KITP program: Physics of Dense Suspensions

Extensional and shear flow material functions of dense suspensions— *microstructure, particle pressure, and N_1*

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JP17K05618

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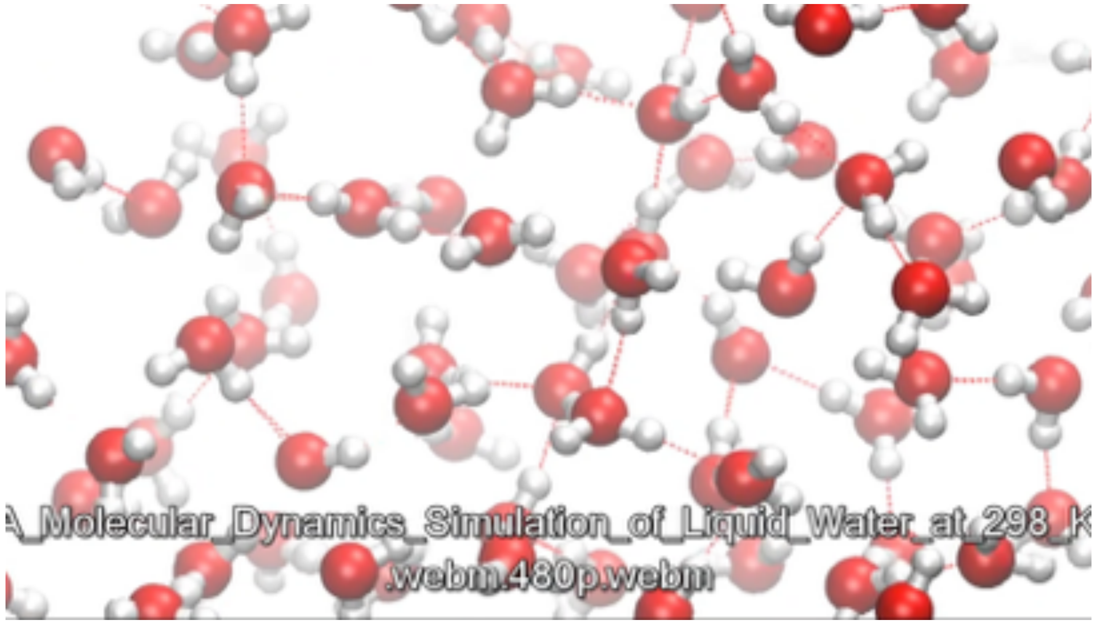
MD simulation



Molecular dynamics

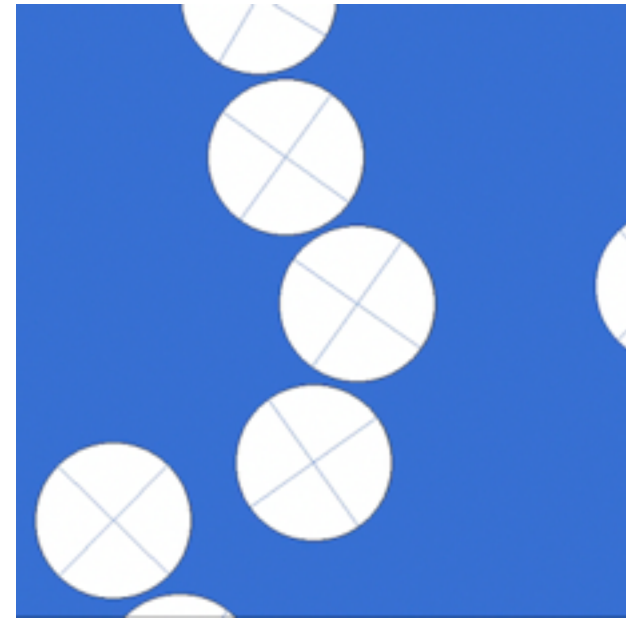
10^{-12}s

$\downarrow$

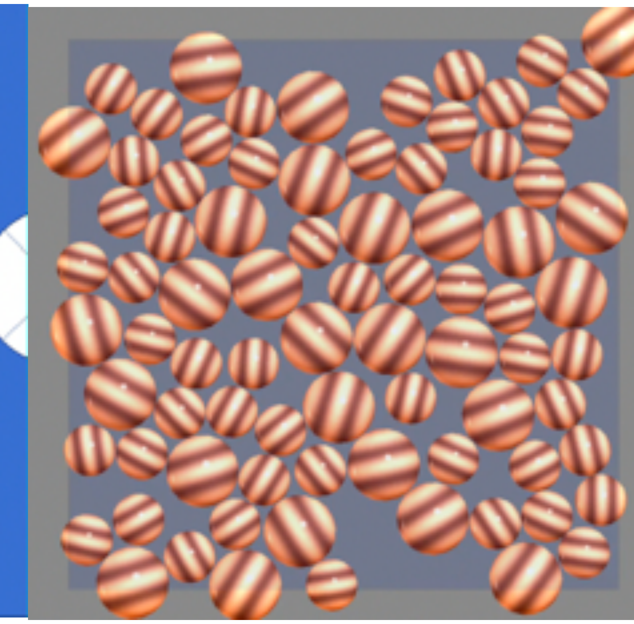
Newtonian

$$\boldsymbol{\sigma} = -p\boldsymbol{I} + 2\eta_0 \boldsymbol{D}$$

Stokesian Dynamics



Discrete Element Method



Dynamics of solid particles in a viscous liquid

$$\frac{6\pi\eta_0 a^3}{k_B T} \sim 1\text{s}$$

$\downarrow$

non-Newtonian

$$\boldsymbol{\sigma} = ???$$

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Rheology

velocity gradient

$$\begin{pmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} & \frac{\partial u_x}{\partial z} \\ \frac{\partial u_y}{\partial x} & \frac{\partial u_y}{\partial y} & \frac{\partial u_y}{\partial z} \\ \frac{\partial u_z}{\partial x} & \frac{\partial u_z}{\partial y} & \frac{\partial u_z}{\partial z} \end{pmatrix}$$

stress tensor

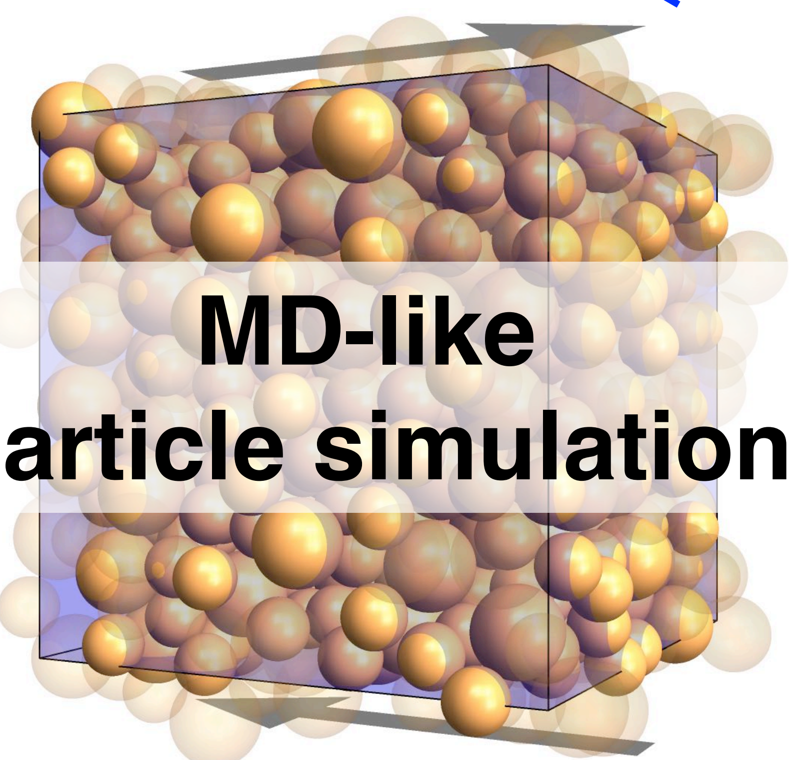
$$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{pmatrix}$$

$$\rho\left\{\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u}\right\} = \nabla\cdot\boldsymbol{\sigma}$$

Constitutive modeling

impose

response



material functions

scalars

(*not* coordinate specific)

physical interpretations

$$\eta(\dot{\gamma}),\ N_1(\dot{\gamma}),\ N_2(\dot{\gamma})$$

Outline

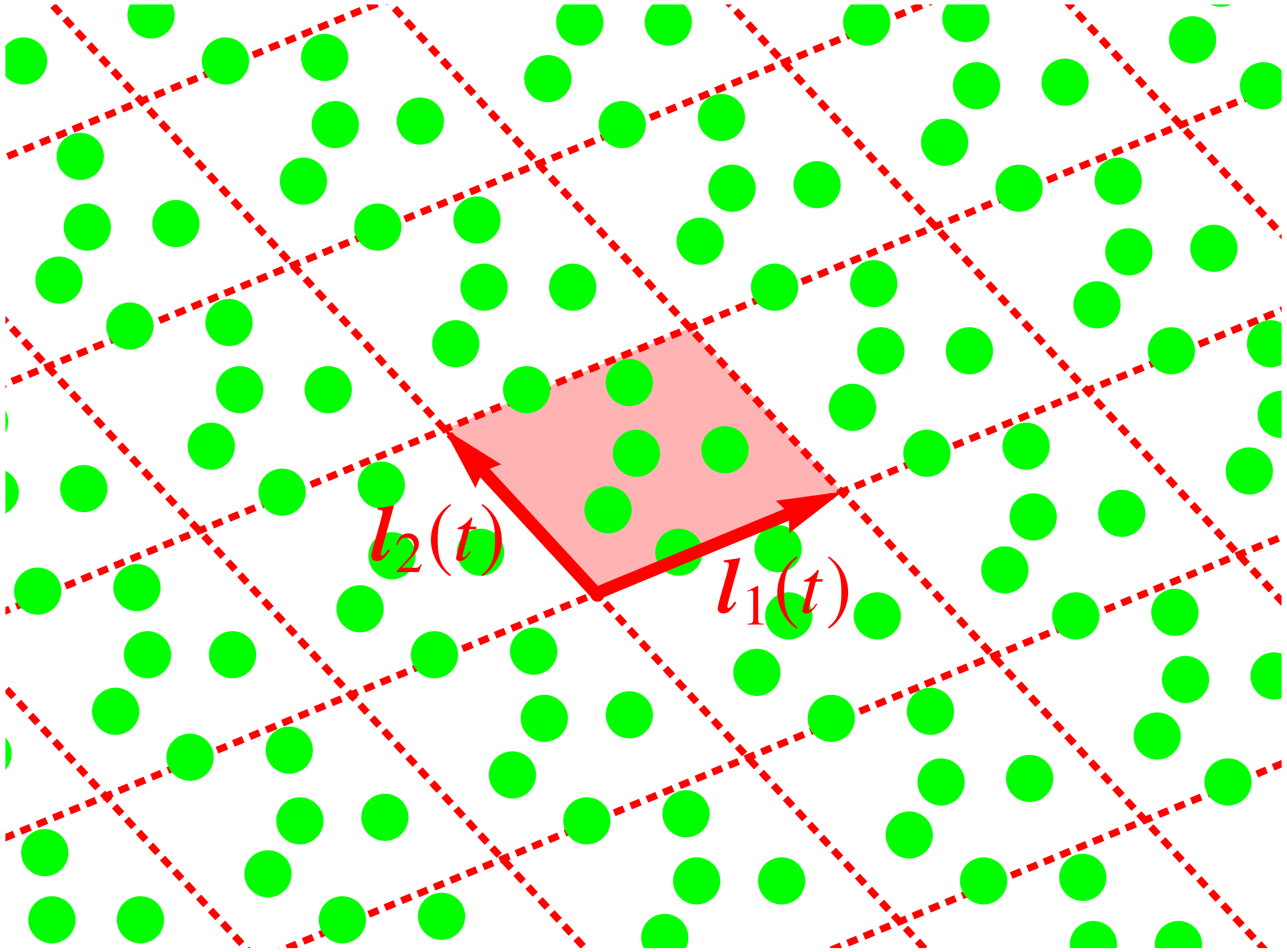
1. Periodic boundary conditions
2. Simulation model for particle dynamics
3. Material functions
4. N_1 issue
5. Extensional rheology

Time-dependent periodic boundary conditions

To impose $\nabla \boldsymbol{u}$ and to evaluate stress tensor $\boldsymbol{\sigma}$

$$\boldsymbol{r}' = \boldsymbol{r} + i\boldsymbol{l}_1(t) + j\boldsymbol{l}_2(t) \qquad \begin{array}{l} i = \pm 1, \pm 2, \ldots \\ j = \pm 1, \pm 2, \ldots \end{array}$$

copies original



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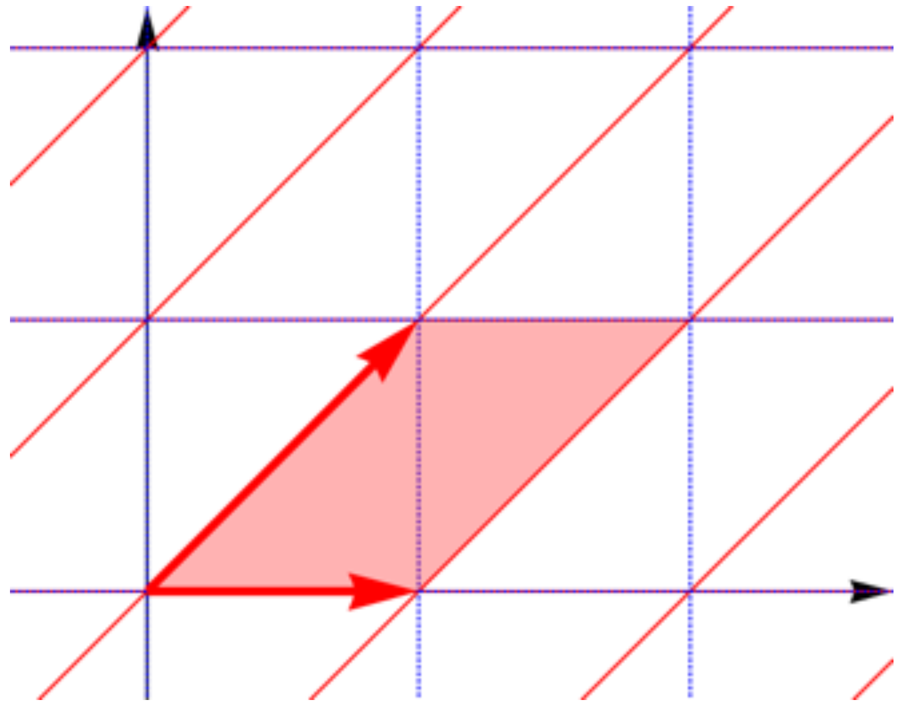
Time-dependent periodic boundary conditions

$$\boldsymbol{r}' = \boldsymbol{r} + i\boldsymbol{l}_1(t) + j\boldsymbol{l}_2(t) \qquad \begin{array}{l} i = \pm 1, \pm 2, \ldots \\ j = \pm 1, \pm 2, \ldots \end{array}$$

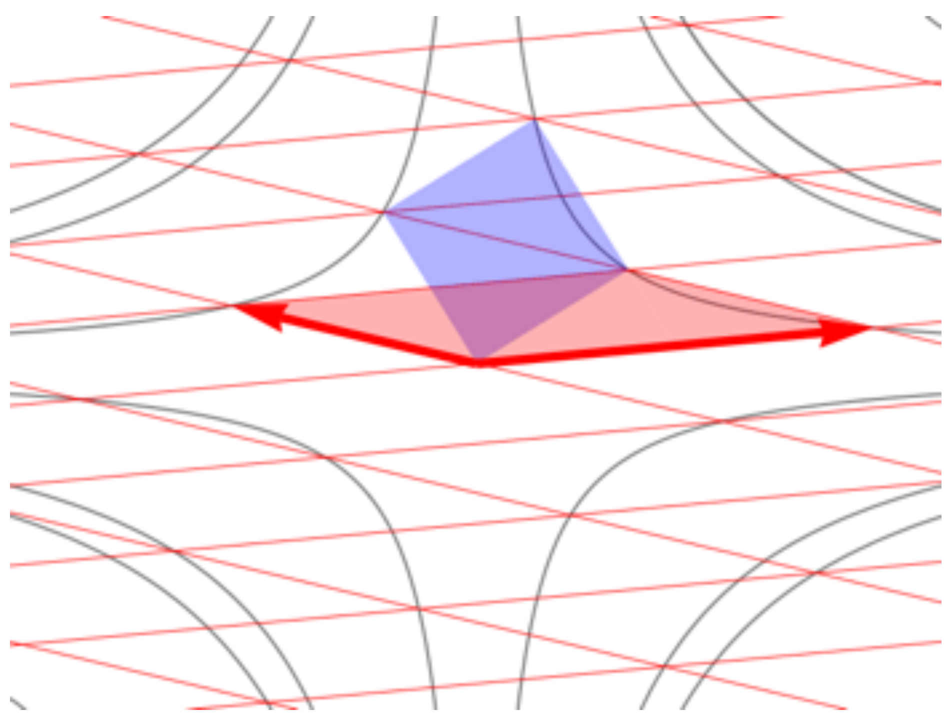
$$\nabla \boldsymbol{u} = \begin{pmatrix} 0 & \dot{\gamma} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



Lees–Edwards P.B.C.

simple shear

$$\nabla \boldsymbol{u} = \begin{pmatrix} \dot{\varepsilon} & 0 & 0 \\ 0 & -\dot{\varepsilon} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



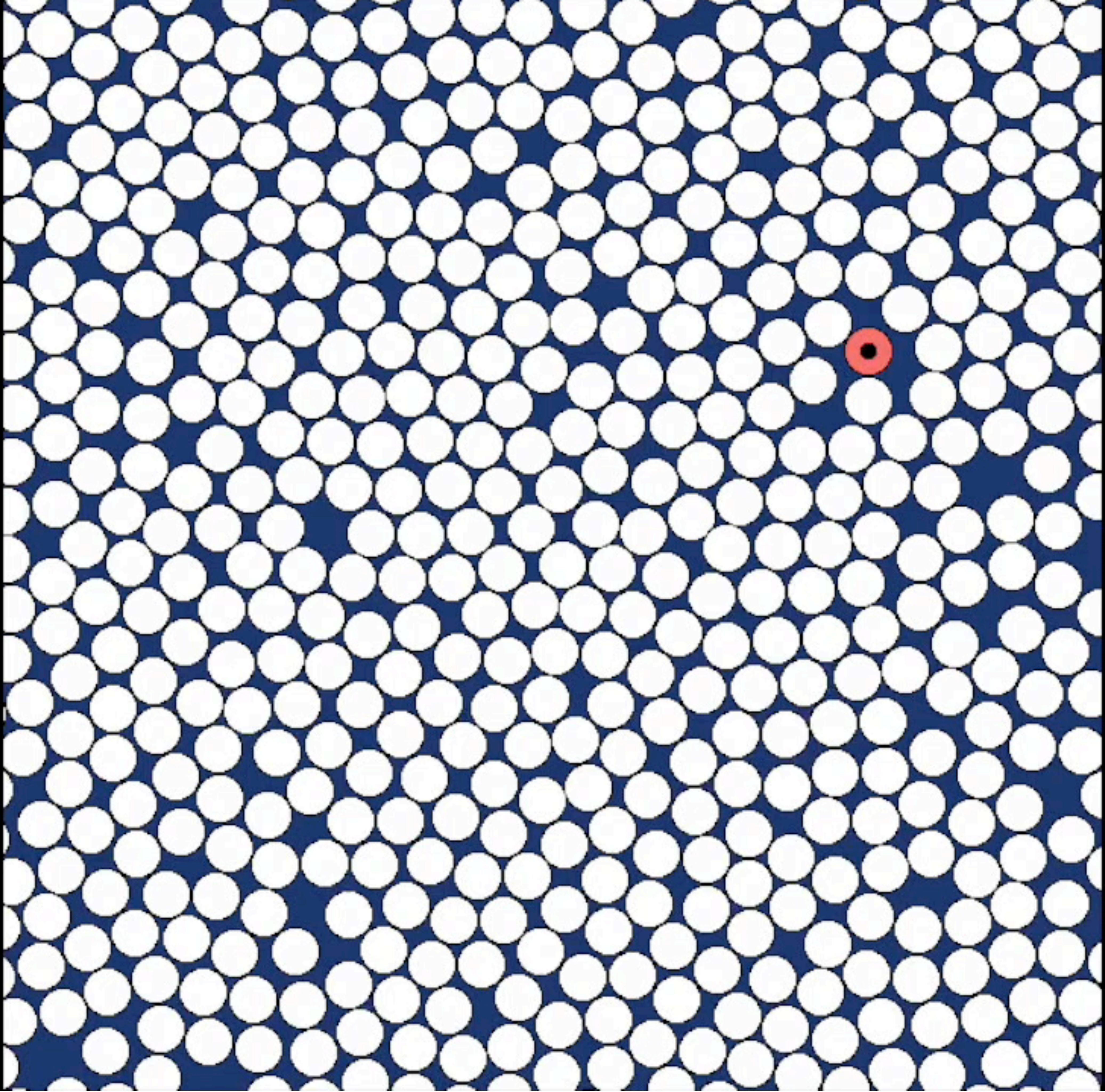
Kraynik–Reinelt P.B.C.

planar extensional (pure shear)

Kraynik and Reinelt (1992)

Sami, Master Thesis (1996) in Brady's group

Todd and Daivis (1998)



Time-dependent periodic boundary conditions

$\begin{cases} \text{no worry for wall slips and migration} \\ \text{little worry for shear banding} \end{cases}$

All good for local rheology!

