Interface Between Topological and Superconducting Qubits

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Motivation

• Conventional Quantum Systems
  – E.g., spins, ions, photons, SC devices, ...
  – Merits: universal gate set, distant entanglement, ...
  – Challenges: vulnerable to various imperfections

• Topological Quantum Systems
  – E.g., Kitaev lattice model, FQHE, Topological Insulators, ...
  – Merits: robust against local decoherence.
  – Challenges: non-universal, hard to build a network ...

• Hybrid Systems
  – Combined merits from both systems
  – Network of topological quantum computers

• Coherent Interface
  – Between topological & conventional quantum systems
  – E.g., Controlled-phase gate
TOPOLOGICAL QUANTUM SYSTEMS
Majorana Fermions

- Majorana Fermions (MFs)
  - Fermion $\gamma_a \gamma_b = -\gamma_b \gamma_a$
  - Own anti-particle $\gamma = \gamma^\dagger$
  - E.g., “half of a Dirac Fermion” $\gamma_{2j-1} = \frac{c_j + c_j^\dagger}{2}$ and $\gamma_{2j} = \frac{c_j - c_j^\dagger}{2i}$

- Search for MFs:
Topological Qubit

• Four MFs encode 1 topological qubit
  – Subspace with odd Dirac fermion
    \{ |1\rangle_{12} |0\rangle_{34}, |0\rangle_{12} |1\rangle_{34} \}.

• Braiding of MFs
  – Non-abelian anyons
    \[ \left| \psi_{\text{final}} \right\rangle = U_{AB} \left| \psi_{\text{init}} \right\rangle \]
    with \( U_{AB} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \)

• Comments:
  – 2N+2 MFs encode N topological qubit
  – Braiding MFs using quasi-1D T-junctions
  – Braiding MFs is not universal (for computation).

Topological Insulators

Interior: gapped insulator

\[ \text{TI (bulk)} \]

Surface: spin-locked conductor

\[ H_{TI} = \psi^\dagger (\nu \hat{\sigma} \cdot \hat{\rho} - \mu) \psi \]

How to create MFs? – Tri-Junction

• Topological Insulator
  – Interior: gapped insulator
  – Surface: spin-locked conductor

\[ H_{TI} = \psi^{\dagger} (\nu \vec{\sigma} \cdot \vec{p} - \mu) \psi \]

• S-wave superconductor

\[ H_S = \Delta \psi_{\uparrow}^{\dagger} \psi_{\downarrow}^{\dagger} + h.c. \]

• Similar to p+ip superconductor
  – Support MFs at vortices
    (e.g., Tri-Junction)

Fu and Kane, PRL 100, 096407 (2008)
Quantum Wire (S-TI-S)

\[ H = H_{TI} + H_{SC} \]

\[ H_{TI} = \psi^\dagger (v \hat{\sigma} \cdot \hat{p} - \mu) \psi \]

\[ H_S = \Delta \psi^\dagger \psi^\dagger + \text{h.c.} \]

With order parameter:

\[ \Delta(x, y) = \begin{cases} 
\Delta_0 e^{i\phi} & \text{for } y > W/2 \\
0 & |y| < W/2 \\
\Delta_0 & \text{for } y < -W/2 
\end{cases} \]

For \( W = \mu = 0 \)

Two branches of bound states

\[ E_{\pm}(p_x) = \pm \left[ v^2 p_x^2 + \Delta_0^2 \cos^2 \left( \phi/2 \right) \right]^{1/2} \]

For \( W \ll v/\Delta_0 \)

Effective low energy theory

\[ H_{\text{wire}} = -i \tilde{v} \tau^x \partial_x + \delta_\phi \tau^z \]

with \( \delta_\phi = \Delta_0 \cos \left( \phi/2 \right) \) and \( \tilde{v} \approx v + \ldots \)

Two MFs with Coupling

- Using two tri-junctions connected by a quantum wire
  - MF wavefunction controlled by $\varepsilon$
  - Interact along quantum wire

\[ H_{12}^{MF} = iE(\varepsilon)\gamma_1\gamma_2 \approx E(\varepsilon)Z_{\text{topo}} \]

