Interface Between Topological and Superconducting Qubits

Liang Jiang

IQI, Caltech

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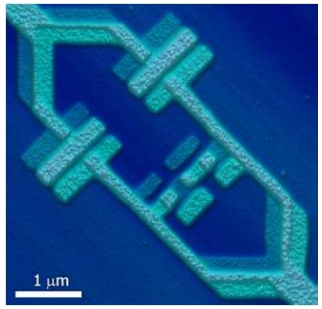
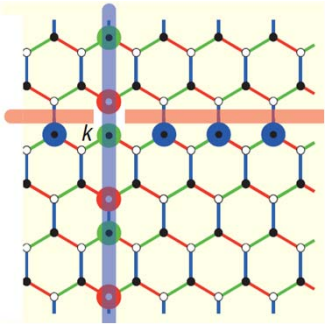
In collaboration with: Charlie Kane and John Preskill

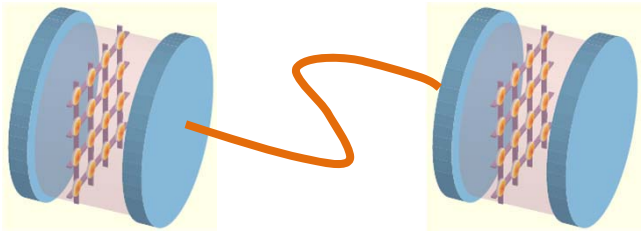
Phys. Rev. Lett. 106, 130504 (2011), arXiv 1010.5862.

Motivation

- Conventional Quantum Systems
  - E.g., spins, ions, photons, SC devices, …
  - Merits: universal gate set, distant entanglement, …
  - Challenges: vulnerable to various imperfections
- Topological Quantum Systems
  - E.g., Kitaev lattice model, FQHE, Topological Insulators, …
  - Merits: robust against local decoherence.
  - Challenges: non-universal, hard to build a network …
- Hybrid Systems
  - Combined merits from both systems
  - Network of topological quantum computers
- **Coherent Interface**
  - Between topological & conventional quantum systems
  - E.g., Controlled-phase gate







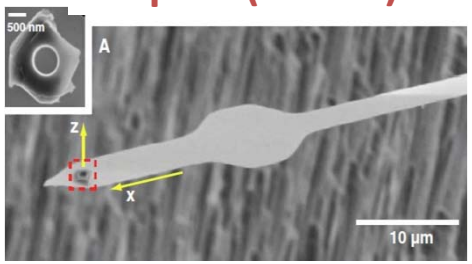
TOPOLOGICAL QUANTUM SYSTEMS

Majorana Fermions

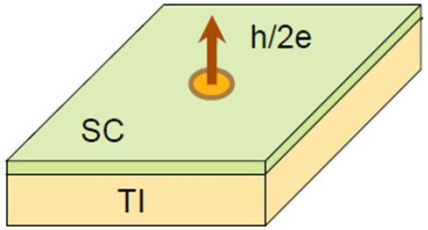
- Majorana Fermions (MFs)
  - Fermion $\gamma_a \gamma_b = -\gamma_b \gamma_a$
  - Own anti-particle $\gamma = \gamma^\dagger$
  - E.g., "half of a Dirac Fermion" $\gamma_{2j-1} = \dfrac{c_j + c_j^\dagger}{2}$ and $\gamma_{2j} = \dfrac{c_j - c_j^\dagger}{2i}$
- Search for MFs:

Ettore Majorana (1937)

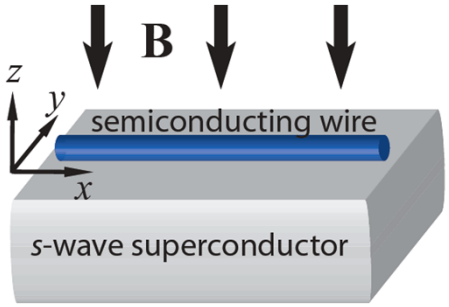
P+ip SC (SrRuO)



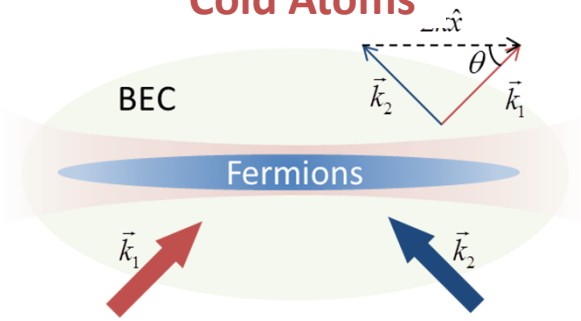
Topological Insulators



Quantum Wires

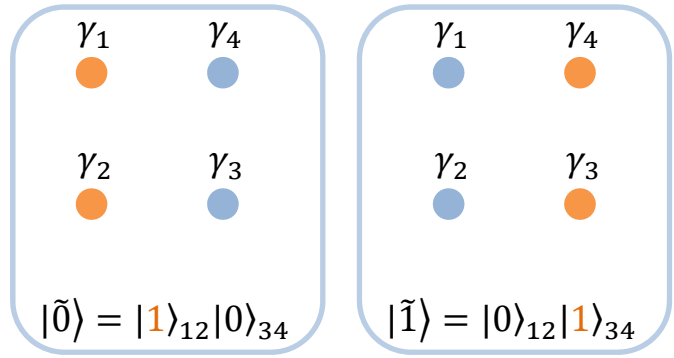


Cold Atoms



Topological Qubit

- Four MFs encode 1 topological qubit
  - Subspace with odd Dirac fermion $\{|1\rangle_{12}|0\rangle_{34}, |0\rangle_{12}|1\rangle_{34}\}$.

