How (and why) do force chains arise in dense amorphous materials

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Support: Science & Engineering Research Board

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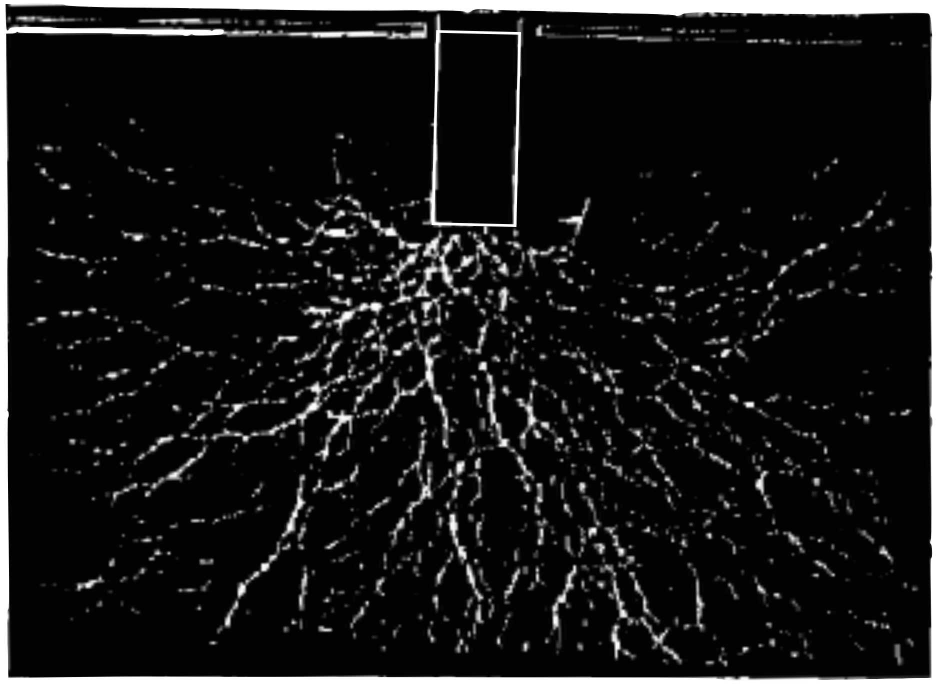
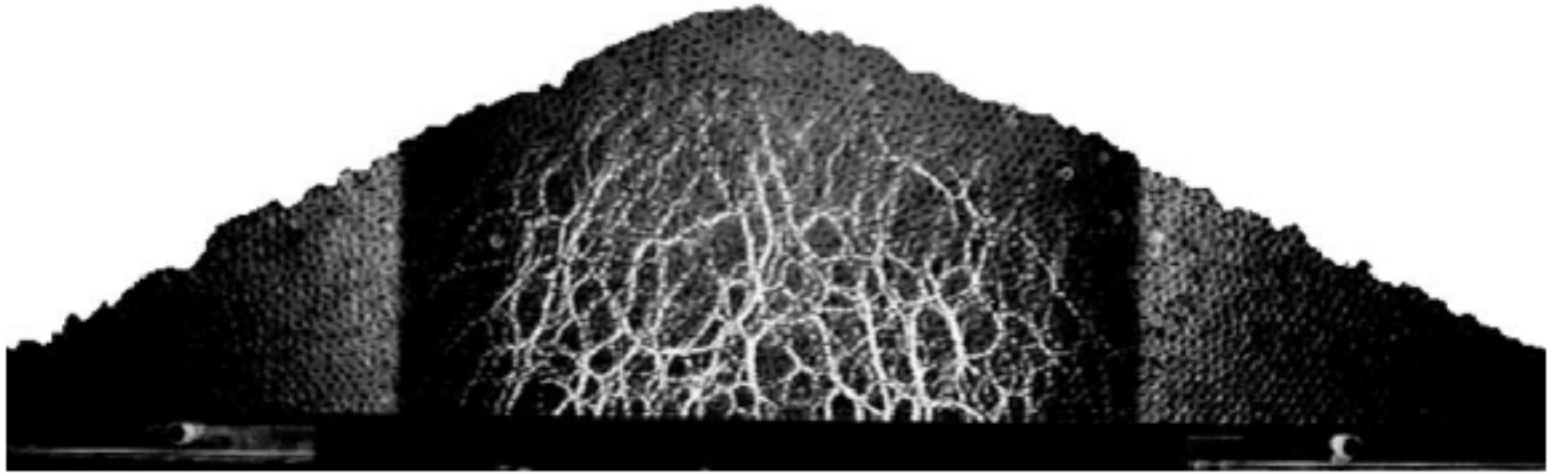
Pile driving



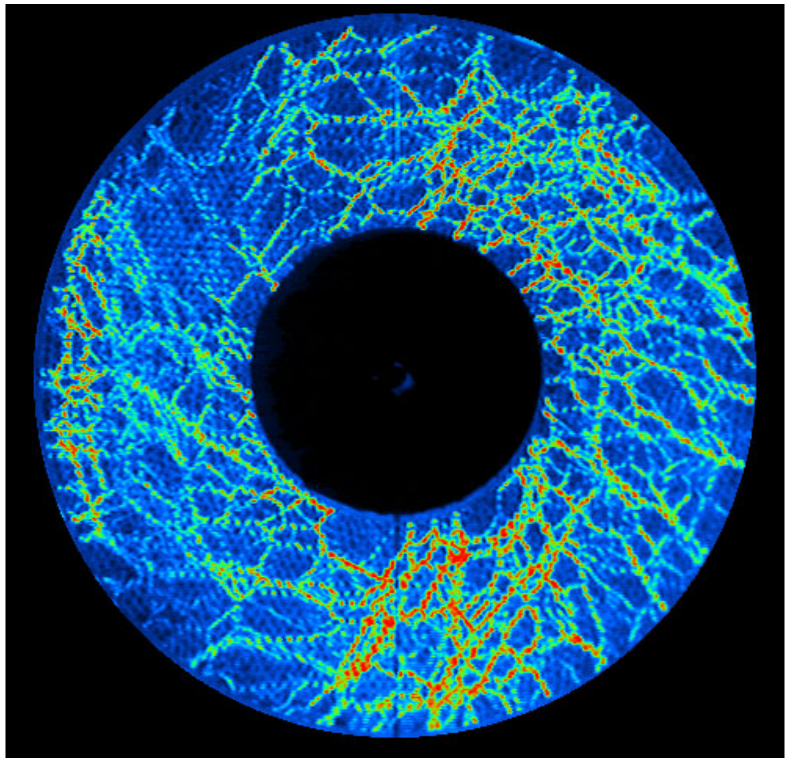
Dantu (1957)

Heap



Vanel et al, PRE (1999)

Shear

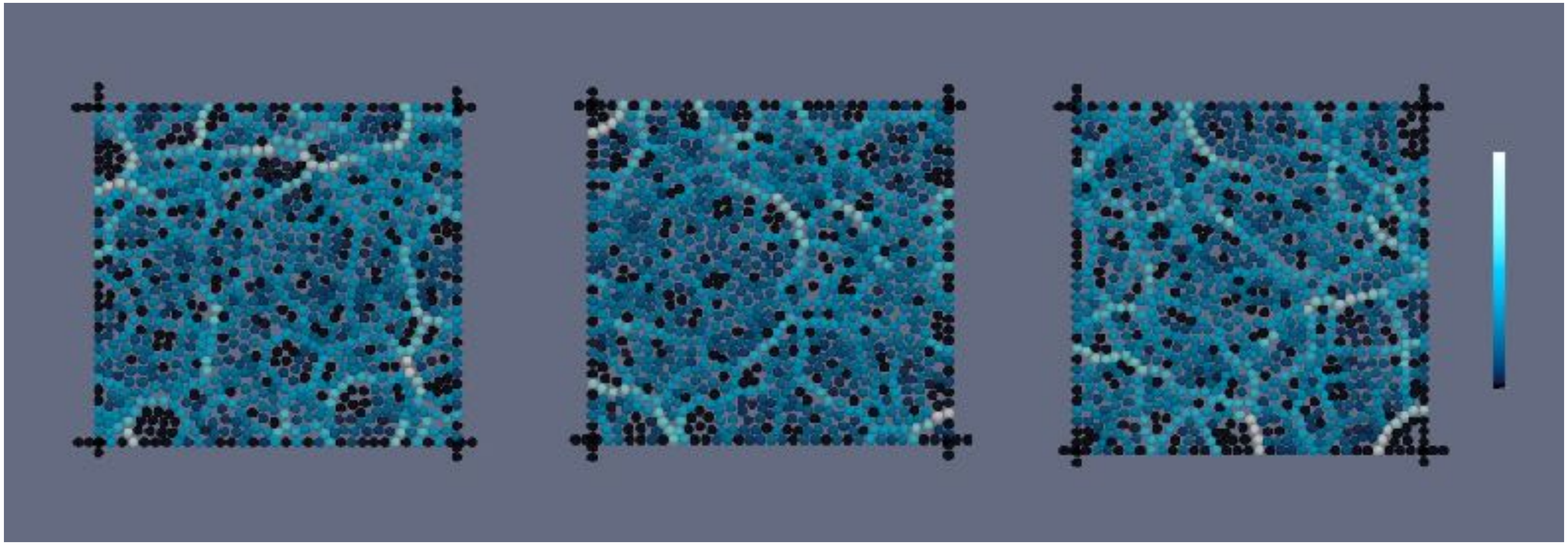


Howell *et al* (2000)

How does a disordered microstructure give rise to an inhomogeneous, but coherent, network of force-bearing chains of particles?

Features of a granular force network

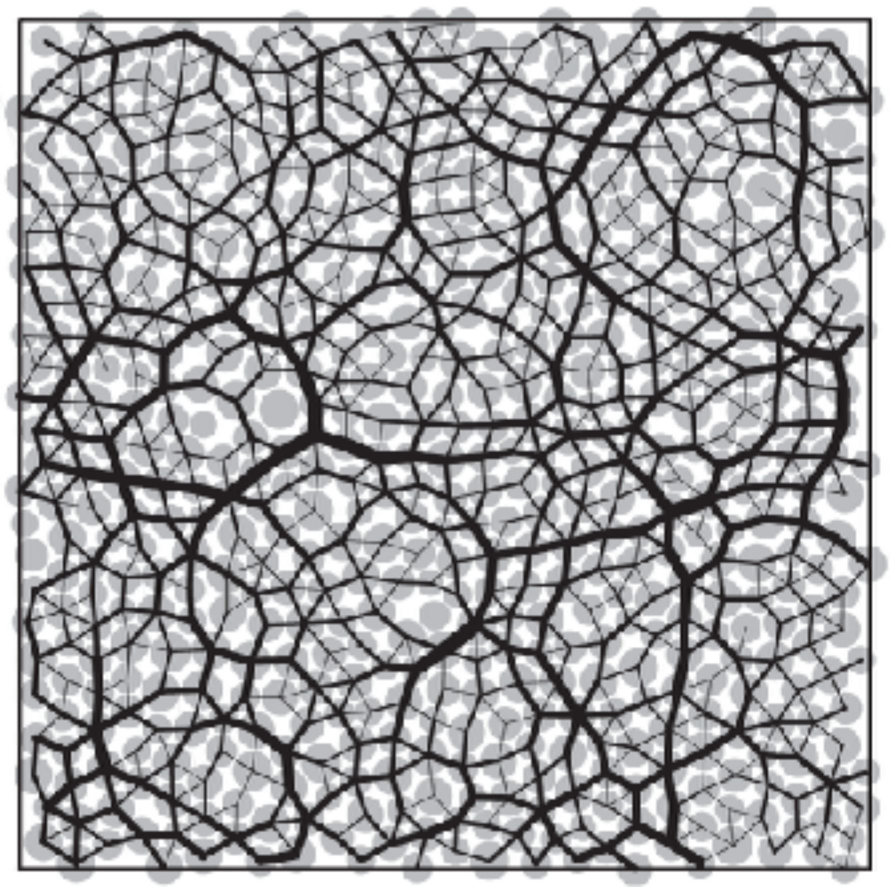
- A small subset of the ensemble carries a large fraction of the applied load, termed as “force chains”
- The linearity of the chains is an important observed featured

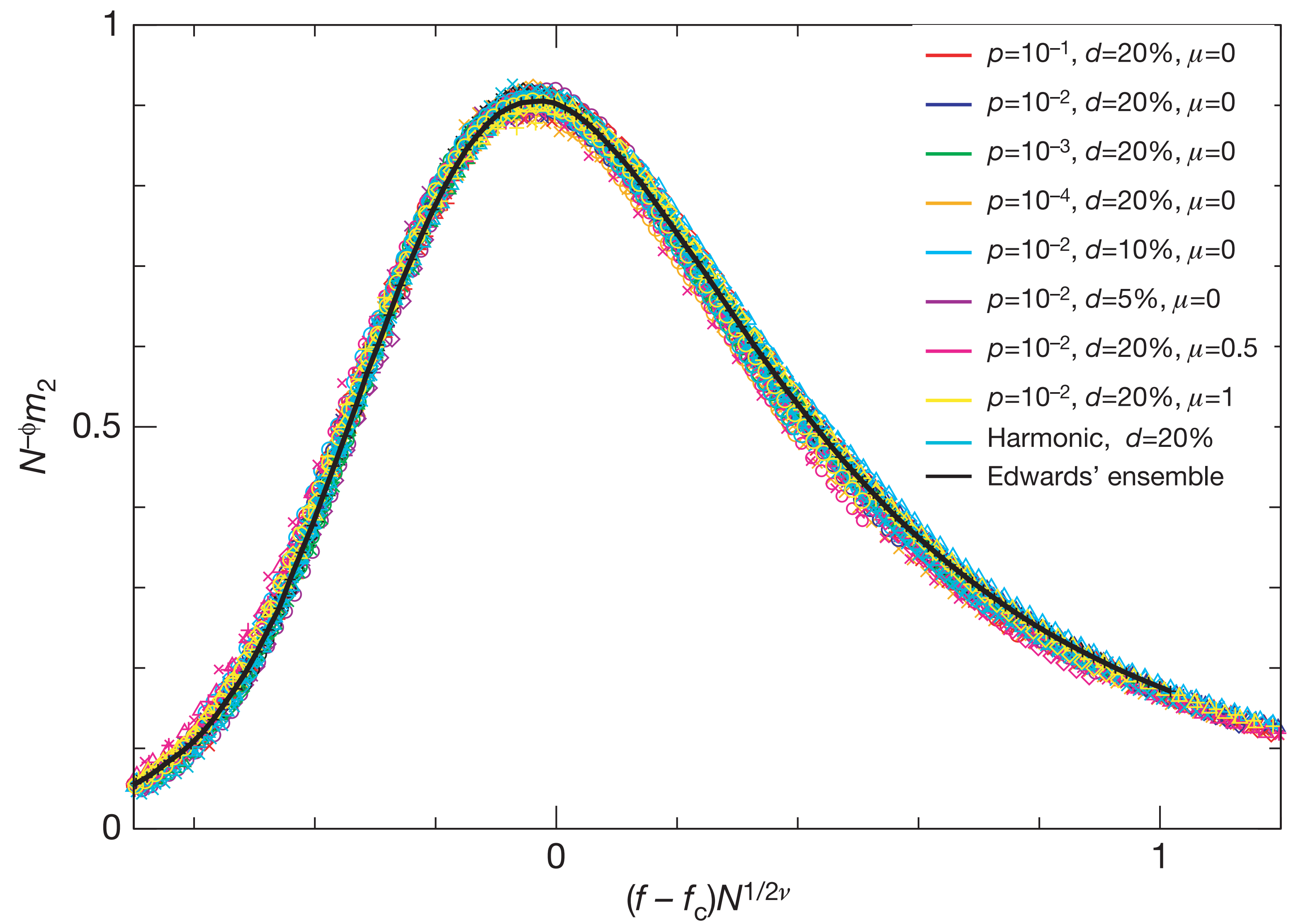


2D Isotropic compression

Features of granular force networks

Ostojic, Somfai & Nienhuis, Nature (2006)



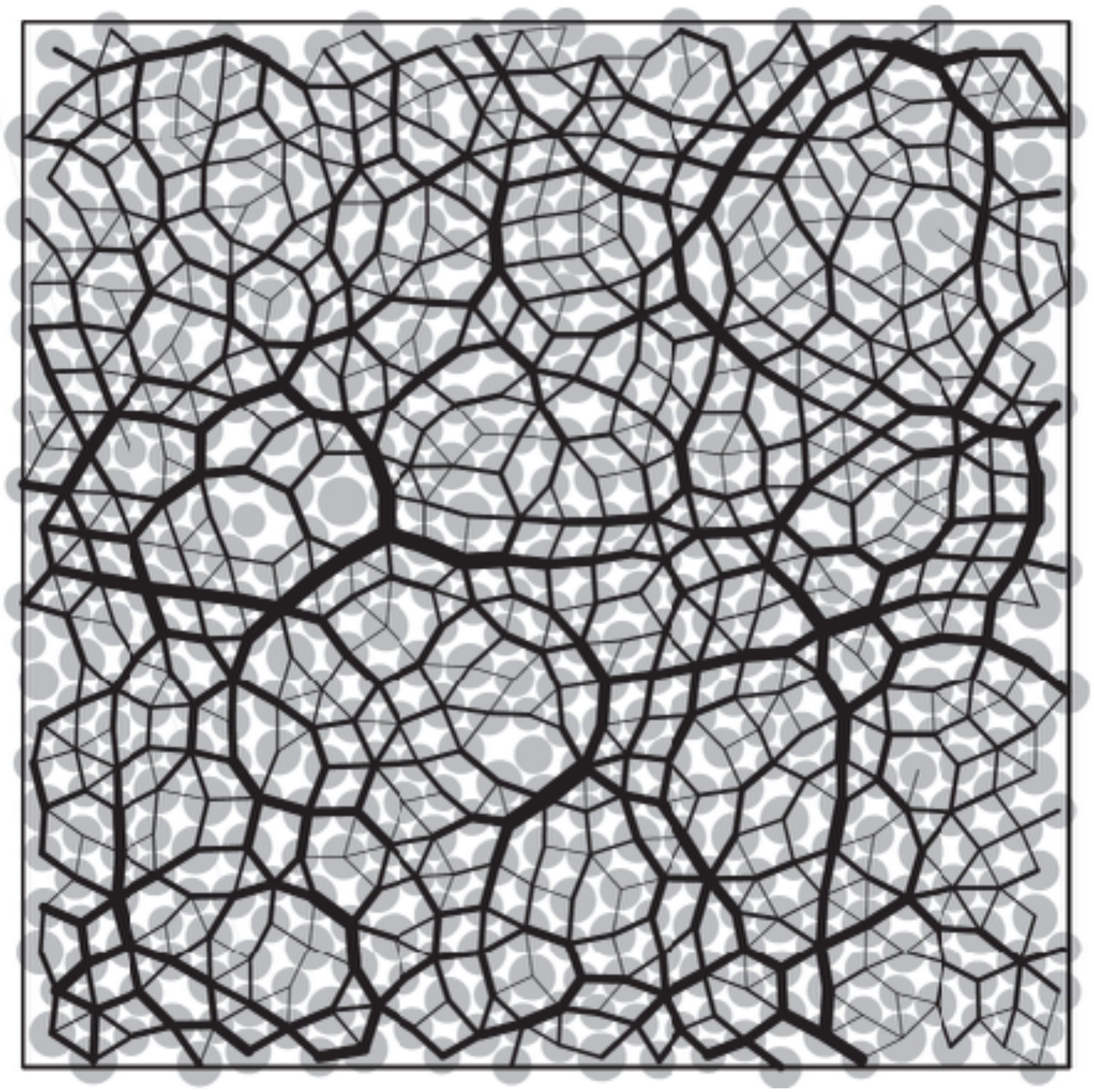


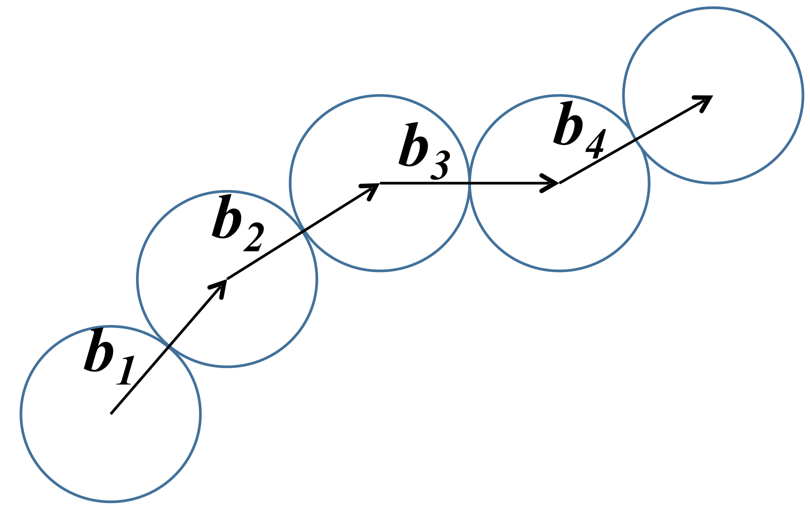
"Our results establish an unexpected connection between static granular matter and equilibrium critical phenomena. The observed scale invariance … defines a novel universality class of isotropic force networks. … This result implies the existence of long-range correlations between forces, a characteristic of structures such as force chains."

