Milestones toward Majorana-based quantum computing

Majorana-based quantum computing

“Majorana mountain”

Exponential degeneracy

Majorana signatures

Device fabrication

Albrecht et al., Nature 531, 206 (2016)

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Physical Review X 6, 031016 (2016)
(Ising) non-Abelian anyons

Excited states

Gap

Ground state with no anyons
(Ising) non-Abelian anyons

Excited states

Gap

Ground states w/ anyons present
(Ising) non-Abelian anyons

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But what about, say, free spin-$1/2$'s?
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Majorana zero modes encode degeneracy

(\text{Ising}) non-Abelian anyons

Excited states

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Locally indistinguishable ground states
(Ising) non-Abelian anyons

Locally indistinguishable ground states

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$\phi_i$
Locally indistinguishable ground states

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$\phi_i$
(Ising) non-Abelian anyons

Non-Abelian statistics

$$\phi_i \rightarrow U_{ij} \phi_j$$

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Locally indistinguishable ground states

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\[ \phi_i \rightarrow U_{ij} \phi_j \]

Nontrivial fusion rules
\[ \sigma \times \sigma = I + \psi \]

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Barkeshli, Bonderson, Cheng, and Wang, arXiv:1410.4540; Rowell and Wang, PRA 93, 030102(R)
Locally indistinguishable ground states

Excited states

Gap

Locally indistinguishable ground states

Qubits $|000\rangle$, $|001\rangle$, ...

Non-Abelian statistics

$\phi_i \rightarrow U_{ij} \phi_j$

Quantum gates

Nontrivial fusion rules

$\sigma \times \sigma = I + \psi$

Readout

$I$

‘$3 million idea’: fault-tolerant topological quantum computation!

Alexei Kitaev
(Ising) non-Abelian anyons

‘$3 million idea’: fault-tolerant topological quantum computation!

Billion $ question: how to build the hardware?

Alexei Kitaev
Inception (late 80's) Moore & Seiberg, Witten, …
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Realizations (1991-2000's)

- Moore-Read state $\gamma_{\text{Moore-Read}}$
  - e/4 Ising anyon
  - Moore & Read (1991)

- 2D $p+ip$ SC $\gamma_{\text{Read & Green}}$
  - h/2e vortex
  - Read & Green (2000)

- 1D $p$-wave SC $\gamma_{\text{Kitaev}}$
  - Kitaev (2001)
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- s-wave SC
  - 3D TI
  - Fu & Kane (2008)

- (many others)

- Lutchyn et al., Oreg et al. (2010)
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- Marcus et al.
- Nadj-Perge et al.
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Majorana control era (coming soon?)
I. New all-electrical Majorana control scheme

II. Majorana-control milestones

Fusion-rule detection

\[ \gamma_1 \rightarrow \leftrightarrow \gamma_2 \]

Topological qubit validation

\[ \sigma \times \sigma = I + \psi \]

Braiding
I. New all-electrical Majorana control scheme

Goal: leverage quantum dot/spin qubit tools for experiments below

II. Majorana-control milestones

\[ \sigma \times \sigma = I + \psi \]

Majorana control scheme: setup

nanowire (e.g., InAs)

$V_g$
Majorana control scheme: setup

- Mesoscopic superconducting island (e.g., Al)
- Nanowire (e.g., InAs)
- Island gate $V_g$
Majorana control scheme: setup

- Magnetic field needed for Majoranas
- Mesoscopic superconducting island (e.g., Al)
- Nanowire (e.g., InAs)
- Island gate

Lutchyn, Sau, Das Sarma PRL 105, 077001 (2010);
Oreg, Refael, von Oppen PRL 105, 177002 (2010)
Majorana control scheme: setup

- Magnetic field needed for Majoranas
- Trivial bulk superconductor
- Mesoscopic superconducting island (e.g., Al)
- Nanowire (e.g., InAs)
- $V_g$: island gate
Majorana control scheme: setup

\[ H = H_J[\hat{\phi}] + E_C (\hat{n} - n_{\text{offset}})^2 \]
Majorana control scheme: setup

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Parity-to-charge conversion

Initialization

Charge eigenstates

Parity eigenstates

Readout
Multi-island extension

Double dot
Multi-island extension

Bulk SC

SC island

SC island

Bulk SC

Double dot

Single dot
Multi-island extension

Double dot
Single dot
Topological superconductor
Multi-island extension

Double dot
Single dot
Topological superconductor
Double topological superconductor
Multi-island extension

Double dot
Single dot
Topological superconductor
Double topological superconductor

Suffice for fusion rules, topological qubit validation.
Multi-island extension

- Double dot
- Single dot
- Topological superconductor
- Double topological superconductor

Suffice for fusion rules, topological qubit validation.