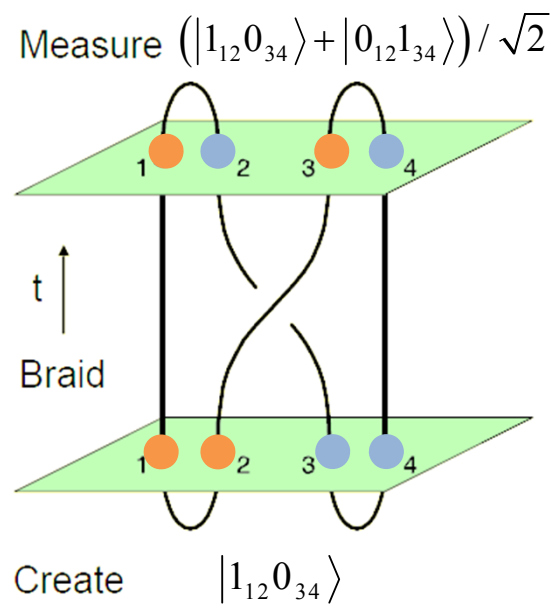


- Braiding of MFs
  - Non-abelian anyons

$$\left|\psi_{final}\right\rangle = U_{AB}\left|\psi_{init}\right\rangle$$

$$\text{with } U_{AB} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

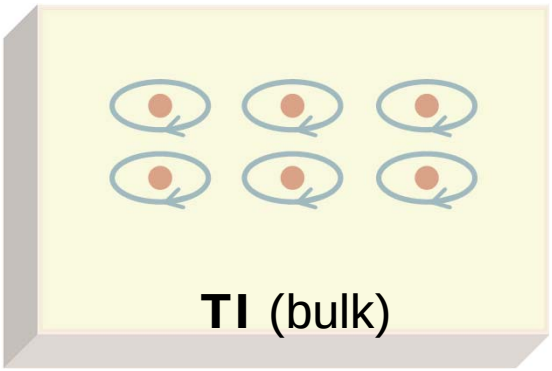
- Comments:
  - 2N+2 MFs encode N topological qubit
  - Braiding MFs using quasi-1D T-junctions
  - Braiding MFs is not universal (for computation).

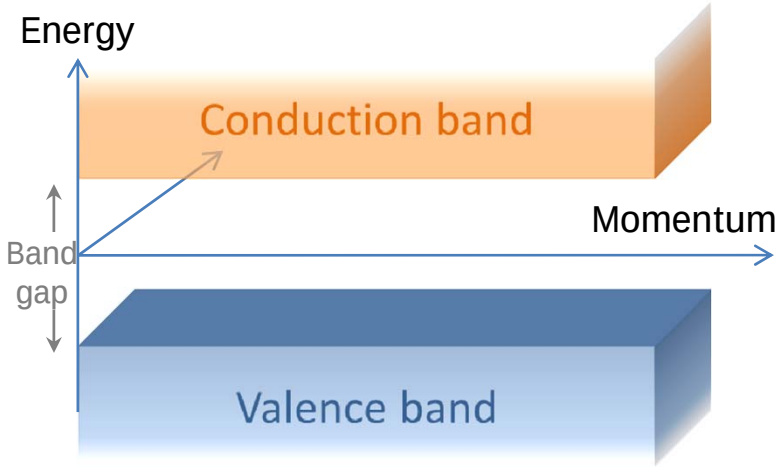


Nayak, et al., RMP 80, 1083 (2008). J. Alicea, et al., Nat. Phys. 7, 412-417 (2010).

Topological Insulators

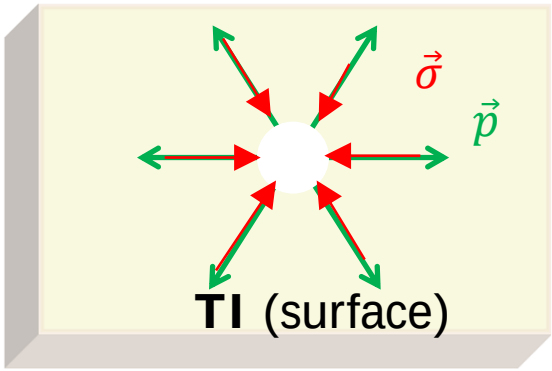
Interior: gapped insulator

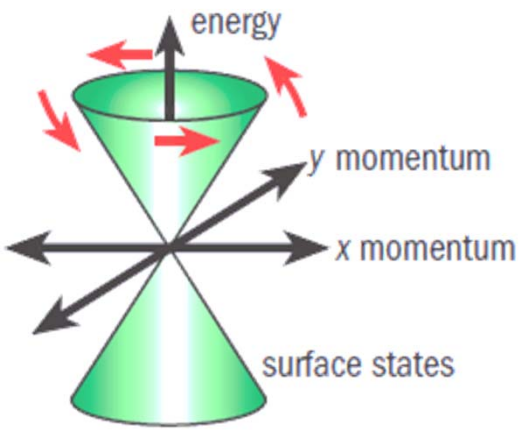




Surface: spin-locked conductor

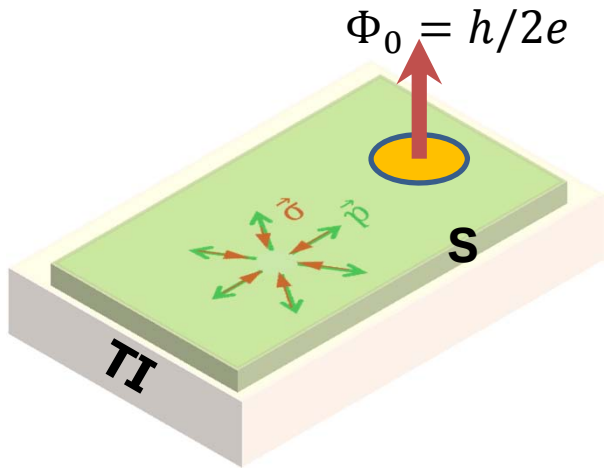
$$H_{TI} = \psi^{\dagger}(v\vec{\sigma}\cdot\vec{p} - \mu)\psi$$





Hasan and Kane, RMP 82, 3045 (2010). C. Brüne, et al., Phys. Rev. Lett. 106, 126803 (2011).