We need reliable ***simulation models*** *for particle dynamics…*

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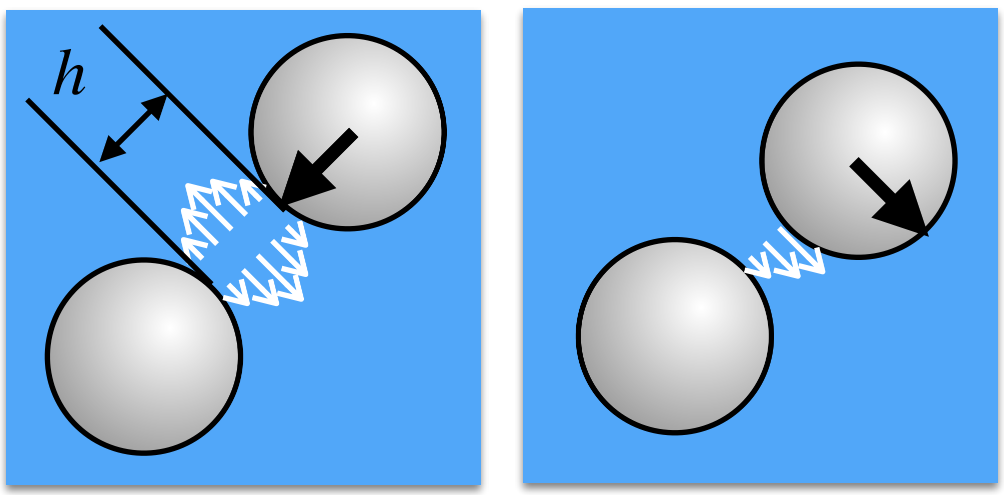
Purely hydrodynamic suspensions

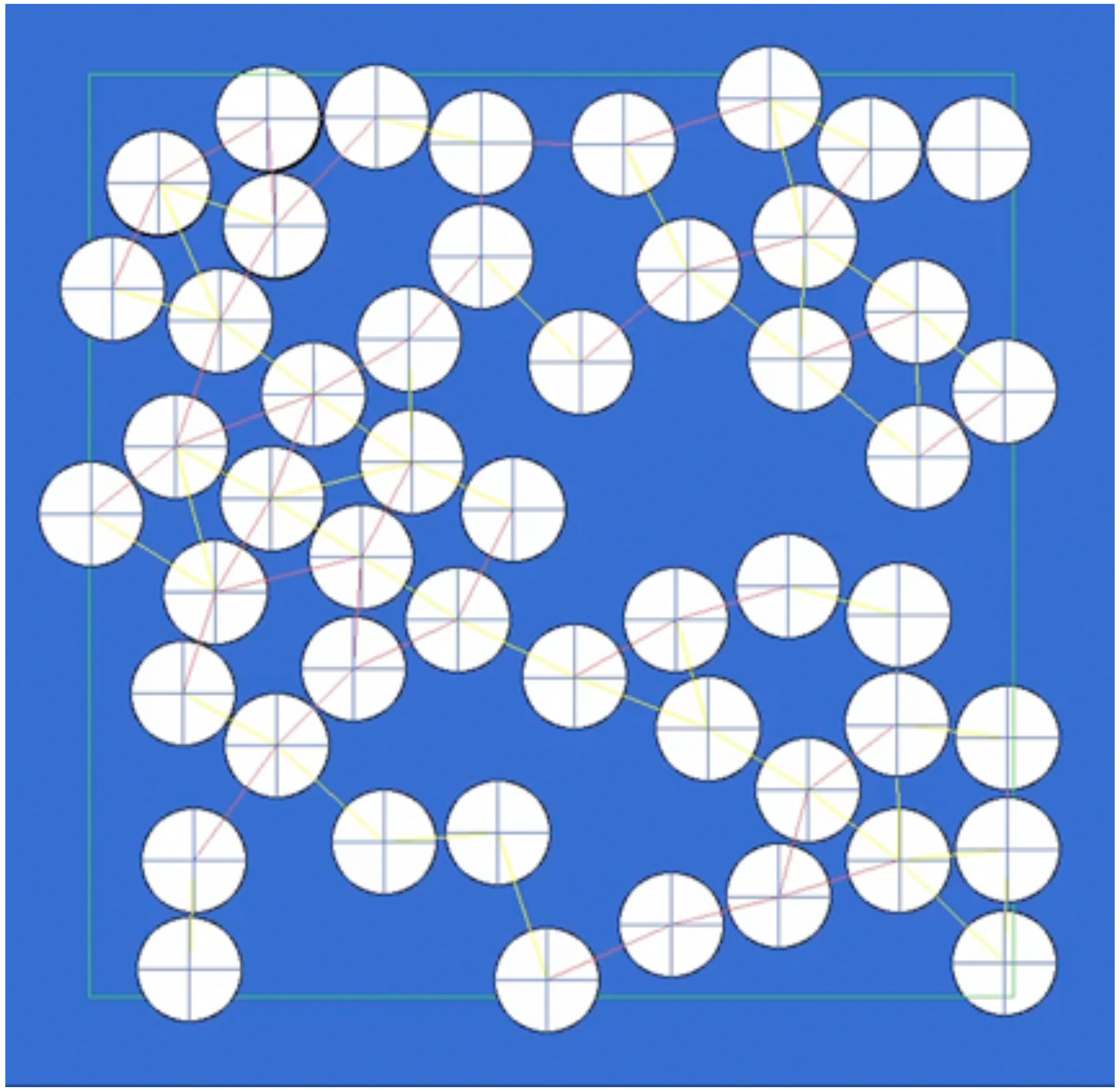
$$\vec{0} = -\nabla p + \nabla^2 \vec{u}$$ Zero Reynolds number

Perfect reversibility ***(memory)***,
if lubrication layers can remain.



$$F^{(\mathrm{nor})}_{\mathrm{Lub}} \sim -\frac{1}{h} \Delta U^{(\mathrm{nor})}$$

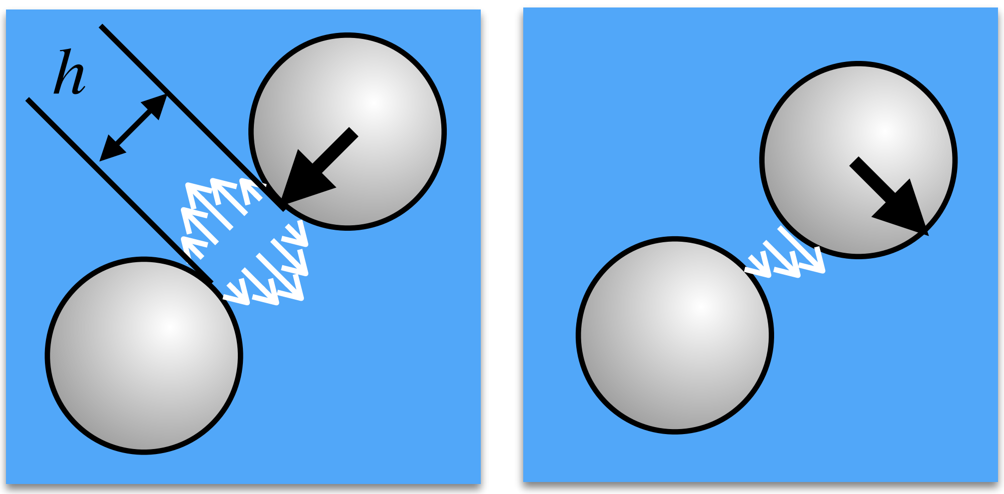
$$F^{(\mathrm{tan})}_{\mathrm{Lub}} \sim -\log\left(\frac{1}{h}\right) \Delta U^{(\mathrm{tan})}$$



shear reversal demo

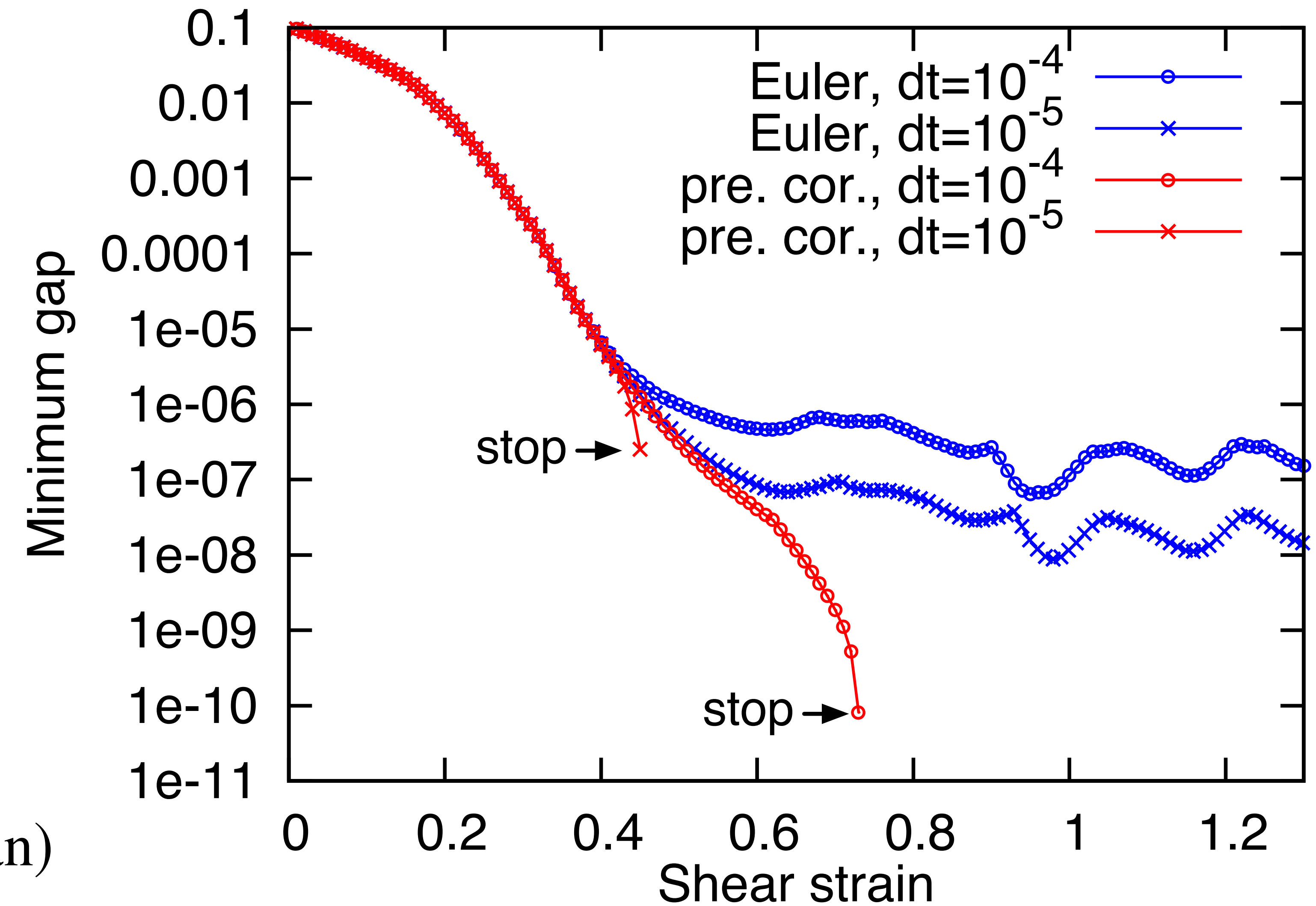
Purely hydrodynamic suspensions

However, infinitesimal resolution is required!!



$$F_{\mathrm{Lub}}^{(\mathrm{nor})} \sim -\frac{1}{h}\Delta U^{(\mathrm{nor})}$$

$$F_{\mathrm{Lub}}^{(\mathrm{tan})} \sim -\log\left(\frac{1}{h}\right)\Delta U^{(\mathrm{tan})}$$



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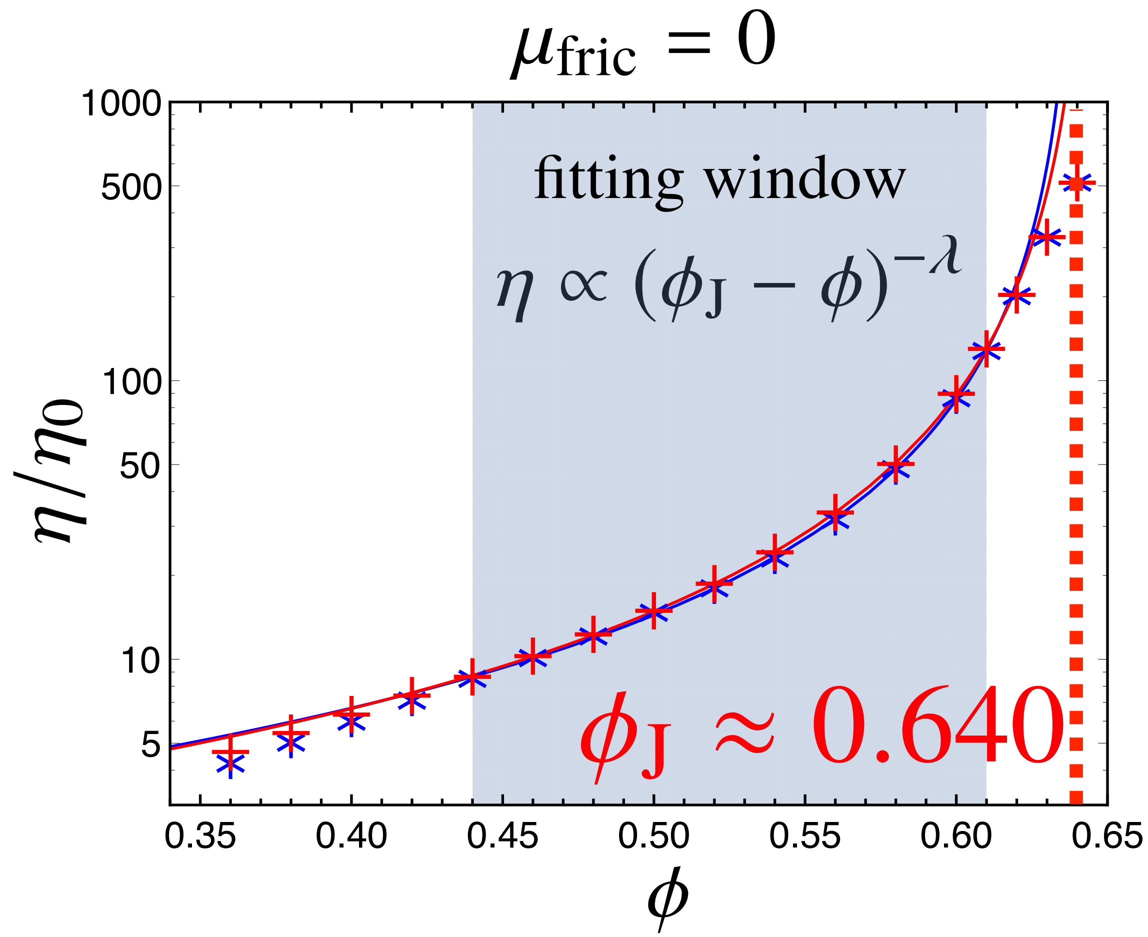
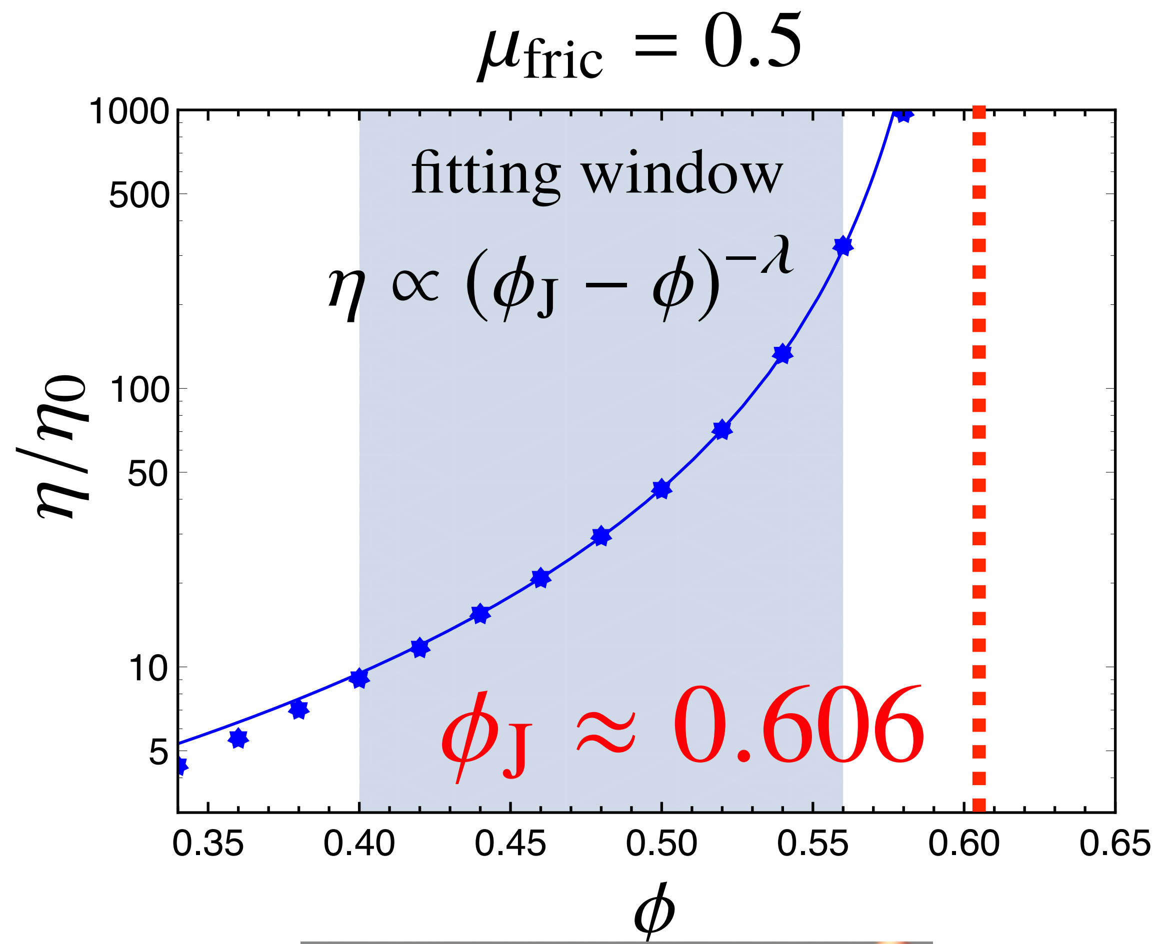
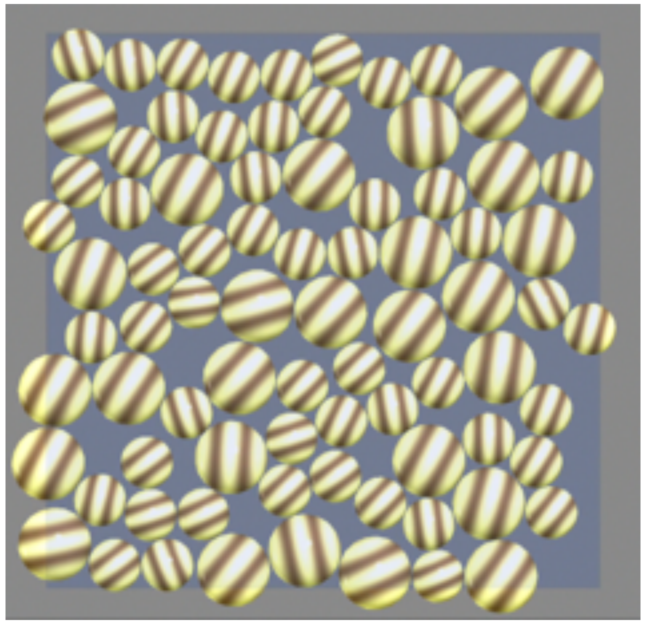
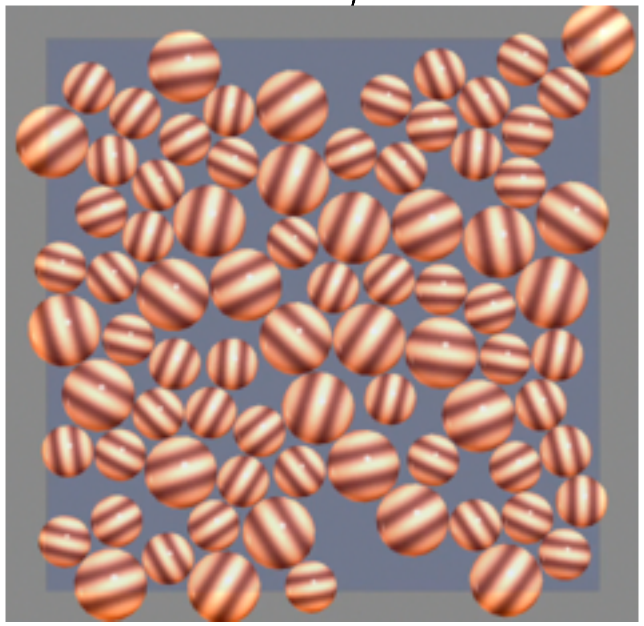
~~Purely~~ hydrodynamic suspensions $F_{\mathrm{Lub}} \sim -\dfrac{1}{h+\delta}\Delta U$

+ contact force (with some sliding constraint)





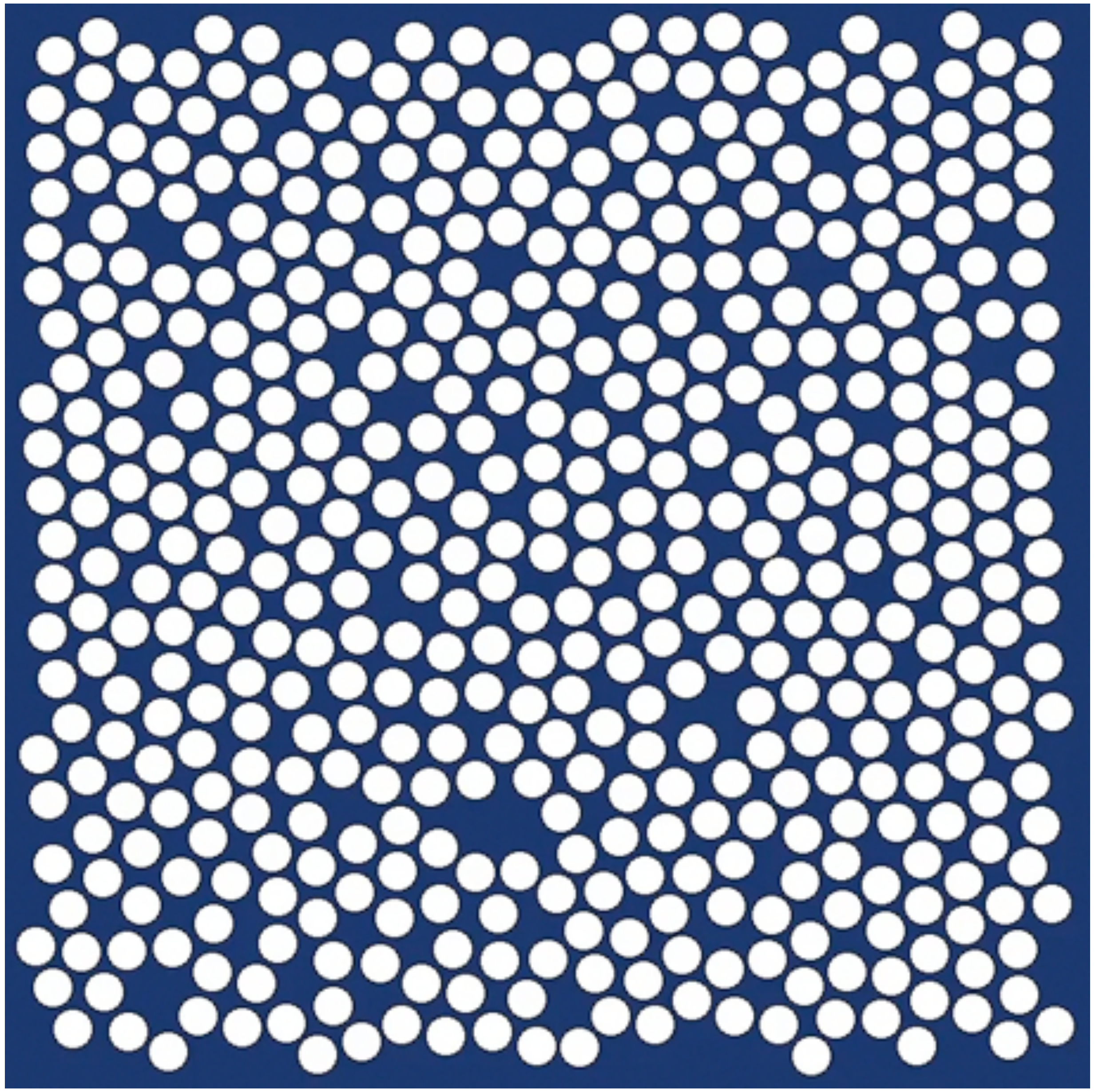




H.I. + Con. + Brownian force or/and repulsive force

shear-induced microstructure ⇆ *relaxed* microstructure

non-equilibrium equilibrium

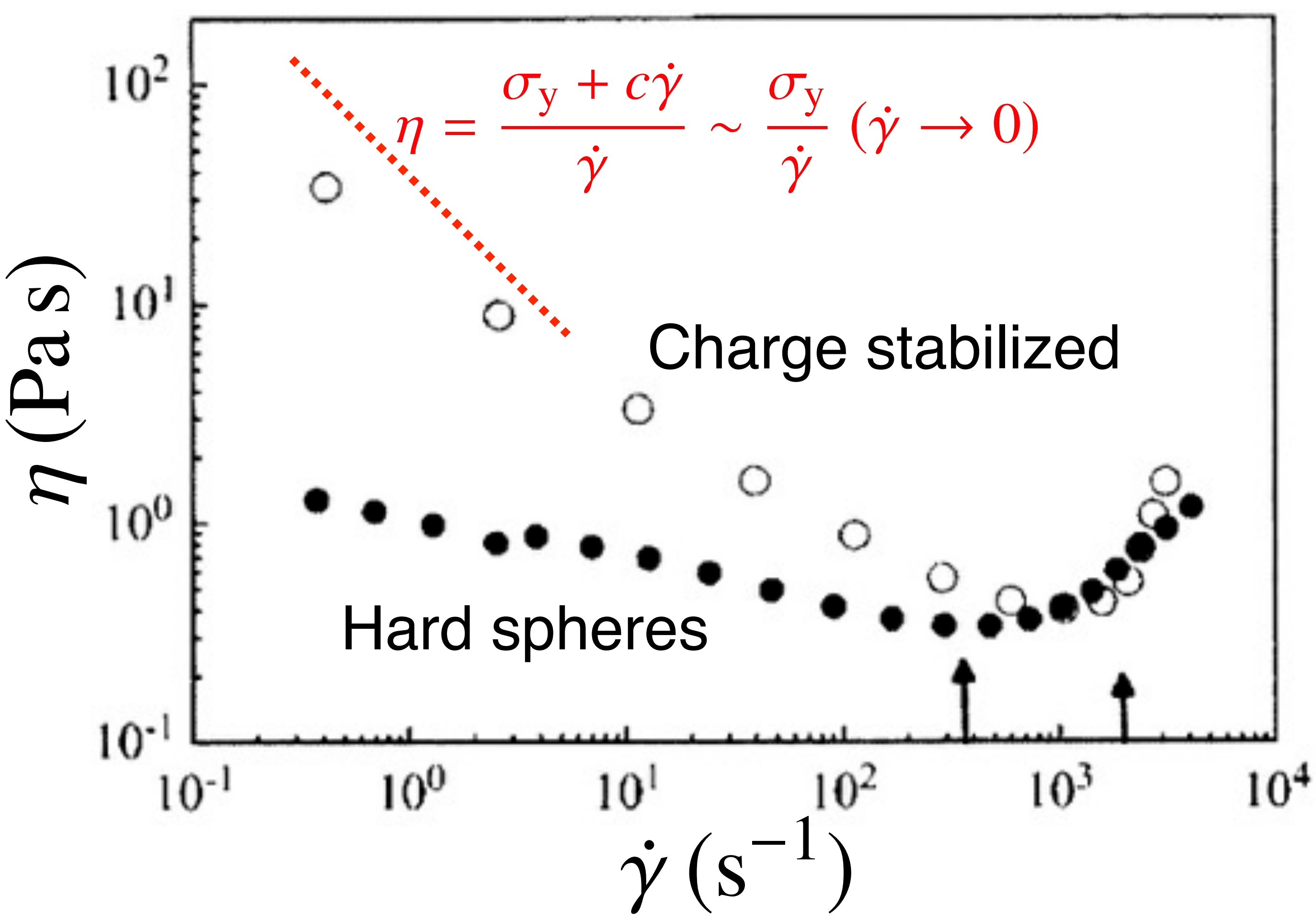


Demo for “flow–stop–flow–stop–…”

