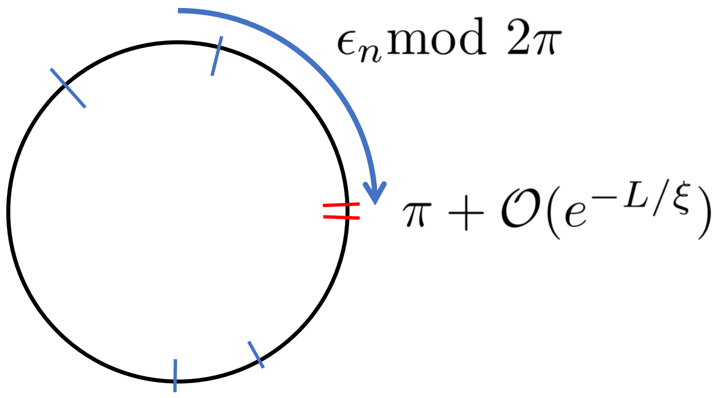
$$H_0 = -i\frac{\pi}{T}\lambda_0 \sum_{n=1}^{N-1} a_n b_{n+1}$$

$$H_1 = -i\frac{\pi}{T}\lambda_1 \sum_{n=1}^{N} a_n b_n, .$$



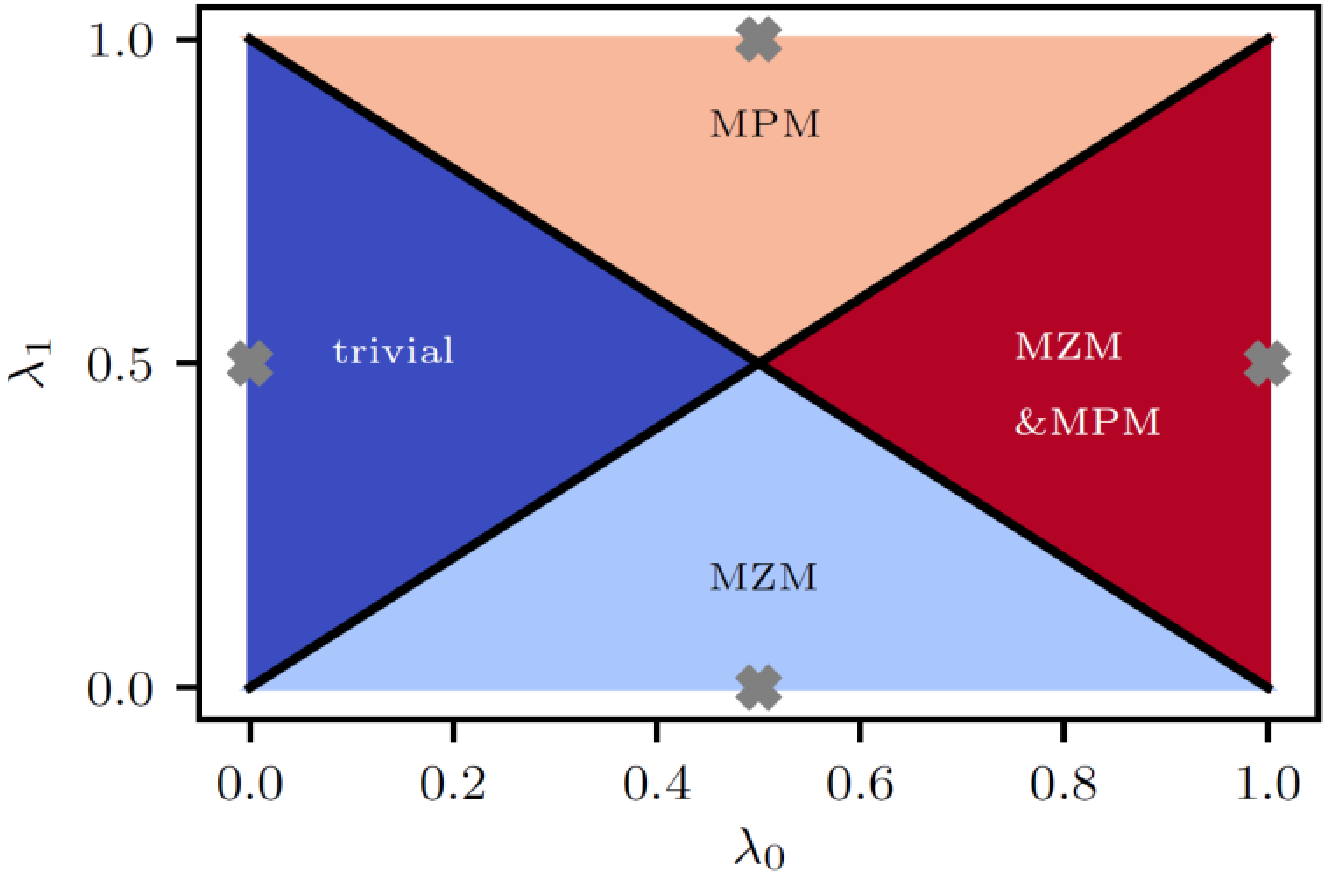
$$\begin{aligned} \lambda_0 &= 1/2 \\ \lambda_1 &= 1 \end{aligned}$$





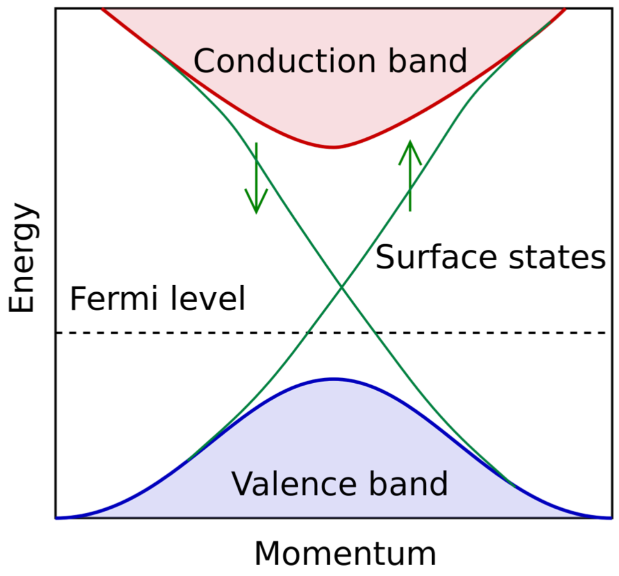
L. Jiang et al, 2011

Phase diagram



C. W. von Keyserlingk, S. L. Sondhi 2016

Floquet topological phases



- Example of a broader class of phase: Symmetry-protected topological phases

- Discrete time-translation symmetry as part of the protecting symmetries

[Von Keyserlingk & Sondhi 2016; Else & Nayak 2016; Potter, Morimoto & Vishwanath 2016; Roy & Harper 2016]