How to create MFs? – Tri-Junction



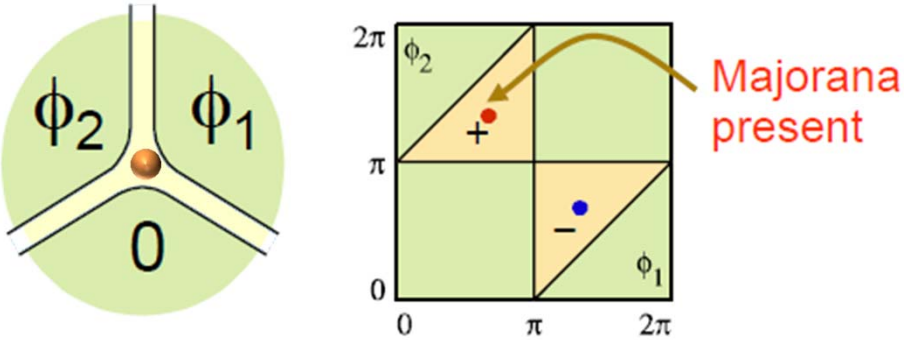
- Topological Insulator
  - Interior: gapped insulator
  - Surface: spin-locked conductor

$$H_{TI} = \psi^\dagger (v\vec{\sigma} \cdot \vec{p} - \mu)\psi$$

- S-wave superconductor

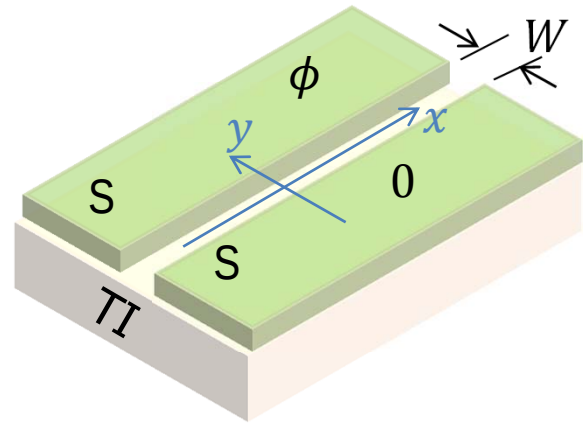
$$H_S = \Delta \psi_\uparrow^\dagger \psi_\downarrow^\dagger + h.c.$$

- Similar to p+ip superconductor
  - Support MFs at vortices (e.g., Tri-Junction)



Fu and Kane, PRL 100, 096407 (2008)

Quantum Wire (S-TI-S)



$$H = H_{TI} + H_{SC}$$

$$H_{TI} = \psi^{\dagger}(v\vec{\sigma}\cdot\vec{p} - \mu)\psi$$

$$H_S = \Delta\psi_{\uparrow}^{\dagger}\psi_{\downarrow}^{\dagger} + h.c.$$

With order parameter: $\Delta(x,y) = \begin{cases} \Delta_0 e^{i\phi} & \text{for } y > W/2 \\ 0 & |y| < W/2 \\ \Delta_0 & \text{for } y < -W/2 \end{cases}$

For $W = \mu = 0$ Two branches of bound states

$$E_{\pm}(p_x) = \pm\left[v^2 p_x^{\,2} + \Delta_0^2\cos^2(\phi/2)\right]^{1/2}$$

For $W \ll v/\Delta_0$ Effective low energy theory

$$\boxed{H_{wire} = -i\tilde{v}\tau^x\partial_x + \delta_\phi\tau^z}$$

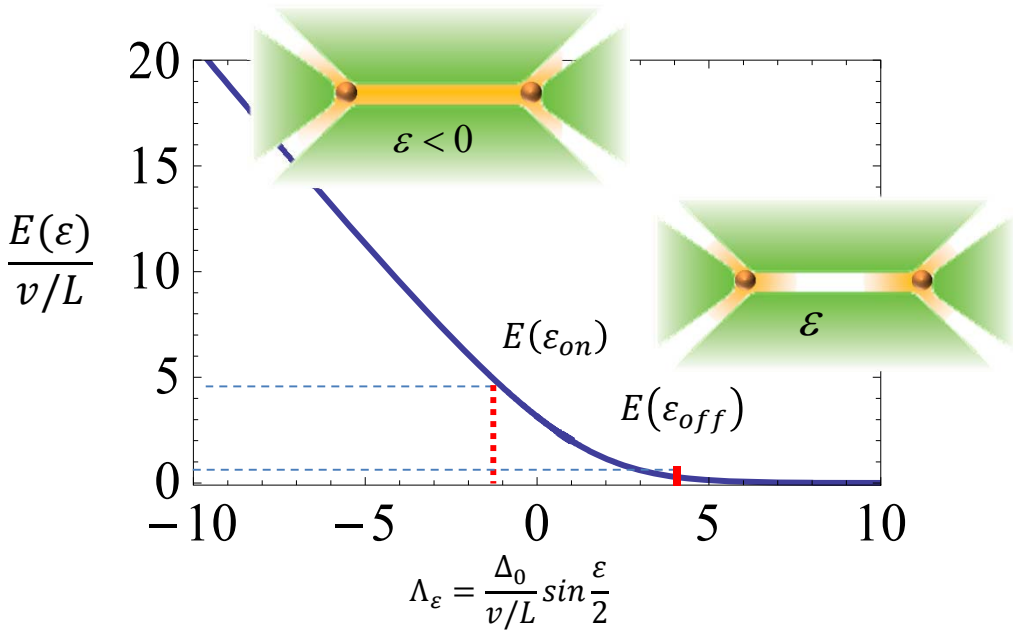
with $\delta_\phi = \Delta_0\cos(\phi/2)$ and $\tilde{v} \approx v + \cdots$