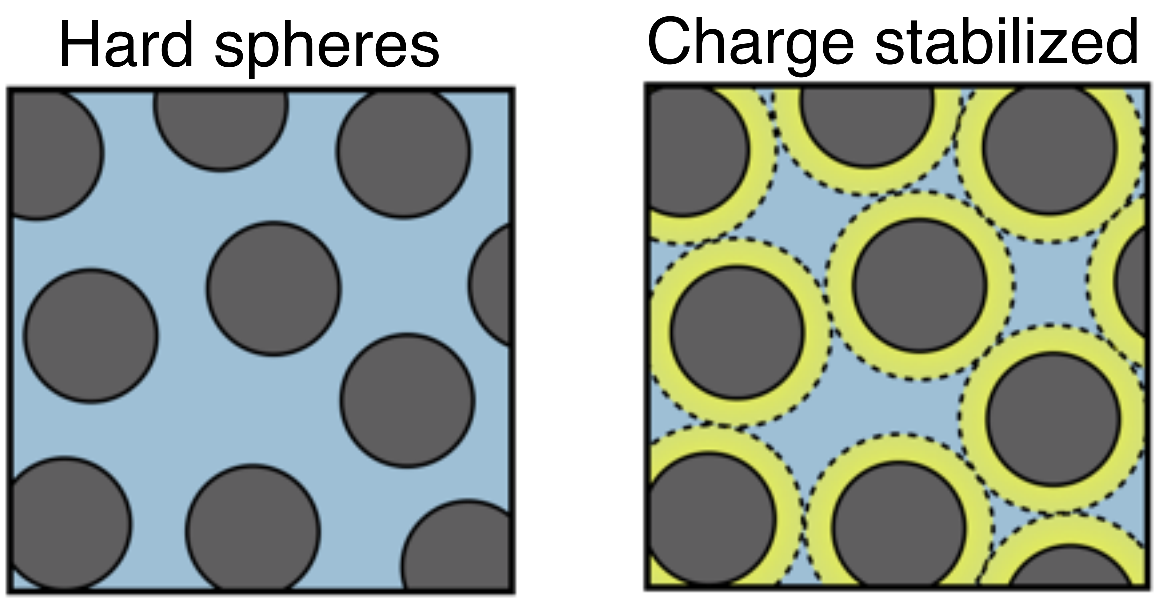
H.I. + Con. + Brownian force or/and repulsive force

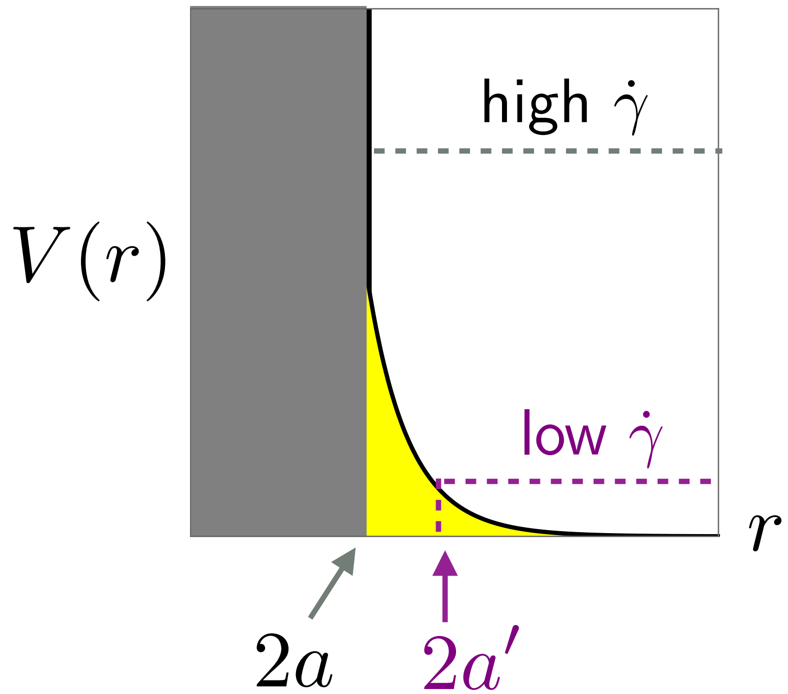
If volume fraction of particles with **extended radii** in the low shear rate exceeds jamming, the viscosity is infinity.

→ **Yield stress**



Maranzano & Wagner 2001





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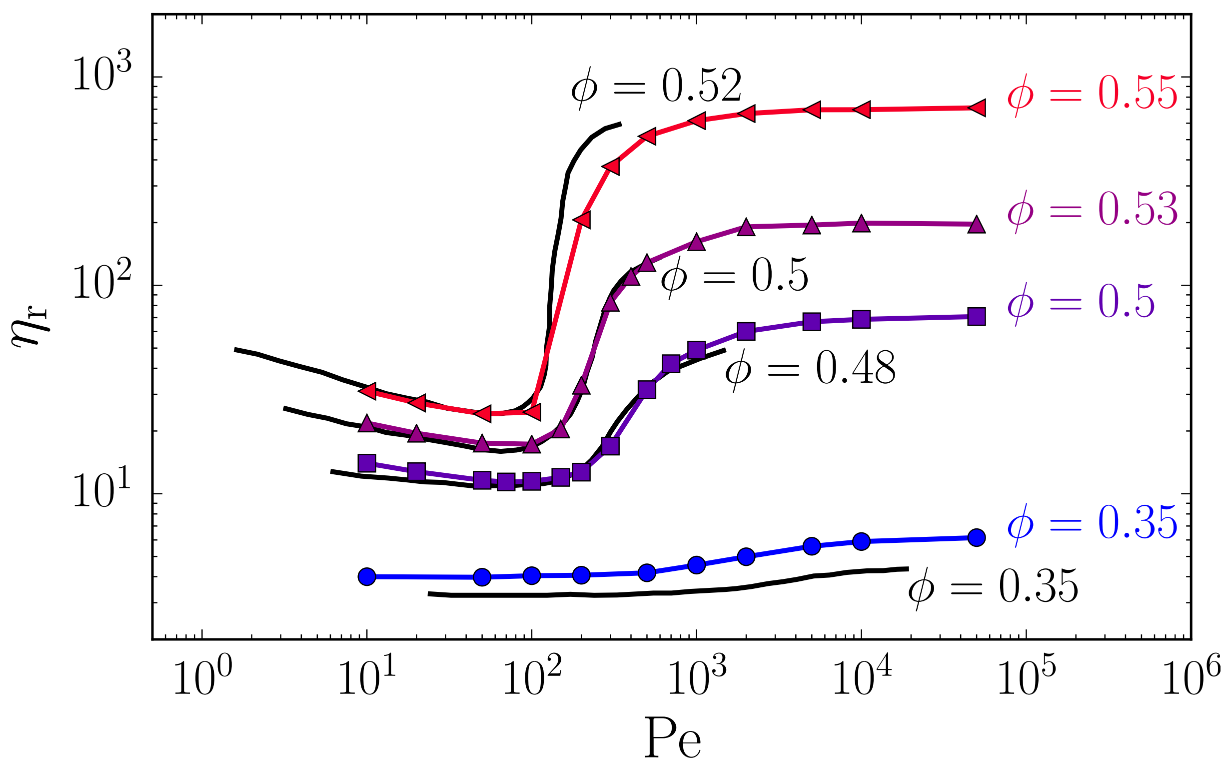
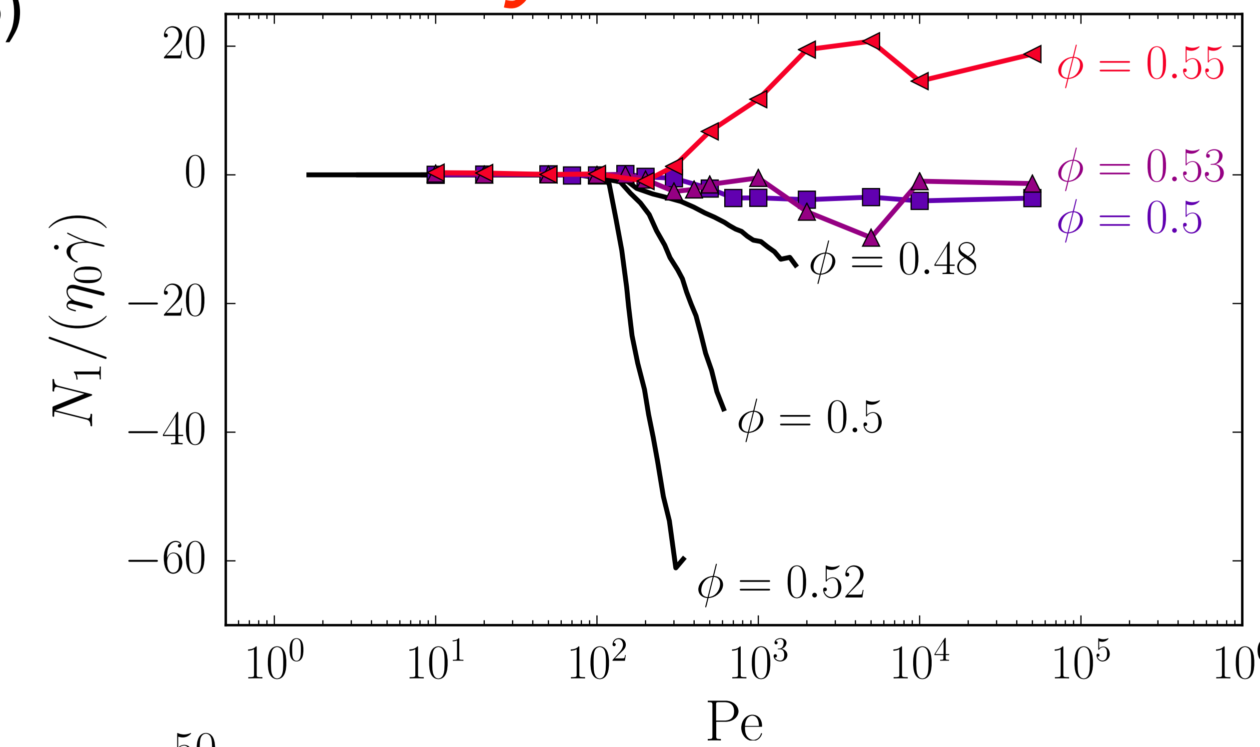
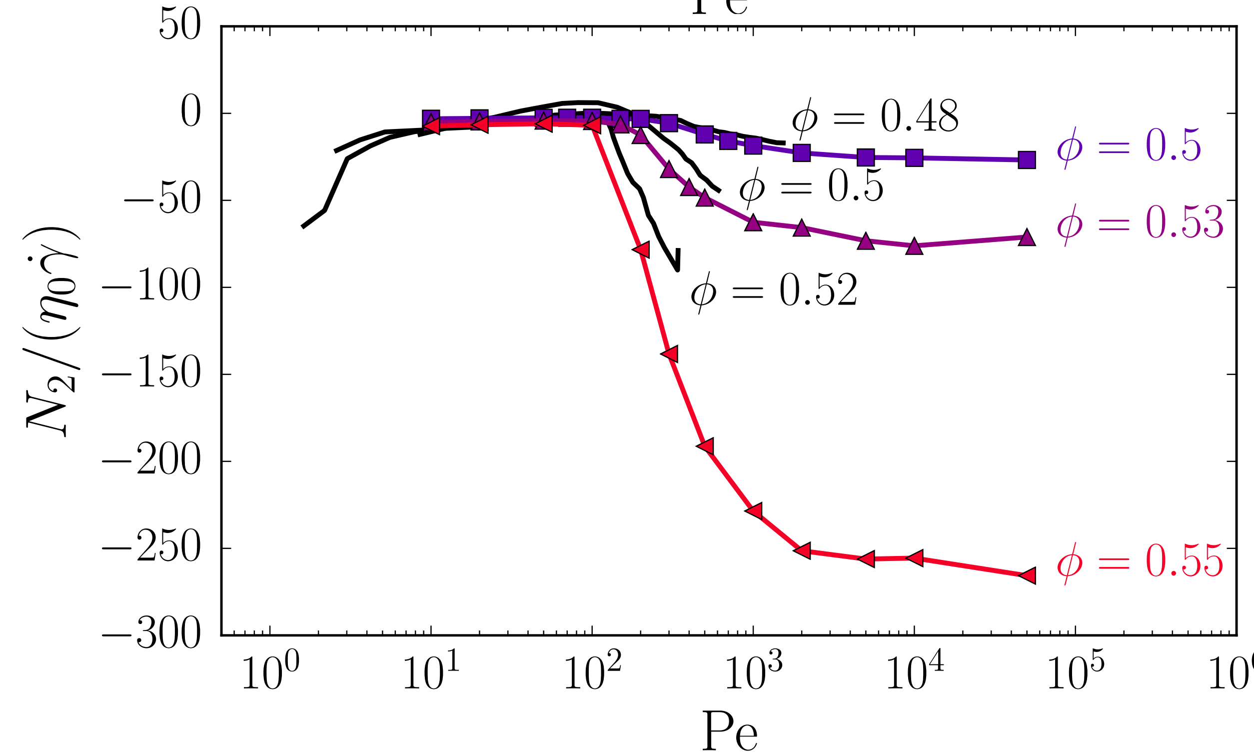
Hydrodynamic interaction + contact force + Brownian force or/and repulsive force

Seto, Mari, Morris, and Denn (PRL 2013)
Mari, Seto, Morris, and Denn (PNAS 2015)

very different!?







Experimental data

black-solid lines

Cwalina and Wagner 2014

Outline

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2. Simulation model for particle dynamics
3. Material functions
4. N_1 issue
5. Extensional rheology

Rheology

velocity gradient

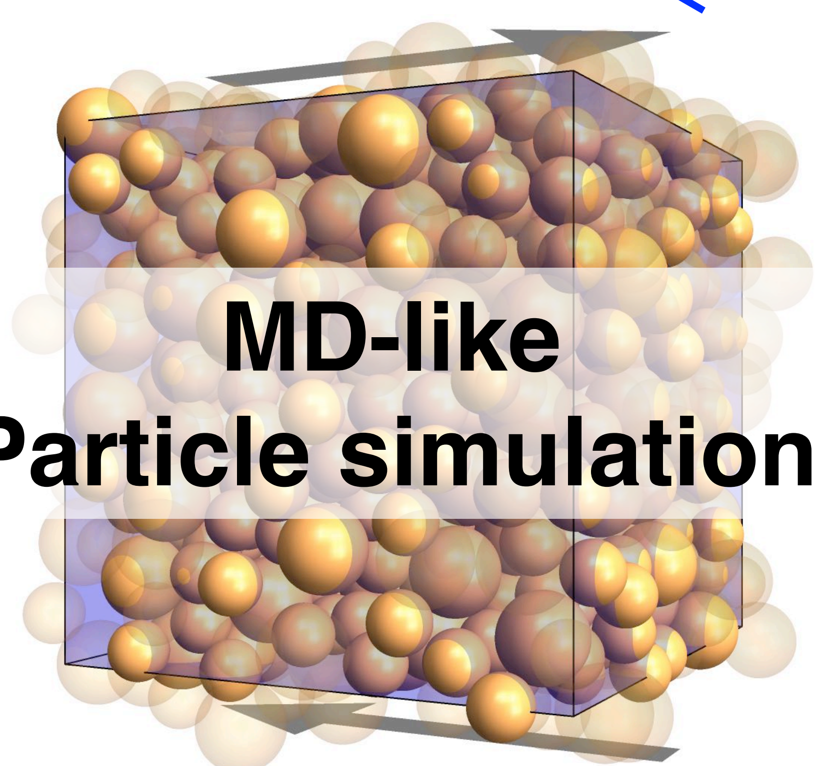
$$\begin{pmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} & \frac{\partial u_x}{\partial z} \\ \frac{\partial u_y}{\partial x} & \frac{\partial u_y}{\partial y} & \frac{\partial u_y}{\partial z} \\ \frac{\partial u_z}{\partial x} & \frac{\partial u_z}{\partial y} & \frac{\partial u_z}{\partial z} \end{pmatrix}$$

stress tensor

$$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{pmatrix}$$

impose

response



material functions

scalars

(*not* coordinate specific)

physical interpretations

$$\eta(\dot{\gamma}),\ N_1(\dot{\gamma}),\ N_2(\dot{\gamma})$$

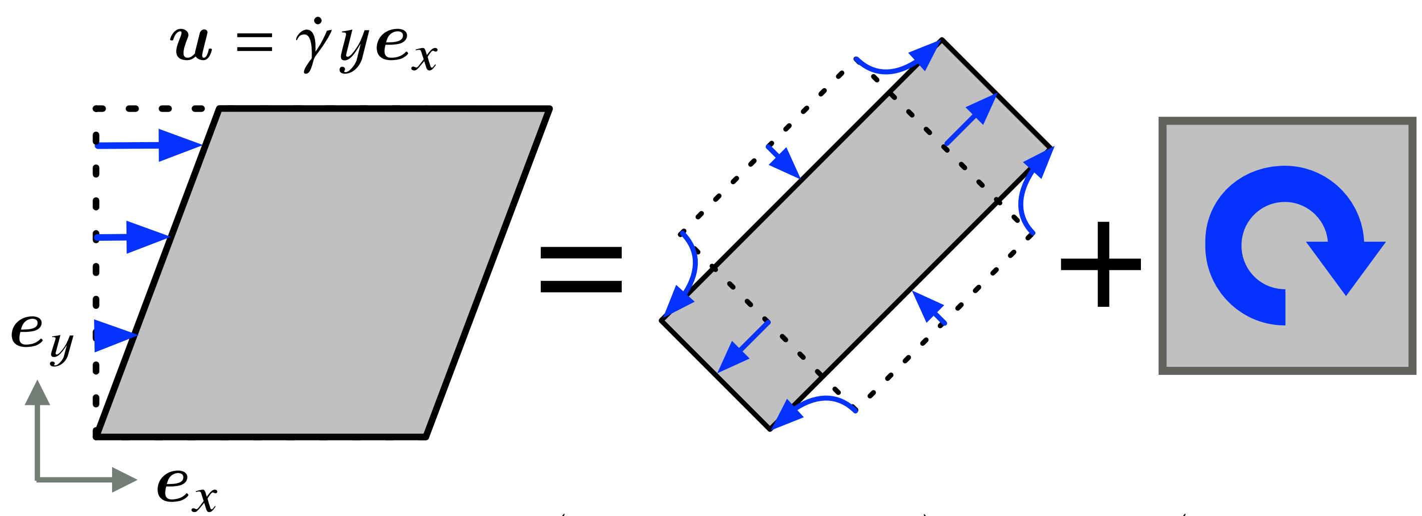
$$\rho\left\{\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u}\right\} = \nabla\cdot\boldsymbol{\sigma}$$

Constitutive modeling

Conventional material functions

simple shear flow → stress tensor

$$\nabla \boldsymbol{u} = \begin{pmatrix} 0 & \dot{\gamma} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \boldsymbol{D} + \boldsymbol{W}$$



$$\boldsymbol{D} = \begin{pmatrix} 0 & \dot{\gamma}/2 & 0 \\ \dot{\gamma}/2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \boldsymbol{W} = \begin{pmatrix} 0 & \dot{\gamma}/2 & 0 \\ -\dot{\gamma}/2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ \sigma_{xy} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$$

$$= -p\boldsymbol{I} + 2\eta(\dot{\gamma})\boldsymbol{D}$$

$$+ \frac{1}{3}\begin{pmatrix} 2N_1 + N_2 & 0 & 0 \\ 0 & -N_1 + N_2 & 0 \\ 0 & 0 & N_1 - 2N_2 \end{pmatrix}$$

viscometric functions

$$\begin{cases} \eta \equiv \sigma_{xy}/\dot{\gamma} \\ N_1 \equiv \sigma_{xx} - \sigma_{yy} \\ N_2 \equiv \sigma_{yy} - \sigma_{zz} \end{cases}$$

Generalized material functions

(Only planar flows in this talk)

$$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{pmatrix} \Rightarrow$$

any (planar) uniform flows

$$\boldsymbol{\sigma} = -p\boldsymbol{I} + 2\eta\boldsymbol{D} + 2\lambda_0\boldsymbol{E} + 2\lambda_3\boldsymbol{G}_3$$

$$\mathcal{B} = \left(\boldsymbol{I}, \boldsymbol{D}, \boldsymbol{E}, \boldsymbol{G}_3\right)$$