Time crystals

Duality

Kitaev model w/ zero modes ⟺ Ising Ferromagnet

$$H = \sum_i \left[-\mu c_i^\dagger c_i - \frac{w}{2} \left(c_i^\dagger c_{i+1} + c_i c_{i+1} + \mathrm{h.c.} \right) \right]$$

$$H = \sum_i \left[h\sigma_i^x - J\sigma_i^z \sigma_{i+1}^z \right]$$

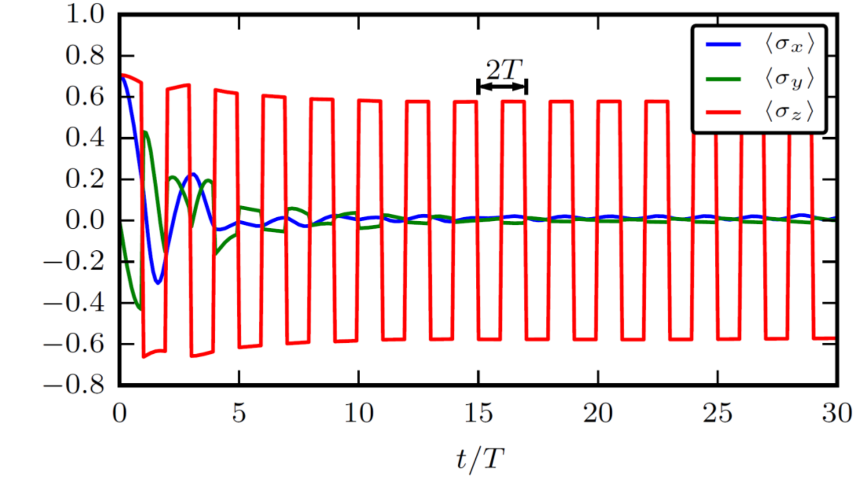
Driven Kitaev model w/ π modes ⟺ Driven Ising model

$$U_F = e^{i\frac{T}{2}H_0} e^{i\frac{T}{2}H_1}$$

$$U_F = e^{i\kappa_2 \sum_i \sigma_i^z \sigma_{i+1}^z} e^{i\kappa_1 \sum_i \sigma_i^x}$$

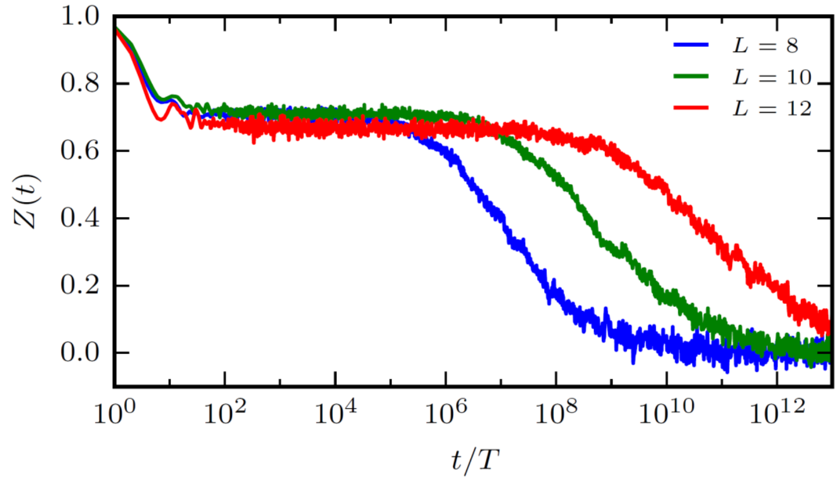
Else, BB, Nayak, 2016

Persistent oscillations



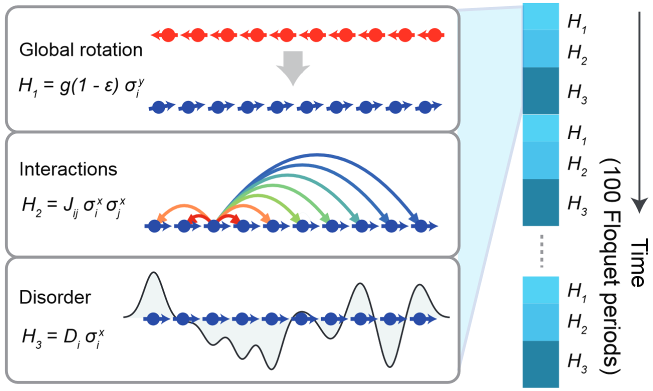
$h = 0.3,\ t_0 = 1,\ t_1 = \pi/2$

TEBD with $L = 200$



Oscillations persist to exponentially long times for generic parameters!

Experiments



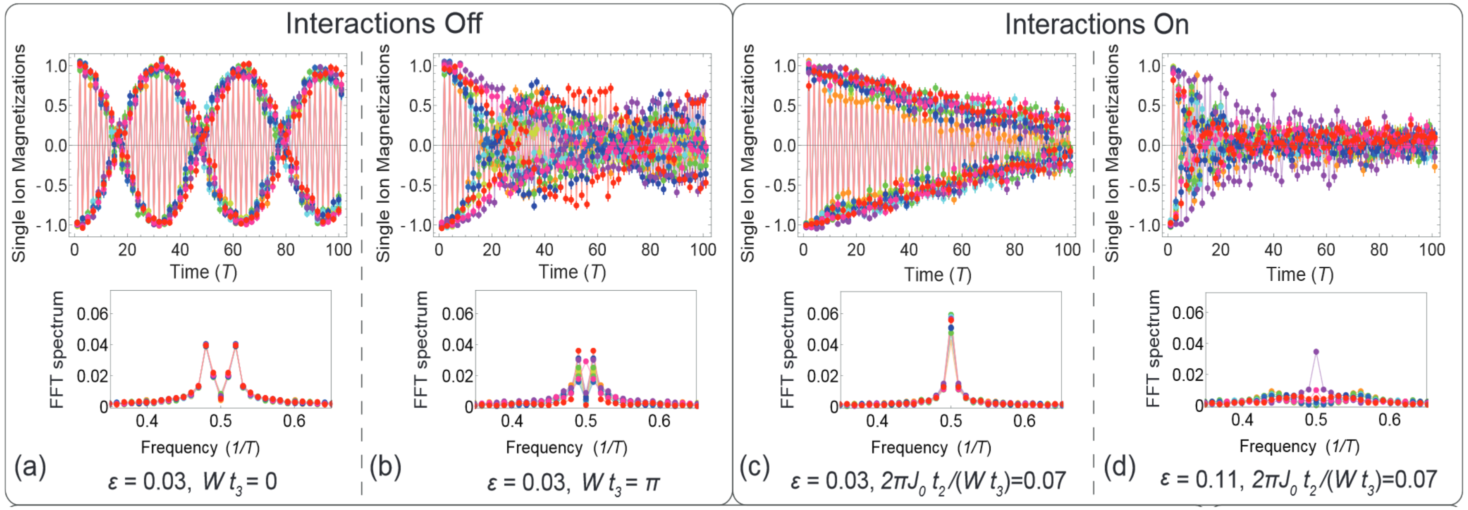
Observation of a Discrete Time Crystal

J. Zhang, P. W. Hess, A. Kyprianidis, P. Becker, A. Lee, J. Smith, G. Pagano,[1] I.-D. Potirniche,[2] A. C. Potter,[2,3] A. Vishwanath,[2,4] N. Y. Yao,[2] and C. Monroe[1]