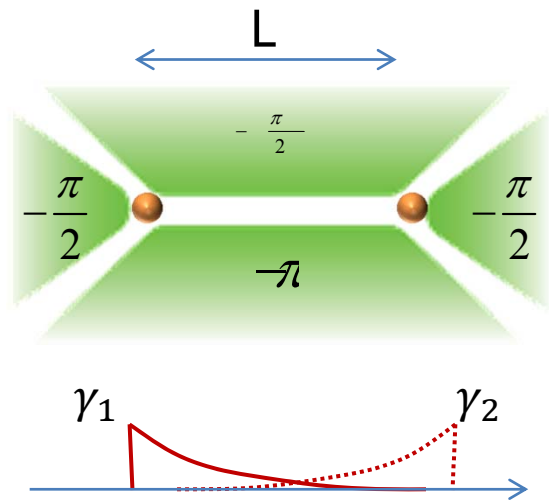
Fu and Kane, PRL 100, 096407 (2008). Titov and Beenakker, PRB 74, 041401(R) (2006)

Two MFs with Coupling

- Using two <u>tri-junctions</u> connected by a <u>quantum wire</u>
  - MF wavefunction controlled by ε
  - Interact along quantum wire

$$H_{12}^{MF} = iE(\varepsilon)\gamma_1\gamma_2 \cong E(\varepsilon)Z_{topo}$$





$$H_{wire} = -i\tilde{v}\tau^x\partial_x + \delta_\phi\tau^z$$

- Single qubit unitary gate:

$$U = e^{iH_{12}^{MF}t} = e^{i\theta Z_{topo}}$$

- Universal set of operations

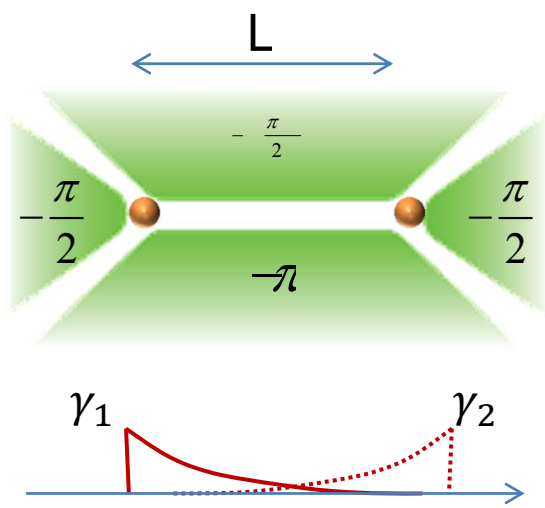
Fu and Kane, PRL 100, 096407 (2008)

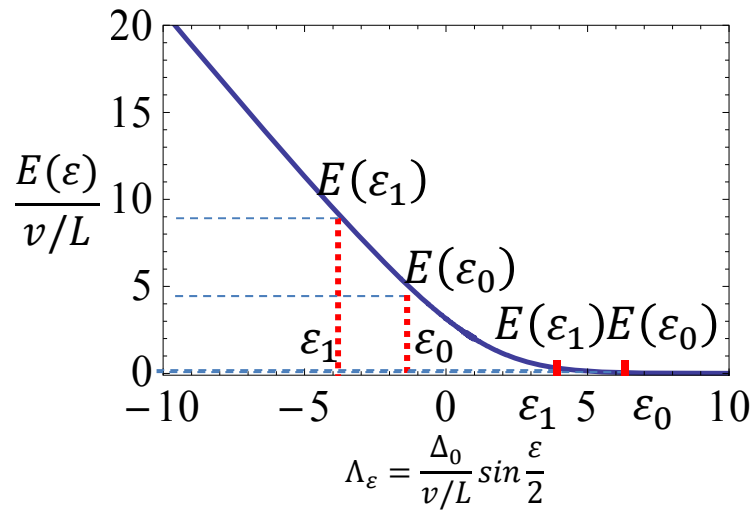
Superposition of Evolutions

- Interaction between Two MFs
  - Overlap along quantum wire

$$H_{12}^{MF} = iE(\varepsilon)\gamma_1\gamma_2 \cong E(\varepsilon)Z_{topo}$$

- Observation
  - $\hat{\varepsilon} \rightarrow |\varepsilon_0\rangle + |\varepsilon_1\rangle$ induces superposition of evolutions (i.e., Ctrl-Phase evolution)
  - Highly non-linear (good for switch on/off)
- How to achieve $|\varepsilon_0\rangle + |\varepsilon_1\rangle$?