Pathak et al, PRE (2017) — random percolation universality class

"The main consequence of our findings is the absence of long-ranged correlations between the magnitudes of the forces in jammed granular packings."

Geometry of the force network



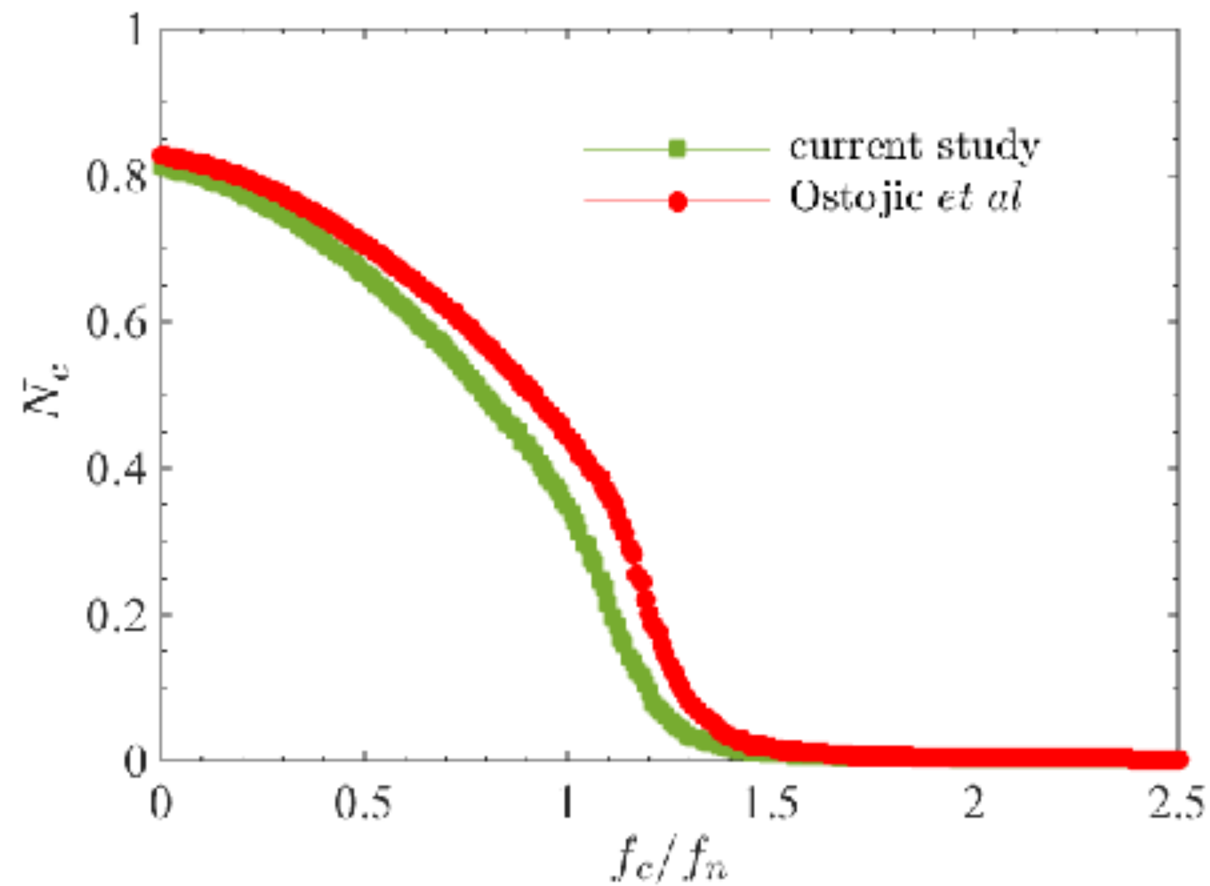


$$b_i . b_j > 0 \; \& \; F_{ij} \geq F_c$$

Starting from a randomly selected pair, all paths which satisfy the above conditions are explored.

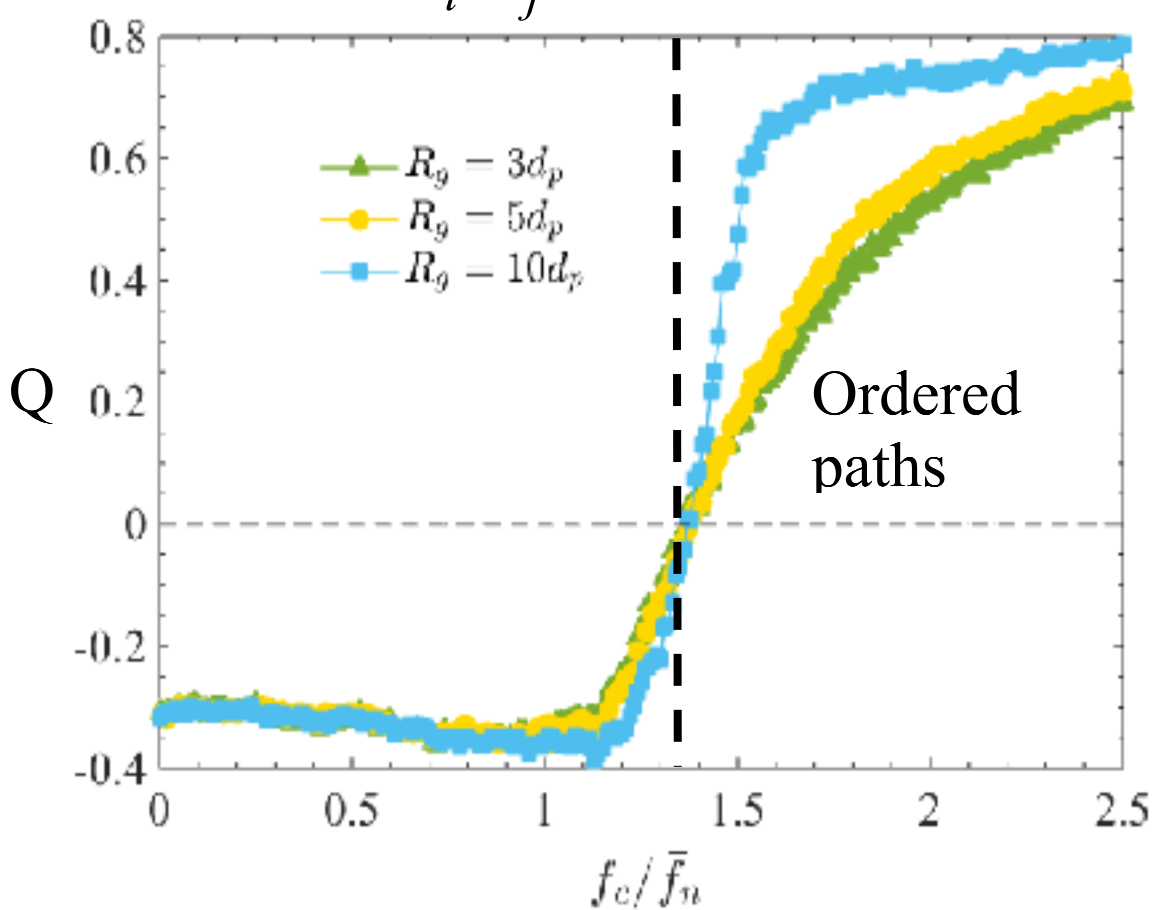
The number of particles in such clusters N_c , and Q $\equiv \langle \min(\boldsymbol{b}_i . \boldsymbol{b}_j) \rangle$ are determined

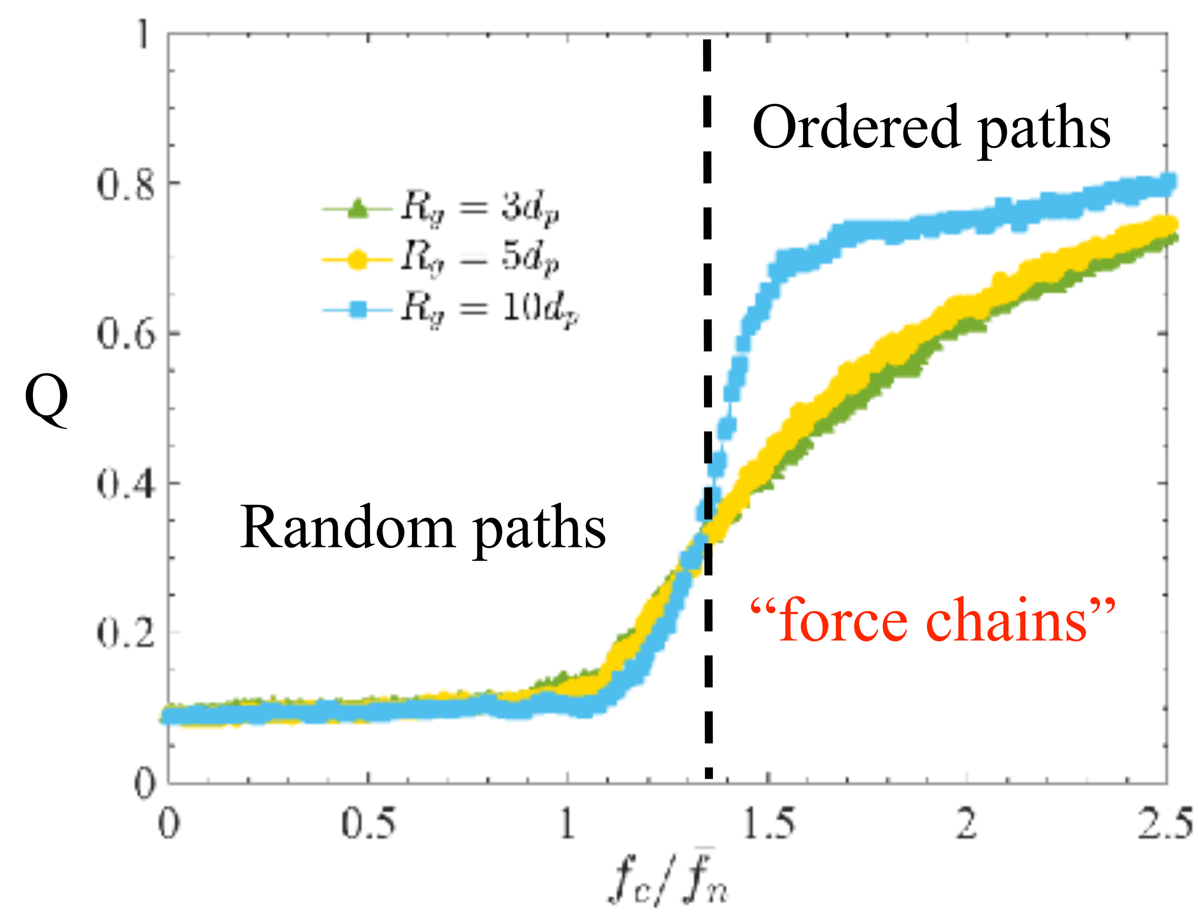
Tortuosity of a path is strongly correlated to the force



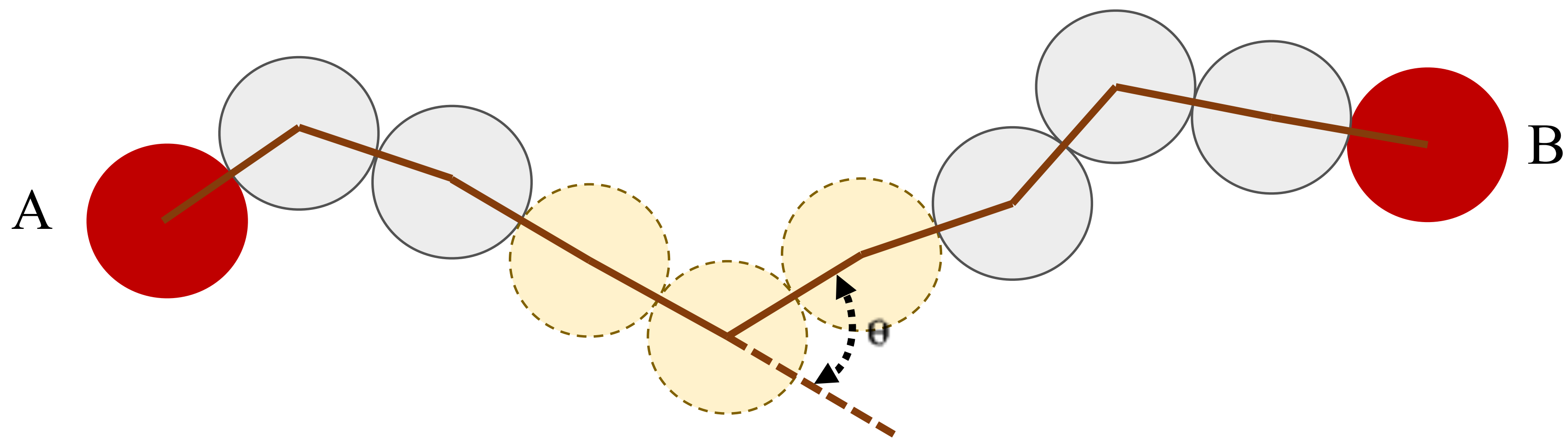
Force percolation threshold ~ 1.33

Relax $\boldsymbol{b}_i . \boldsymbol{b}_j > 0$





Definition a linearity parameter for a connected cluster



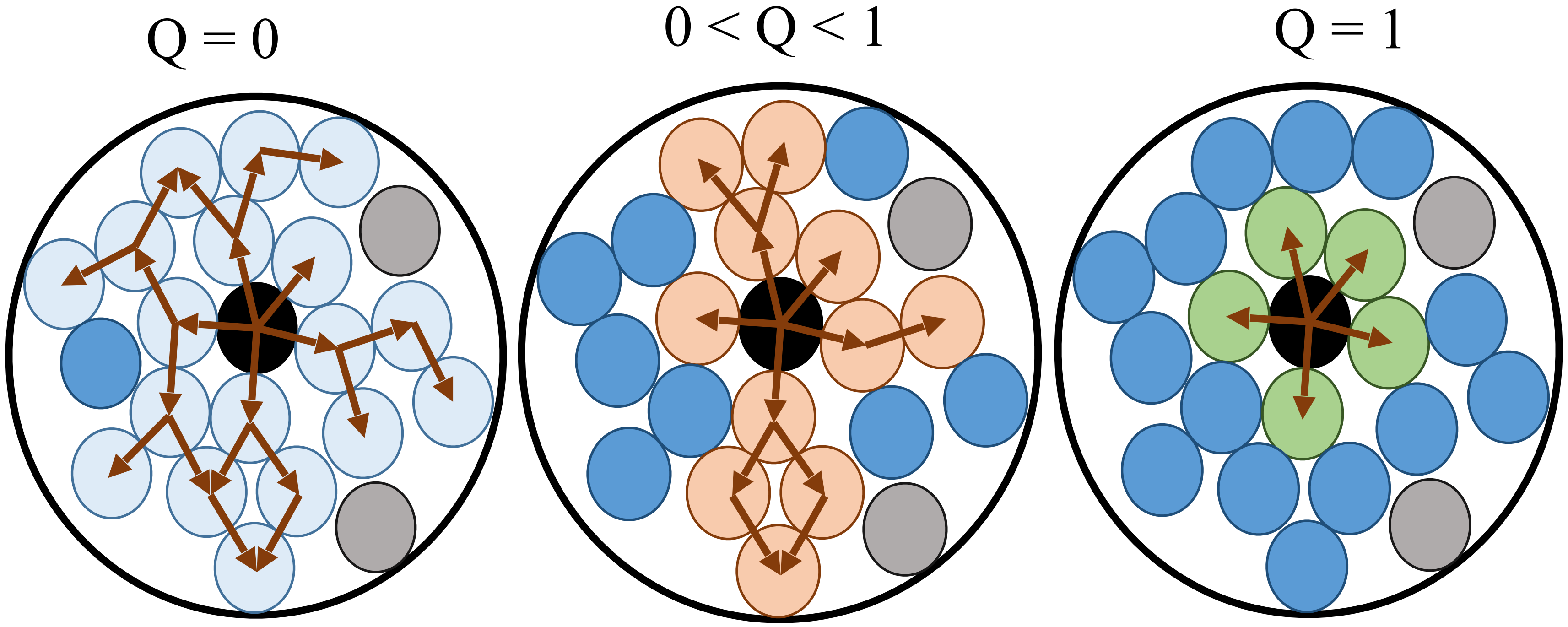
$$Q = \min(b_i . b_j \mid b_i . b_j > 0)_{\text{cluster}}$$