[1]Joint Quantum Institute, University of Maryland Department of Physics and National Institute of Standards and Technology, College Park, MD 20742
[2]Department of Physics, University of California Berkeley, Berkeley, CA 94720, USA
[3]Department of Physics, University of Texas at Austin, Austin, TX 78712, USA
[4]Department of Physics, Harvard University, Cambridge, MA 02138, USA

(Dated: September 29, 2016)

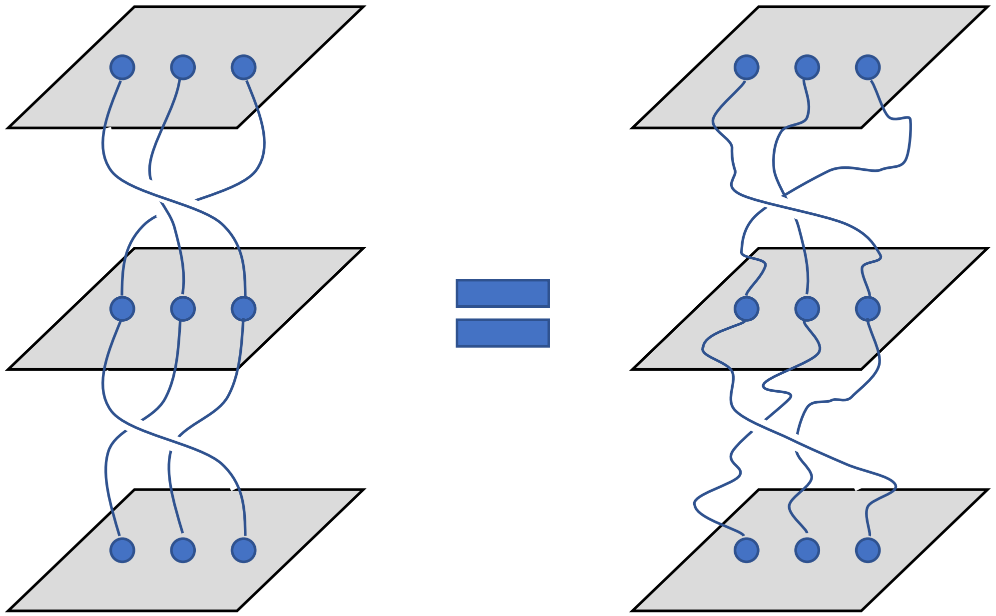
Nature 2017;
also Choi et al, Nature 2017



Braiding

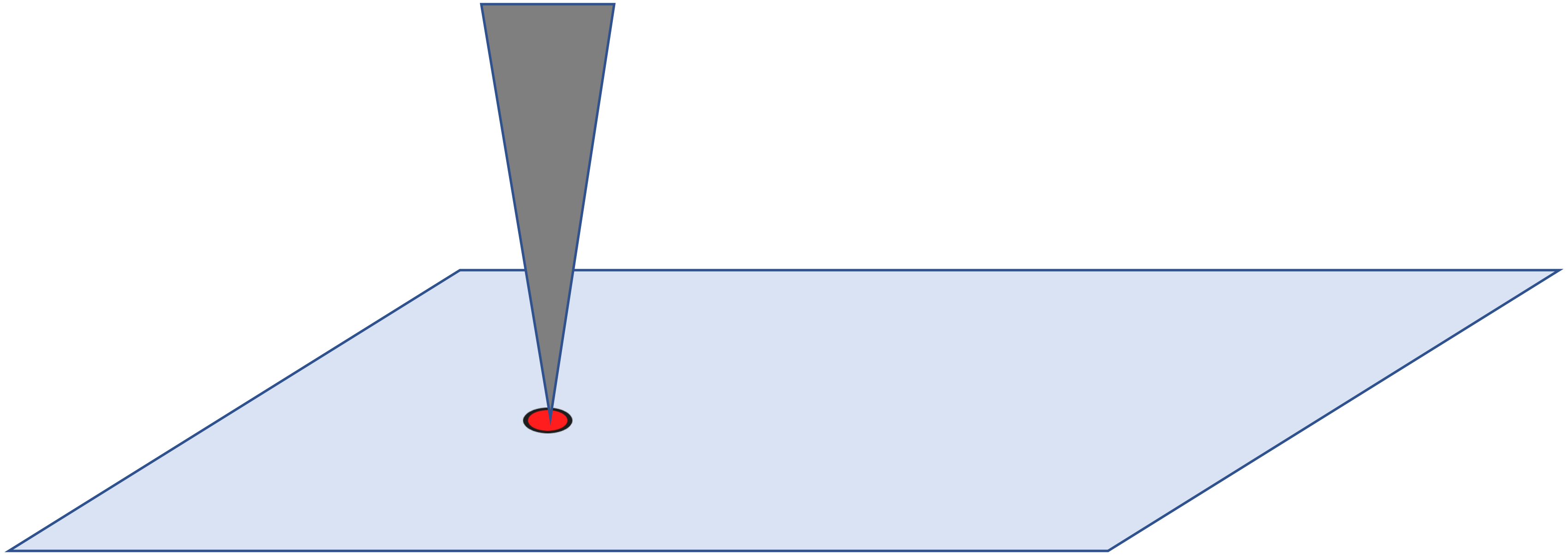
Tying their world-lines into knots:
Unitary operations within ground state manifold