Readily extends to networks (braiding).
I. New all-electrical Majorana control scheme

II. Majorana-control milestones

Fusion-rule protocol

Control (boring)
Fusion-rule protocol

Control (boring)

\(|Q_L, Q_R\rangle\)
Fusion-rule protocol

Control (boring)

$|Q_L, Q_R\rangle$

$\gamma_1 \gamma_2 \gamma_3 \gamma_4$
Fusion-rule protocol

Control (boring)

\[ |Q_L, Q_R\rangle \]

\[ \gamma_1 \quad \gamma_2 \quad \gamma_3 \quad \gamma_4 \]

Fuse

\[ |Q_L, Q_R\rangle \]

**Deterministic charge measurement**

\[ \sigma \times \sigma = I + \psi \]
Fusion-rule protocol

\[ |Q_L, Q_R\rangle \]

Control (boring)

|Q_L, Q_R\rangle

Nontrivial

\[ \gamma_1 \quad \gamma_4 \]

Fuse

\[ |Q_L, Q_R\rangle \]

Deterministic charge measurement

\[ \sigma \times \sigma = I + \psi \]
Fusion-rule protocol

Control (boring)

Nontrivial

Islands entangled!

Deterministic charge measurement

$\sigma \times \sigma = I + \psi$
Fusion-rule protocol

Deterministic charge measurement
\[ \sigma \times \sigma = I + \psi \]

Probabilistic charge measurement!
\[ \sigma \times \sigma = I + \psi \]
Fusion-rule protocol

Comments
1. Experiment is “non-Abelian”
2. Control distinguishes from probabilistic outcome due to noise
3. Charge sensing unnecessary—can use “Majorana-mediated pump”!

Deterministic charge measurement
\[ \sigma \times \sigma = I + \psi \]

Probabilistic charge measurement!
\[ \frac{1}{\sqrt{2}} (|Q_L, Q_R\rangle + |Q_L - 1, Q_R + 1\rangle) \]
I. New all-electrical Majorana control scheme

II. Majorana-control milestones

\[ \gamma_1 \to \gamma_2 \]

\[ \sigma \times \sigma = I + \psi \]
Prototype topological qubit

Locally indistinguishable ground states

Excited states

Gap

$|0\rangle$  $|1\rangle$
Prototype topological qubit

Topological protection requires
1. Time scales $\ll$ poisoning time
2. System confined to ground states w/high probability (i.e., low T, low frequency noise)
Prototype topological qubit

Topological protection requires
1. Time scales << poisoning time
2. System confined to ground states w/high probability (i.e., low T, low frequency noise)

Fundamental question:
Assuming these hold, how to verify topological protection in above device??
Prototype topological qubit

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Excited states
Locally indistinguishable ground states
|0⟩ |1⟩

“Accidental” degeneracies

Conventional zero-energy bound states
Prototype topological qubit

Fundamental question: Assuming these hold, how to verify topological protection in above device??

Answer must sharply distinguish these qubits!

Conventional zero-energy bound states

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Conventional zero-energy bound states

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Gap

|0⟩  |1⟩
Peculiar noise sensitivity

\[ \omega_0 \propto \cos(\kappa L) e^{-L/\xi} \]

\[ |1\rangle \quad \Downarrow \quad |0\rangle \]
Peculiar noise sensitivity

$$\omega_0 \propto \cos(\kappa L) e^{-L/\xi}$$

$$|1\rangle \quad \mid$$

$$|0\rangle \quad \uparrow$$

$e^{-L/\xi(\mu)}$

chem pot $\mu$
Peculiar noise sensitivity

\[ \omega_0 \propto \cos(\kappa L)e^{-L/\xi} \]

\[ |1\rangle \begin{array}{c} \downarrow \\ |0\rangle \end{array} \]