Cf. Conventional material functions

$$\begin{cases} \eta \equiv \sigma_{xy}/\dot{\gamma} \\ N_1 \equiv \sigma_{xx} - \sigma_{yy} \\ N_2 \equiv \sigma_{yy} - \sigma_{zz} \end{cases}$$

simple shear flows

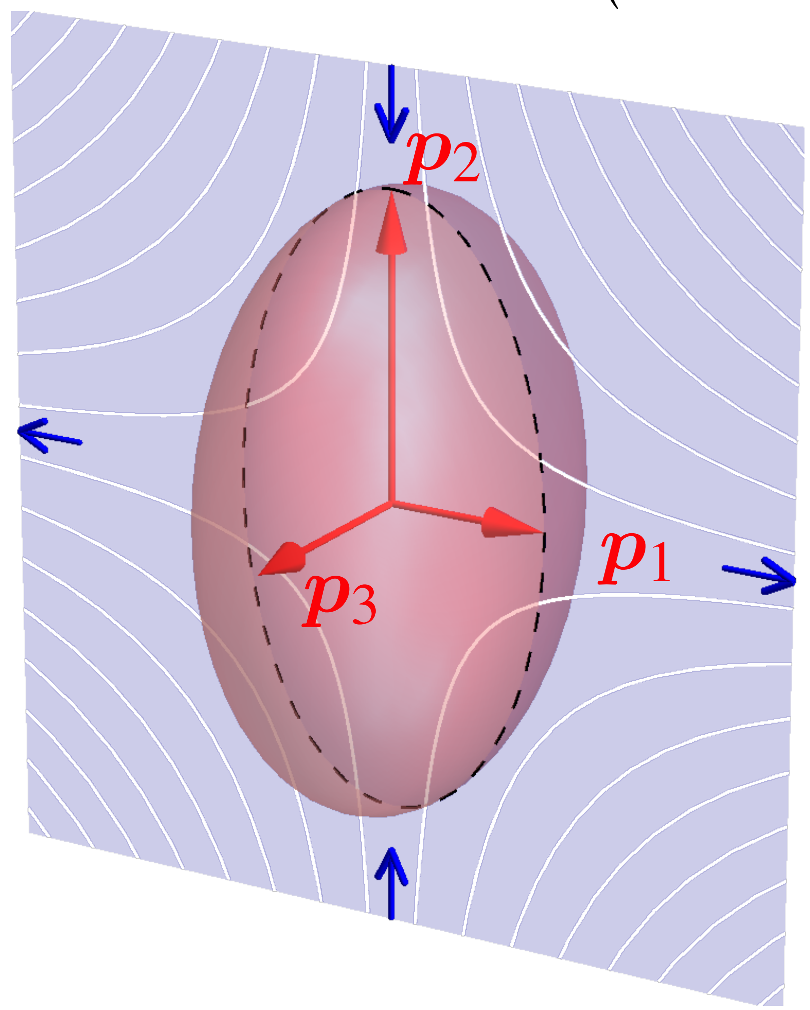
$$\begin{cases} p = -\dfrac{\boldsymbol{\sigma} : \boldsymbol{I}}{\boldsymbol{I} : \boldsymbol{I}} = -\dfrac{\mathrm{Tr}\,\boldsymbol{\sigma}}{3} \\ \eta = \dfrac{\boldsymbol{\sigma} : \boldsymbol{D}}{2\boldsymbol{D} : \boldsymbol{D}} \\ \lambda_0 = \dfrac{\boldsymbol{\sigma} : \boldsymbol{E}}{2\boldsymbol{E} : \boldsymbol{E}} \\ \lambda_3 = \dfrac{\boldsymbol{\sigma} : \boldsymbol{G}_3}{2\boldsymbol{G}_3 : \boldsymbol{G}_3} \end{cases}$$

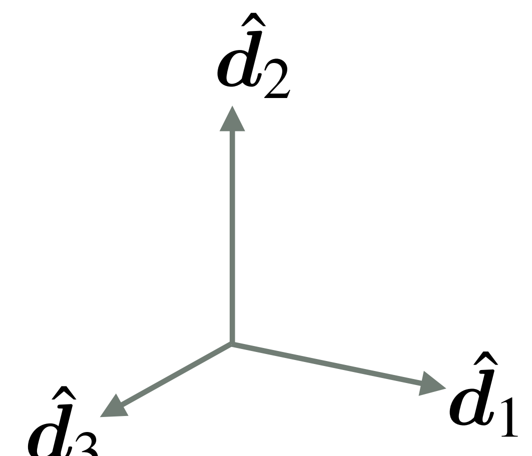
Giulio G. Giusteri & Seto 2017 arXiv:1702.02745

$$\boldsymbol{\sigma} = -p\boldsymbol{I} + 2\eta\boldsymbol{D}$$

$$\boldsymbol{D} = \dot{\varepsilon}(\hat{\boldsymbol{d}}_1\hat{\boldsymbol{d}}_1 - \hat{\boldsymbol{d}}_2\hat{\boldsymbol{d}}_2) = \begin{pmatrix} \dot{\varepsilon} & 0 & 0 \\ 0 & -\dot{\varepsilon} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

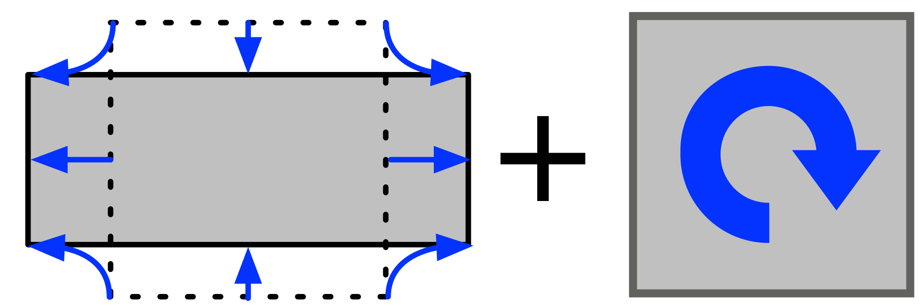
stress
to keep flowing





Eigenvectors of **D**

any uniform planar flows



(Eigenvectors and eigenvalues of $\mathbf{P} = -\boldsymbol{\sigma}$)

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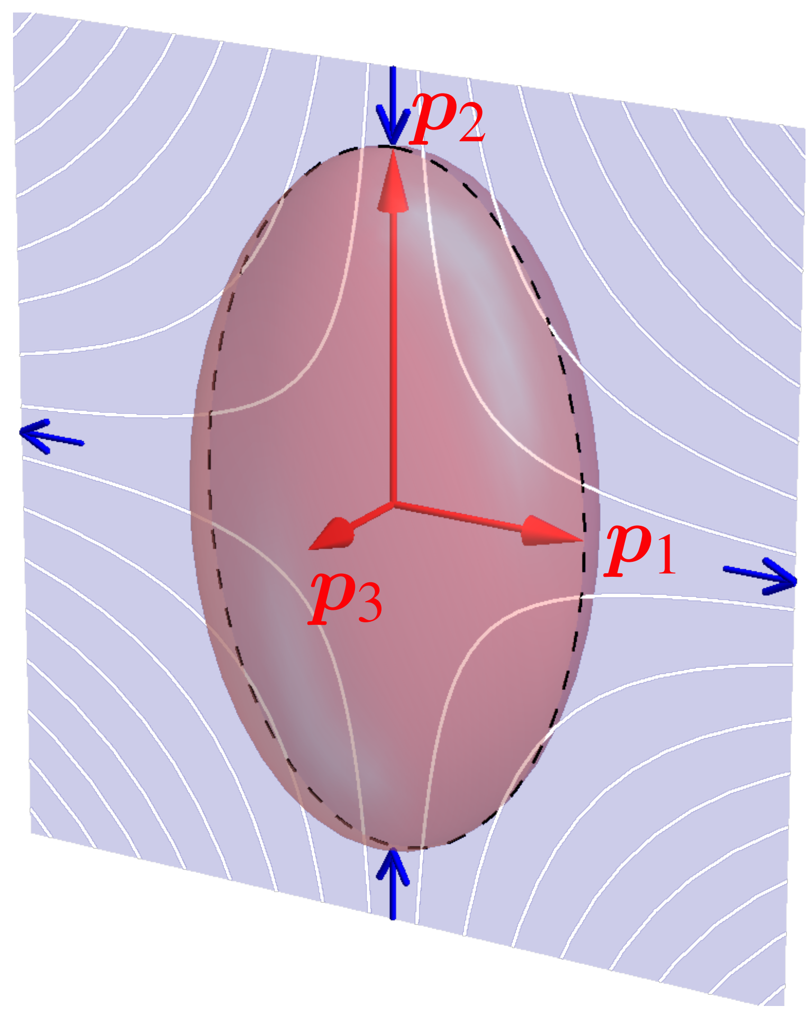
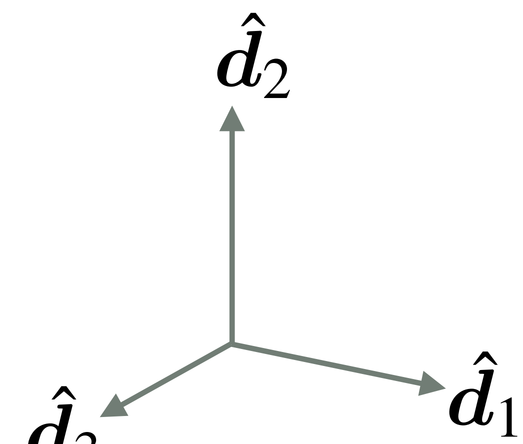
$$\boldsymbol{\sigma} = -p\boldsymbol{I} + 2\eta\boldsymbol{D} + 2\lambda_0\boldsymbol{E}$$

$$\boldsymbol{E} = \dot{\varepsilon}\left(-\frac{1}{2}\hat{\boldsymbol{d}}_1\hat{\boldsymbol{d}}_1 - \frac{1}{2}\hat{\boldsymbol{d}}_2\hat{\boldsymbol{d}}_2 + \hat{\boldsymbol{d}}_3\hat{\boldsymbol{d}}_3\right) = \begin{pmatrix} -\dot{\varepsilon}/2 & 0 & 0 \\ 0 & -\dot{\varepsilon}/2 & 0 \\ 0 & 0 & \dot{\varepsilon} \end{pmatrix}$$

in-plane
out-of-plane
anisotropy





Eigenvectors of **D**

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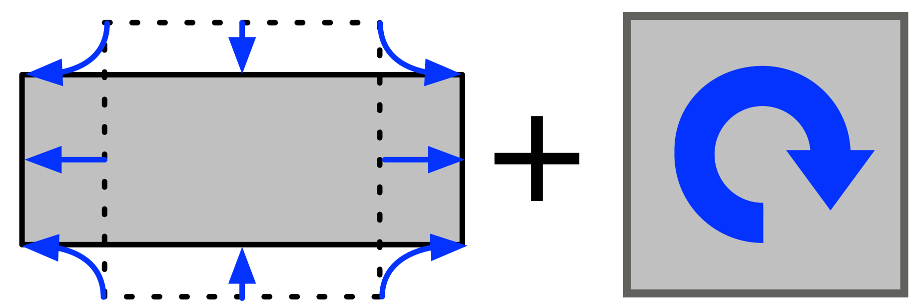
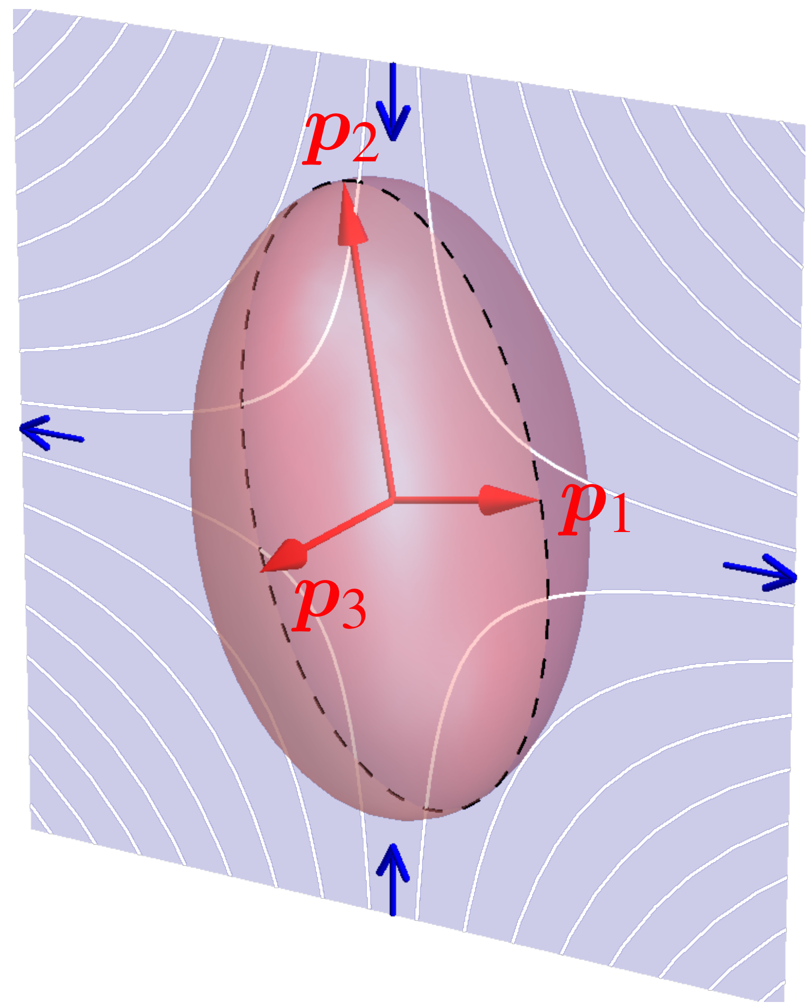


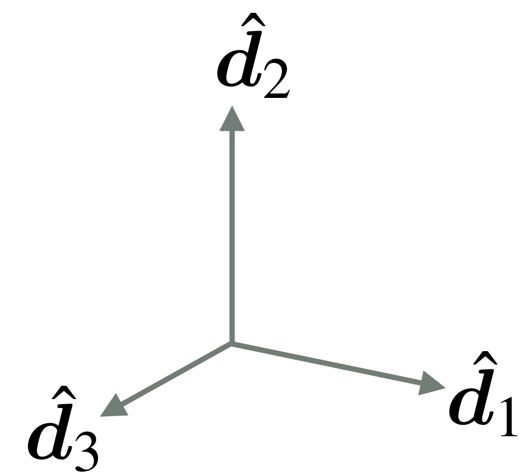
(Eigenvectors and eigenvalues of $\mathbf{P} = -\boldsymbol{\sigma}$)

$$\boldsymbol{\sigma} = -p\boldsymbol{I} + 2\eta\boldsymbol{D} + 2\lambda_3\boldsymbol{G}_3$$

$$\boldsymbol{G}_3 = \dot{\varepsilon}(\hat{\boldsymbol{d}}_1\hat{\boldsymbol{d}}_2 + \hat{\boldsymbol{d}}_2\hat{\boldsymbol{d}}_1) = \begin{pmatrix} 0 & \dot{\varepsilon} & 0 \\ \dot{\varepsilon} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

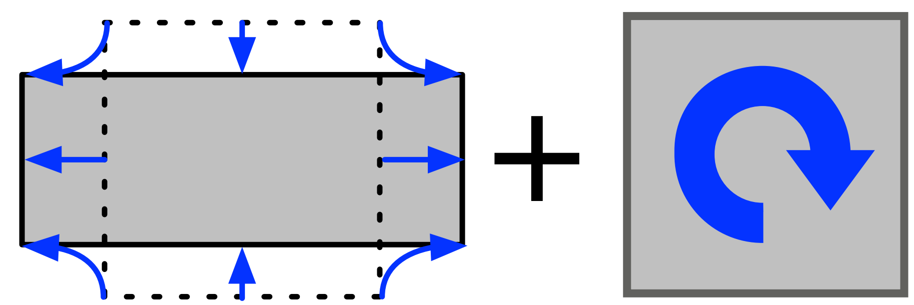
reorientation of principal directions





Eigenvectors of **D**

any uniform planar flows



(Eigenvectors and eigenvalues of $\mathbf{P} = -\boldsymbol{\sigma}$)

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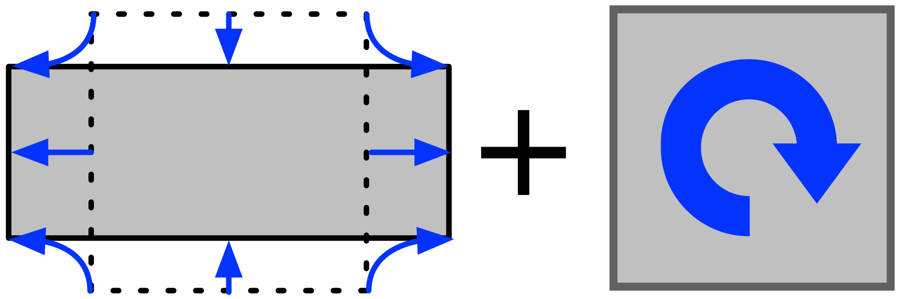
Generalized material functions

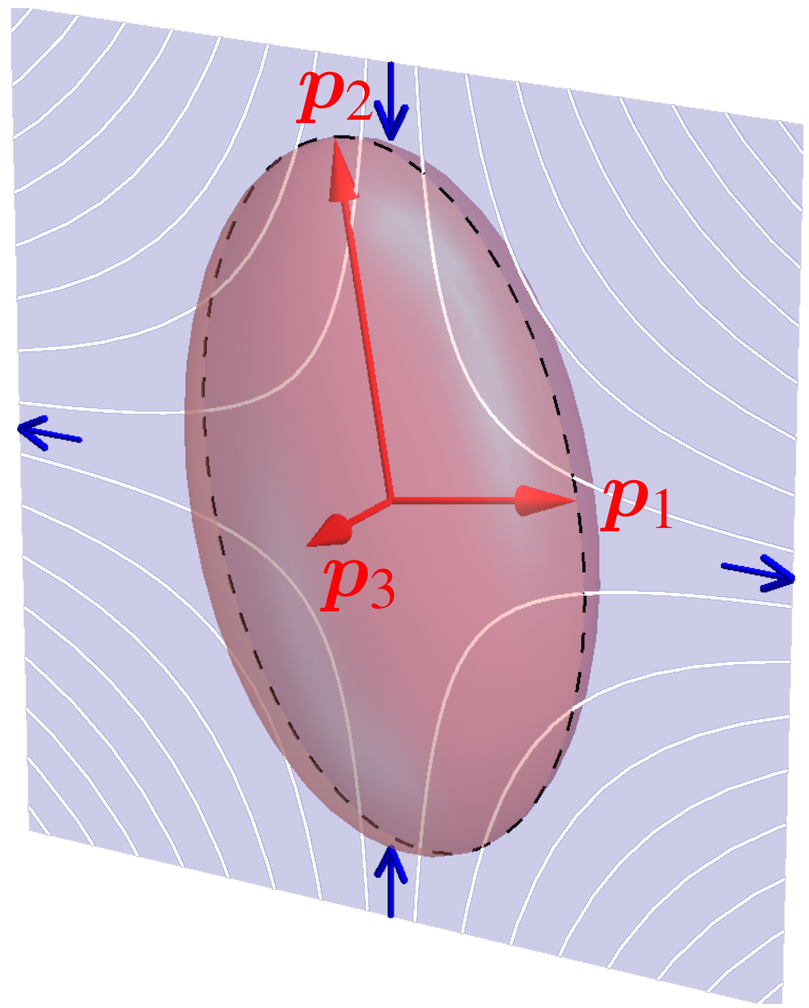
(Only planar flows in this talk.)

$$\boldsymbol{\sigma} = -p\boldsymbol{I} + 2\eta\boldsymbol{D} + 2\lambda_0\boldsymbol{E} + 2\lambda_3\boldsymbol{G}_3$$

$$\mathcal{B} = \left(\boldsymbol{I}, \boldsymbol{D}, \boldsymbol{E}, \boldsymbol{G}_3\right)$$ orthogonal tensorial basis

any uniform planar flows





$$p = -\frac{\boldsymbol{\sigma} : \boldsymbol{I}}{\boldsymbol{I} : \boldsymbol{I}} = -\frac{\operatorname{Tr}\boldsymbol{\sigma}}{3}$$

$$\eta = \frac{\boldsymbol{\sigma} : \boldsymbol{D}}{2\boldsymbol{D} : \boldsymbol{D}}$$

$$\lambda_0 = \frac{\boldsymbol{\sigma} : \boldsymbol{E}}{2\boldsymbol{E} : \boldsymbol{E}}$$

$$\lambda_3 = \frac{\boldsymbol{\sigma} : \boldsymbol{G}_3}{2\boldsymbol{G}_3 : \boldsymbol{G}_3}$$

(Eigenvectors and eigenvalues of $\mathbf{P} = -\boldsymbol{\sigma}$)