Collection of non-Abelian anyons:
Topological ground state degeneracy



Moving quasiparticles

- “Grab and drag,” e.g. use an AFM tip for electrically charged quasiparticles

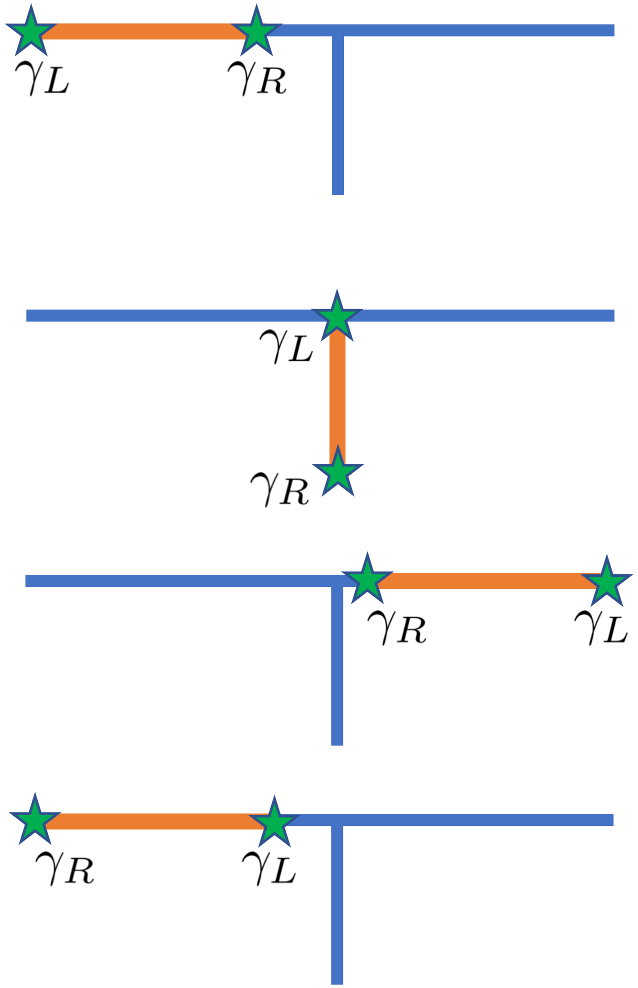


Braiding (exchanging) in wires

MZMs hybridize when close to each other:
Topological protection broken

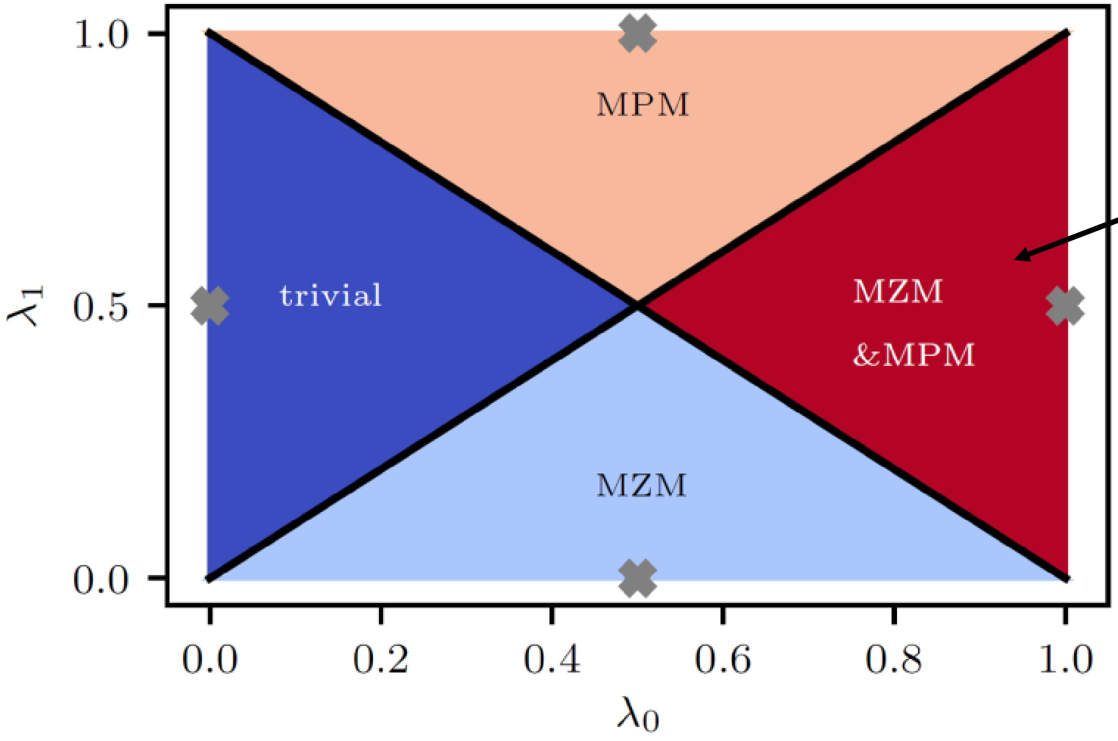
(see also Chiu, Vazifeh and M. Franz, EPL 2015)

Braiding in wire networks



- Any wire-based approach needs T-junctions (or several parallel, connected wires)
- Can we get around this?

Braiding in one wire?



0 and π phase: 4 topologically protected modes in one wire!

Can we braid them?

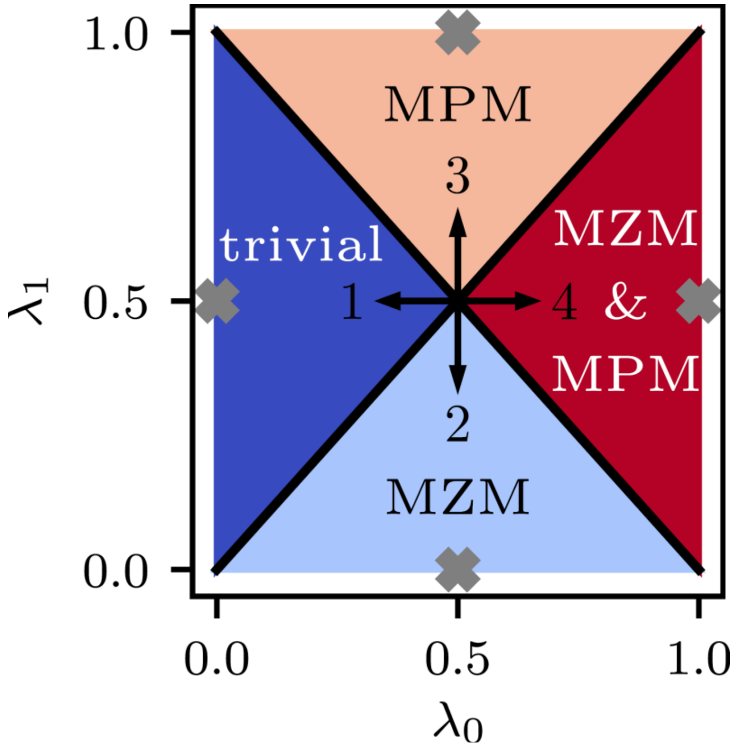
BB et al, 2018; Bomantara & Gong 2017

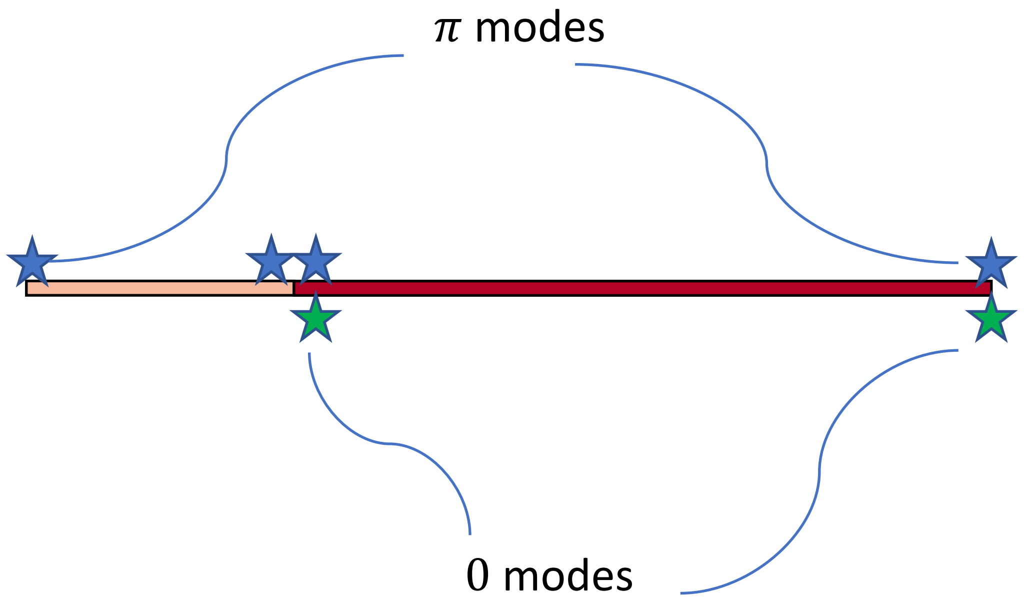
Braiding in one wire?