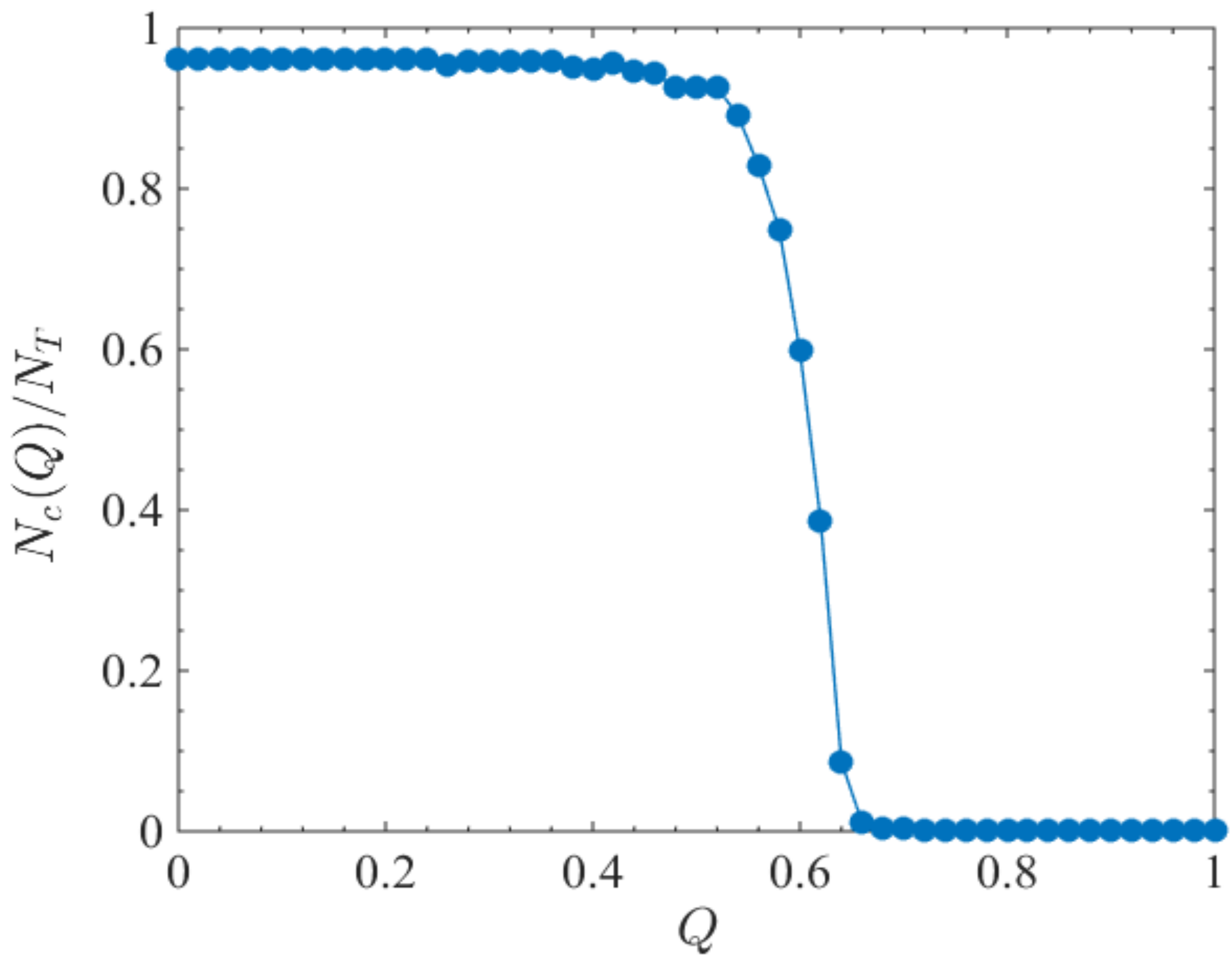
We generate configurations using DEM (and random packs) for a variety of geometries and boundary conditions: isotropic compression, heaps, silos, steady shear …, and demonstrate that at a critical value of Q, there is a sharp transition between ordered and disordered subsets of clusters.

We show that clusters of linearity $\geq Q_c$ are mechanically relevant in all these geometries

Computational protocol

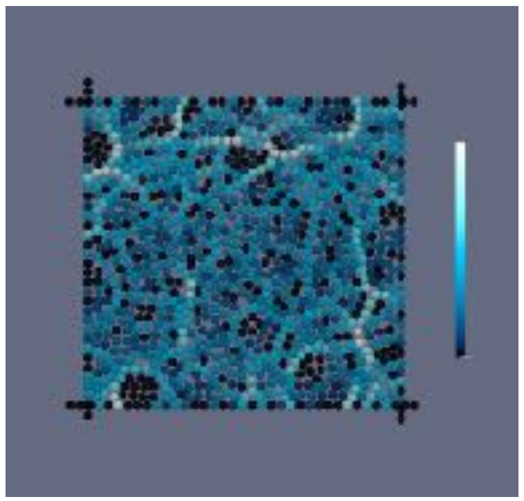
Choose a value of Q, and determine the number of particles N(Q) that reside in clusters of linearity ≥ Q





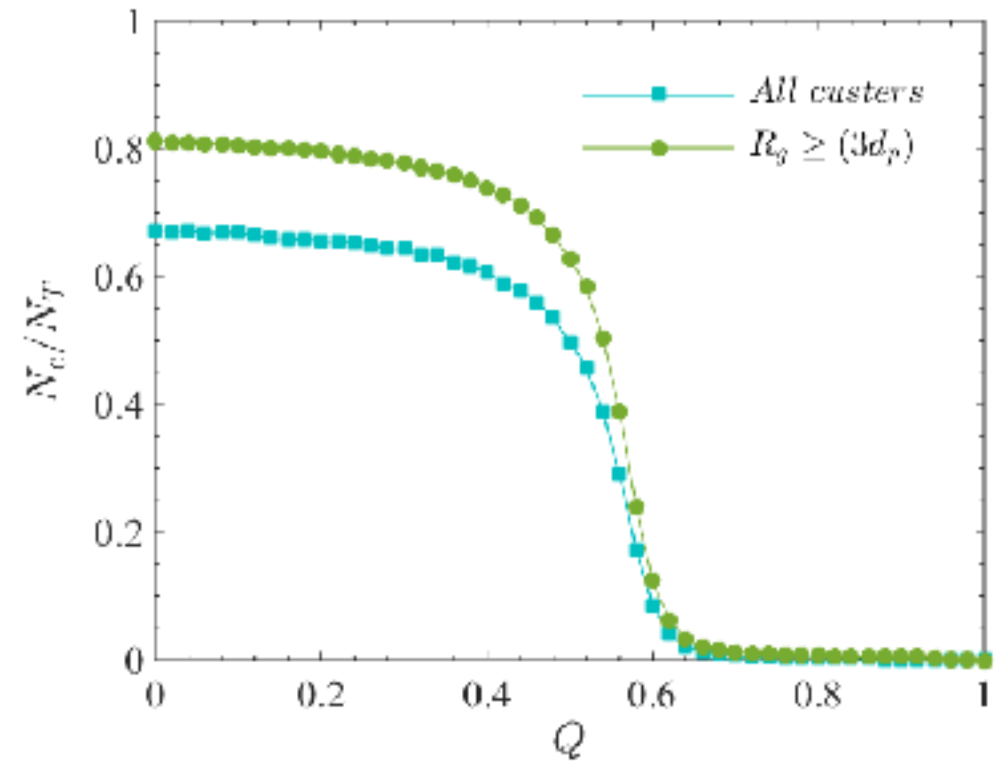
3D Isotropic compression

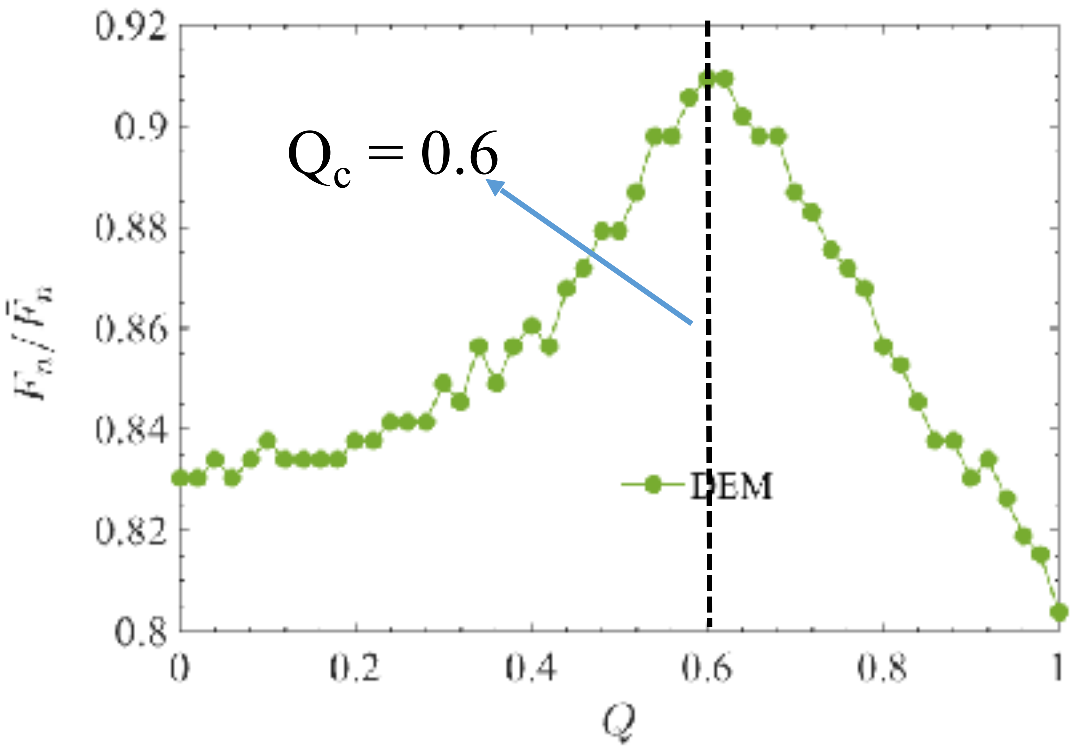
Contact force correlates with linearity

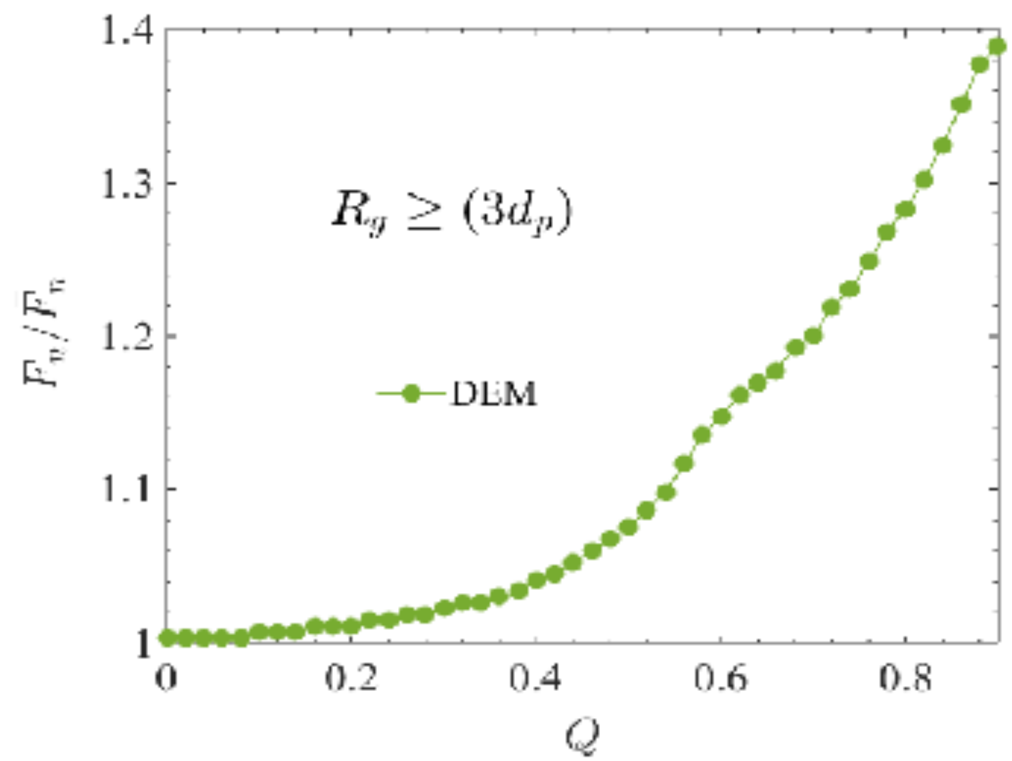


2D isotropic compression,
$N_T = 3000$, $\phi = 0.839$.