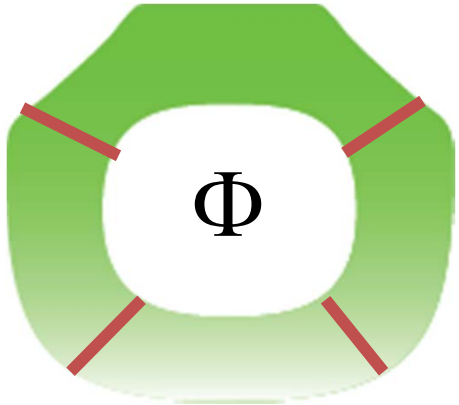


Switch on Interaction

Switch off Interaction

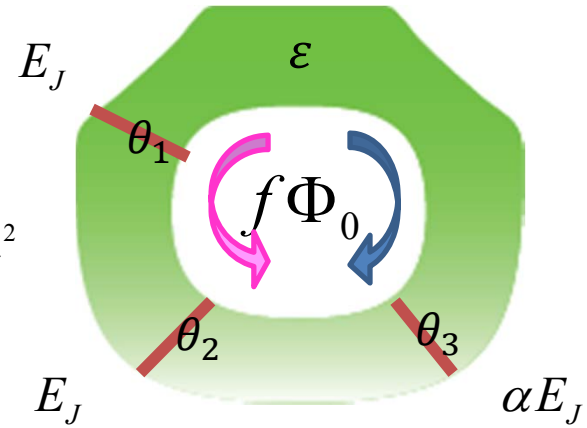
L. Jiang, C. L. Kane and J. Preskill, PRL 106, 130504 (2011).

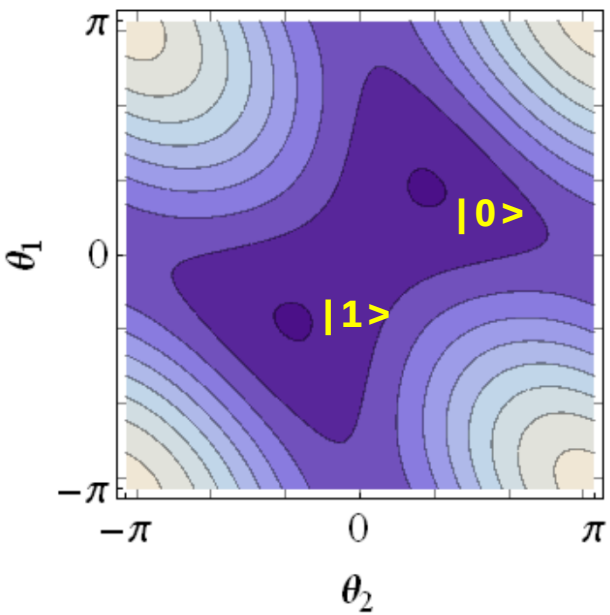
SUPERCONDUCTING FLUX QUBITS



Flux Qubit

- Series of (three) Josephson Junctions
  - Josephson (potential) energy $U = -\sum_i E_{J,i} \cos\theta_i$
  - Charging (kinetic) energy $T = \frac{1}{2}\sum_i C_i V_i^2 = \frac{\Phi_0^2}{8\pi^2}\sum_i C_i \dot{\theta}_i^2$
  - Phase constraint $\sum_i \theta_i + 2f\pi \equiv 0 \pmod{2\pi}$
  - Two potential minimum (1/2<α<1, f=1/2)

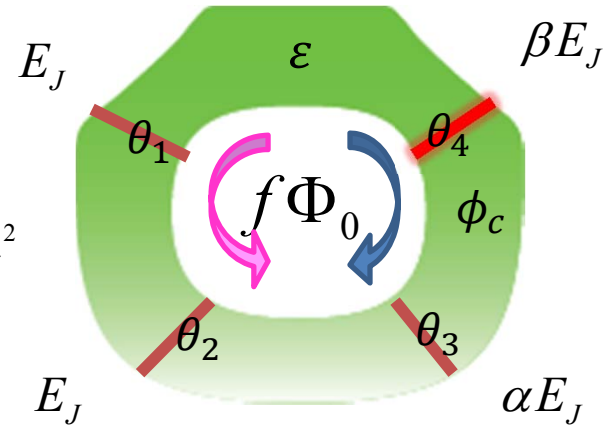


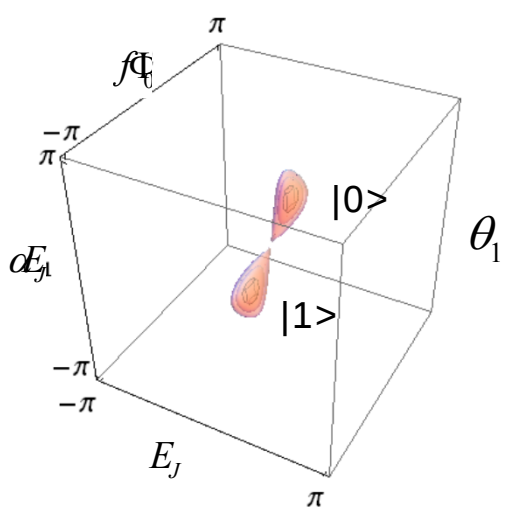


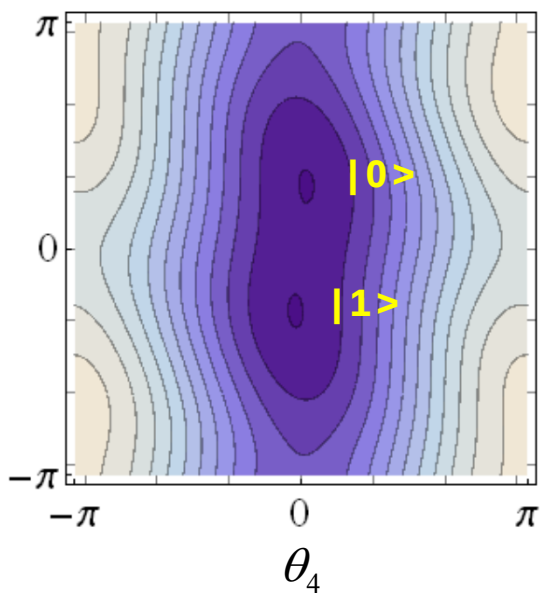
- SC Flux Qubit
  - CW/CCW current
  - Superposition of two values of ε
  - But, too large difference