Two 0 modes hybridize

0 and π modes don’t hybridize!

Inhomogeneous systems





Moving phase boundaries

- Need to adiabatically move phase boundaries in real space
- What does adiabaticity mean in a driven system?
- Consider $H_F = -i \ \log U_F$:
  - Slowly changing parameters of drive implies slow change in H_F

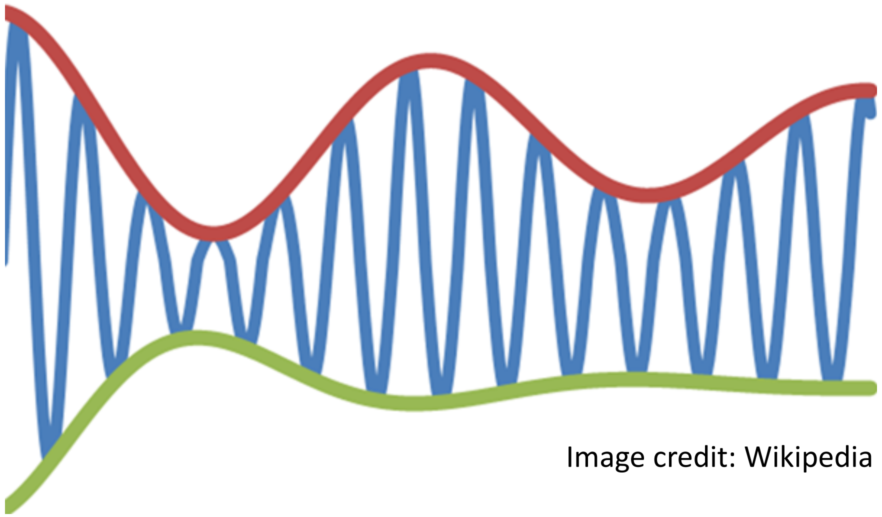
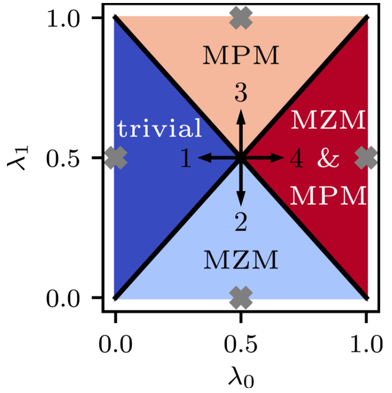
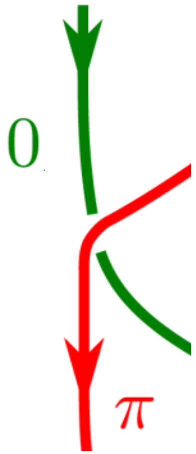
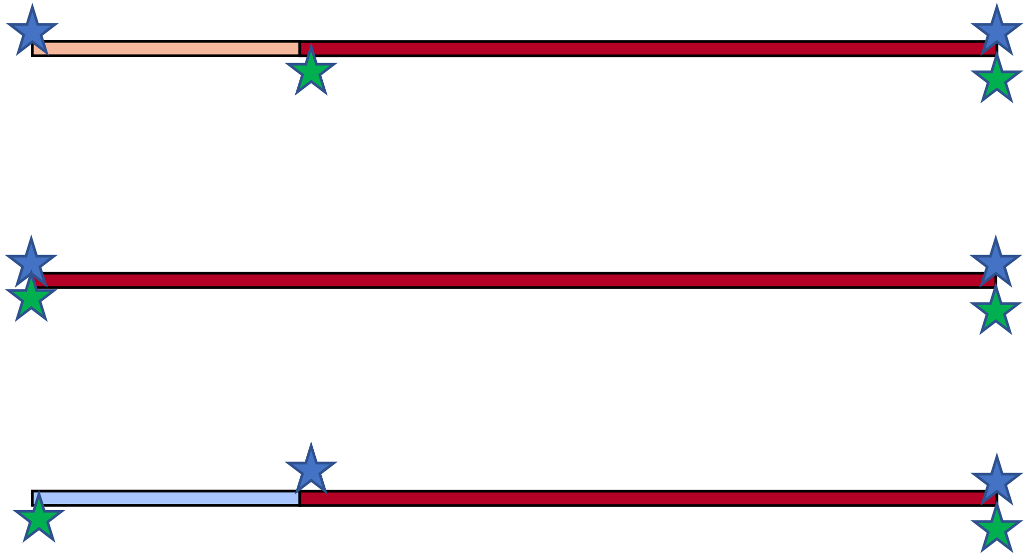