- Single qubit unitary gate:
  \[ U = e^{iH_{12}^{MF}t} = e^{i\theta Z_{\text{topo}}} \]

- Universal set of operations

Fu and Kane, PRL 100, 096407 (2008)
Superposition of Evolutions

• Interaction between Two MFs
  – Overlap along quantum wire
  \[ H_{12}^{MF} = iE(\varepsilon)\gamma_1\gamma_2 \approx E(\varepsilon)Z_{topo} \]

• Observation
  – \( \hat{\varepsilon} \rightarrow |\varepsilon_0\rangle + |\varepsilon_1\rangle \) induces superposition of evolutions (i.e., Ctrl-Phase evolution)
  – Highly non-linear (good for switch on/off)

• How to achieve \( |\varepsilon_0\rangle + |\varepsilon_1\rangle \) ?

SUPERCONDUCTING FLUX QUBITS
**Flux Qubit**

- **Series of (three) Josephson Junctions**
  - Josephson (potential) energy \( U = -\sum_i E_{J,i} \cos \theta_i \)
  - Charging (kinetic) energy \( T = \frac{1}{2} \sum_i C_i V_i^2 = \frac{\Phi_0^2}{8\pi^2} \sum_i C_i \dot{\theta}_i^2 \)
  - Phase constraint \( \sum_i \theta_i + 2f\pi \equiv 0 \pmod{2\pi} \)
  - Two potential minimum \( (1/2<\alpha<1, \ f=1/2) \)

- **SC Flux Qubit**
  - CW/CCW current
  - Superposition of two values of \( \epsilon \)
  - But, too large difference
**Flux Qubit**

- Add a **fourth** junction ($\beta \gg 1$)
  - Josephson (potential) energy $U = -\sum_i E_{J,i} \cos \theta_i$
  - Charging (kinetic) energy $T = \frac{1}{2} \sum_i C_i V_i^2 = \frac{\Phi_0^2}{8\pi^2} \sum_i C_i \dot{\theta}_i^2$
  - Phase constraint $\sum_i \theta_i + 2f\pi \equiv 0 \pmod{2\pi}$
  - Two potential minimum ($1/2 < \alpha < 1$, $f=1/2$)

- **SC Flux Qubit**
  - CW/CCW current
  - Superposition of two values of $\varepsilon$
    $$\varepsilon = \phi_c + \Delta \varepsilon \cdot Z_{\text{flux}}$$
    $$\Delta \varepsilon \approx \frac{1}{\beta} \sqrt{1 - \frac{1}{4\alpha^2}}$$ for $\beta \gg 1$. 
HYBRID SYSTEM
Hybrid System

- Topological Quantum Wire & Flux Qubit
- $\varepsilon$ coherently controls the coupling between MFs:

$$H_{12}^{MF} \Rightarrow \begin{cases} 
E(\varepsilon_0)Z_{\text{topo}} & \text{for } \varepsilon = \varepsilon_0 \text{ with } |0\rangle_{\text{flux}} \\
E(\varepsilon_1)Z_{\text{topo}} & \text{for } \varepsilon = \varepsilon_1 \text{ with } |1\rangle_{\text{flux}}
\end{cases}$$

\[ \Lambda_{\varepsilon} = \frac{\Delta_0}{v/L} \sin \frac{\varepsilon}{2} \]

Switch on Interaction
Switch off Interaction
QUANTUM FLUCTUATIONS
FLUX QUBIT
Quantum Fluctuations

- Harmonic Oscillator model

\[ H \simeq \frac{1}{2} C_4 V^2 + E_{J,4} \left( 1 - \cos \left( \theta_4 - \Delta \varepsilon \cdot Z_{\text{flux}} \right) \right) \]

\[ \approx \frac{\hat{p}_e^2}{2 \left( \beta / 8 E_C \right)} + \frac{\beta E_J}{2} \left( \hat{\varepsilon} - \phi_c - \Delta \varepsilon \cdot Z_{\text{flux}} \right)^2 \]