Flux Qubit

- Add a **fourth** junction ($\beta \gg 1$)
  - Josephson (potential) energy $U = -\sum_i E_{J,i} \cos\theta_i$
  - Charging (kinetic) energy $T = \frac{1}{2}\sum_i C_i V_i^2 = \frac{\Phi_0^2}{8\pi^2}\sum_i C_i \dot{\theta}_i^2$
  - Phase constraint $\sum_i \theta_i + 2f\pi \equiv 0 \pmod{2\pi}$
  - Two potential minimum (1/2<α<1, f=1/2)







- SC Flux Qubit
  - CW/CCW current
  - Superposition of two values of ε

$$\varepsilon = \phi_c + \Delta\varepsilon \cdot Z_{flux}$$

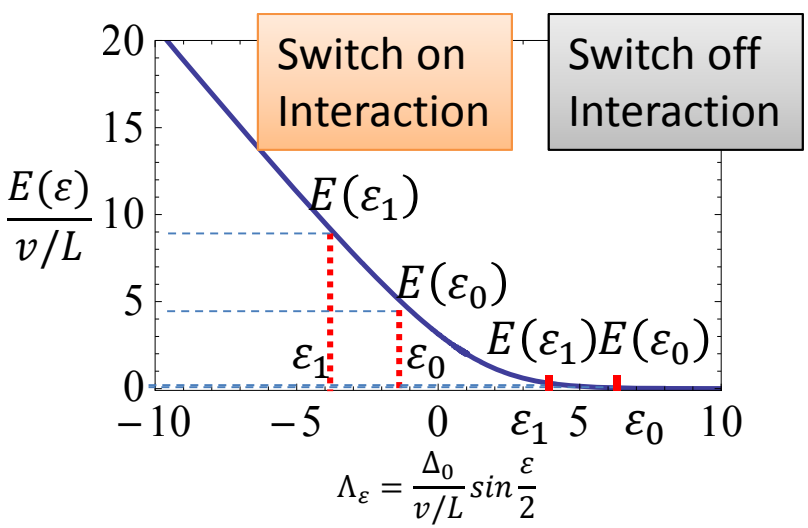
$$\Delta\varepsilon \approx \frac{1}{\beta}\sqrt{1-\frac{1}{4\alpha^2}} \text{ for } \boxed{\beta \gg 1.}$$

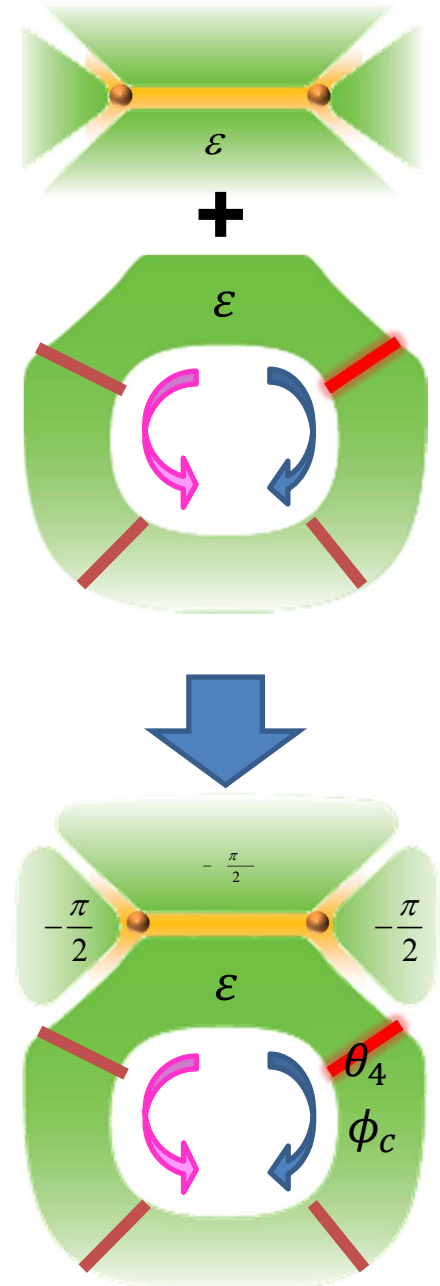
HYBRID SYSTEM

Hybrid System

- Topological Quantum Wire & Flux Qubit
- ε coherently controls the coupling between MFs:

$$H_{12}^{MF} \Rightarrow \begin{cases} E(\varepsilon_0) Z_{topo} & \text{for } \varepsilon = \varepsilon_0 \text{ with } |0\rangle_{flux} \\ E(\varepsilon_1) Z_{topo} & \text{for } \varepsilon = \varepsilon_1 \text{ with } |1\rangle_{flux} \end{cases}$$