Image credit: Wikipedia

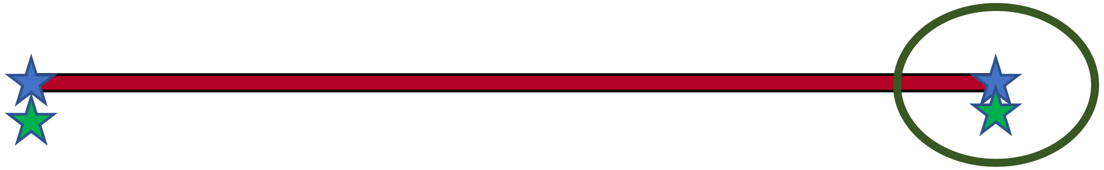
Braiding 0 and π modes

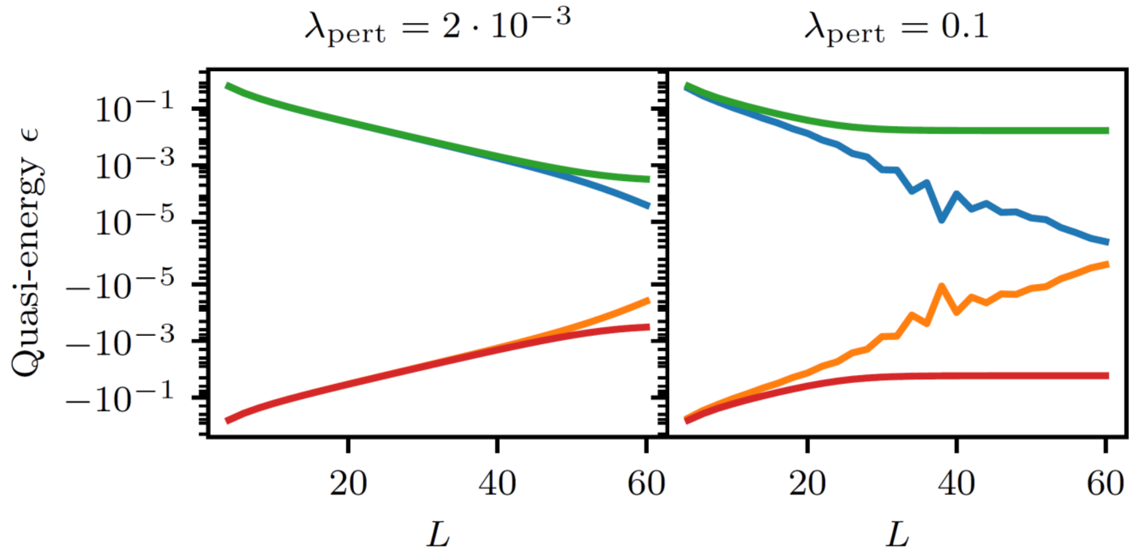






Half-frequency perturbations

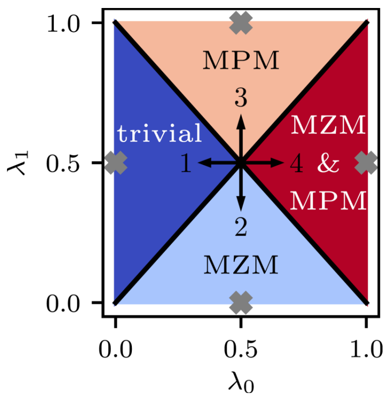


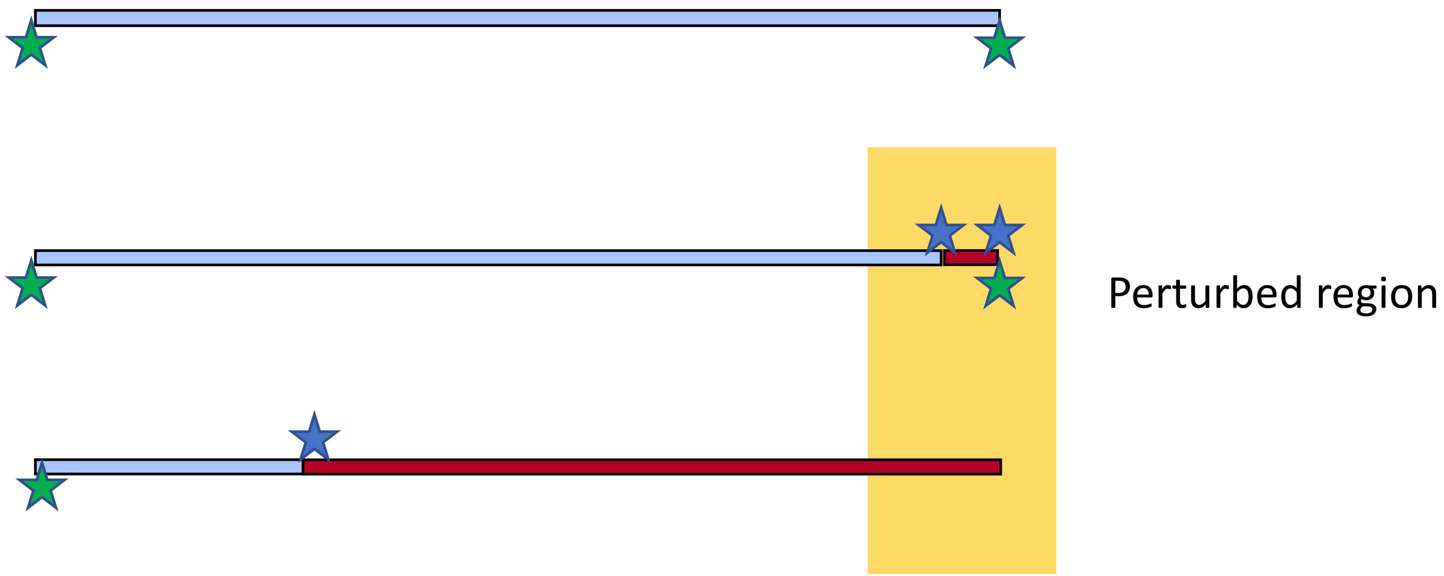


Apply local perturbation every other cycle:

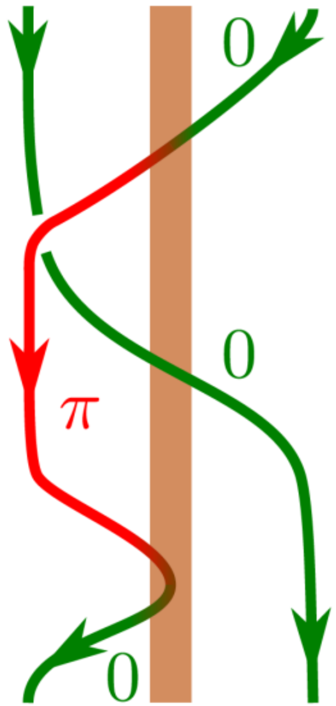
$$U_F \to U_F^2 U_{\mathrm{pert}}$$

Converting 0 and π modes





Braiding protocol



Simulations

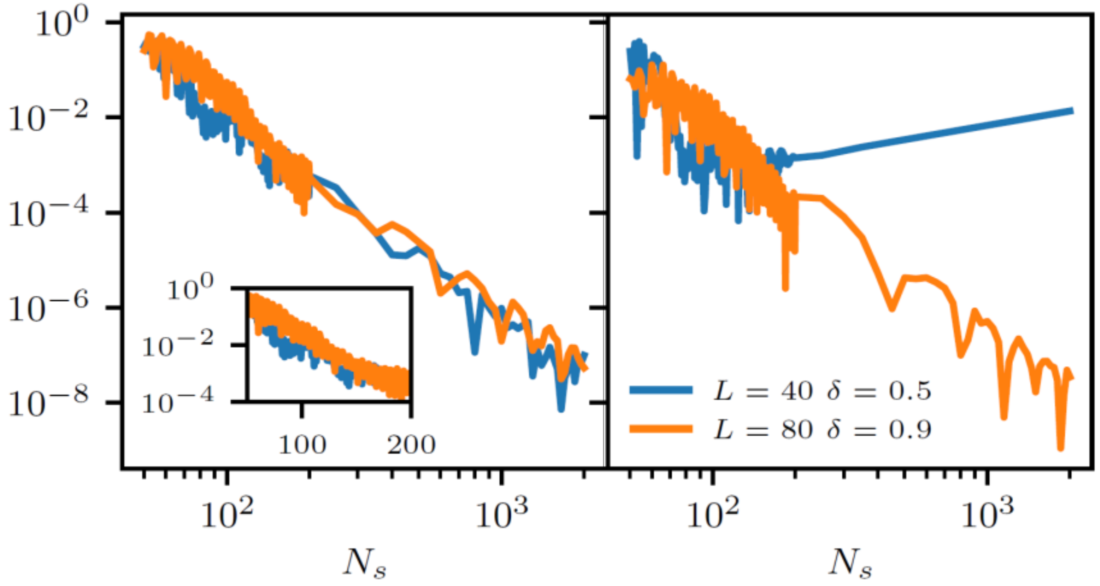
- Numerically simulate time evolution using fermionic Gaussian formalism
- Find effect of time evolution operator on initial Majorana zero modes:

$$\begin{pmatrix} \gamma_1^f \\ \gamma_2^f \end{pmatrix} = U_r \begin{pmatrix} \gamma_1^i \\ \gamma_2^i \end{pmatrix}$$