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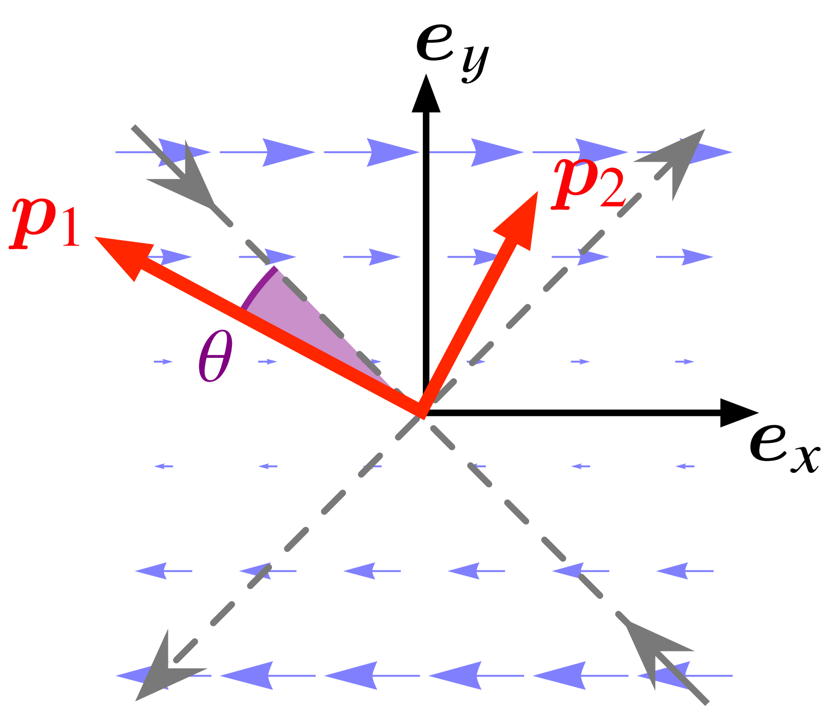
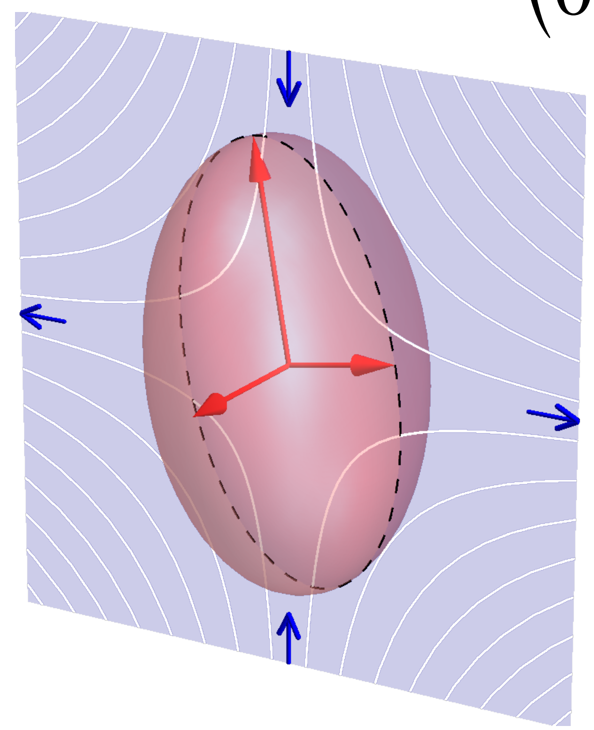
$$\boldsymbol{G}_3 = \dot{\varepsilon}(\hat{\boldsymbol{d}}_1\hat{\boldsymbol{d}}_2 + \hat{\boldsymbol{d}}_2\hat{\boldsymbol{d}}_1) = \begin{pmatrix} 0 & \dot{\varepsilon} & 0 \\ \dot{\varepsilon} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$





reorientation of principal directions

(Eigenvectors and eigenvalues of $\mathbf{P} = -\boldsymbol{\sigma}$)

$$N_1 = -2\dot{\varepsilon}\lambda_3$$

$$\theta \equiv \tan^{-1}\left(\frac{-N_1/2\sigma}{1 + \sqrt{1 + (N_1/2\sigma)^2}}\right) \approx -\frac{N_1}{4\sigma}$$

$$\sigma = \eta\dot{\gamma}$$

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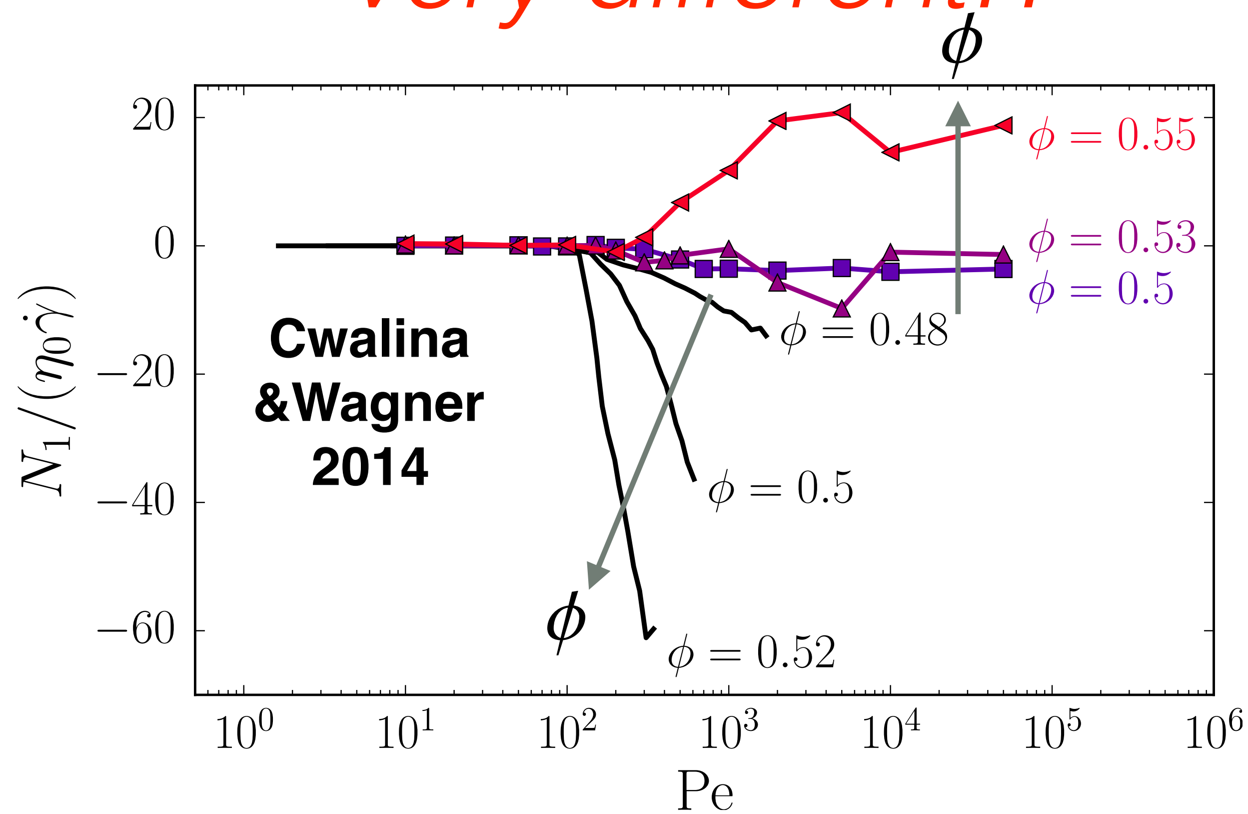
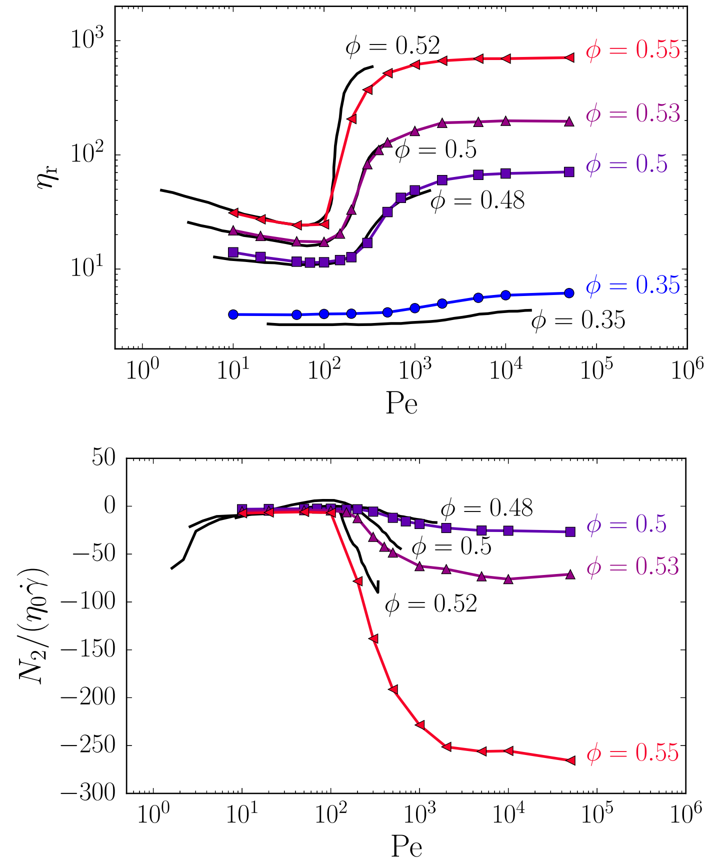
N_1 issue

very different!?

$$\frac{N_1}{\eta_0 \dot{\gamma}}$$



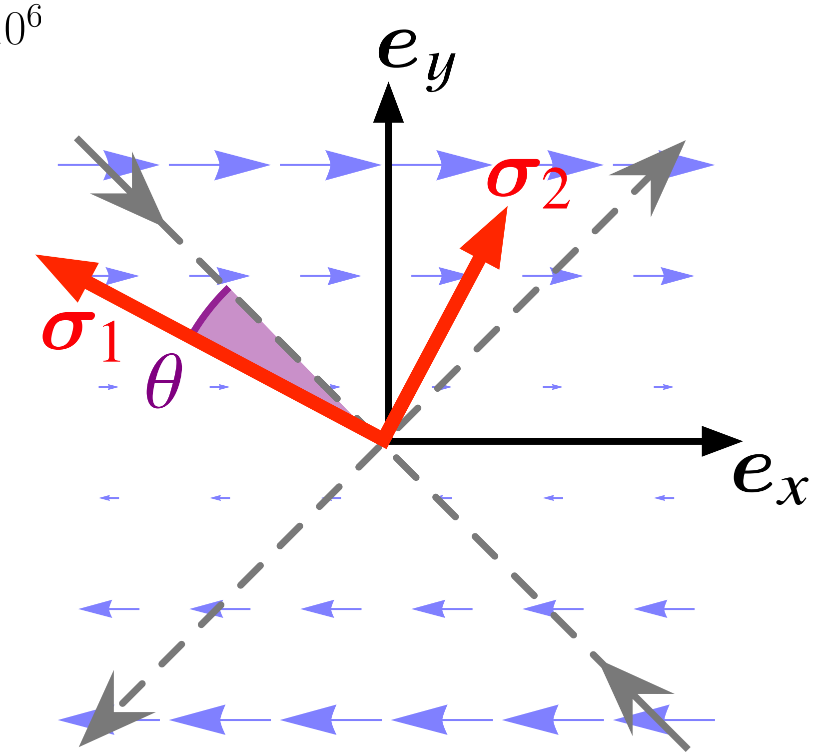
Mari et. al. 2015



reorientation angle

$$\theta \equiv \tan^{-1}\left(\frac{-N_1/2\sigma}{1+\sqrt{1+(N_1/2\sigma)^2}}\right) \approx -\frac{N_1}{4\sigma}$$

$$N_1 = -2\dot{\varepsilon}\lambda_3$$



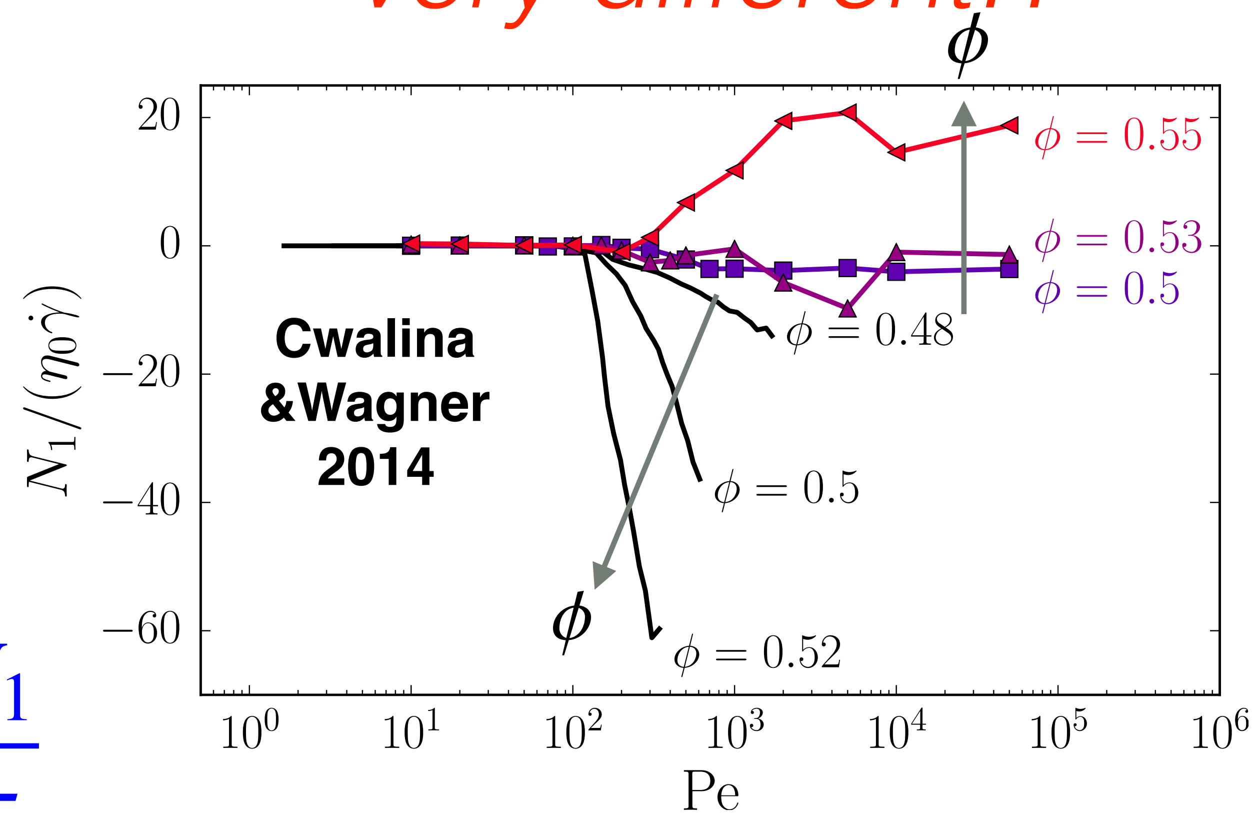
OIST

N_1 issue

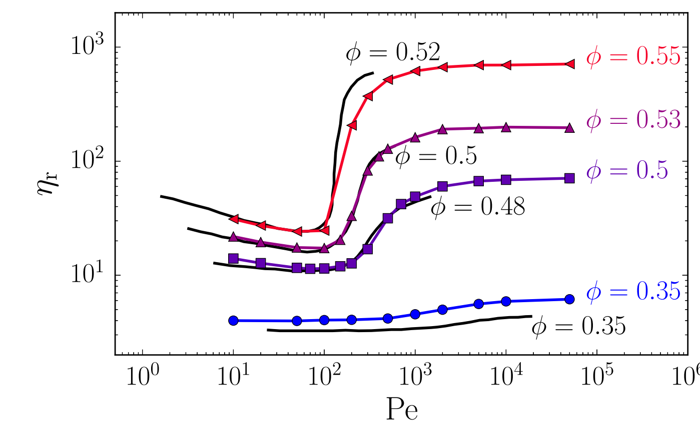
very different!?

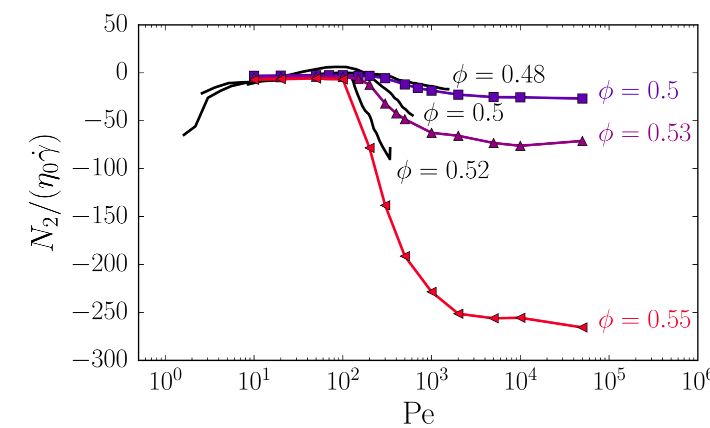
$$\frac{N_1}{\eta_0 \dot{\gamma}}$$

$$\frac{N_1}{\eta \dot{\gamma}} = \frac{N_1}{\sigma}$$

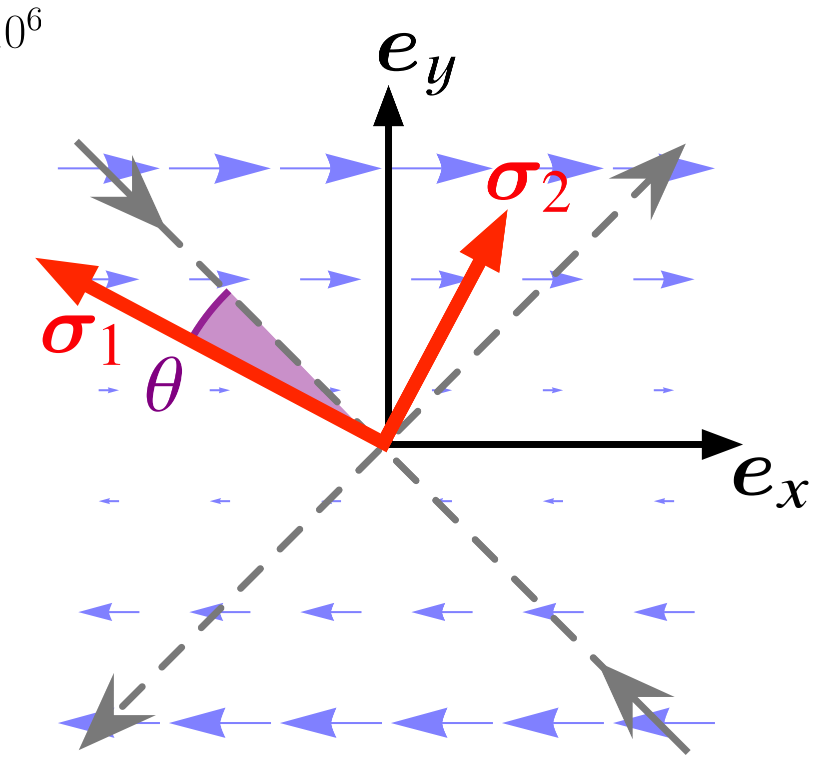


Mari et. al. 2015





reorientation angle

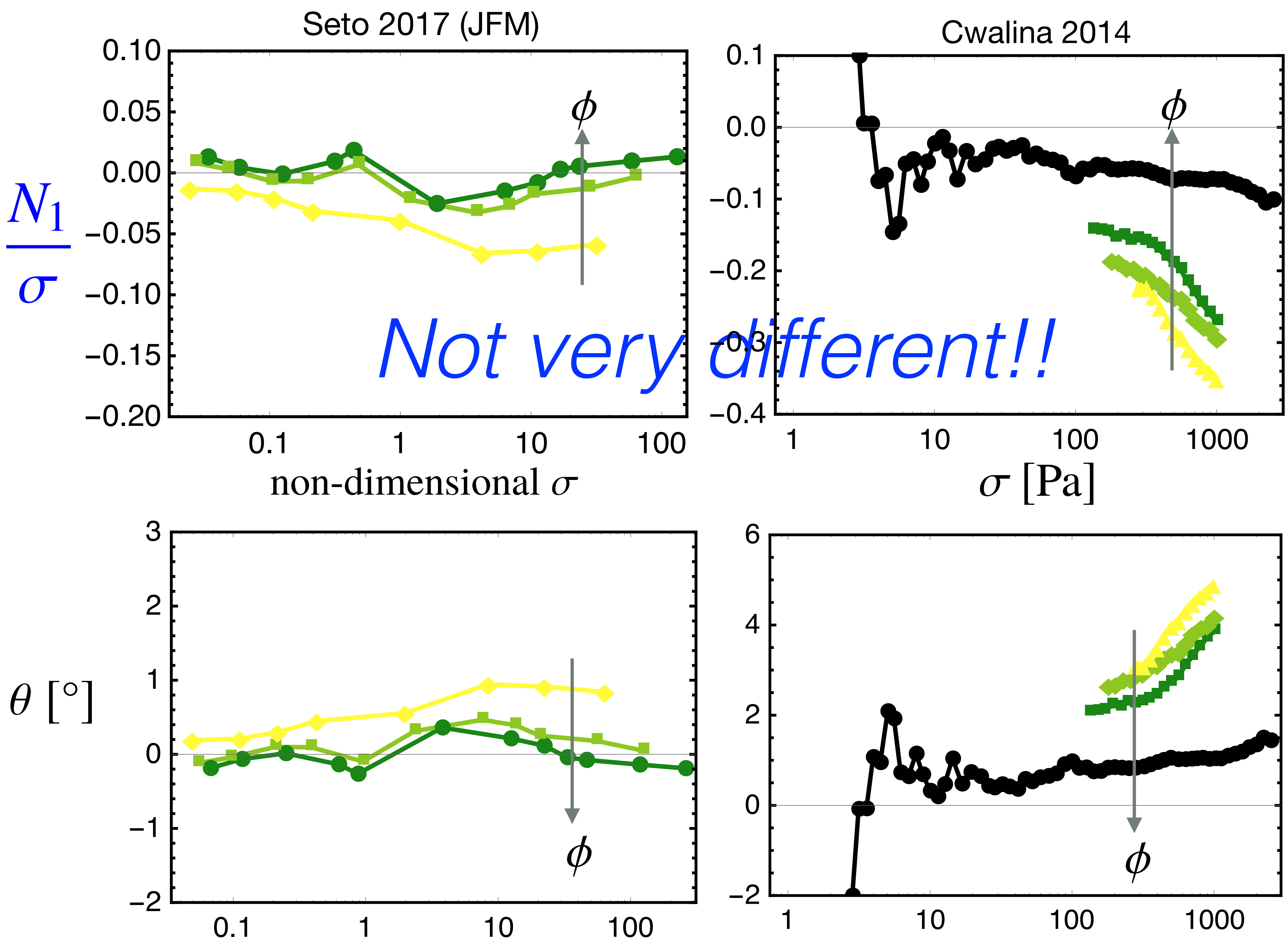


$$\theta \equiv \tan^{-1}\left(\frac{-N_1/2\sigma}{1+\sqrt{1+(N_1/2\sigma)^2}}\right) \approx -\frac{N_1}{4\sigma}$$