Time-averaged quantities & fluctuations deeply linked for topological qubit!
Peculiar noise sensitivity

\[ \omega_0 \propto \cos(\kappa L) e^{-L/\xi} \]

\[
\begin{align*}
|1\rangle & \quad \downarrow \\
|0\rangle & \quad \uparrow
\end{align*}
\]

\[ \omega_0 \sim \Delta E_{\text{typical}} \]

\[ T_2 \sim \hbar/\Delta E_{\text{typical}} \quad \rightarrow \quad 1/T_2 \sim \omega_0 \]

Time-averaged quantities & fluctuations deeply linked for topological qubit!
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\[ |1\rangle \downarrow \\
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\[ \omega_0 \sim \Delta E_{\text{typical}} \]

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\[ 1/T_2 \sim \omega_0 \]

Time-averaged quantities & fluctuations deeply linked for topological qubit!
Measuring oscillation frequency & $T_2$

Initialize double dot

Evolution on Bloch sphere:

Initialize qubit into $|0\rangle = |0_{12}, 0_{34}\rangle$

Apply $\pi/2$ pulse

Unitary evolution for time $t$

Apply $\pi/2$ pulse

Readout

$\omega_0$

$e^{-L/\xi(\mu)}$

$1/T_2$

Time-averaged quantities & fluctuations deeply linked for topological qubit!
I. New all-electrical Majorana control scheme

II. Majorana-control milestones

\[ \gamma_1 \rightarrow \gamma_2 \]

\[ \sigma \times \sigma = I + \psi \]
Initialize

$|Q_L, Q_R\rangle$
Braiding protocol

Initialize

$|Q_L, Q_R\rangle \rightarrow |0\rangle$
Braiding protocol

$\gamma_1 \quad \gamma_3 \quad \gamma_4$

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\[ |Q_L, Q_R\rangle \rightarrow |0\rangle \]
Braiding protocol

\[ |Q_L, Q_R\rangle \rightarrow |0\rangle \]

\[
\begin{bmatrix}
|0\rangle + i|1\rangle \\
\sqrt{2}
\end{bmatrix}
\]
Braiding protocol

Initialize

\[ |Q_L, Q_R\rangle \rightarrow |0\rangle \]

\[ \frac{|0\rangle + i|1\rangle}{\sqrt{2}} \]

\[ |Q_L, Q_R\rangle + i|Q_L - 1, Q_R + 1\rangle \]

\[ \frac{1}{\sqrt{2}} \]

Readout (Probabilistic)
Braiding protocol

\[ |Q_L, Q_R\rangle \rightarrow |0\rangle \]
\[ |0\rangle + i|1\rangle \]
\[ \frac{1}{\sqrt{2}} \]
\[ |Q_L, Q_R\rangle + i|Q_L - 1, Q_R + 1\rangle \]

Readout (Probabilistic)

\[ |Q_L - 1, Q_R + 1\rangle \]

Readout (Deterministic!)
Milestones for the Majorana control era

Fusion-rule detection

$\gamma_1 \rightarrow \gamma_2$

$\sigma \times \sigma = I + \psi$

Topological qubit validation

Braiding
Milestones for the Majorana control era

Realizable with single-wire devices!
(May generalize to other platforms/schemes)

\[ \gamma_1 \xrightarrow[\sim]} \gamma_2 \]

\[ \sigma \times \sigma = I + \psi \]
Milestones for the Majorana control era

Realizable with single-wire devices!
(May generalize to other platforms/schemes)

Fusion-rule detection
\[ \gamma_1 \quad \gamma_2 \]
\[ \sigma \times \sigma = I + \psi \]

Topological qubit validation

Braiding

Reveals defining property of non-Abelian anyons; natural braiding precursor
Milestones for the Majorana control era

Realizable with **single-wire devices**!
(May generalize to other platforms/schemes)

- **Fusion-rule detection**
  - $\gamma_1 \rightarrow \gamma_2$
  - $\sigma \times \sigma = I + \psi$

- **Topological qubit validation**

- **Braiding**

Reveals **defining** property of non-Abelian anyons; natural braiding precursor

Verify basic tenets of topological quantum computing

All protocols very difficult to mimic in non-topological setups.