- Ideally, $U_r = i\sigma^y$
- Dependence on many parameters:
  - System size L
  - Adiabaticity N_s
  - Detuning from fixed point δ
  - Strength of perturbation λ_{pert}

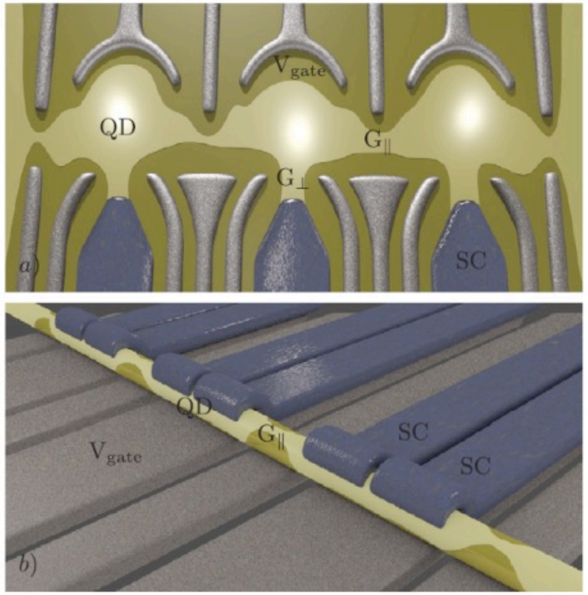
$$\Delta_{\text{diab}} = |U_r^\dagger U_r - 1|$$

$$\begin{aligned} U_r &\sim \operatorname{diag}\left(r_1 e^{i\phi_1}, r_2 e^{i\phi_2}\right) \\ \Delta_{\text{phase}} &= |\phi_1 + \pi/2| + |\phi_2 - \pi/2| \end{aligned}$$

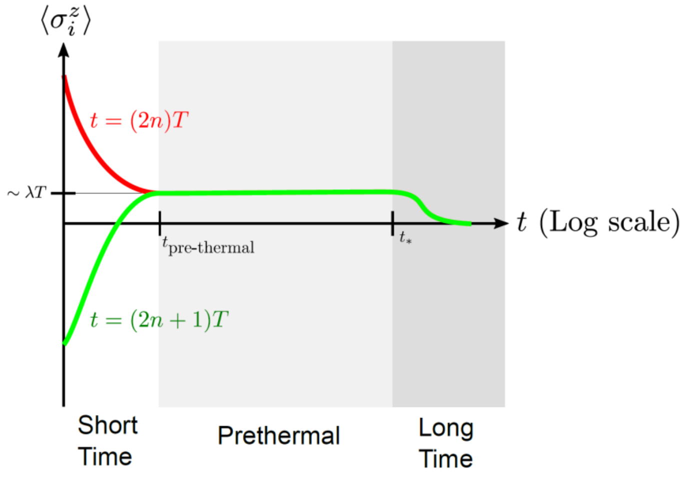


Outlook

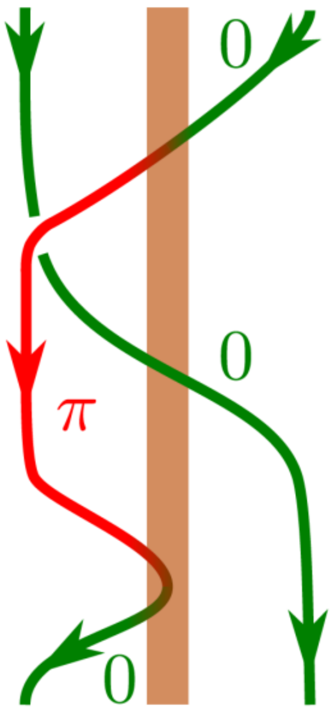
- Experimental realization:
  - Kitaev chain realized in quantum dot chains?
    Ion C Fulga et al, New J. Phys. 15, 045020 (2013)
  - Necessary phases realized by only driving chemical potential



- Preventing heating
  - Prethermal: tune to regime where heating time $\gg$ braiding time
  - Borrowing ideas from prethermal time crystals
    Else, BB, Nayak PRX 2017



Summary

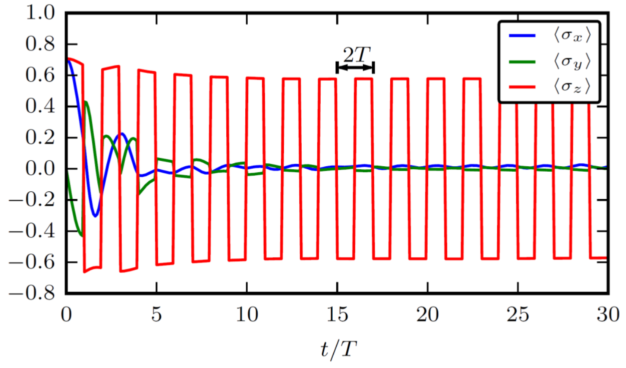


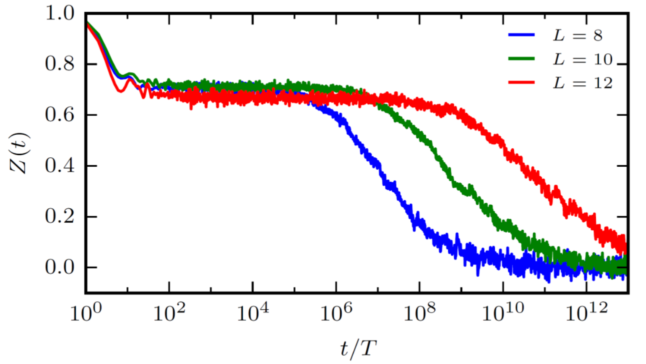
- Many new phases in driven systems can be understood as resulting from discrete time-translation symmetry

- Time crystals become possible in driven systems

- Driven topological superconductors can be used to braid in strictly one-dimensional systems, using quasi-energy as “extra dimension”

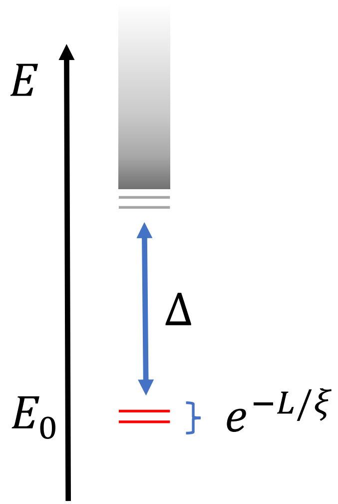
- Such protocols are topologically protected as long as time-translation symmetry is not broken

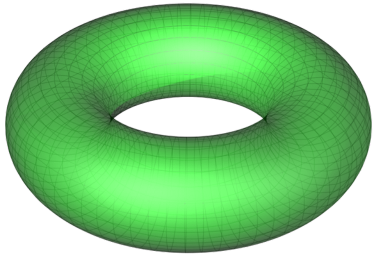


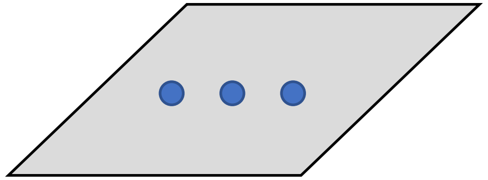


Thank you for your attention!