QUANTUM FLUCTUATIONS
FLUX QUBIT

Quantum Fluctuations

- Harmonic Oscillator model

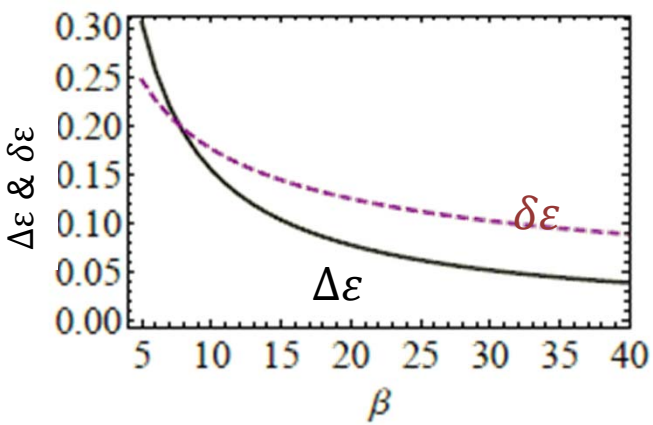
$$H \simeq \frac{1}{2} C_4 V_4^2 + E_{J,4}\left(1-\cos\left(\theta_4 - \Delta\varepsilon \cdot Z_{flux}\right)\right)$$

$$\simeq \frac{\hat{p}_\varepsilon^2}{2\left(\beta / 8E_C\right)} + \frac{\beta E_J}{2}\left(\hat{\varepsilon} - \phi_c - \Delta\varepsilon \cdot Z_{flux}\right)^2$$

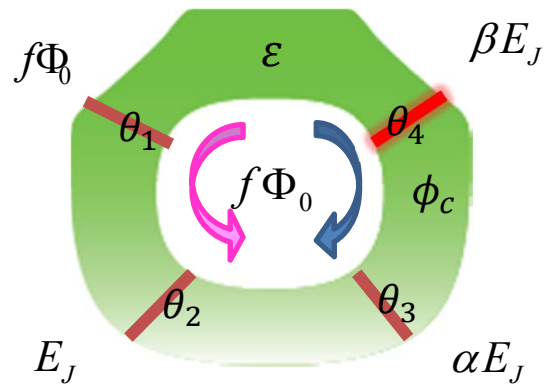
- Oscillator frequency: $\omega = \sqrt{8E_J E_C}$,

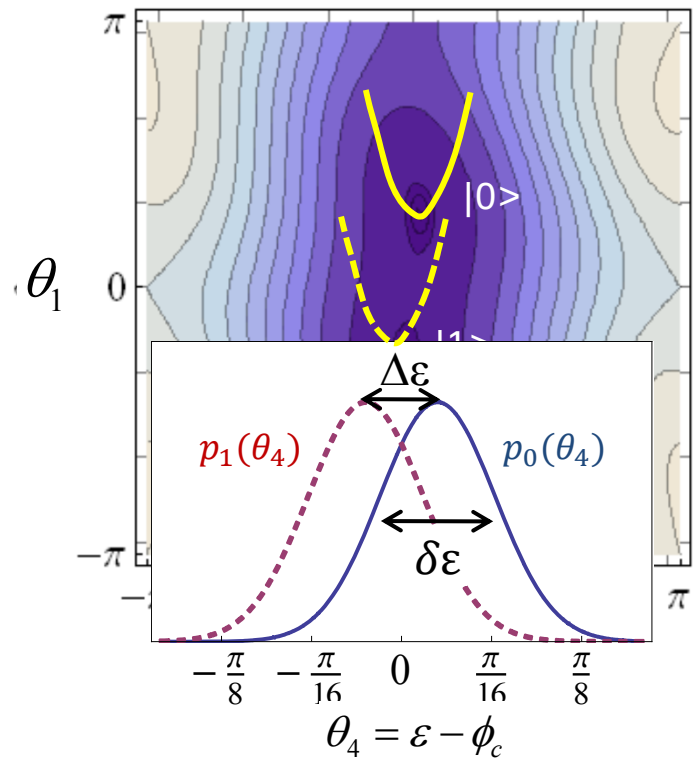
  Quantum fluctuation: $\delta\varepsilon \approx \frac{1}{\sqrt{\beta}}\left(\frac{8E_C}{E_J}\right)^{1/4} \propto \beta^{-1/2}$

  Phase separation: $\Delta\varepsilon \approx \frac{1}{\beta}\sqrt{1-\frac{1}{4\alpha^2}} \propto \beta^{-1}$



$$\hat{\varepsilon} \approx \phi_c + \Delta\varepsilon \cdot Z_{flux} + \delta\varepsilon \cdot \left(a^\dagger + a\right)$$





Coupling Hamiltonian

- Quantum Description of SC phase

$$\hat{\varepsilon} \approx \phi_c + \Delta\varepsilon \cdot Z_{flux} + \delta\varepsilon \cdot \left(a^\dagger + a\right)$$

- MF Hamiltonian

$$H = E\left(\hat{\varepsilon}\right) Z_{topo} \approx \left\langle E\left(\hat{\varepsilon}\right)\right\rangle_{G.S.} Z_{topo}$$

$$= \left(\left\langle E_0\right\rangle |0\rangle\langle 0| + \left\langle E_1\right\rangle |1\rangle\langle 1|\right)_{flux} \otimes Z_{topo}$$

with $\left\langle E_{0/1}\right\rangle \equiv \int E\left(\varepsilon\right) p_{0/1}\left(\varepsilon - \phi_c\right) d\varepsilon$