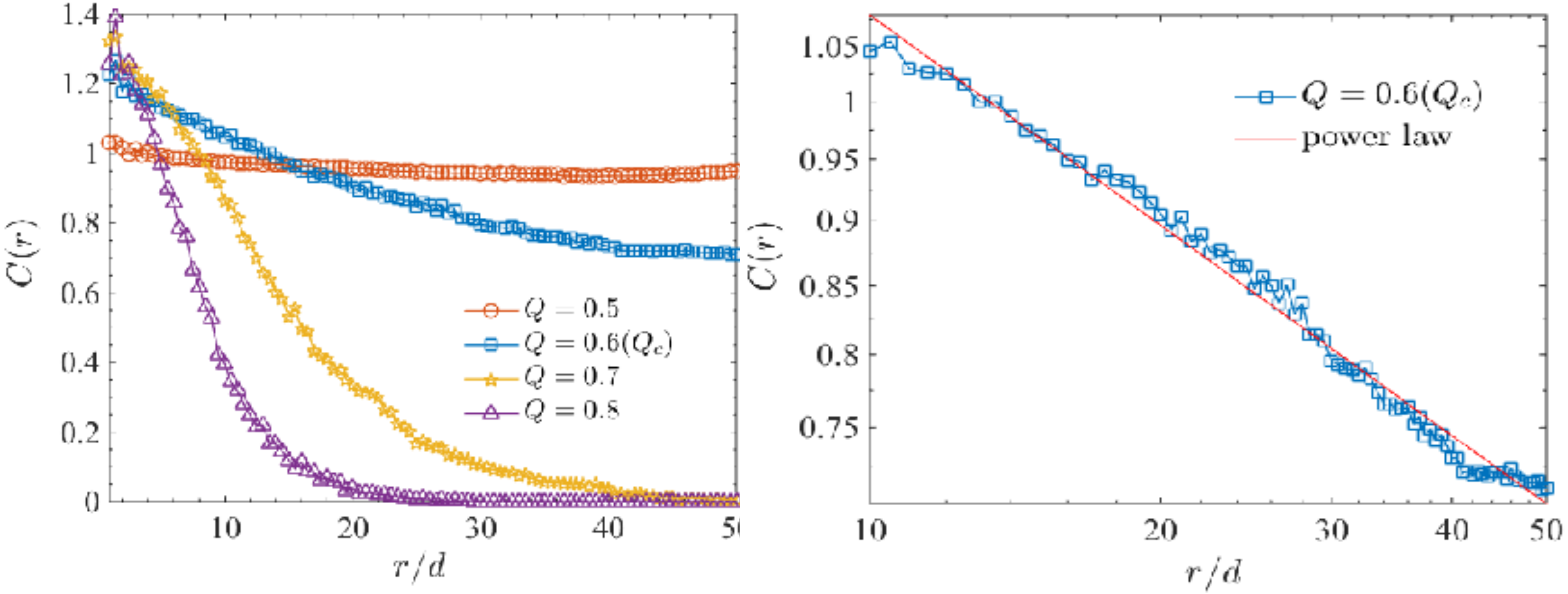
z = 2.58







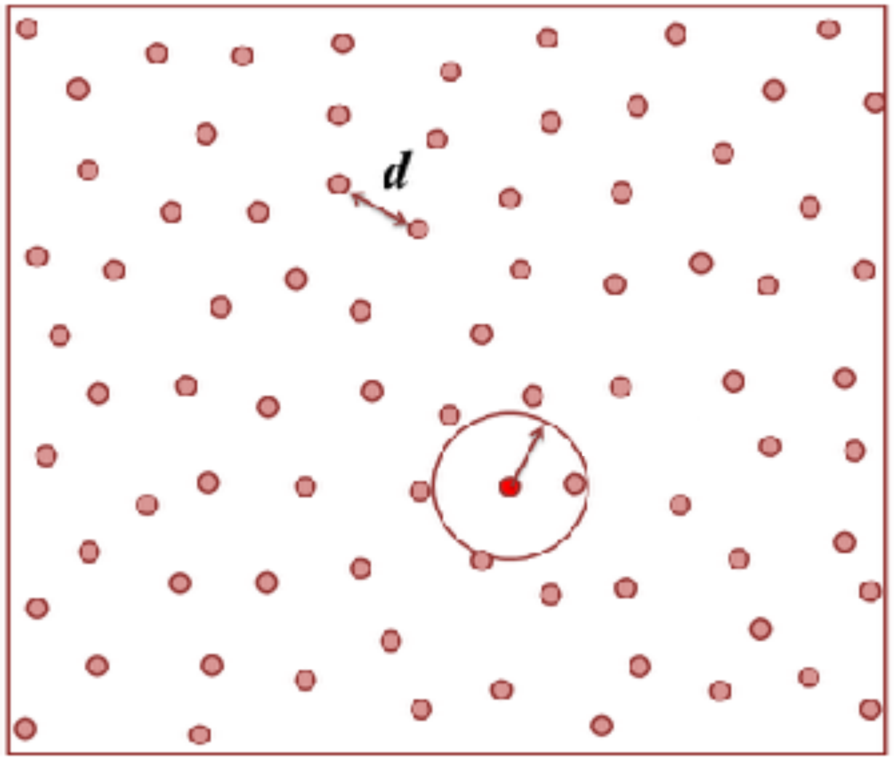
Spatial force correlation

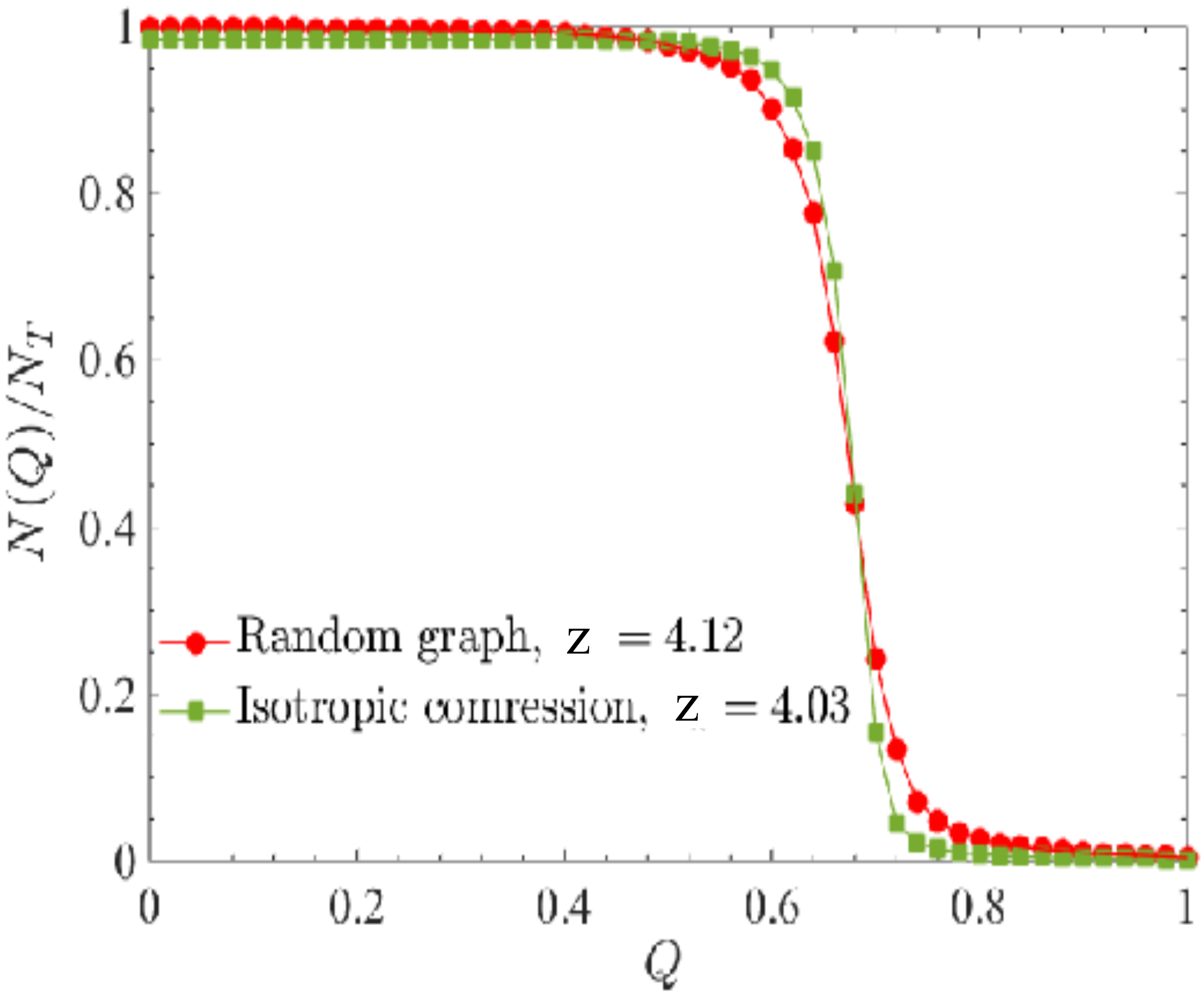


Force correlations are long ranged at $Q = Q_c$

Origin of criticality

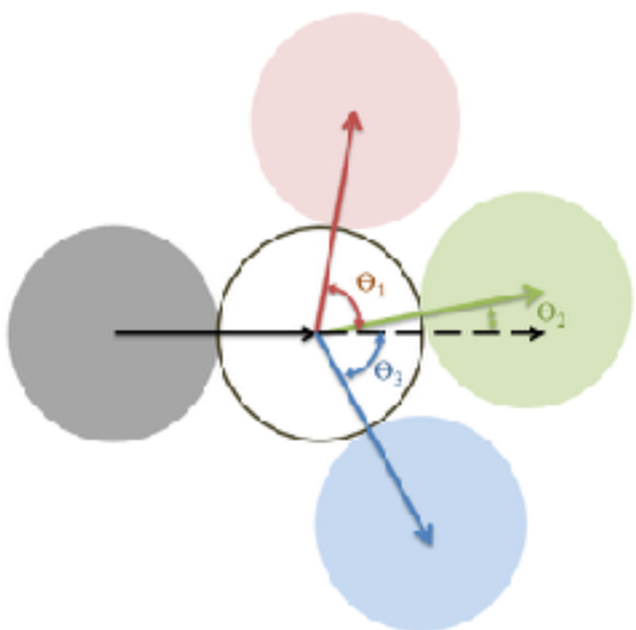
Construct a random network with nodes intially placed a distance at least d apart.
Particle grown: a pair is considered connected if nodes are separated by distance less than $\boldsymbol{\alpha d}$ – $\boldsymbol{\alpha}$ varied to get the required coordination number





Critical linearity is purely a consequence of geometric/ packing constraints

Random walk models



Random walks are generally used to study dynamical processes on networks.

We introduce correlations in random walks to show how it surprisingly replicates important features of the granular force network

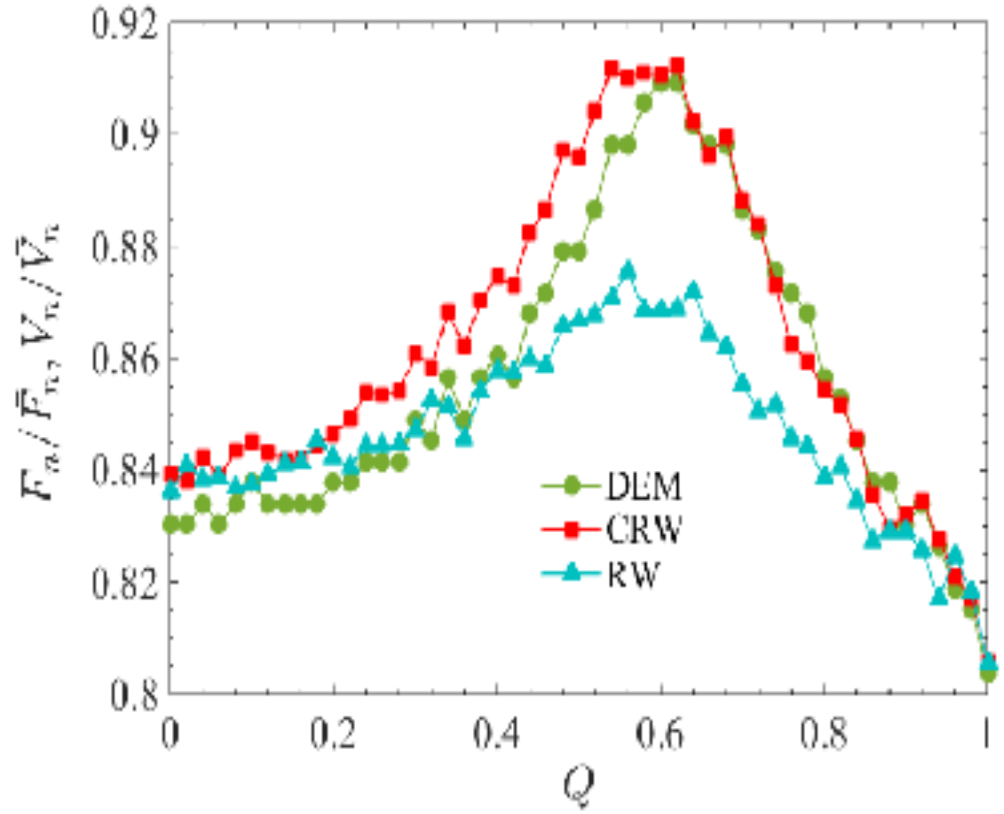
$\boldsymbol{b}_i . \boldsymbol{b}_j > 0$, P_i random Random Walk (RW)

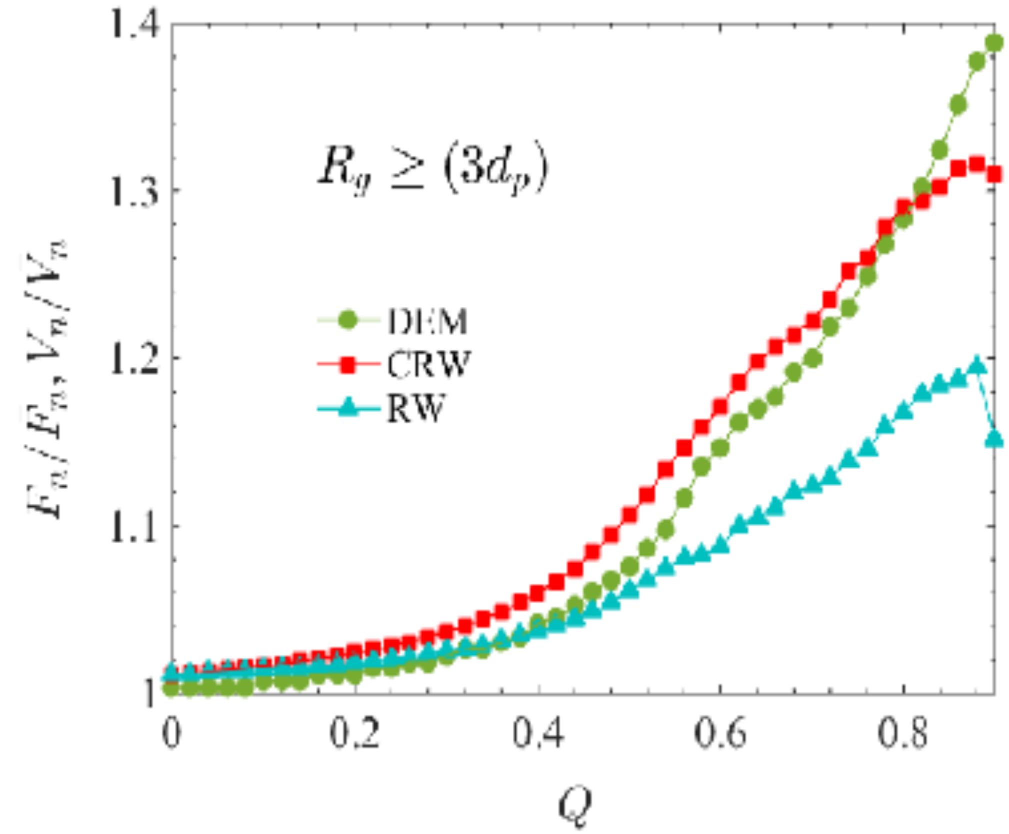
Similar to the q-model of Liu et al (Science, 1995)

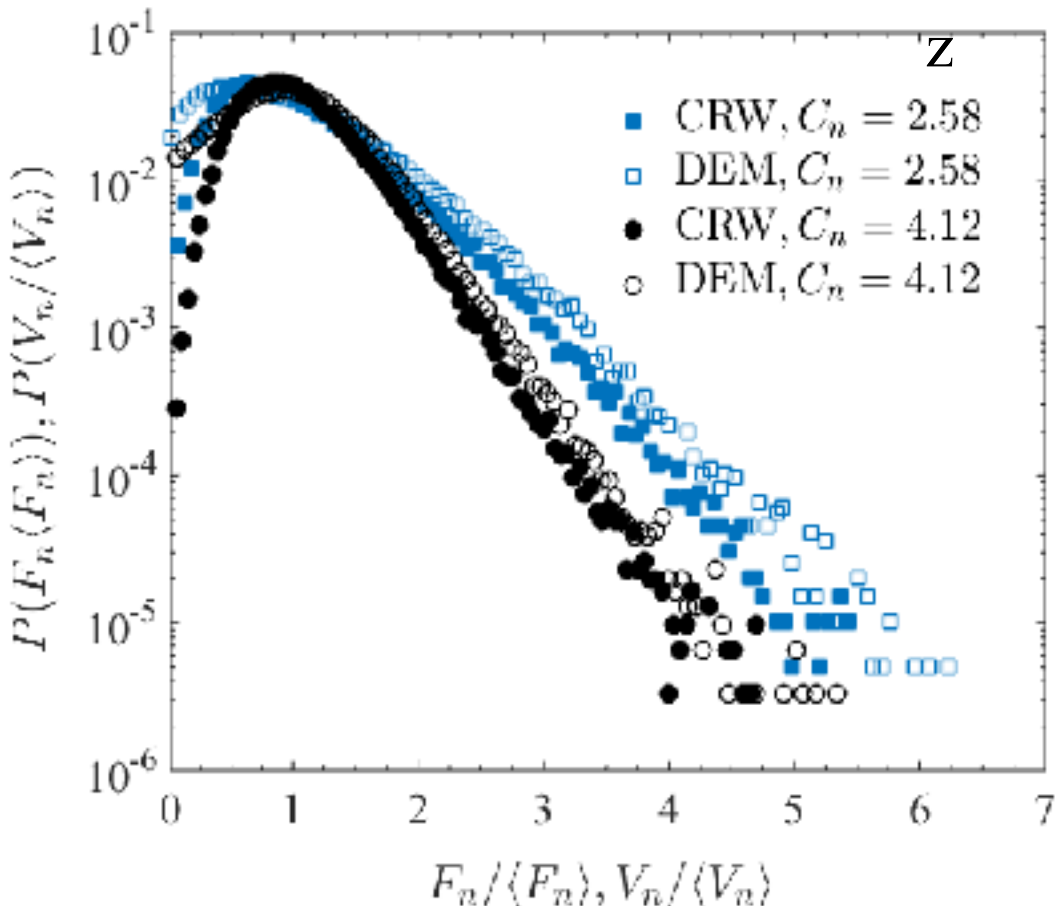
$\boldsymbol{b}_i . \boldsymbol{b}_j > 0$ & $P_i = \frac{\cos\Theta_i}{\sum_{j=1}^{n} \cos\Theta_j}$ Correlated Random Walk (CRW)

Significance of linearity-based correlations

Average cluster force



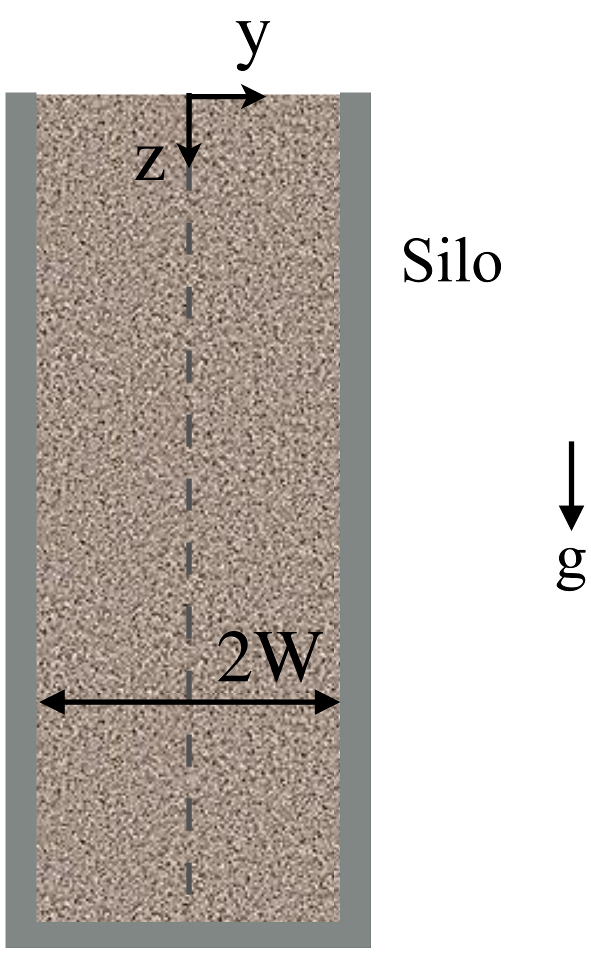


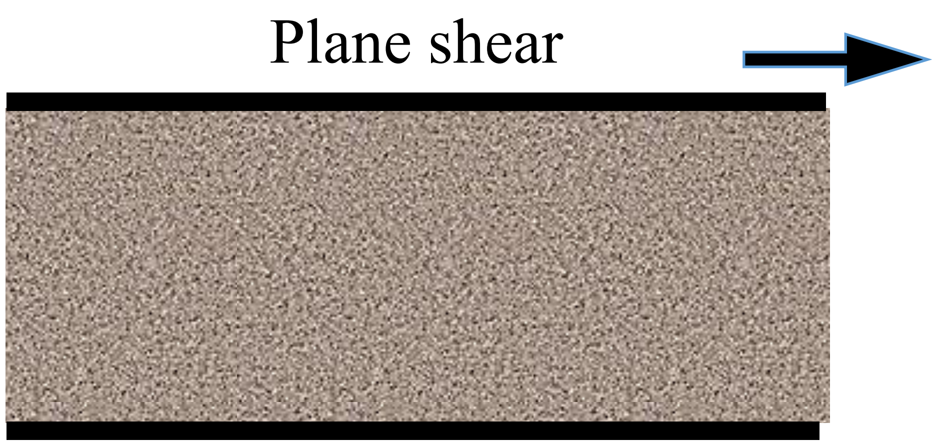


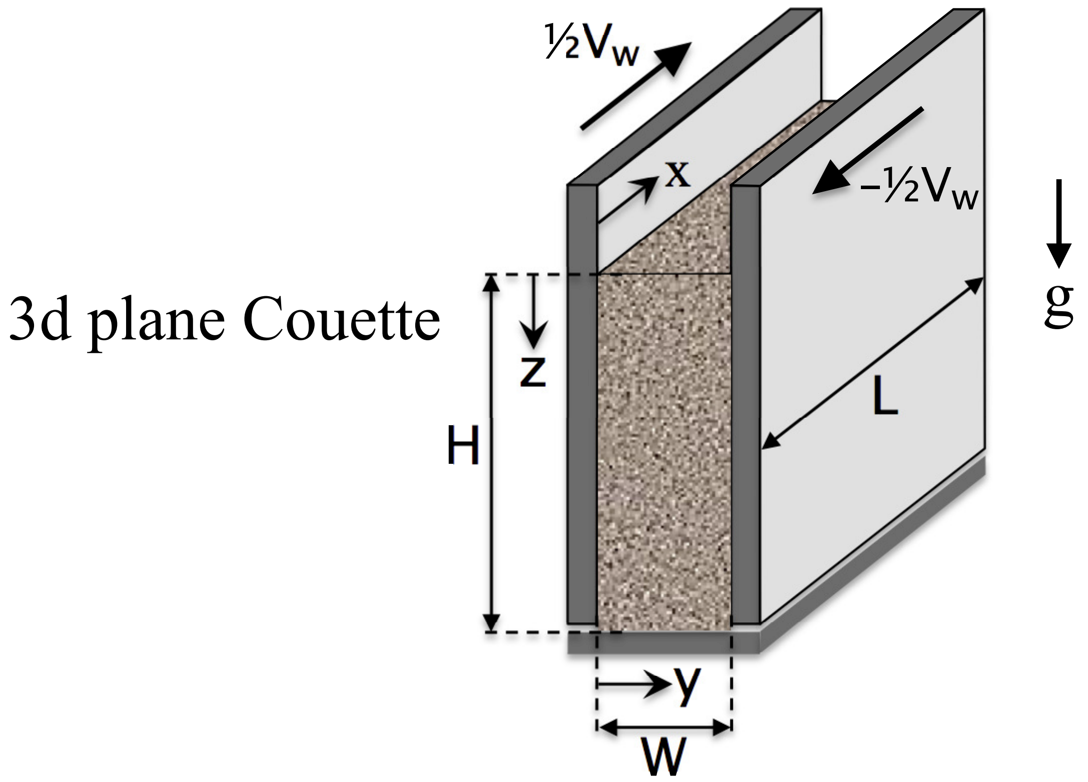
PDF (F_n)