Topologically ordered phases



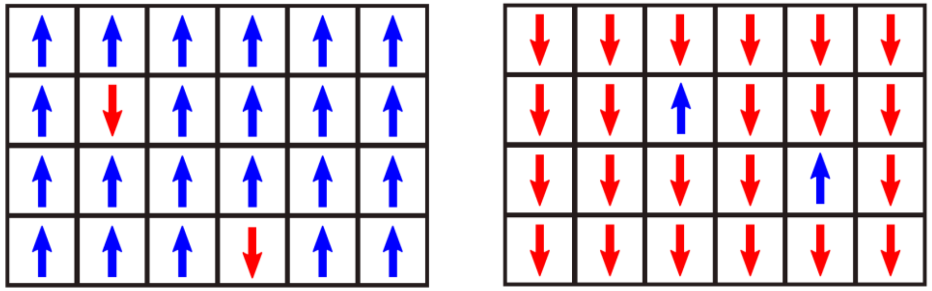




- Topological degeneracy
- Exponentially small splitting:
  $H|n\rangle = (E_0 + O(e^{-L/\xi}))|n\rangle$
- Robust:
  $\langle n|O_{loc}|m\rangle = o\delta_{mn} + O(e^{-L/\xi})$

Spontaneous symmetry-breaking

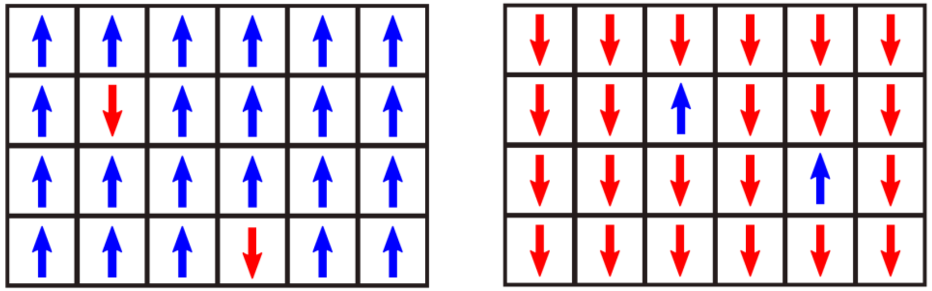
- This argument is too strong: rules out *any* symmetry breaking
- Example: Ising ferromagnet



$$|\pm\rangle = \frac{1}{\sqrt{2}}(|\uparrow\ldots\uparrow\rangle \pm |\downarrow\ldots\downarrow\rangle) \qquad |E_+ - E_-| \sim e^{-L/\xi}$$

$$[H, \prod \sigma_x] = 0 \rightarrow \langle n|\sigma_z|n\rangle = 0$$

Spontaneous symmetry-breaking



$$|\pm\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\ldots\uparrow\rangle \pm |\downarrow\ldots\downarrow\rangle\right) \qquad |E_+ - E_-| \sim e^{-L/\xi}$$

- Long-range correlated: $\langle O(x)O(y)\rangle - \langle O(x)\rangle\langle O(y)\rangle \neq 0$
- Eigenstates are unphysical “cat” states

- Physical states: $|+\rangle \pm |-\rangle$ have finite order parameter and are metastable with mixing time $\tau \sim e^{L/\xi}$

Definitions of SSB

Eigenstates are long-range correlated:

$$\langle O(x)O(y)\rangle - \langle O(x)\rangle\langle O(y)\rangle \neq 0$$

Physical initial states with finite order parameter remain stable to times exponentially divergent in system size.

Definitions of TTSB

Eigenstates are long-range correlated:

$$\langle O(x)O(y)\rangle - \langle O(x)\rangle\langle O(y)\rangle \neq 0$$

Time-independent states are long-range correlated:

$$\langle O(x)O(y)\rangle - \langle O(x)\rangle\langle O(y)\rangle \neq 0$$

Physical initial states with finite order parameter remain stable to times exponentially divergent in system size.

Physical initial states exhibit oscillations to times exponentially divergent in system size.

Floquet time crystal

$$U_f = e^{-it_0 H_{\mathrm{MBL}}} e^{-it_1 H_1}$$

$$H_{\mathrm{MBL}} = \sum_i \left(J_i \sigma_i^z \sigma_{i+1}^z + h_i^z \sigma_i^z + \cancel{h_i^x \sigma_i^x}\right)$$

$$J_i, h_i^z \in [1/2, 1] \qquad h_i^x \in [0, h]$$

$$H_1 = \sum_i \sigma_i^x$$

$$e^{-i\pi H_1/2} = \prod \sigma_i^x$$

Khemani et al 2015

Solvable limit:

$h = 0$

$t_1 = \pi/2$

Else, BB, Nayak 2016

Floquet time crystal

- Solvable limit: $h = 0$

$$U_f = e^{-it_0 H_{\mathrm{MBL}}} \prod_i \sigma_i^x$$

$$H_{\mathrm{MBL}} = \sum_i \left(\sigma_i^z \sigma_{i+1}^z + \sigma_i^z\right)$$

- Eigenstates of H_{MBL}: product states $|\sigma\rangle = |\uparrow\uparrow\downarrow\uparrow\downarrow\downarrow \cdots\rangle$
- Eigenstates of U_f: $|\sigma^{\pm}\rangle = |\sigma\rangle \pm e^{i\alpha}|\bar{\sigma}\rangle$

Time-independent states are long-range correlated:

$$\langle O(x)O(y)\rangle - \langle O(x)\rangle\langle O(y)\rangle \neq 0$$

✔

Else, BB, Nayak 2016

Floquet time crystal

- Solvable limit: $h = 0$

$$U_f = e^{-it_0 H_{\mathrm{MBL}}} \prod_i \sigma_i^x$$

$$H_{\mathrm{MBL}} = \sum_i \left(\sigma_i^z \sigma_{i+1}^z + \sigma_i^z\right)$$

- Short-range correlated initial state $|\psi_0\rangle = |\sigma\rangle$:

$$\langle\psi_0|(U_f^\dagger)^n \sigma_i^z U_f^n|\psi_0\rangle = (-1)^n \langle\psi_0|\sigma_i^z|\psi_0\rangle$$

Physical initial states exhibit oscillations to times exponentially divergent in system size. ✓

Else, BB, Nayak 2016

Floquet time crystal

$$U_f = e^{-it_0 H_{\mathrm{MBL}}} e^{-it_1 H_1}$$

$$H_{\mathrm{MBL}} = \sum_i \left(J_i \sigma_i^z \sigma_{i+1}^z + h_i^z \sigma_i^z + h_i^x \sigma_i^x \right)$$

$$J_i, h_i^z \in [1/2, 1] \qquad h_i^x \in [0, h]$$

$$H_1 = \sum_i \sigma_i^x$$

Khemani et al 2015

Away from solvable limit:

$h \neq 0$

$t_1 \neq \pi/2$

Else, BB, Nayak 2016