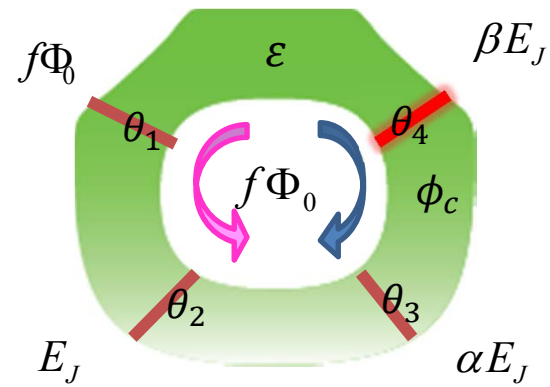
- Coupling for Controlled-Phase Gate

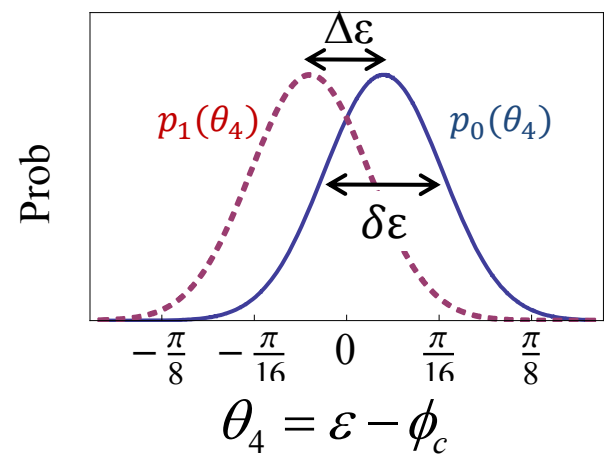
$$H_I = \frac{g}{4} Z_{flux} Z_{topo}$$

with coupling strength

$$g \approx \left(E\left(\varepsilon_1\right) - E\left(\varepsilon_0\right)\right) + \frac{1}{4}\left(E''\left(\varepsilon_1\right) - E''\left(\varepsilon_0\right)\right)\delta\varepsilon^2 + \ldots$$

- Transfer quantum information between topological and flux-qubits …



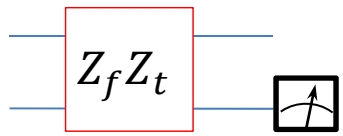


Transfer Quantum Information

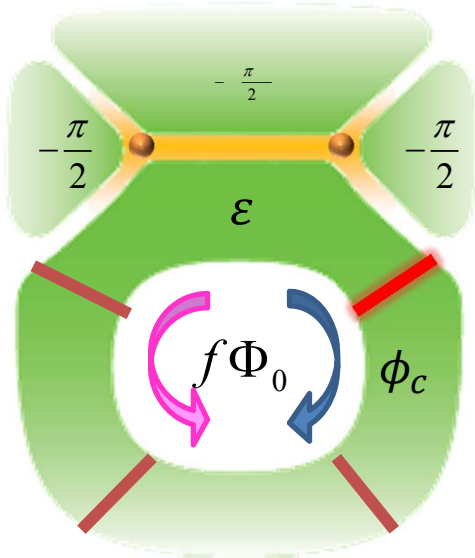
- QND Repetitive measurement

Topo qubit:. $|0\rangle, |1\rangle$

Flux qubit: $|0\rangle + |1\rangle$



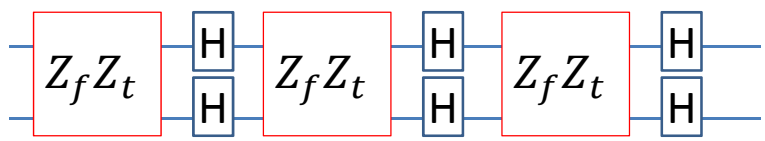
$|0\rangle \pm |1\rangle$



- **SWAP** quantum state between topological qubit to flux qubit

Topo qubit:. $|\psi_1\rangle$

Flux qubit: $|\psi_2\rangle$

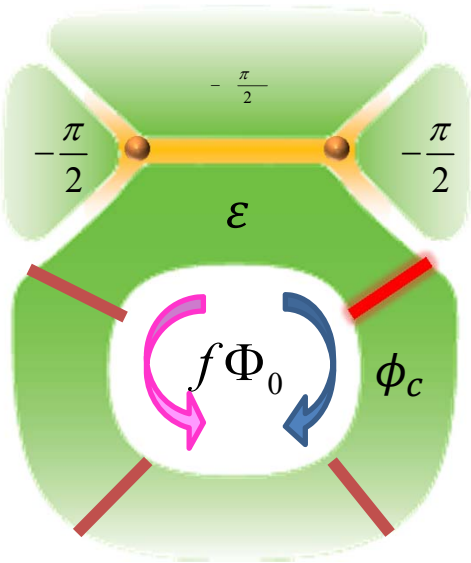


$|\psi_2\rangle$

$|\psi_2\rangle$

Imperfections

- Flux qubit tunneling $\eta_{tunnel} \approx (t/g)^2$
- Oscillator excitation $\eta_{exc} \approx (g/\omega)^2$
- Finite length L $\eta_L \approx e^{-KL}$
- Thermal excitation $\eta_{th} \approx e^{-v_F/(Lk_BT)}$



Possible to have $\eta < 10^{-2}$, with parameters:

$$E_J = 200(2\pi)GHz, E_C = 2.5(2\pi)GHz, \alpha = 0.8, \beta = 10$$
$$\Rightarrow \Delta\varepsilon \sim \delta\varepsilon \sim 0.1\, rad$$

$$L \approx 5\mu m,\ v_F \approx 10^5 m/s,\ \Delta_0 \approx 0.1 meV, T = 20mK$$
$$\Rightarrow \mathrm{g} \approx 0.2{\sim}2\ (2\pi)GHz,$$
$$\Rightarrow \omega \gg g \gg t \gg 1/T_2$$
with $\omega \approx 60(2\pi)GHz, t \approx 70(2\pi)MHz, T_2 \approx 4\mu s.$

Summary & Outlook

- Hybrid system of topological and flux qubits
  - Measure, probe anyonic statics, connect different topological systems, …
- Various related proposals
  - Semiconductor quantum wire + SC flux qubit using Aharonov-Casher effect
    - Hassler et al., NJP 12, 125002 (2010)
    - Bonderson, Lutchyn, PRL 106, 130505 (2011)
  - Toric code & cavity QED
    - Jiang, et al., Nature Physics 4, 482 (2008)
- Topological quantum networks
  - Optically connect topological systems