By introducing correlations in a random walk, important features, such as the large force fluctuations are captured

“Critical” clusters (of linearity Q_c) are mechanically relevant



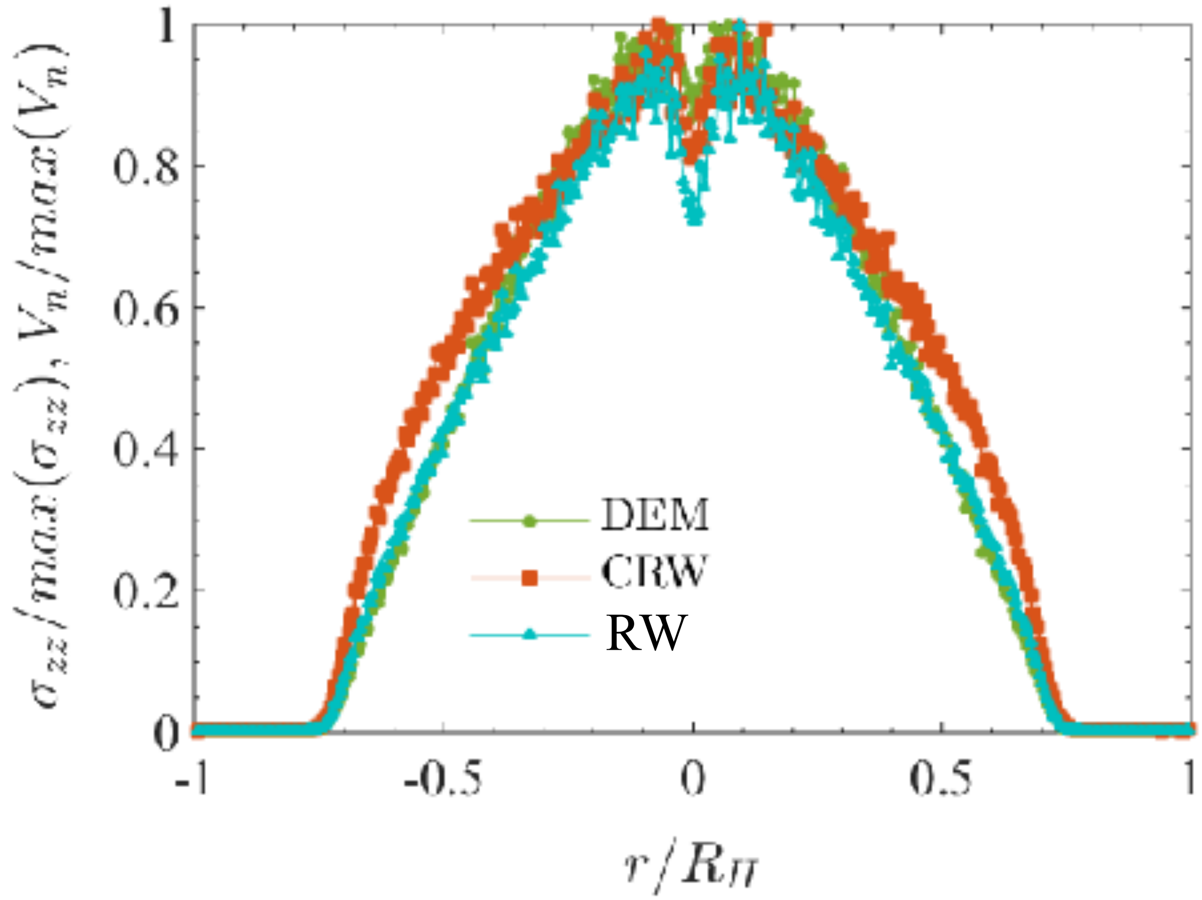




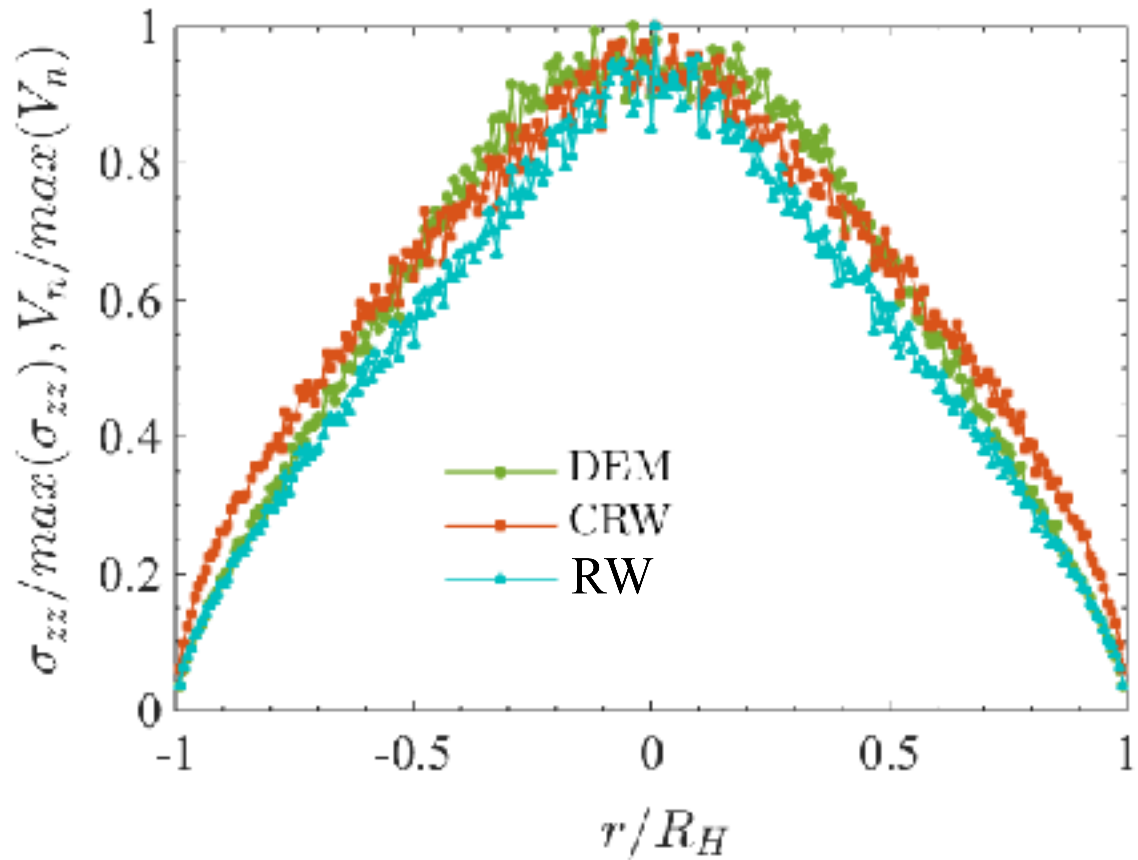
Mechanical significance of linearity-based correlations

2D heap

Funnelled

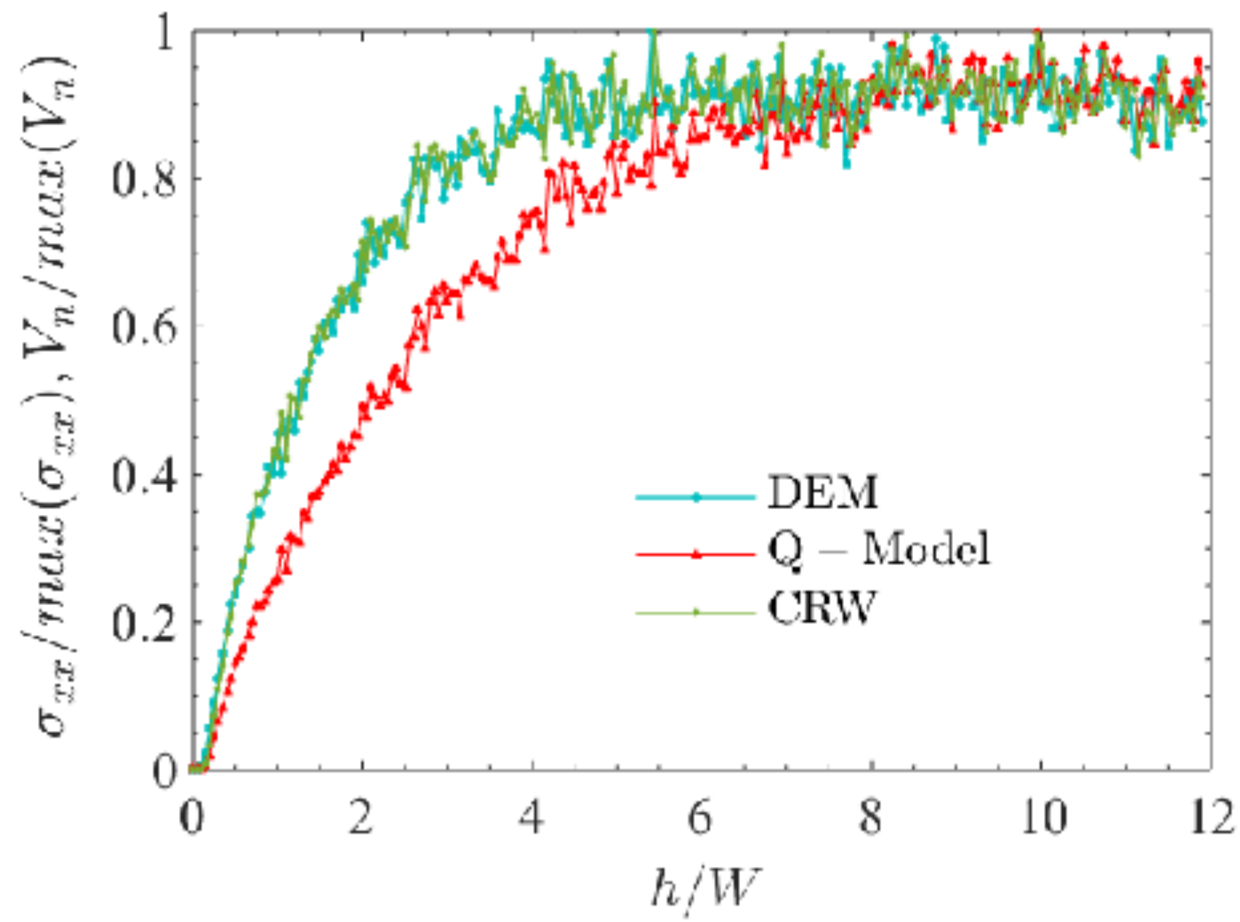


Rained



Mechanical significance of linearity-based correlations

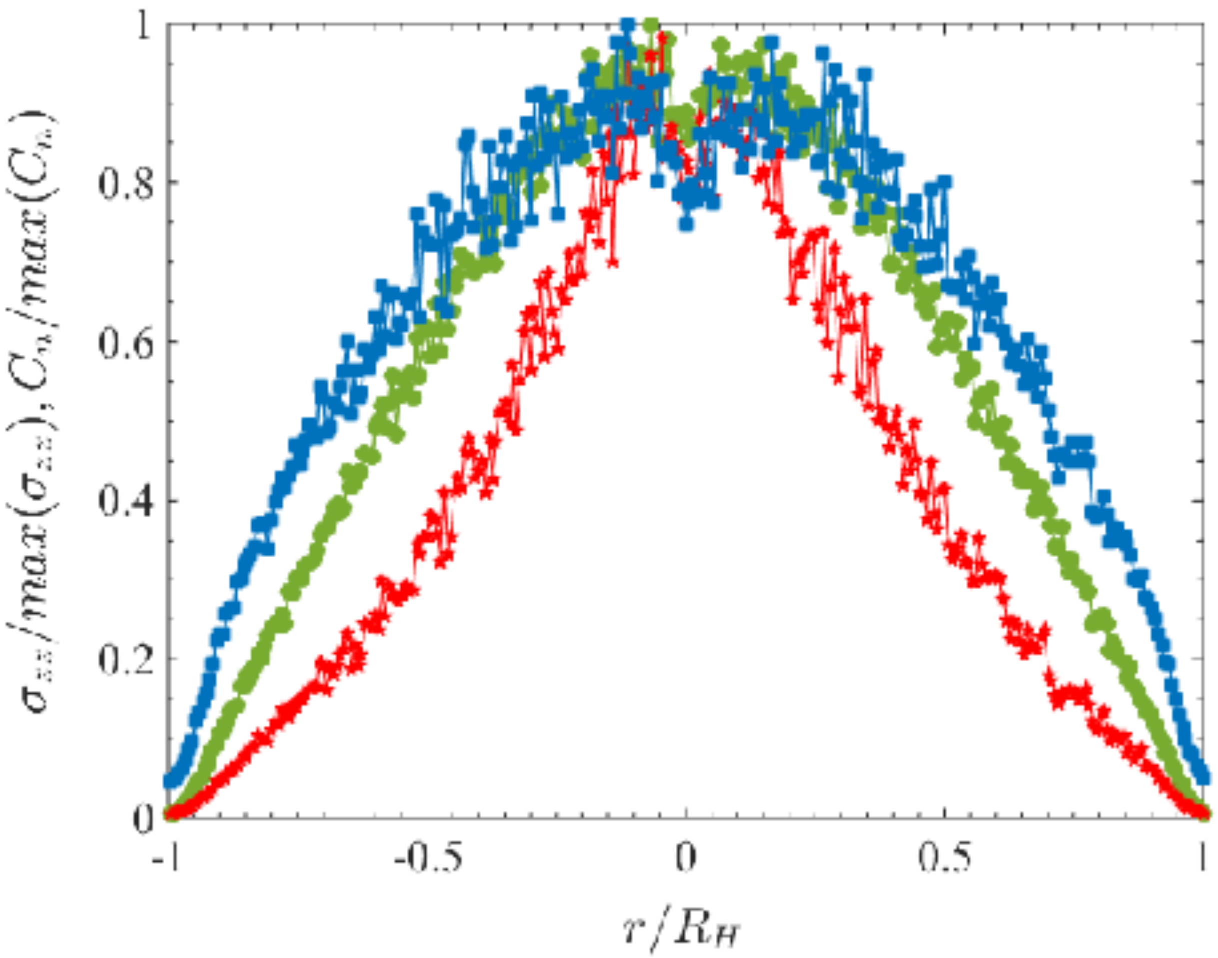
2D silo



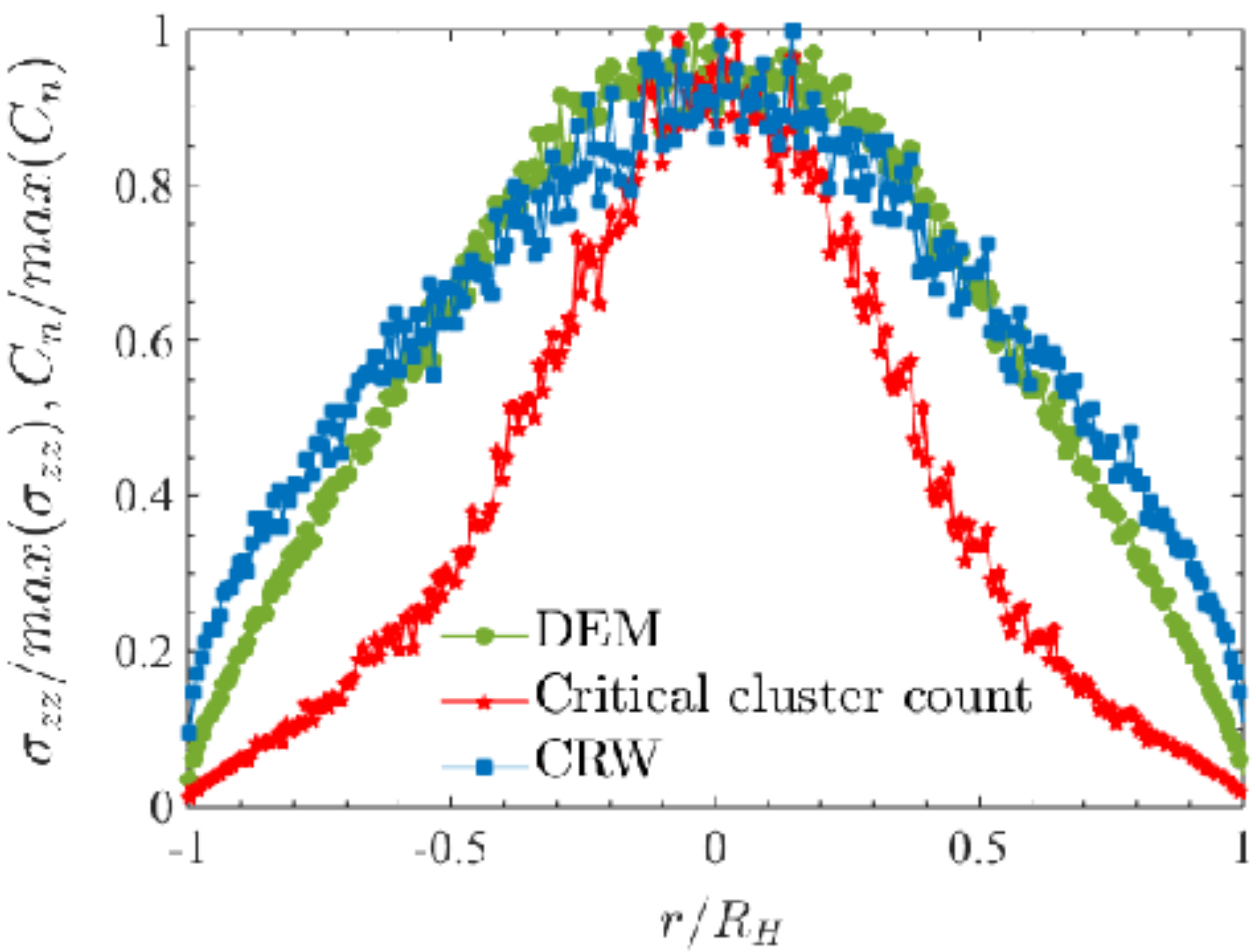
Mechanical significance of critical clusters