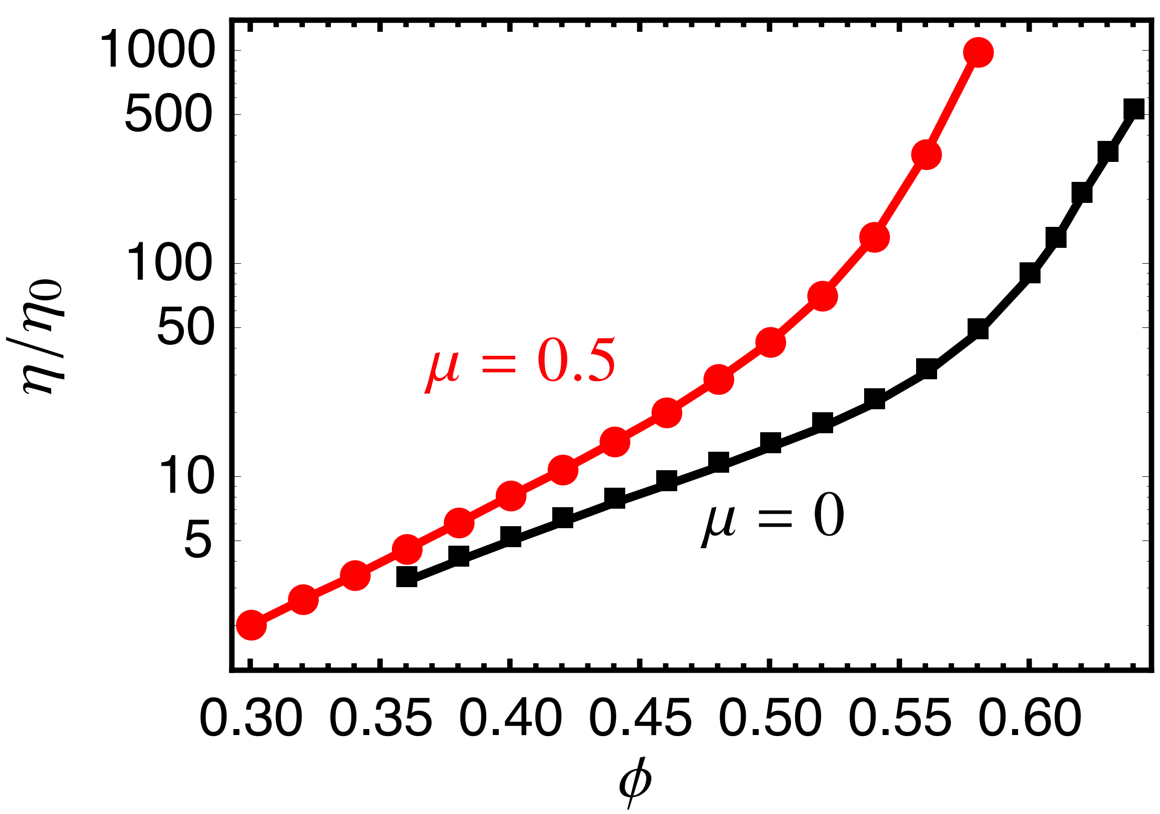
$$N_1 = -2\dot{\varepsilon}\lambda_3$$

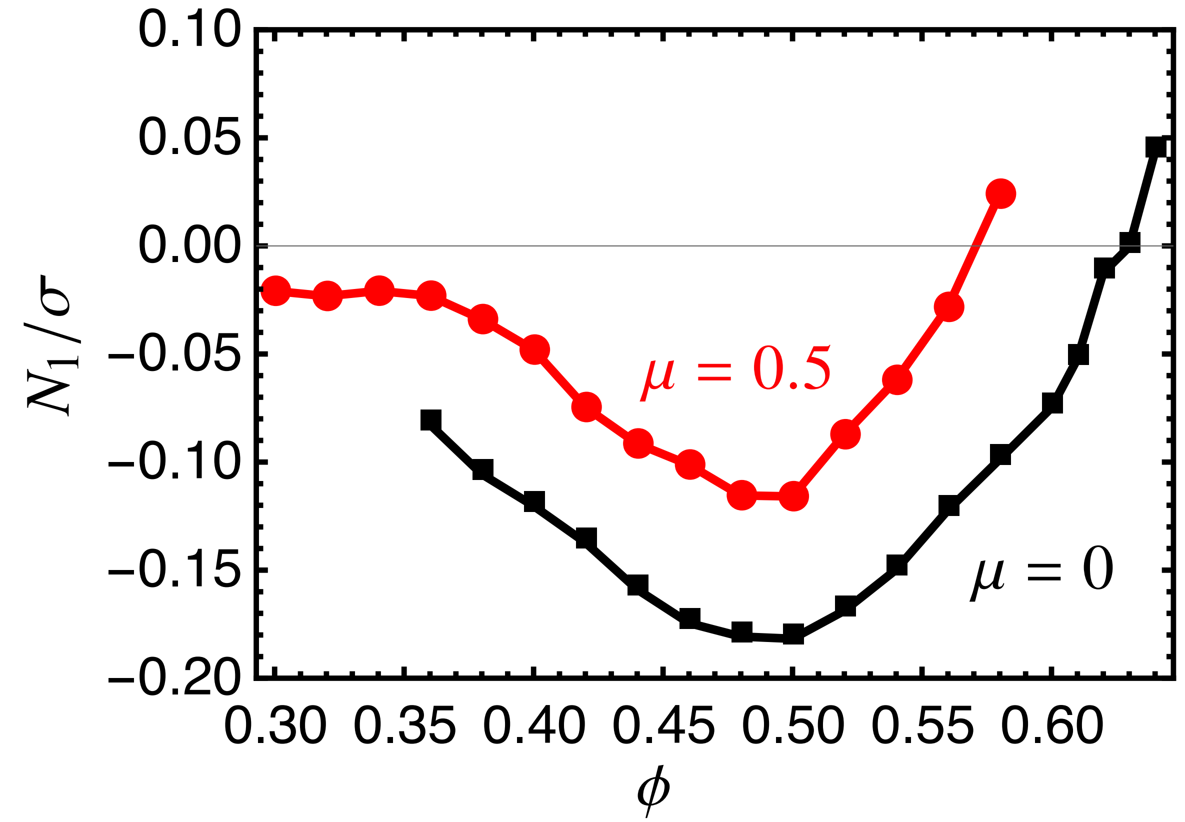
N_1 issue

$$\frac{N_1}{\eta\dot{\gamma}} = \frac{N_1}{\sigma}$$

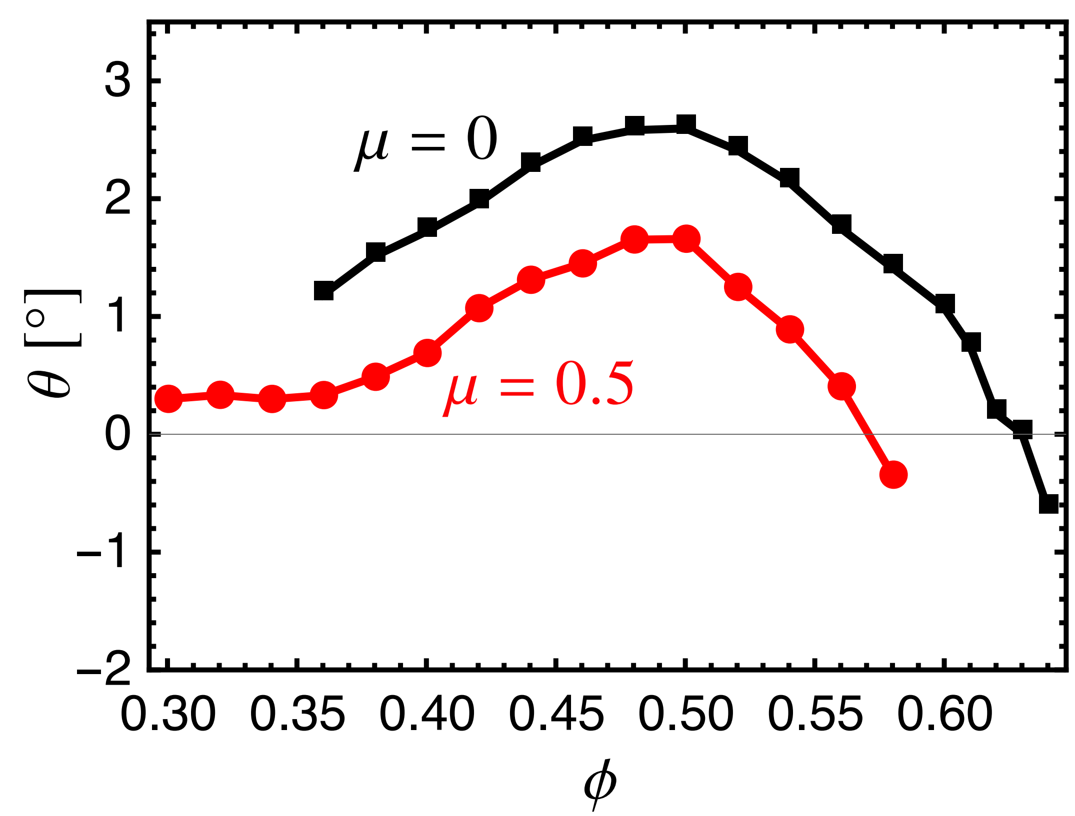


The change of sign is not surprising!





$$\theta \equiv \tan^{-1}\left(\frac{-N_1/2\sigma}{1+\sqrt{1+(N_1/2\sigma)^2}}\right) \approx -\frac{N_1}{4\sigma}$$



OIST

Outline

1. Periodic boundary conditions
2. Simulation model for particle dynamics
3. Material functions
4. N_1 issue
5. Extensional rheology

Planar flows

$$\boldsymbol{\sigma} = -\left[p_{\mathrm{f}} + \Pi(\dot{\varepsilon}, \beta_3)\right]\boldsymbol{I} + 2\eta(\dot{\varepsilon}, \beta_3)\boldsymbol{D} + 2\lambda_0(\dot{\varepsilon}, \beta_3)\boldsymbol{E} + 2\lambda_3(\dot{\varepsilon}, \beta_3)\boldsymbol{G}_3$$

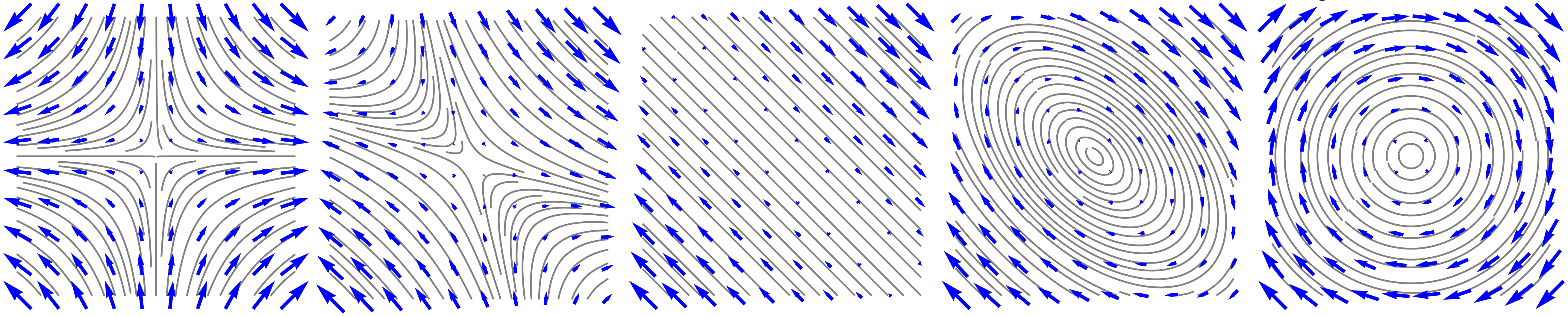
$$\nabla \boldsymbol{u} \begin{cases} \boldsymbol{D} \equiv \dfrac{\nabla \boldsymbol{u} + \nabla \boldsymbol{u}^{\mathrm{T}}}{2} \\ \boldsymbol{W} \equiv \dfrac{\nabla \boldsymbol{u} - \nabla \boldsymbol{u}^{\mathrm{T}}}{2} \end{cases}$$

strain rate $\dot{\varepsilon} \equiv \sqrt{\dfrac{\boldsymbol{D} : \boldsymbol{D}}{2}}$

flow type $\beta_3 \equiv \dfrac{1}{2\dot{\varepsilon}} \hat{\boldsymbol{d}}_3 \cdot \nabla \times \boldsymbol{u}$

objective flow type: $\bar{\beta}_3 = \beta_3 - \delta_3$

extensional | mixed | simple shear | mixed | rigid rotation

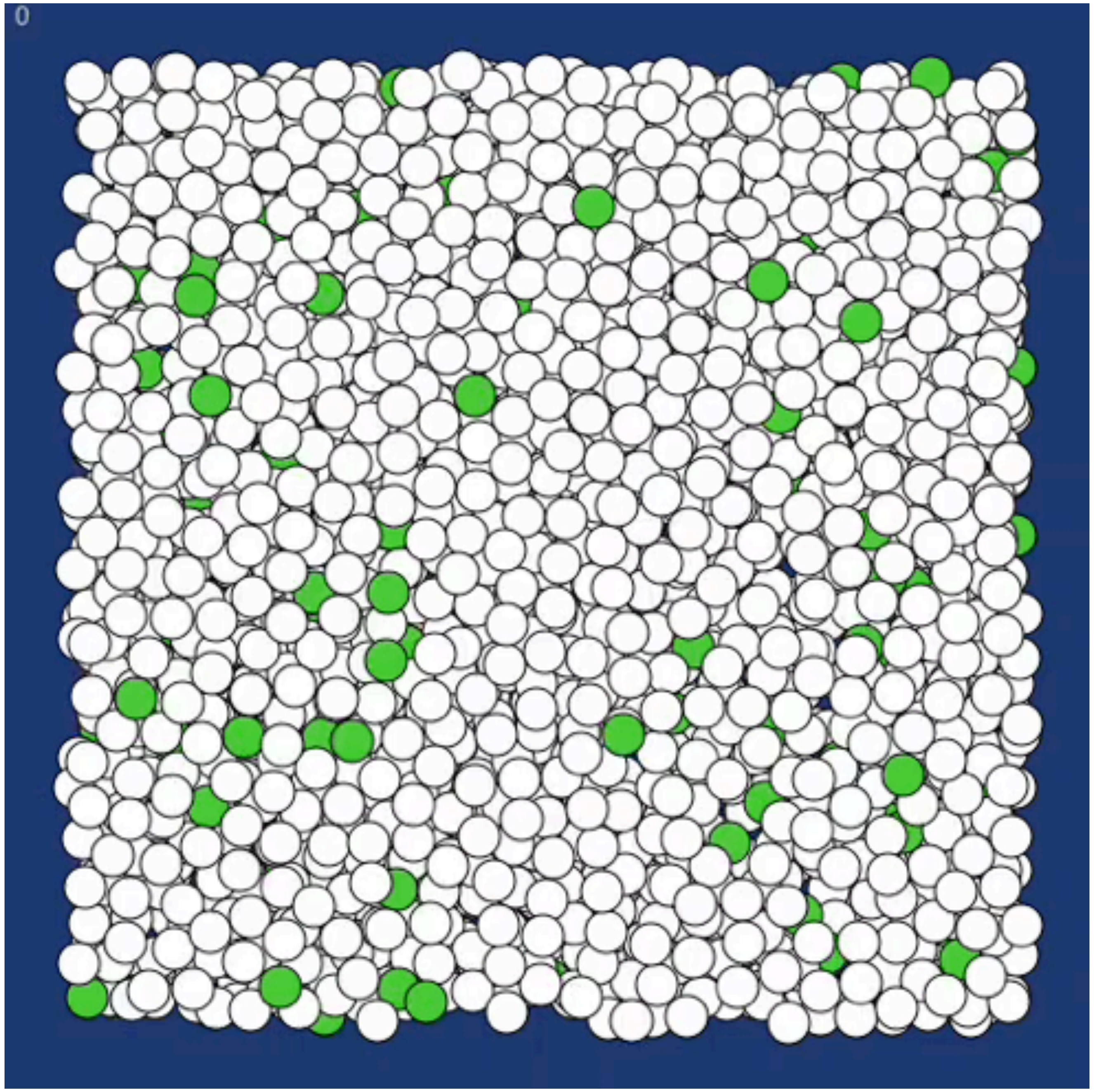


$\beta_3 = 0$ | $\beta_3 = 0.5$ | $\beta_3 = 1$ | $\beta_3 = 2$ | $\beta_3 = \infty$

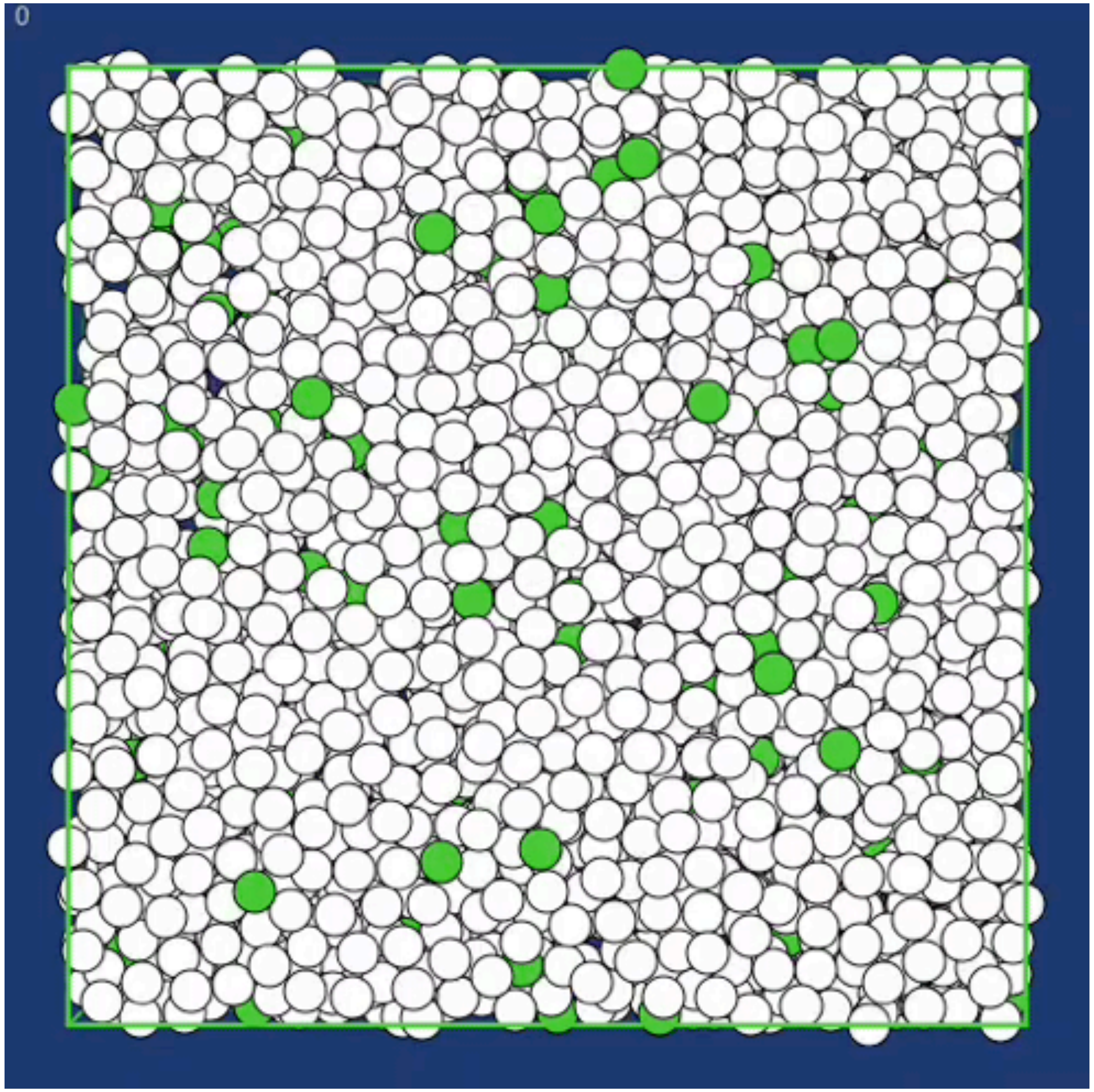
Planar flows

flow type $\beta_3 \equiv \frac{1}{2\dot{\varepsilon}} \hat{d}_3 \cdot \nabla \times u$

$\beta_3 = 1$



$\beta_3 = 0$



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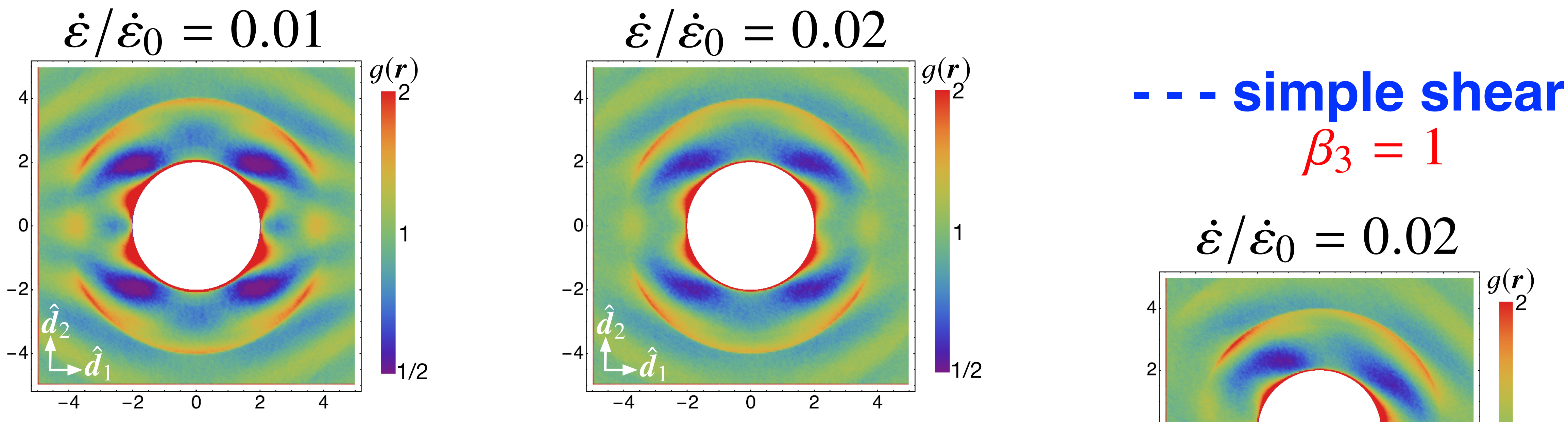
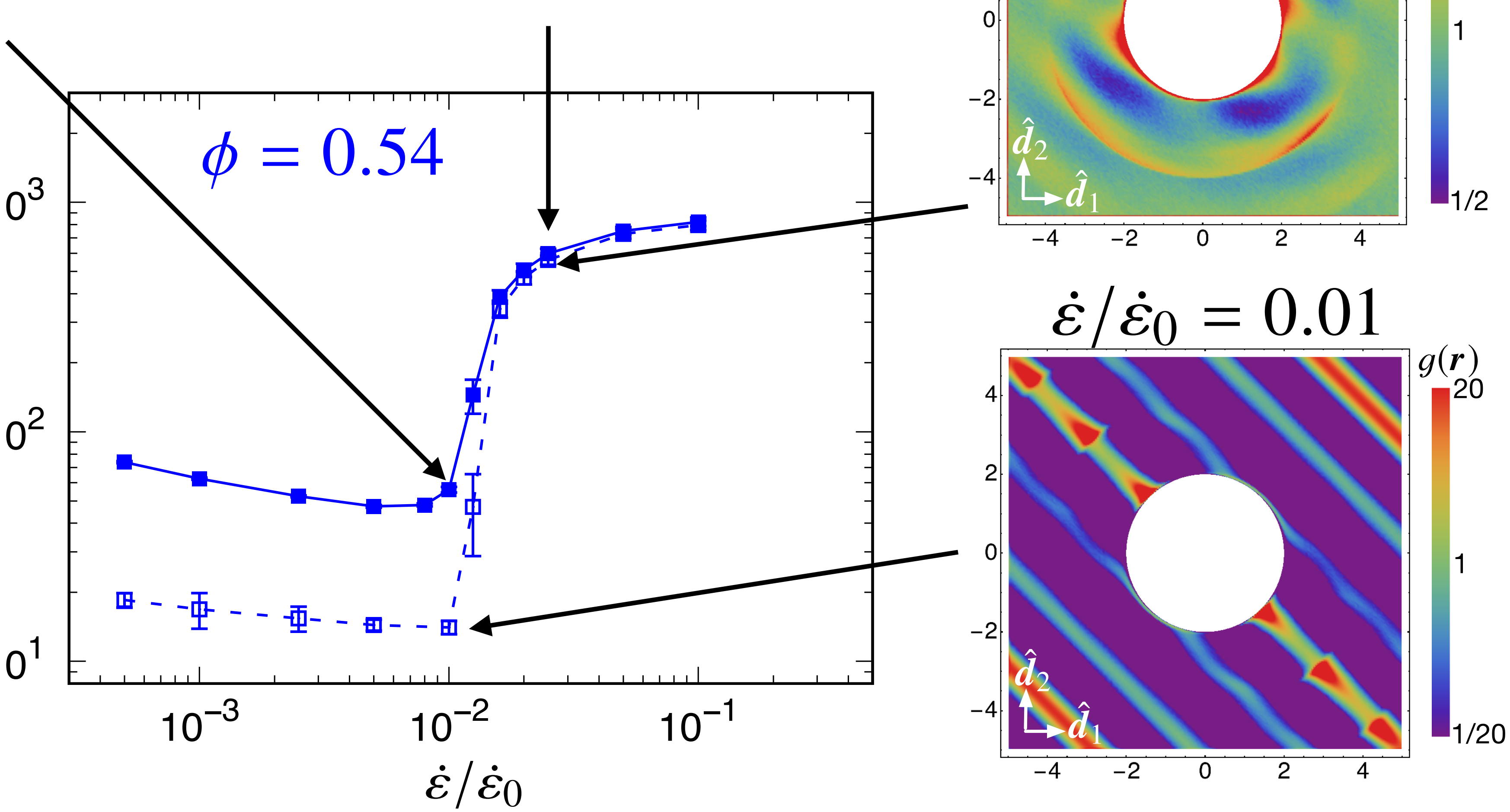
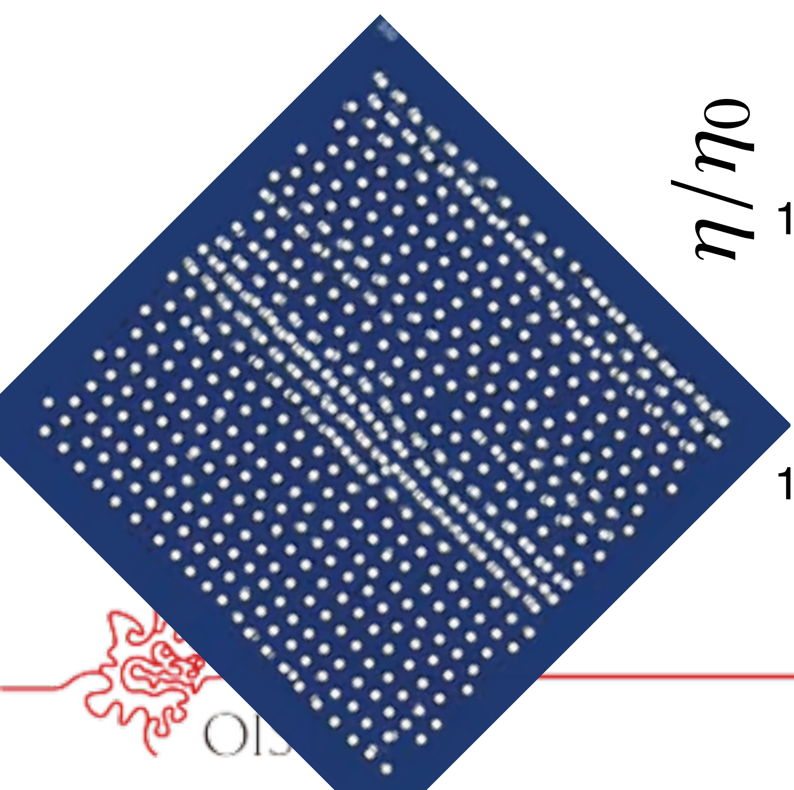
$$\eta(\dot{\varepsilon}, \beta_3) \equiv \frac{1}{2}\frac{\boldsymbol{\sigma} : \boldsymbol{D}}{\boldsymbol{D} : \boldsymbol{D}}$$