- Oscillator frequency: \( \omega = \sqrt{8E_J E_C} \),

Quantum fluctuation: \( \delta \varepsilon \approx \frac{1}{\sqrt{\beta}} \left( \frac{8E_C}{E_J} \right)^{1/4} \propto \beta^{-1/2} \)

Phase separation: \( \Delta \varepsilon \approx \frac{1}{\beta} \sqrt{1 - \frac{1}{4\alpha^2}} \propto \beta^{-1} \)

\[ \hat{\varepsilon} \approx \phi_c + \Delta \varepsilon \cdot Z_{\text{flux}} + \delta \varepsilon \left( a^{\dagger} + a \right) \]
Coupling Hamiltonian

- Quantum Description of SC phase
  \[ \hat{\mathcal{E}} \approx \phi_c + \Delta \epsilon \cdot Z_{\text{flux}} + \Delta \epsilon \cdot (a^\dagger + a) \]

- MF Hamiltonian
  \[ H = E(\hat{\mathcal{E}}) Z_{\text{topo}} \approx \langle E(\hat{\mathcal{E}}) \rangle_{G.S.} Z_{\text{topo}} \]
  \[ = \left( \langle E_0 \rangle |0\rangle\langle 0| + \langle E_1 \rangle |1\rangle\langle 1| \right)_{\text{flux}} \otimes Z_{\text{topo}} \]
  with \( \langle E_{0/1} \rangle \equiv \int E(\epsilon) p_{0/1}(\epsilon - \phi_c) \, d\epsilon \)

- Coupling for Controlled-Phase Gate
  \[ H_I = \frac{g}{4} Z_{\text{flux}} Z_{\text{topo}} \]
  with coupling strength
  \[ g \approx \left( E(\epsilon_1) - E(\epsilon_0) \right) + \frac{1}{4} \left( E''(\epsilon_1) - E''(\epsilon_0) \right) \Delta \epsilon^2 + ... \]

- Transfer quantum information between topological and flux-qubits...

- Diagram showing the coupling between topological and flux states with parameters \( \epsilon, \phi_c, \theta_1, \theta_2, \theta_3, \theta_4, \alpha E_j, \beta E_j \).
Transfer Quantum Information

- **QND Repetitive measurement**

  Topo qubit: $|0\rangle, |1\rangle$
  Flux qubit: $|0\rangle + |1\rangle$

- **SWAP** quantum state between topological qubit to flux qubit

  Topo qubit: $|\psi_1\rangle$
  Flux qubit: $|\psi_2\rangle$
Imperfections

- Flux qubit tunneling \( \eta_{\text{tunnel}} \approx (t / g)^2 \)
- Oscillator excitation \( \eta_{\text{exc}} \approx (g / \omega)^2 \)
- Finite length L \( \eta_L \approx e^{-KL} \)
- Thermal excitation \( \eta_{\text{th}} \approx e^{-\nu_F/(Lk_BT)} \)

Possible to have \( \eta < 10^{-2} \), with parameters:

\[
E_j = 200(2\pi)GHz, E_C = 2.5(2\pi)GHz, \alpha = 0.8, \beta = 10
\Rightarrow \Delta \varepsilon \sim \delta \varepsilon \sim 0.1 \text{ rad}
\]

\[
L \approx 5\mu m, \nu_F \approx 10^5 m/s, \Delta_0 \approx 0.1 \text{meV}, T = 20mK
\Rightarrow g \approx 0.2\sim2 (2\pi)GHz,
\Rightarrow \omega \gg g \gg t \gg 1/T_2
\]

with \( \omega \approx 60(2\pi)GHz \), \( t \approx 70(2\pi)MHz \), \( T_2 \approx 4\mu s \).
Summary & Outlook

• Hybrid system of topological and flux qubits
  – Measure, probe anyonic statics, connect different topological systems, ...

• Various related proposals
  – Semiconductor quantum wire + SC flux qubit using Aharonov-Casher effect
    • Hassler et al., NJP 12, 125002 (2010)
    • Bonderson, Lutchyn, PRL 106, 130505 (2011)
  – Toric code & cavity QED
    • Jiang, et al., Nature Physics 4, 482 (2008)

• Topological quantum networks
  – Optically connect topological systems