2D heap

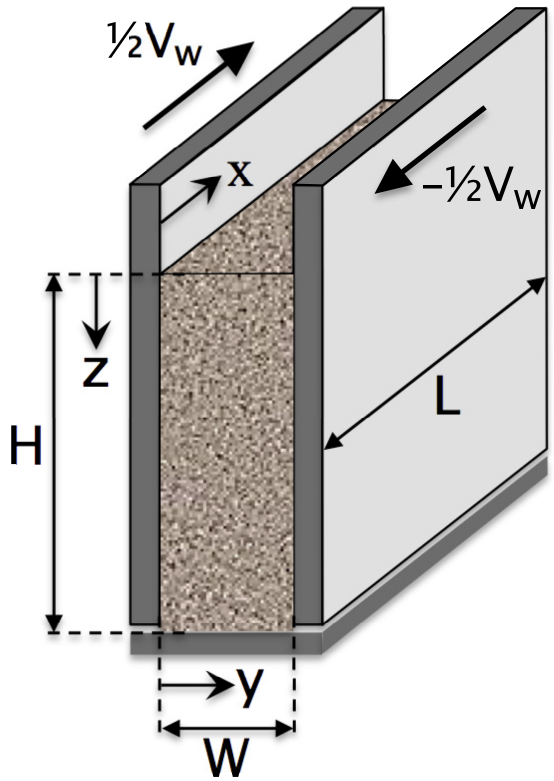
Narrow



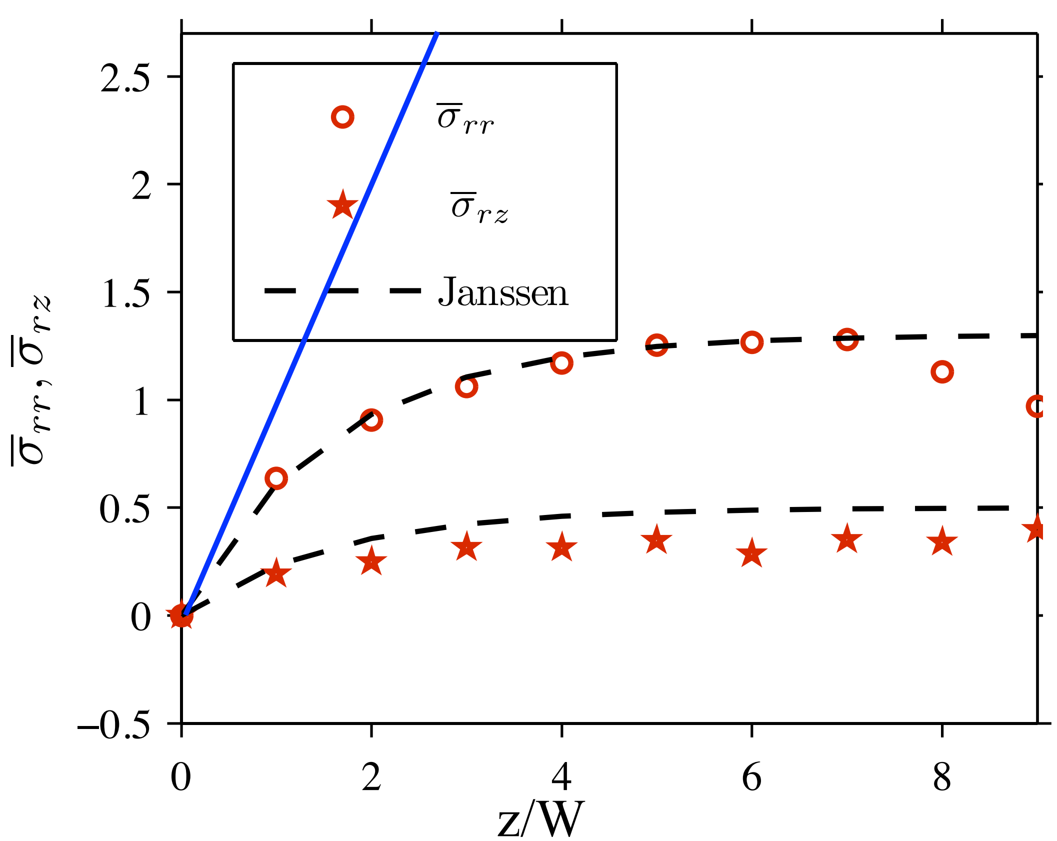
Rained

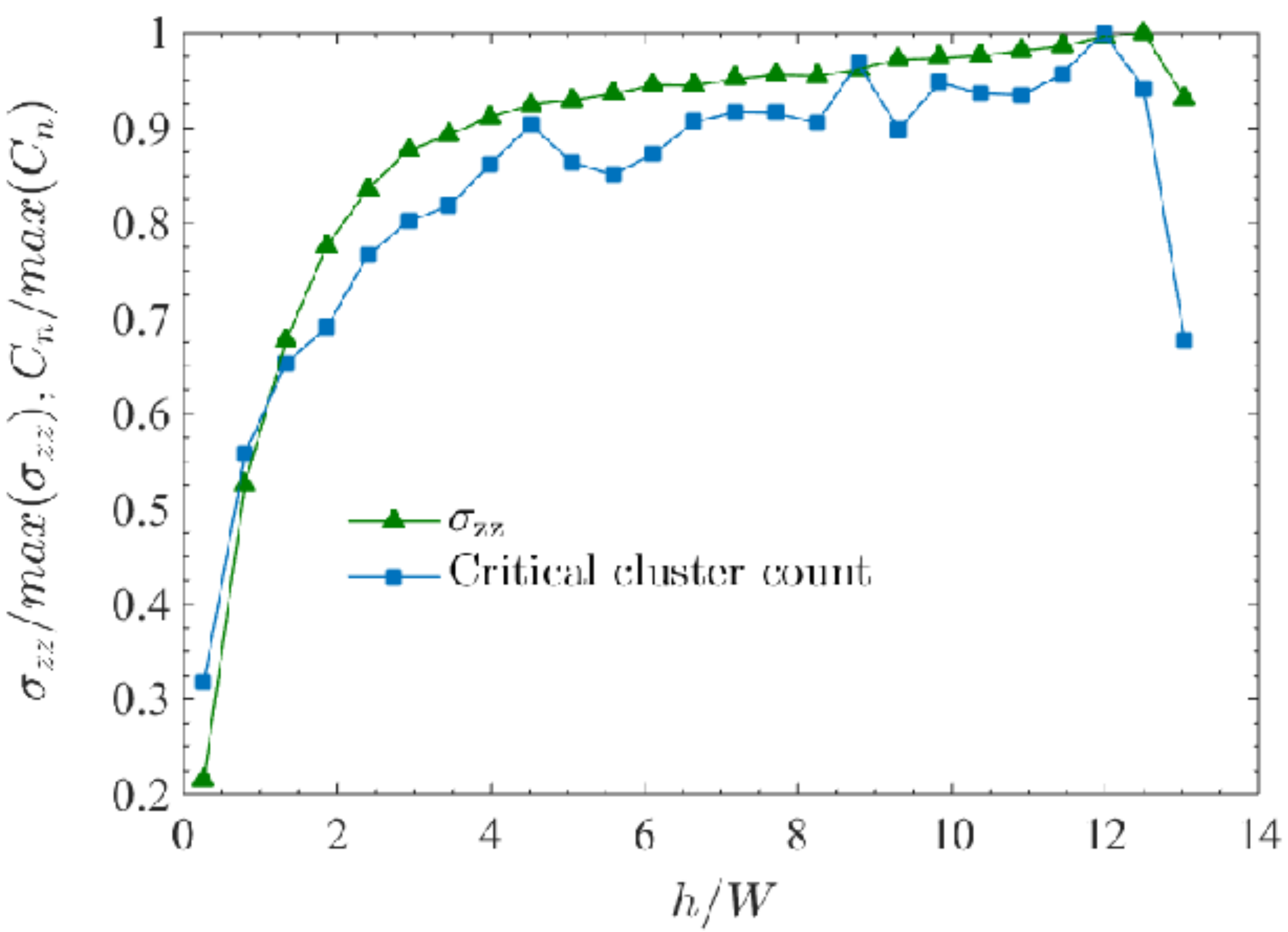


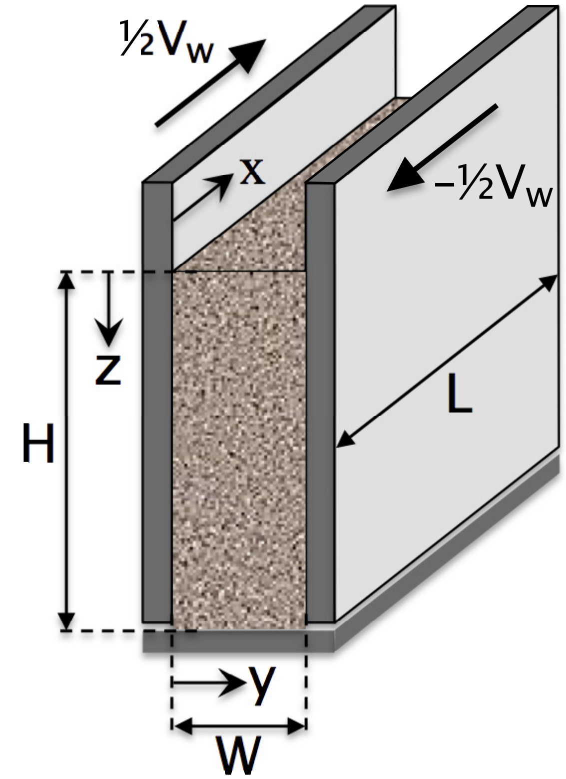
Mechanical significance of critical clusters



3D Silo
$V_w = 0$





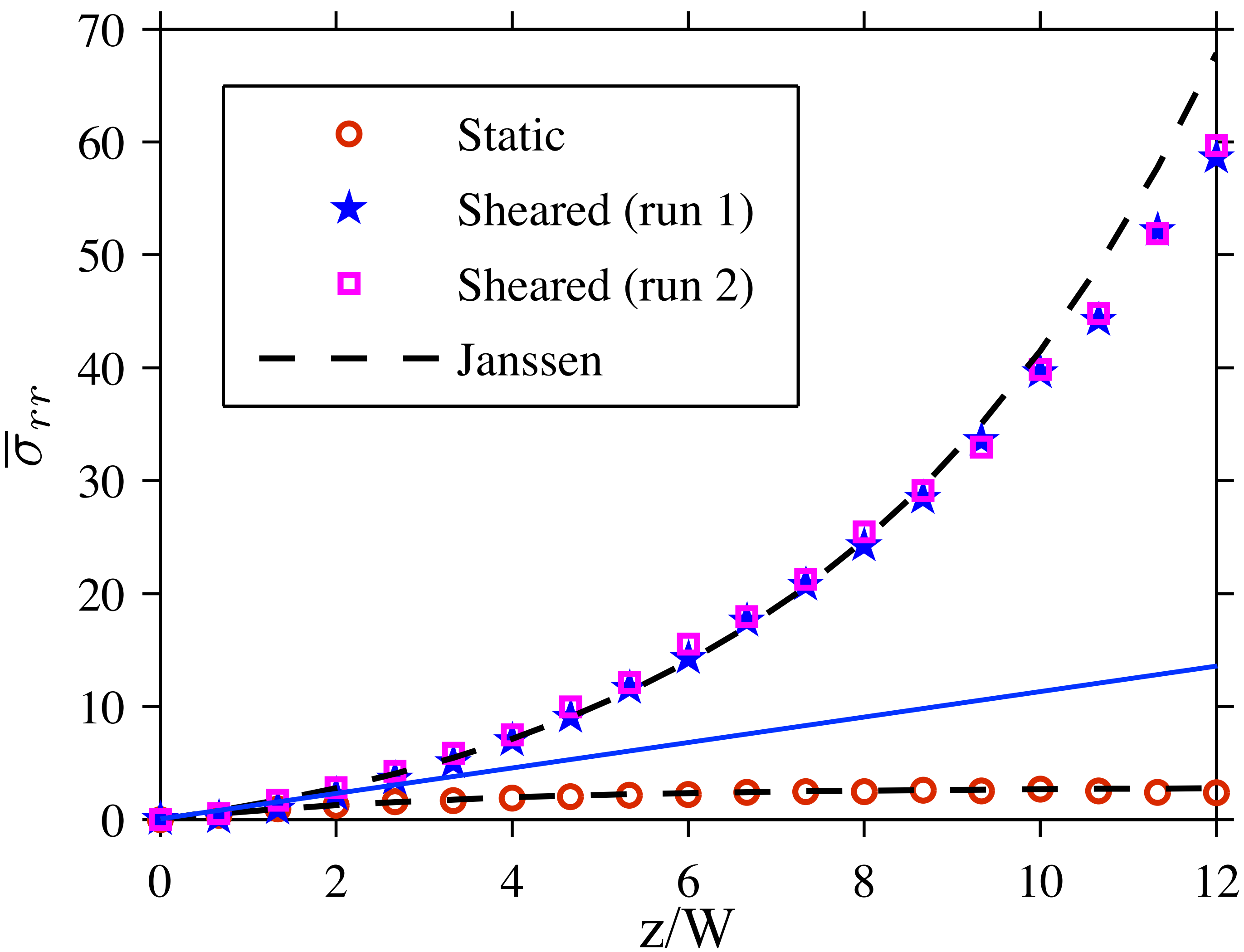


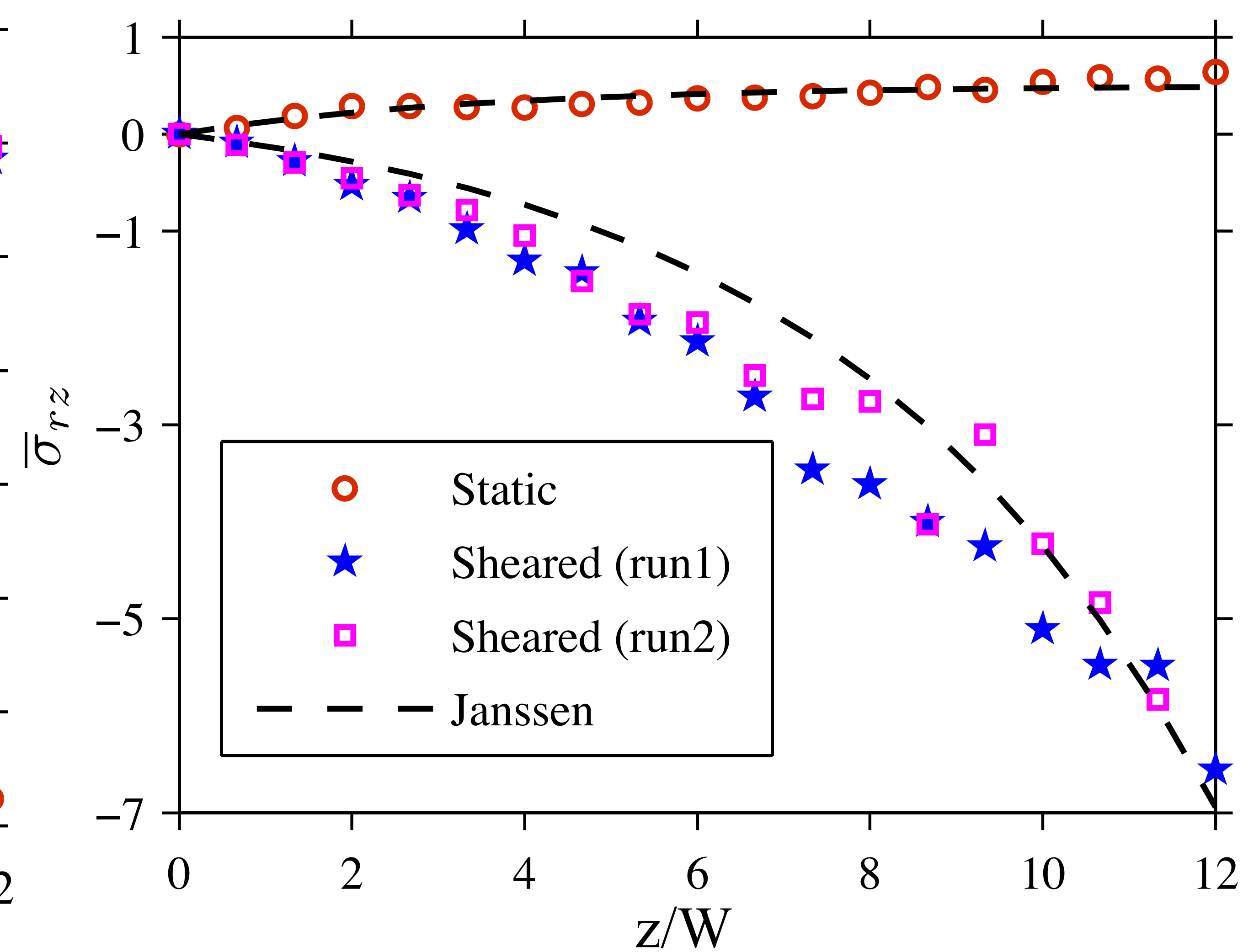
3D plane Couette

$V_w \neq 0$

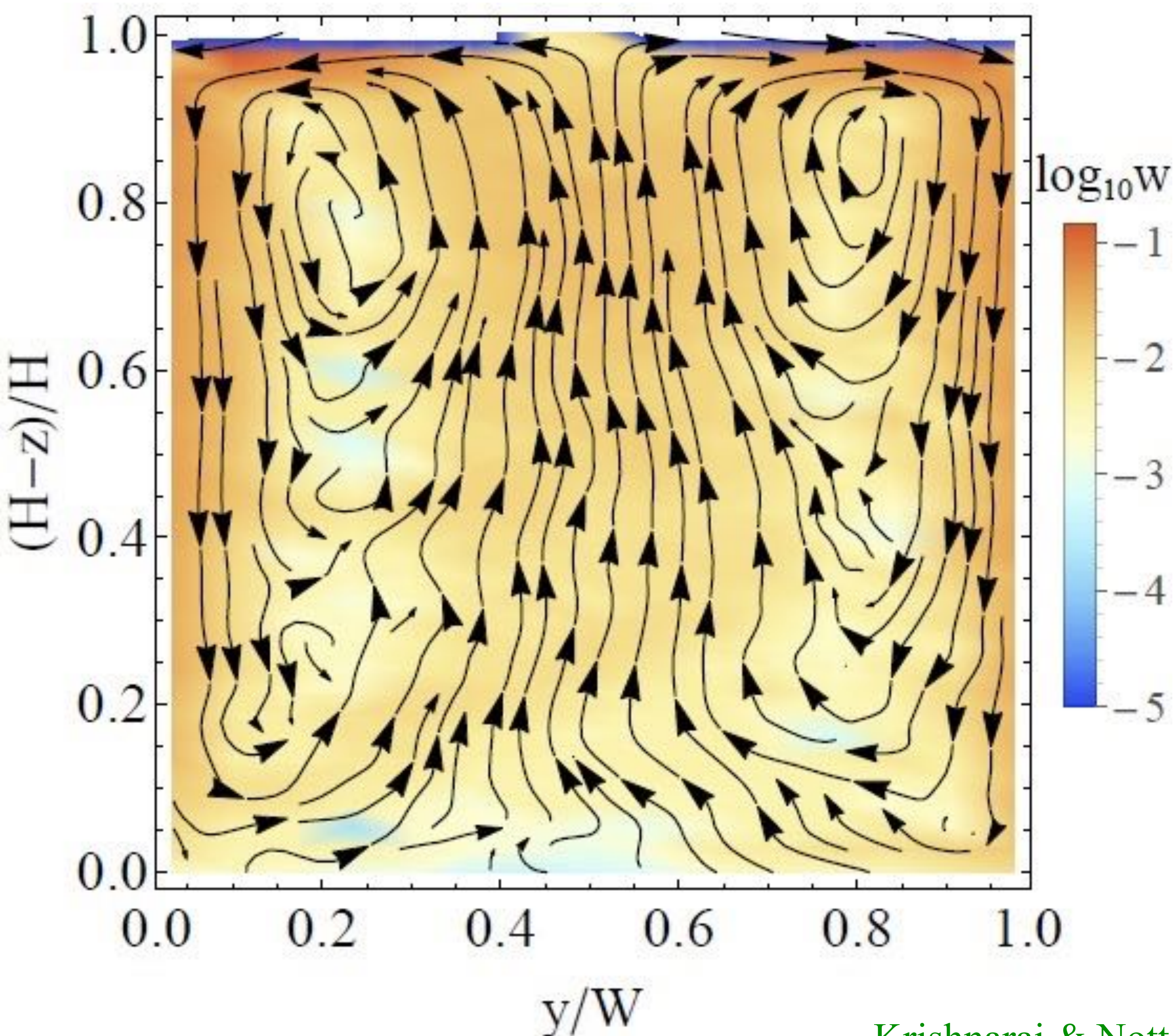
Mehandia, Gutam & Nott, *PRL* (2012)