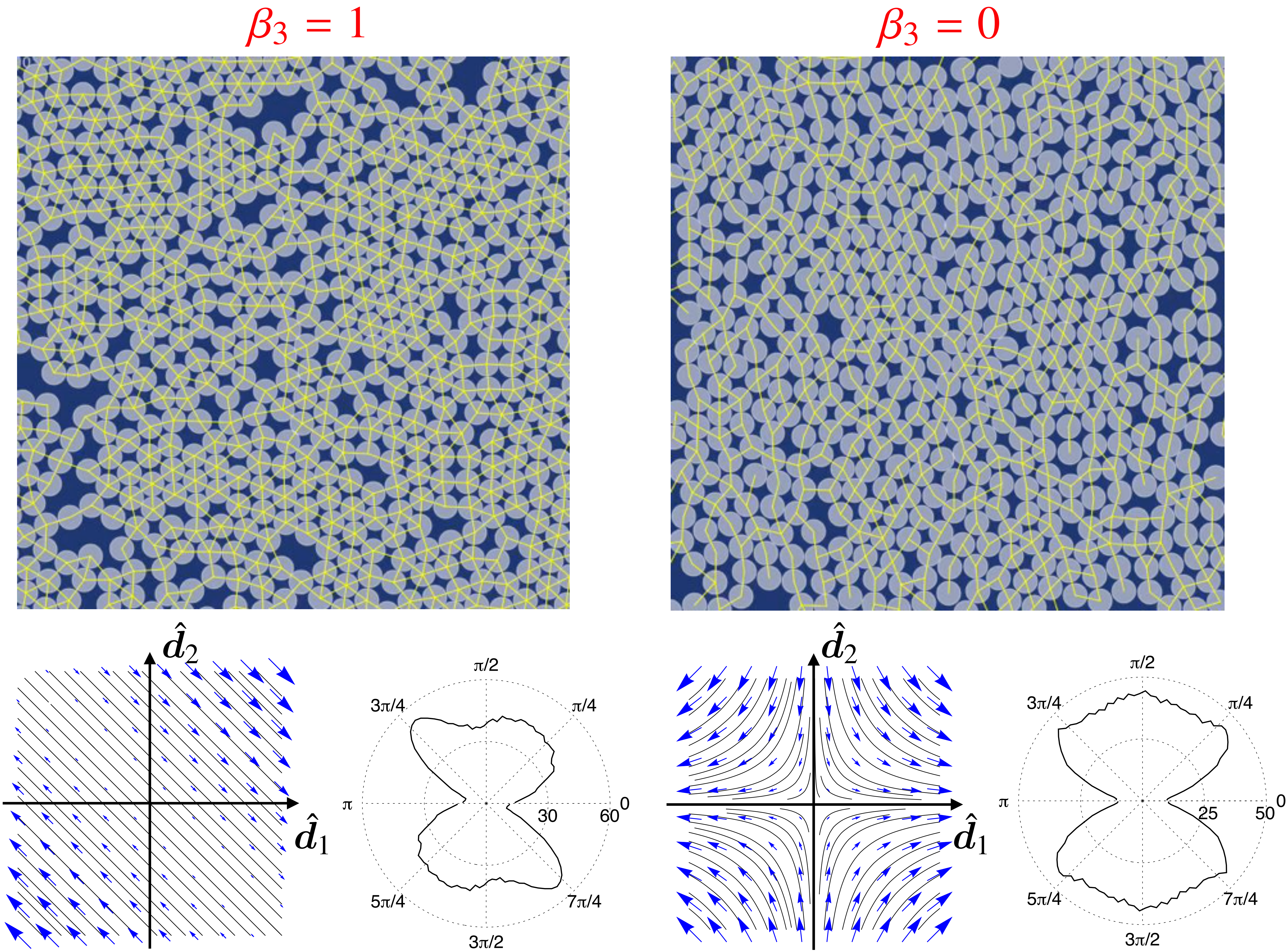
strain rate $\dot{\varepsilon} \equiv \sqrt{\dfrac{\boldsymbol{D} : \boldsymbol{D}}{2}}$

—— extensional $\beta_3 = 0$







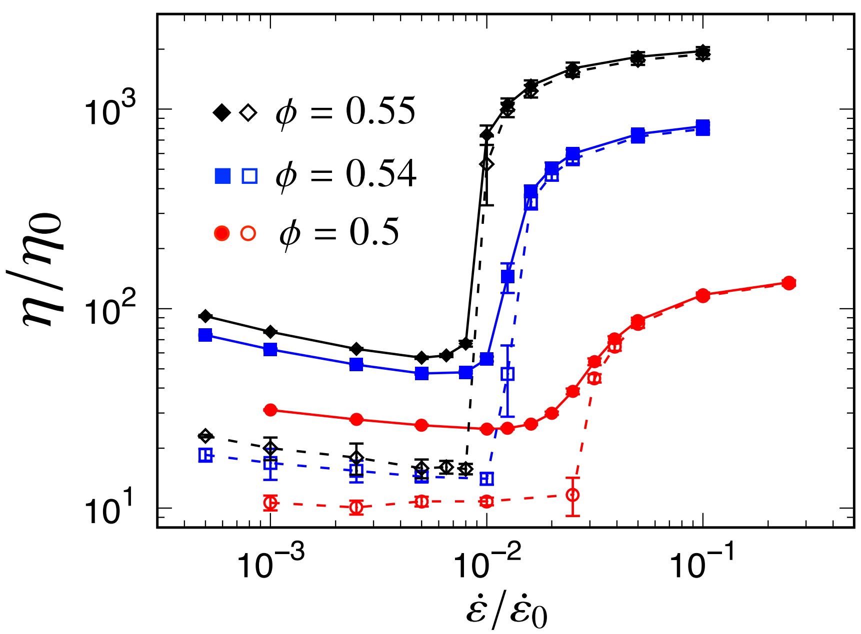


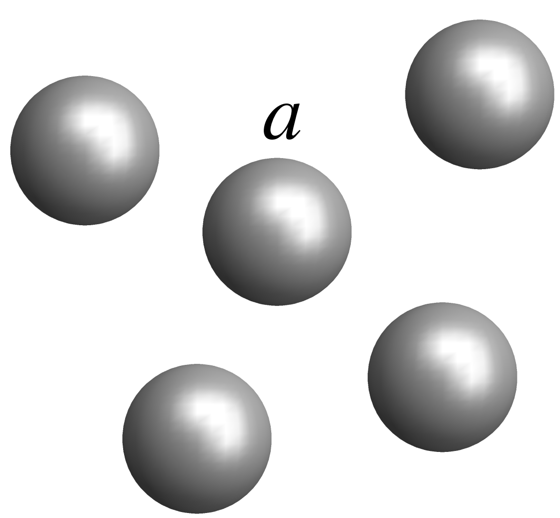
$\beta_3 = 1$
$\beta_3 = 0$
$\hat{d}_2$
$\hat{d}_1$
π/2
3π/4
π/4
π
0
30
60
5π/4
7π/4
3π/2
25
50

If ordering is suppressed, flow type dependence is subtle

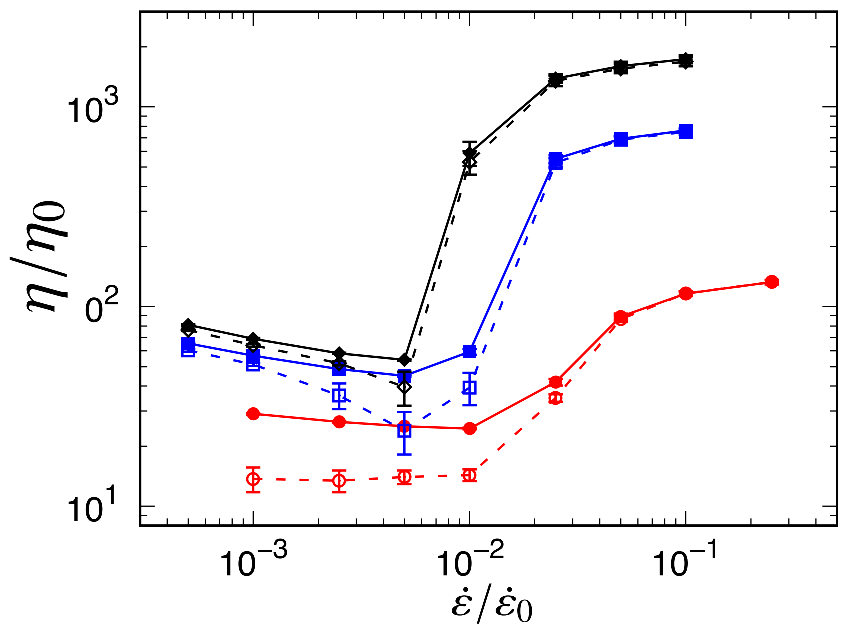
—— extensional $\beta_3 = 0$ - - - simple shear $\beta_3 = 1$

monodisperse

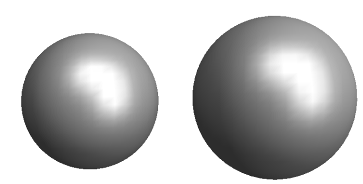




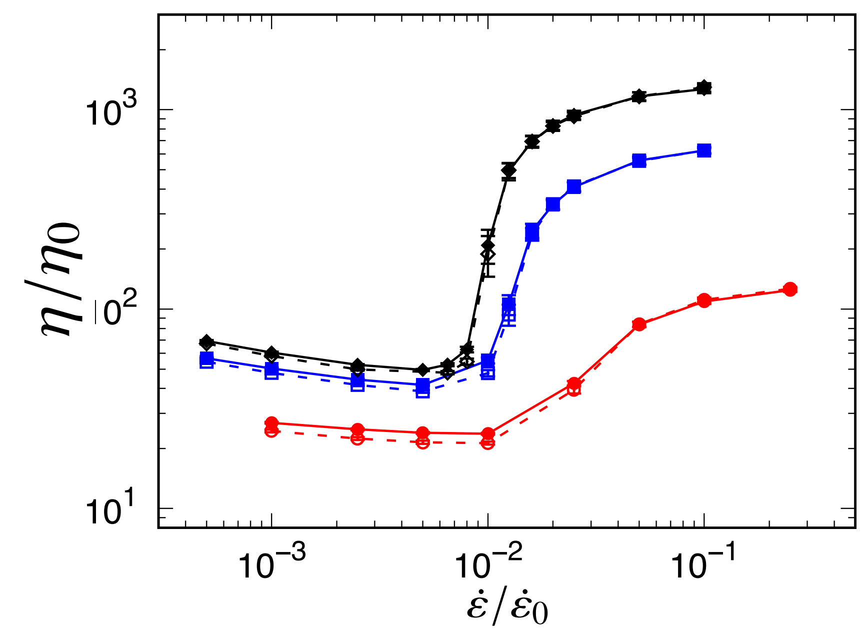
bidisperse



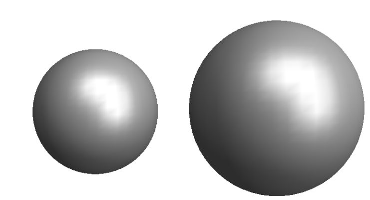
$$\frac{a_2}{a_1} = 1.2$$



bidisperse

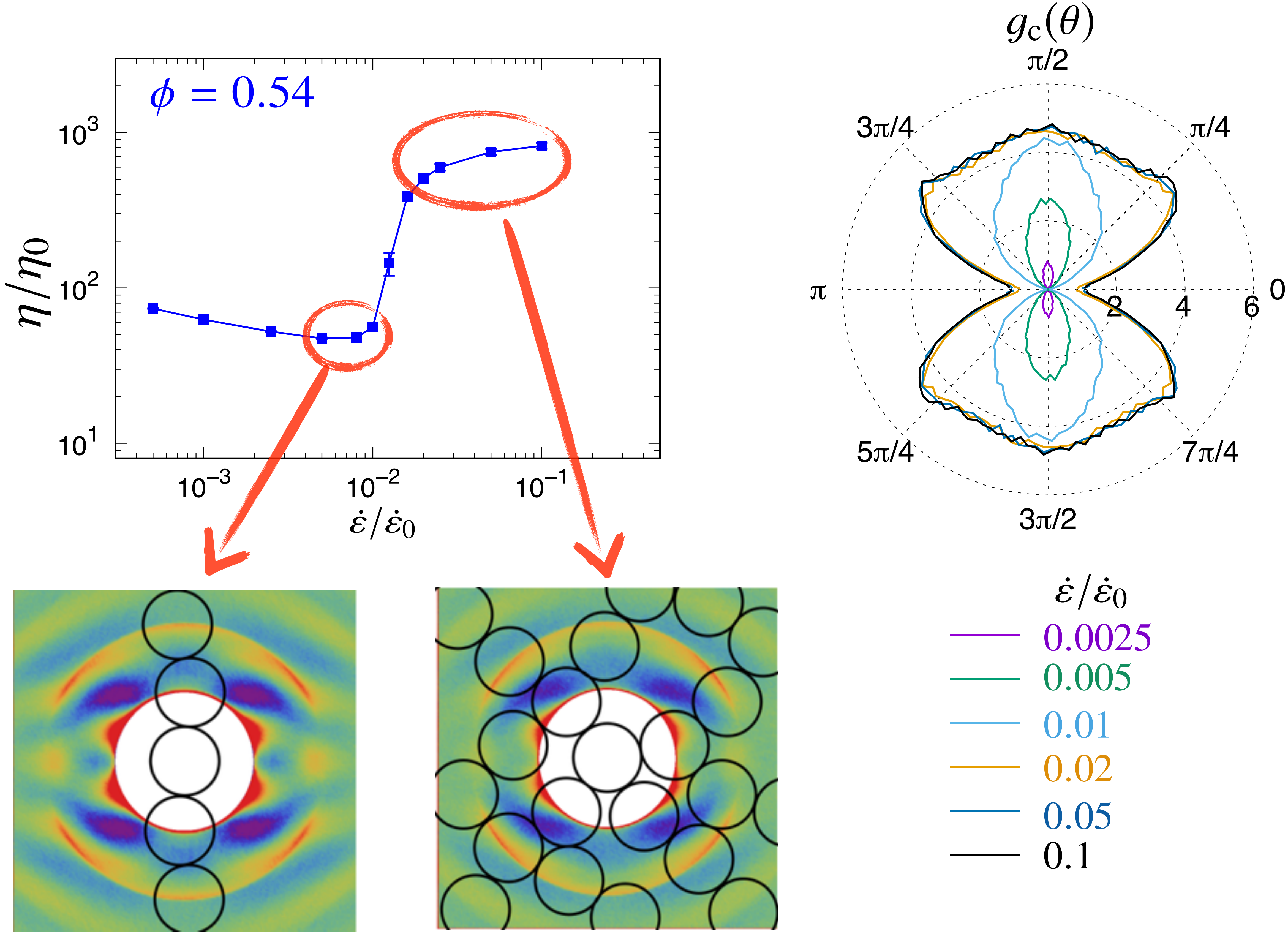


$$\frac{a_2}{a_1} = 1.4$$



OIST

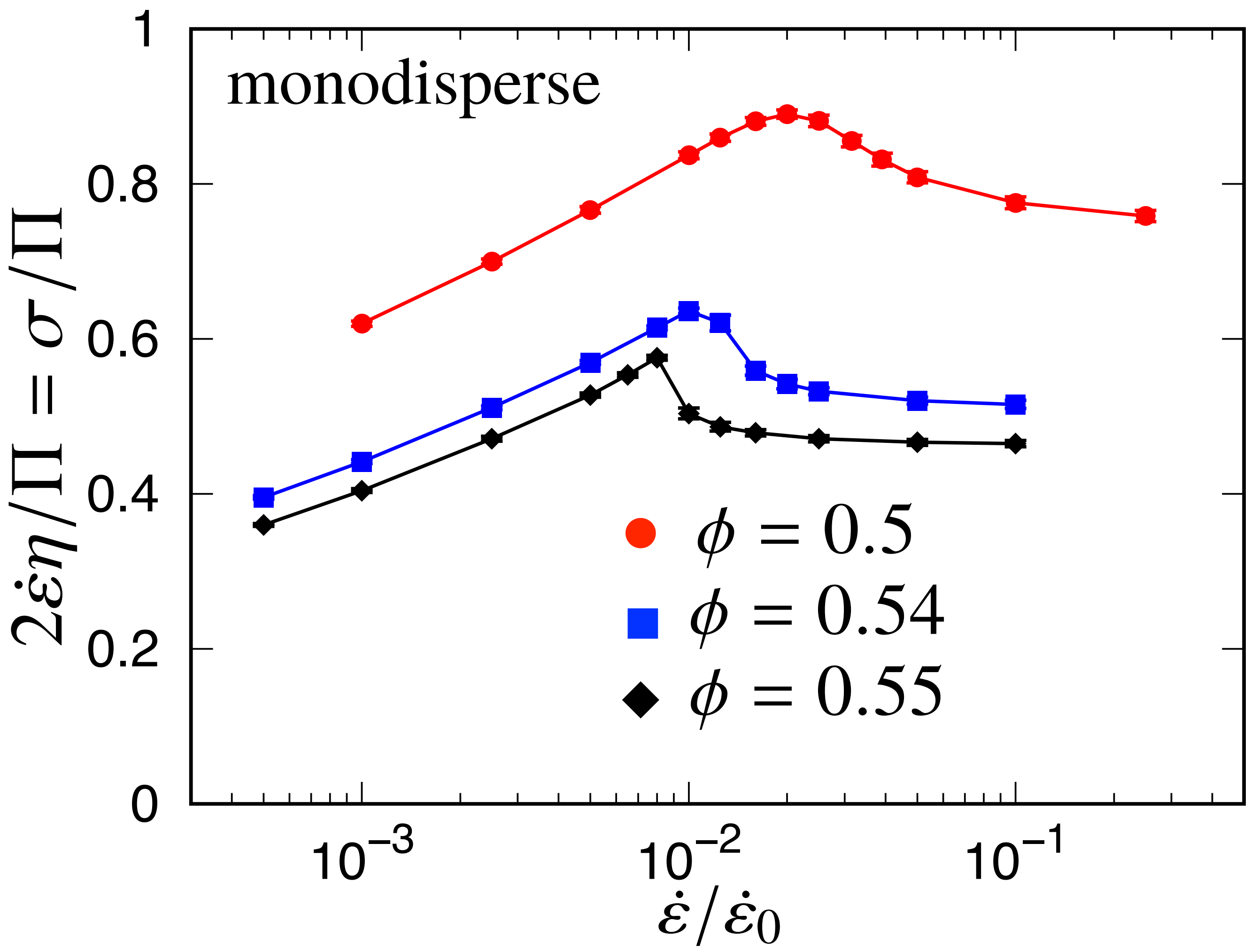
Angular distributions for frictional contacts

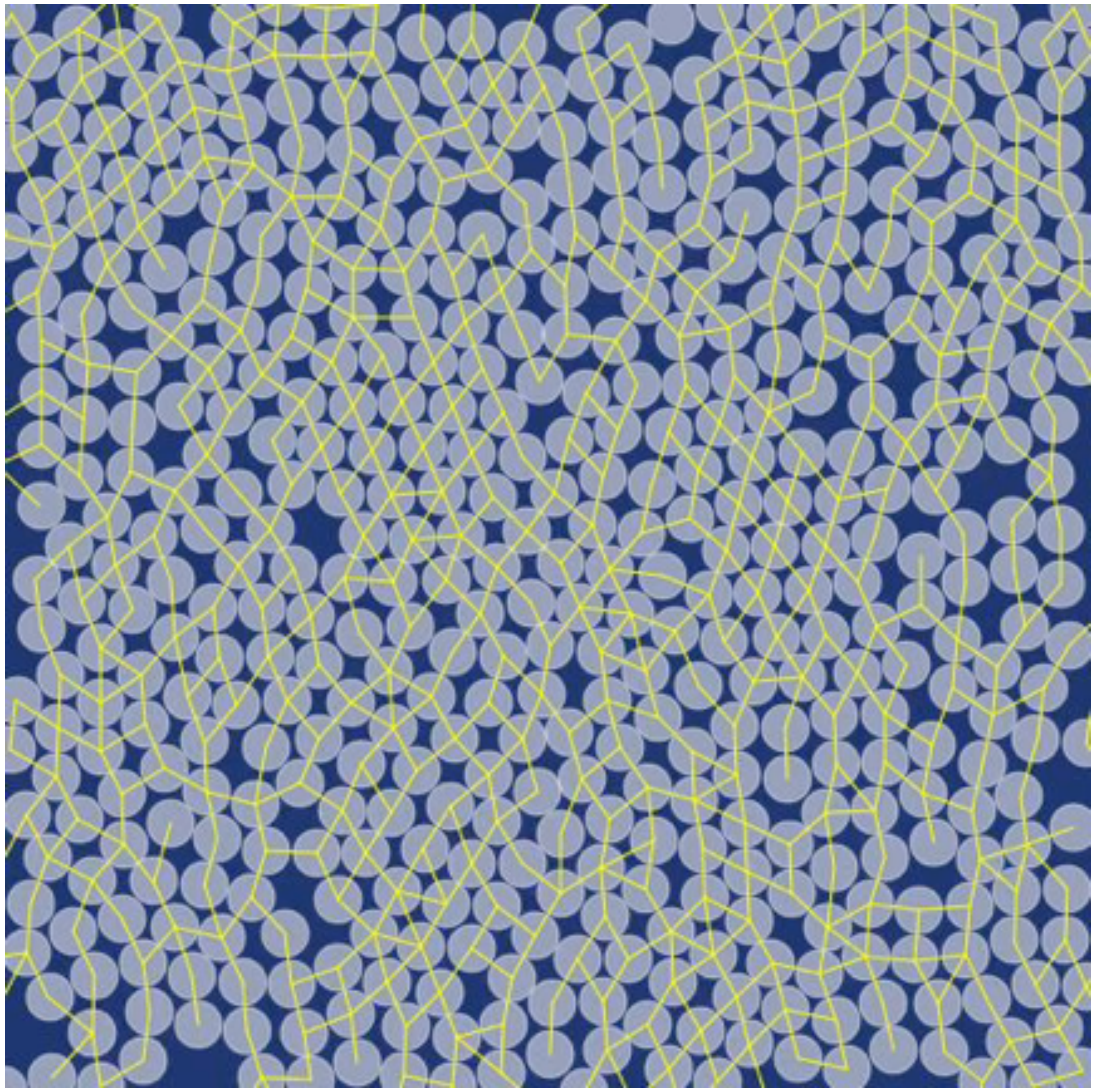


Particle pressure

$$\boldsymbol{\sigma} = -\Pi(\dot{\varepsilon}, \beta_3)\boldsymbol{I} + 2\eta(\dot{\varepsilon}, \beta_3)\boldsymbol{D} + 2\lambda_0(\dot{\varepsilon}, \beta_3)\boldsymbol{E} + 2\lambda_3(\dot{\varepsilon}, \beta_3)\boldsymbol{G}_3$$