Gutam, Mehandia & Nott, *Phys. Fluids* (2013)





3D plane Couette: Anomalous caused by a dilation-driven secondary flow

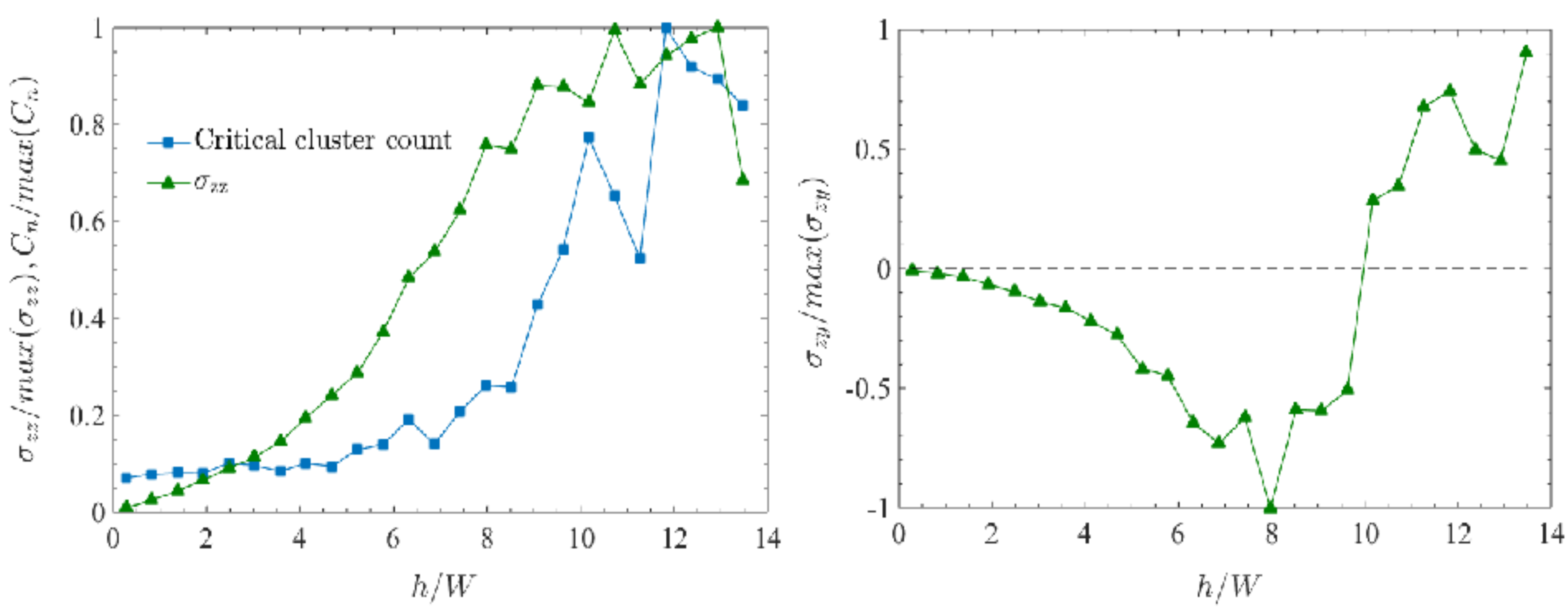


w – speed in the y-z plane

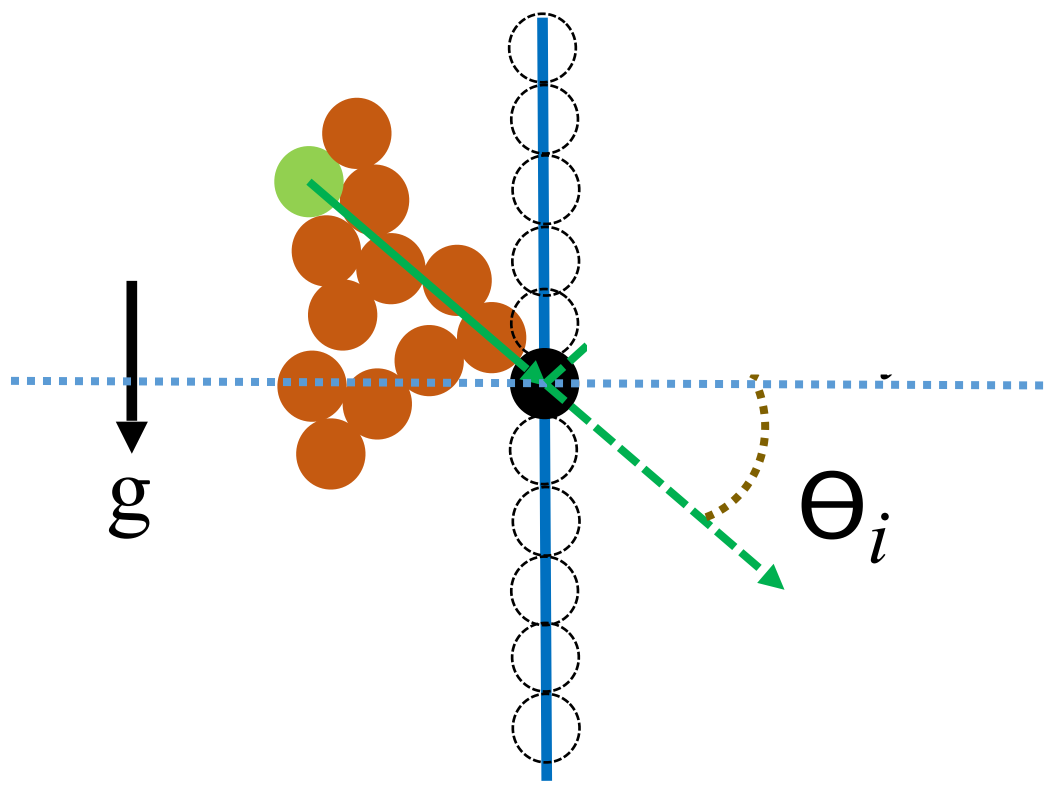
Krishnaraj & Nott (Nature Comm, 2017)

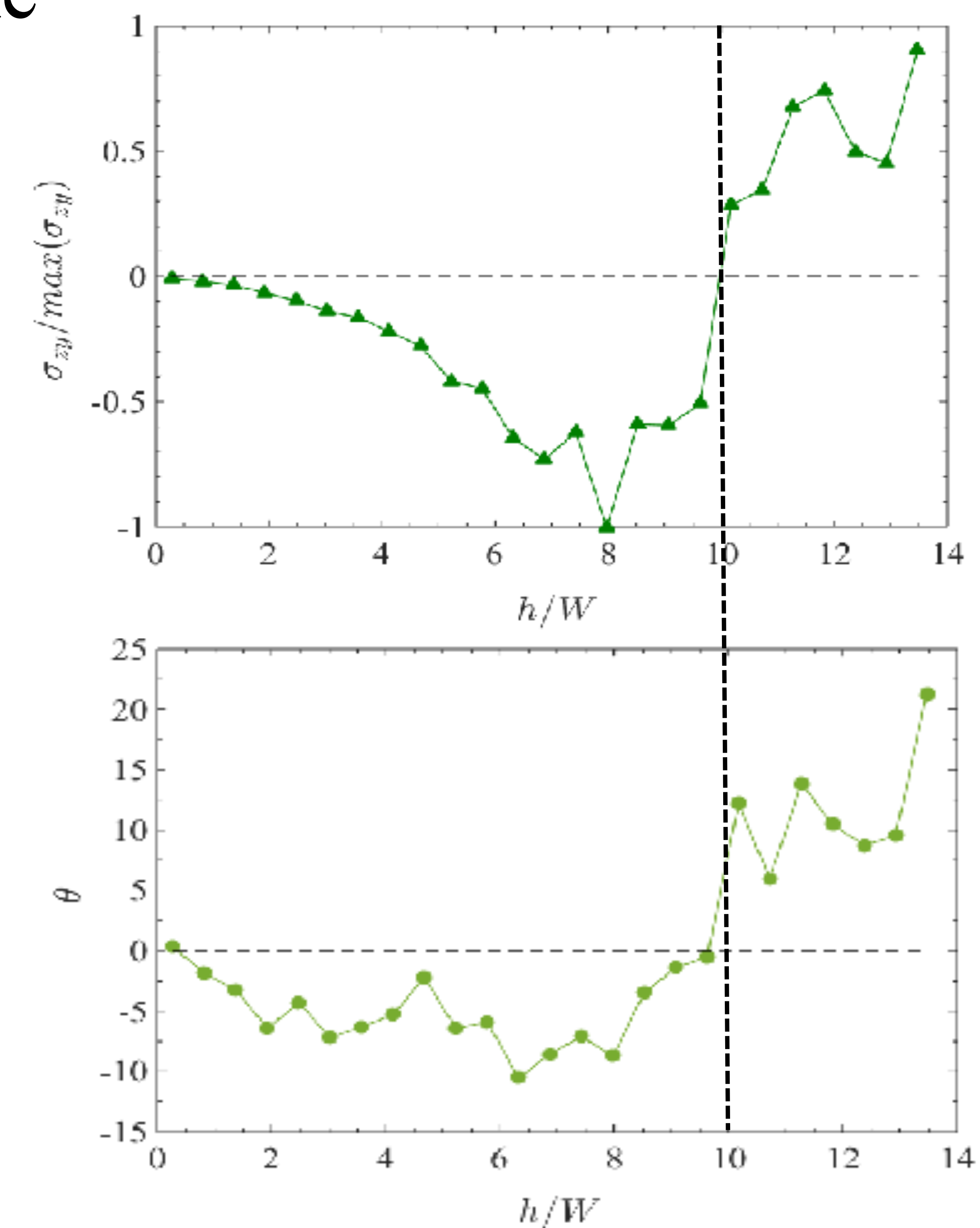
Mechanical significance of critical clusters in 3D plane Couette

Sheared



Orientation of clusters in 3D plane Couette

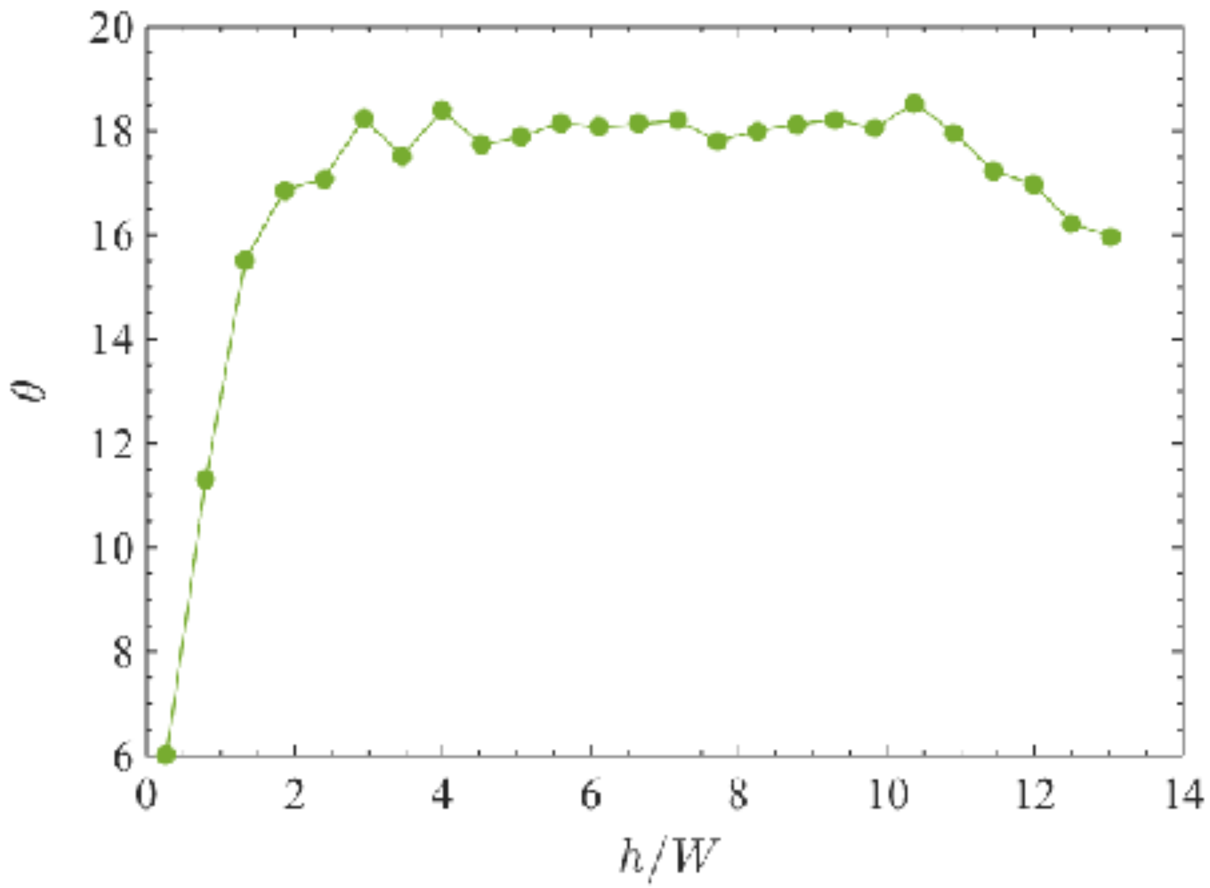




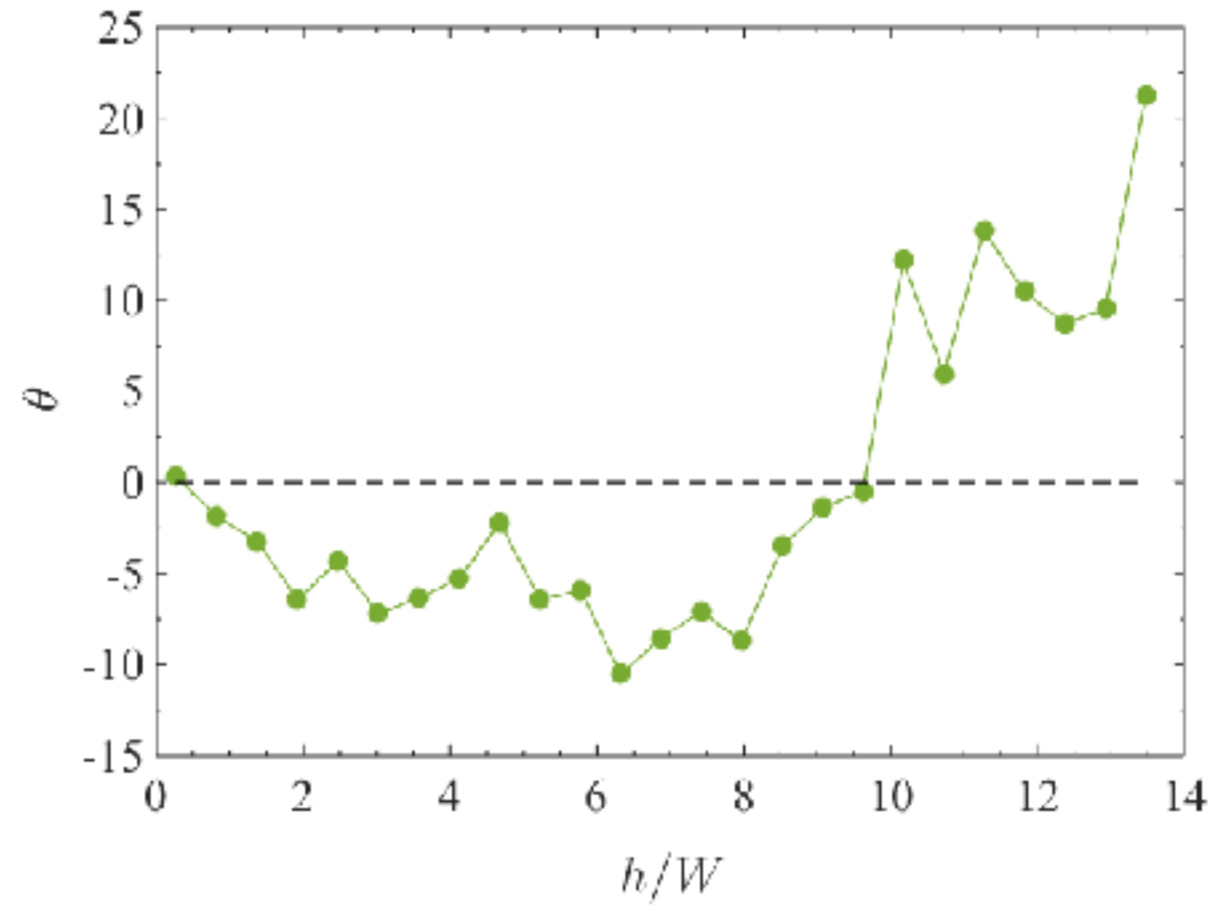
Vertical variation of the orientation of critical clusters mimics stress reversal