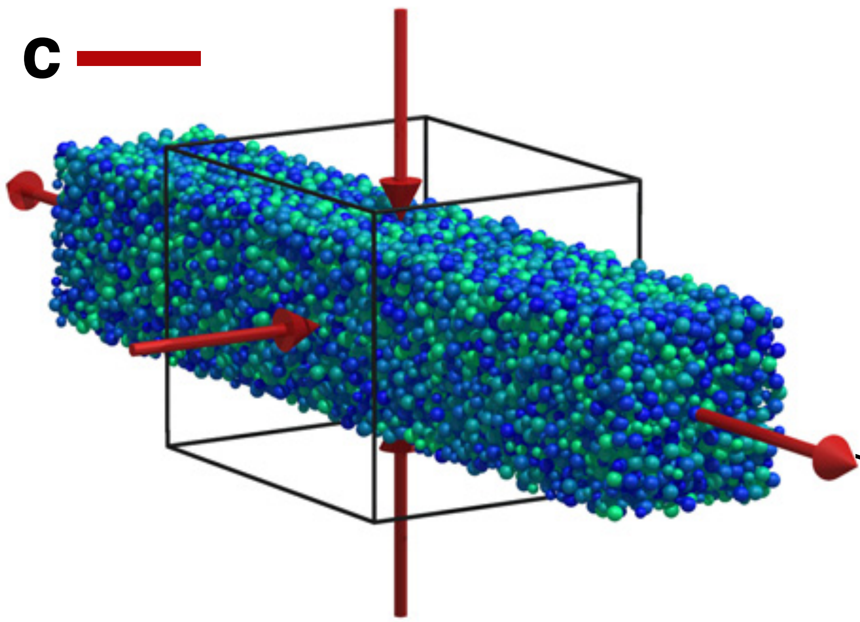
$$\Pi = -\frac{\boldsymbol{\sigma} : \boldsymbol{I}}{\boldsymbol{I} : \boldsymbol{I}} = -\frac{\mathrm{Tr}\,\boldsymbol{\sigma}}{3}$$



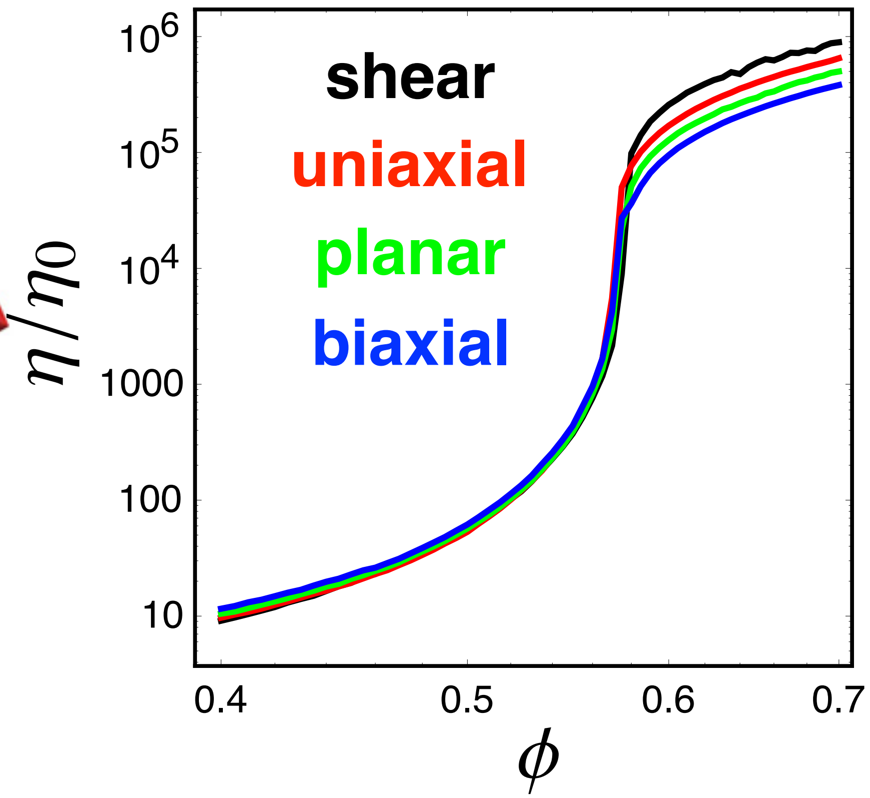


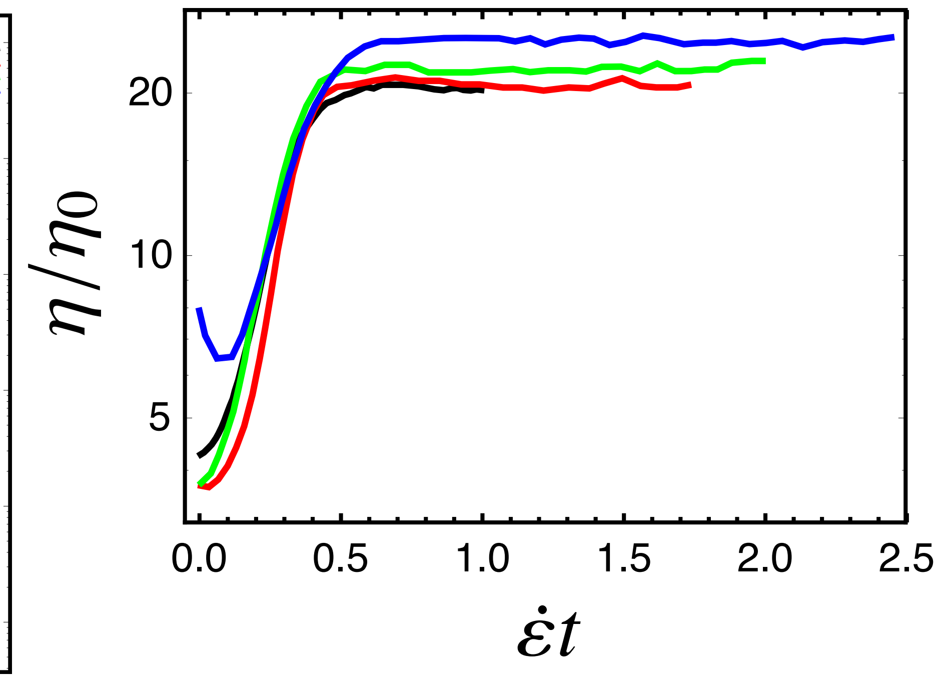
Uniaxial, planar, and biaxial extensional flows

Cheal & Ness (JOR 2018)

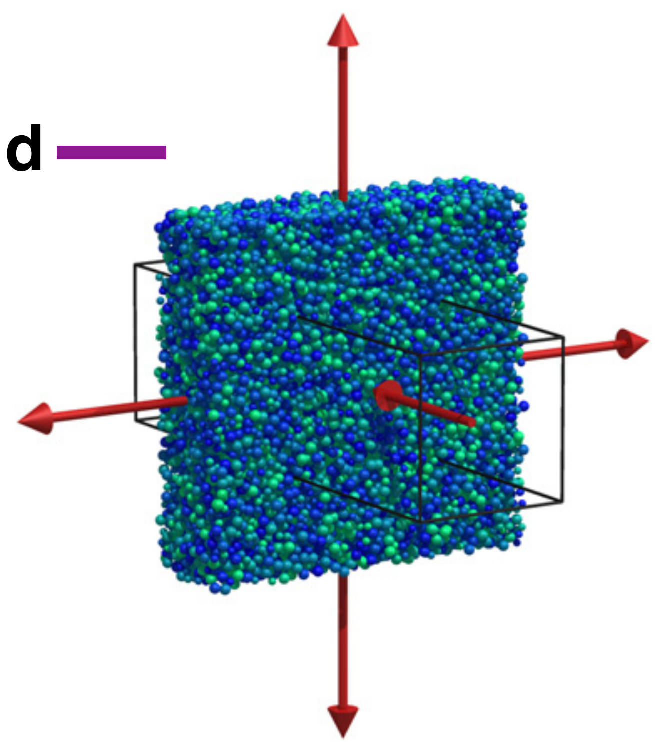


In our representation for the data from C. Ness





$$\dot{\varepsilon} \equiv \sqrt{\boldsymbol{D} : \boldsymbol{D}/2}$$



$$\boldsymbol{D} = \frac{2\dot{\varepsilon}}{\sqrt{3+4\alpha^2}}\left[\hat{\boldsymbol{d}}_1\hat{\boldsymbol{d}}_1 - (1/2+\alpha)\hat{\boldsymbol{d}}_2\hat{\boldsymbol{d}}_2 - (1/2-\alpha)\hat{\boldsymbol{d}}_3\hat{\boldsymbol{d}}_3\right]$$

$$\eta(\dot{\varepsilon}, \alpha) \equiv \frac{1}{2}\frac{\boldsymbol{\sigma} : \boldsymbol{D}}{\boldsymbol{D} : \boldsymbol{D}}$$

$\alpha = 0$ & $\dot{\varepsilon} > 0 \longrightarrow$ uniaxial
$\alpha = 0$ & $\dot{\varepsilon} < 0 \longrightarrow$ biaxial
$\alpha = 1/2 \longrightarrow$ planar

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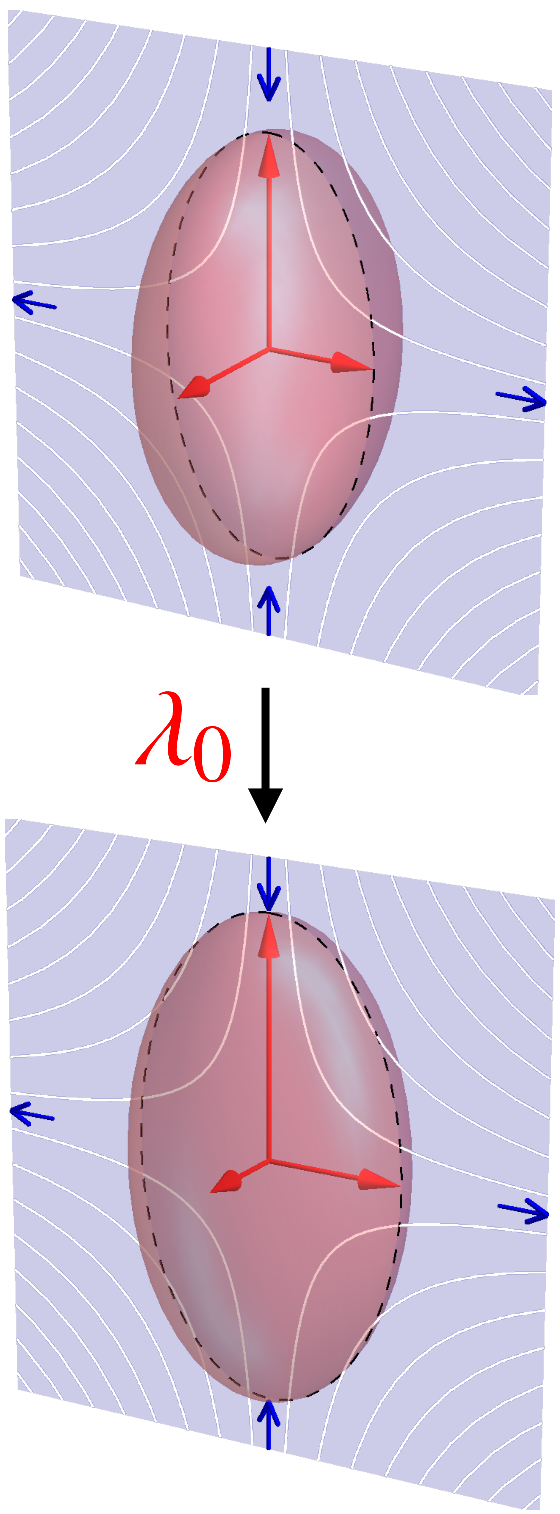
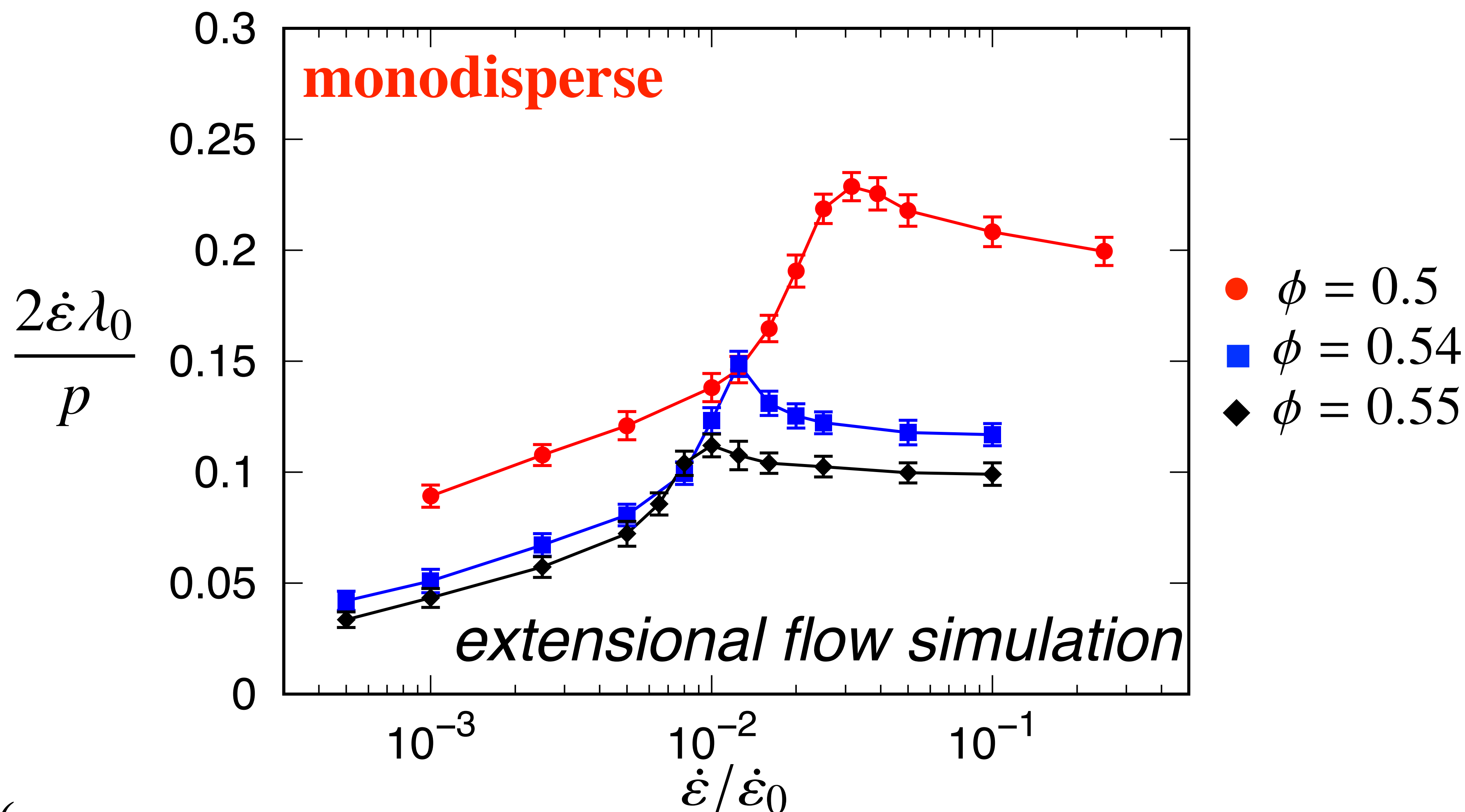
The material function λ_0

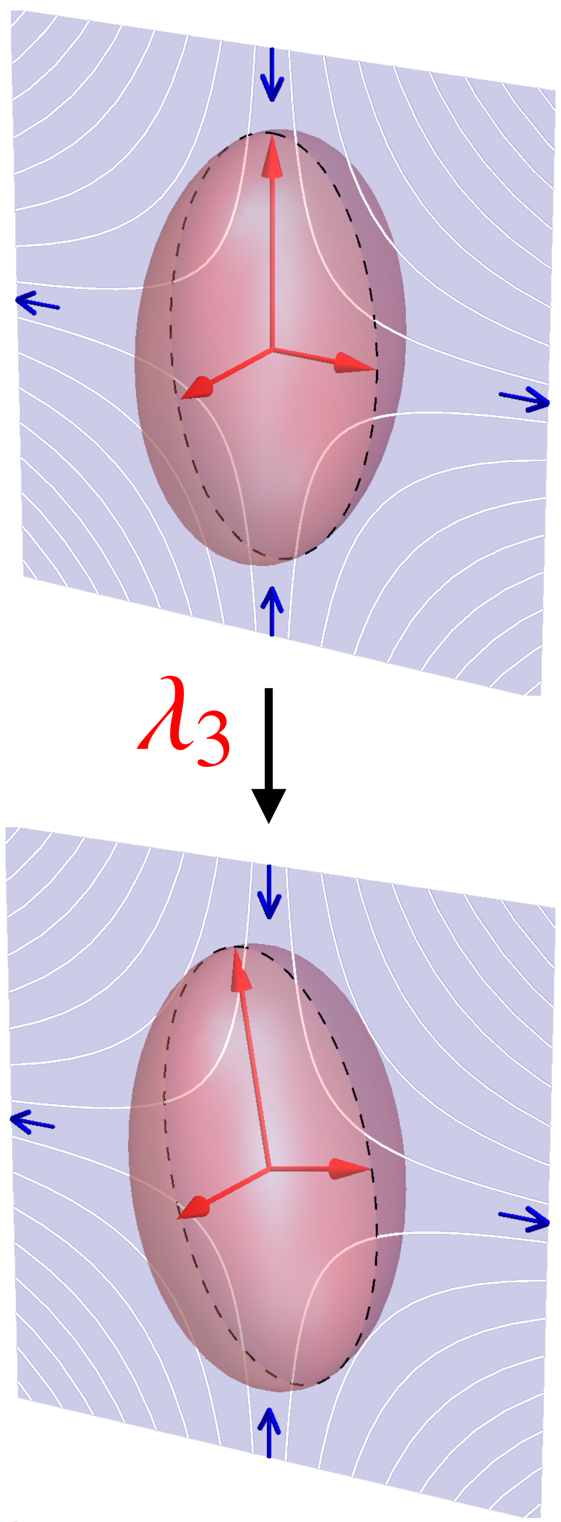
$$\lambda_0(\dot{\varepsilon}, \beta_3) = \frac{\boldsymbol{\sigma} : \boldsymbol{E}}{2\boldsymbol{E} : \boldsymbol{E}} \longrightarrow \textbf{ellipsoidal factor} \quad \frac{2\dot{\varepsilon}\lambda_0}{p}$$





$$\begin{cases} p_{\text{in}} \equiv p + \frac{1}{2}\dot{\varepsilon}\lambda_0 \\ p_{\text{out}} \equiv p - \dot{\varepsilon}\lambda_0 \end{cases}$$

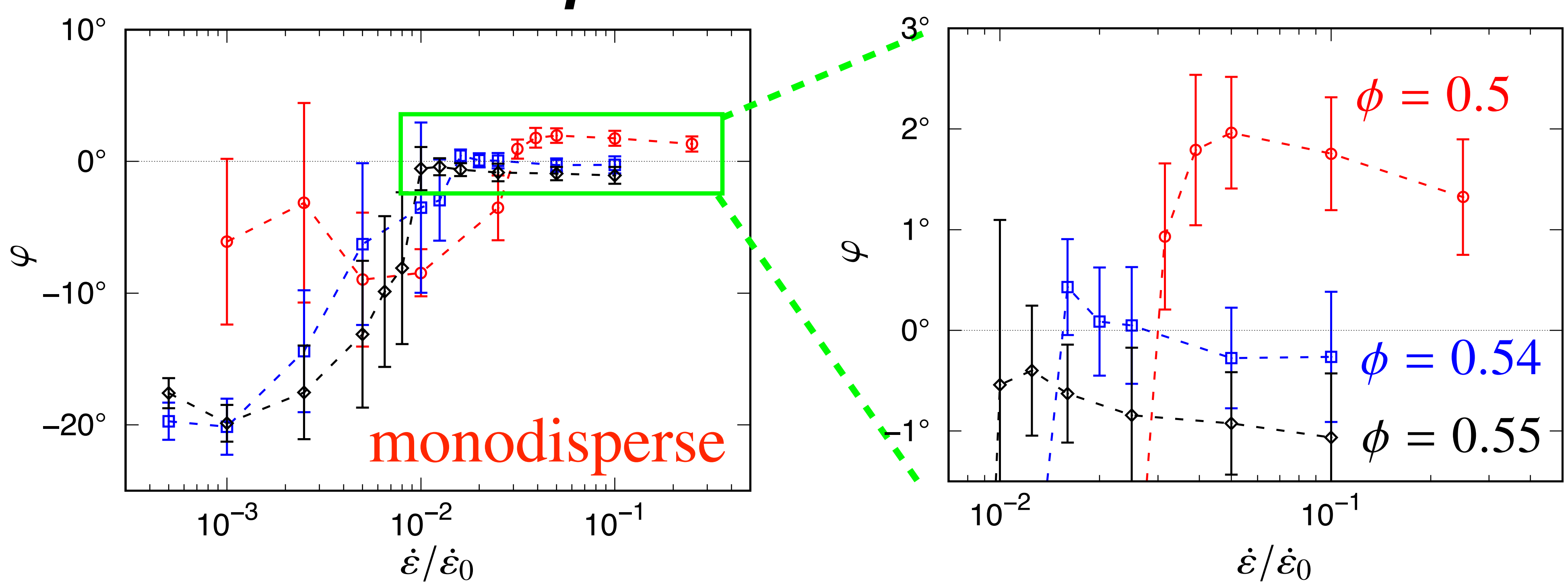
The material function λ_3



$$\lambda_3(\dot{\varepsilon}, \beta_3) = \frac{\boldsymbol{\sigma} : \boldsymbol{G_3}}{2\boldsymbol{G_3} : \boldsymbol{G_3}} \longrightarrow \textbf{reorientation factor} \quad \frac{\lambda_3}{2\eta}$$

$$\tan\theta \equiv \frac{\lambda_3}{\eta + \sqrt{\eta^2 + \lambda_3^2}} \approx \frac{\lambda_3}{2\eta}$$

simple-shear simulation



$$N_1 = -2\dot{\varepsilon}\lambda_3$$

$$\tan\theta = \frac{-N_1/2\sigma}{1 + \sqrt{1 + (N_1/2\sigma)^2}} \approx -\frac{N_1}{4\sigma} \qquad \sigma\text{: shear stress}$$

OIST

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We are ready
to tackle macroscopic flow problems
of dense suspensions!