3D silo (static) & plane Couette

Static



Sheared

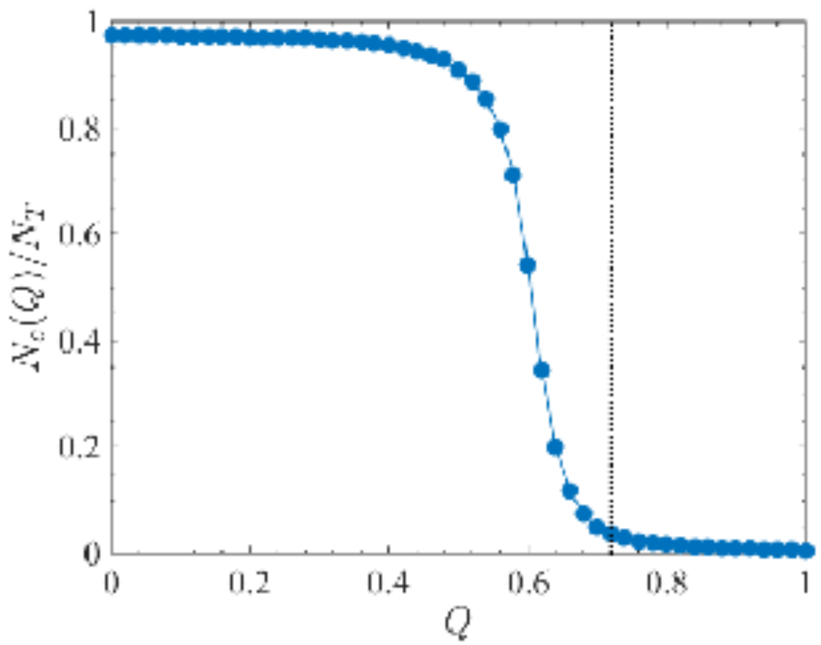


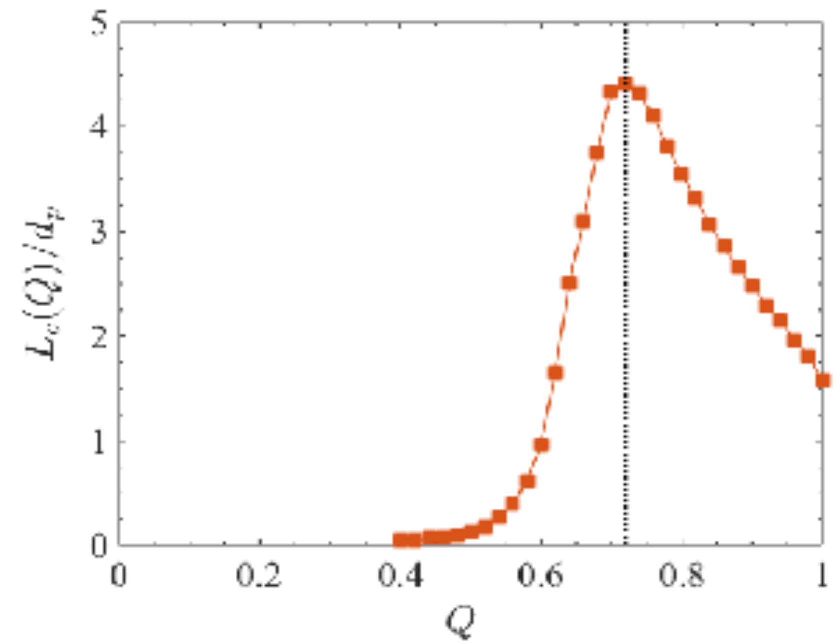
Conclusions

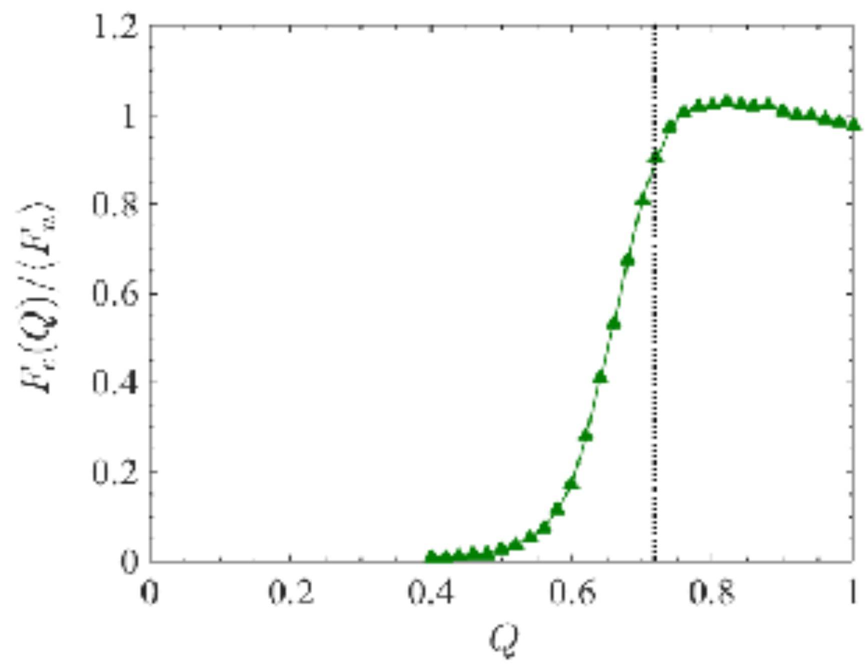
- We have identified an linearity based "order" parameter characterizing the strong force network in granular materials.
- Strong force networks in granular materials are emergent phenomena associated with a connectivity transition in constrained spatial networks
- The pair force correlations in clusters with critical linearity exhibit a power-law decay, indicating long ranged force correlations
- The linearity parameter explains the micromechanical origin of several non-trivial phenomena in a granular assemblies: pile, Janssen stress saturation in a granular silo, and stress reversal in 3d plane Couette

Supplementary slides

Structure of a granular force network (sheared)



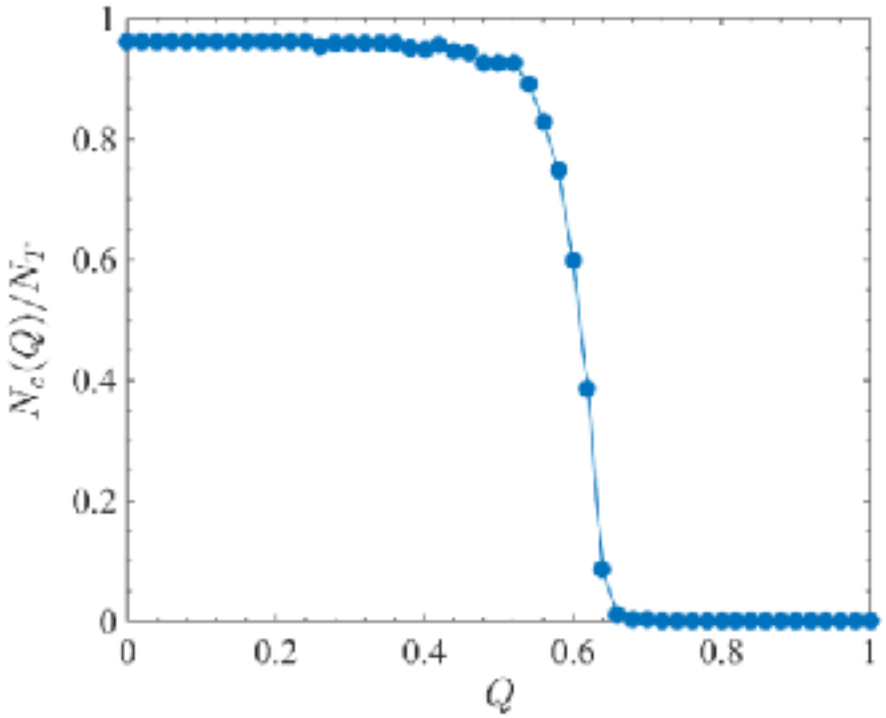


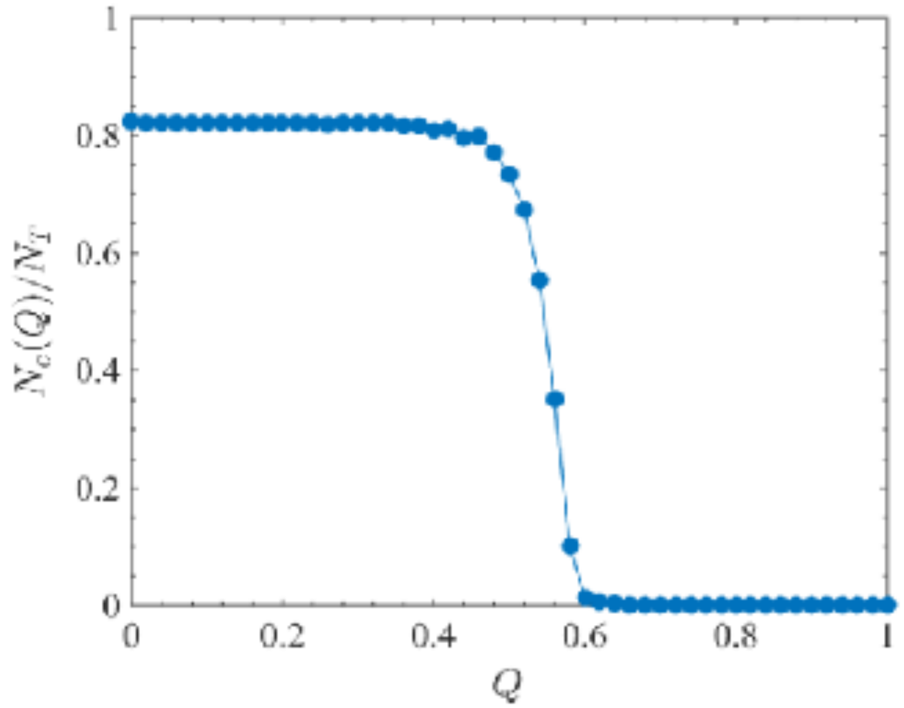


2D Plane shear, N = 1000. Averaged over 100 independent configurations.

Note that global clusters are neglected

Criticality of 3D granular packing's

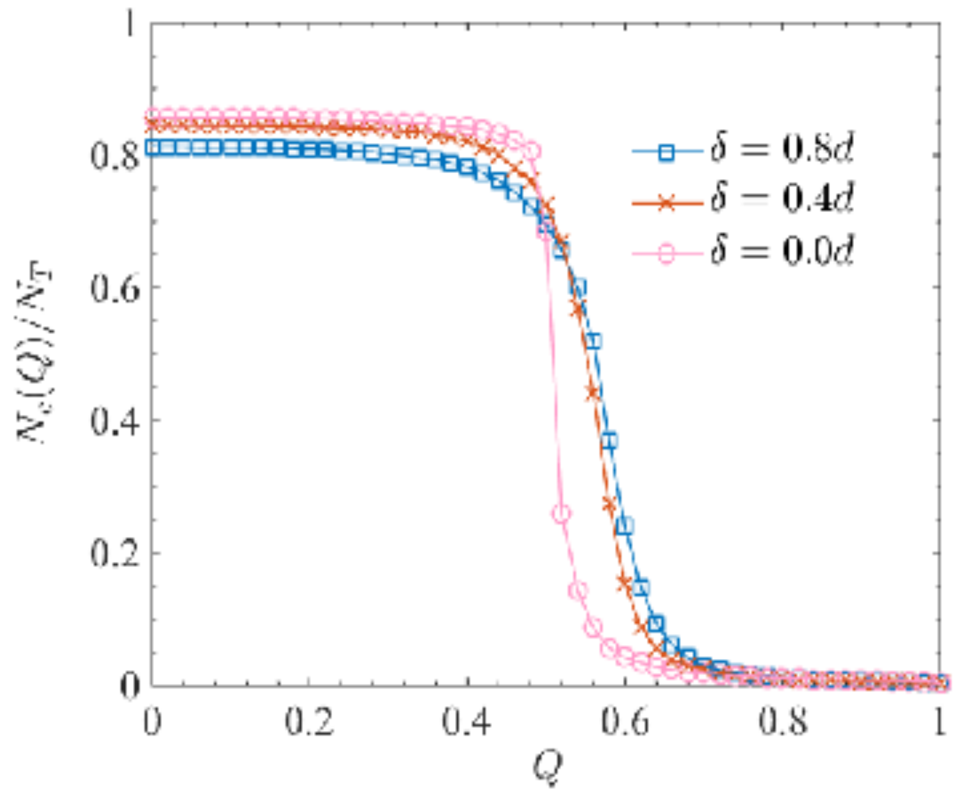


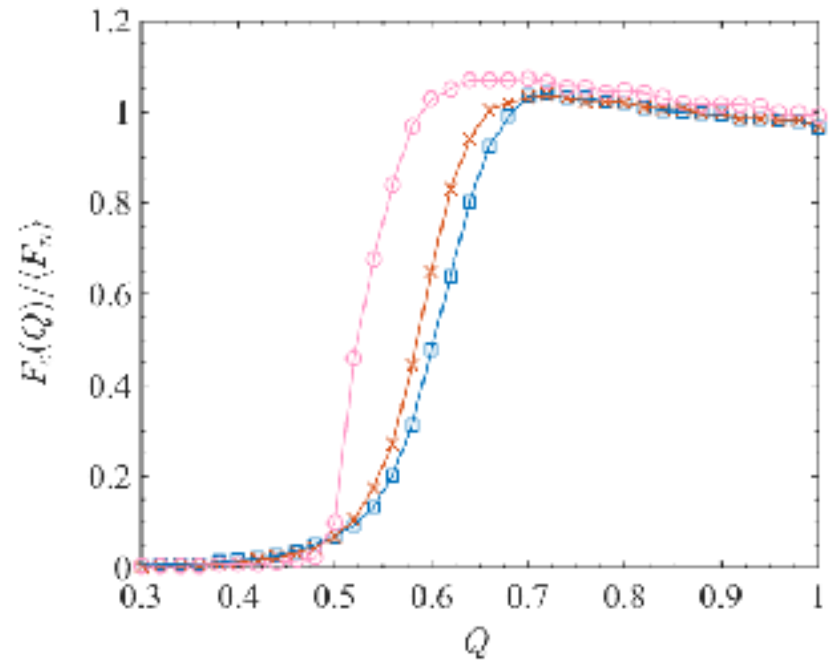


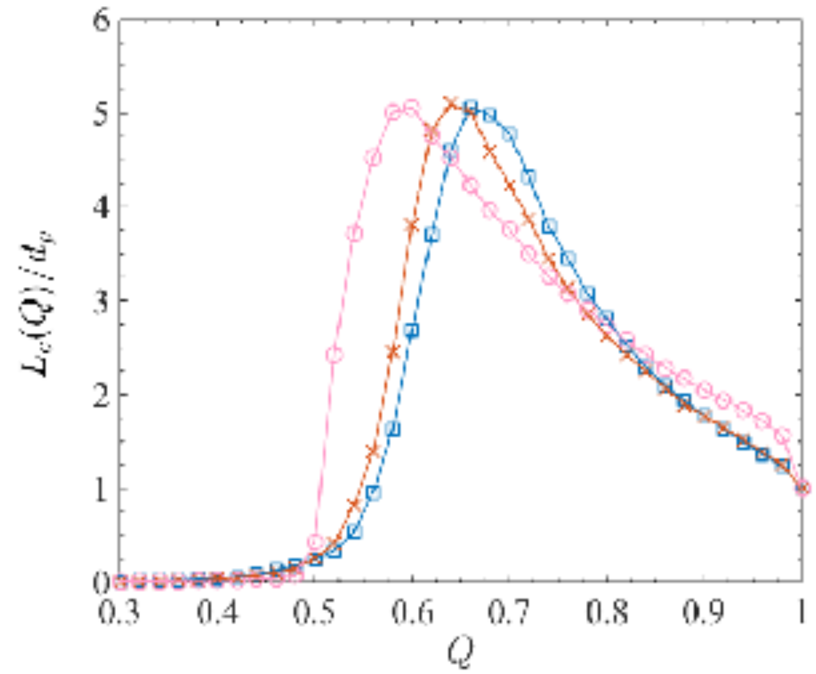
Variation of cluster sizes with Q for 3D plane shear (N_T = 13635)

Variation of cluster sizes with Q for 3D isotropic compression (N_T = 10000)

Effect of polydispersity







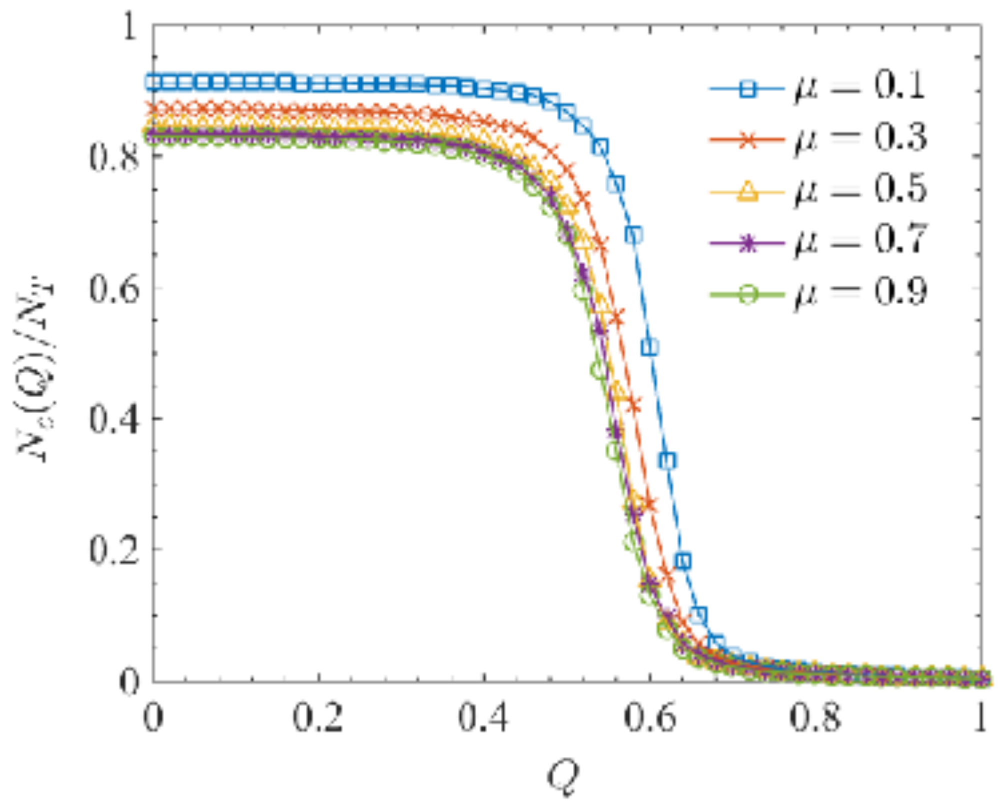
$N_c(Q)$ = Number of particles in the cluster

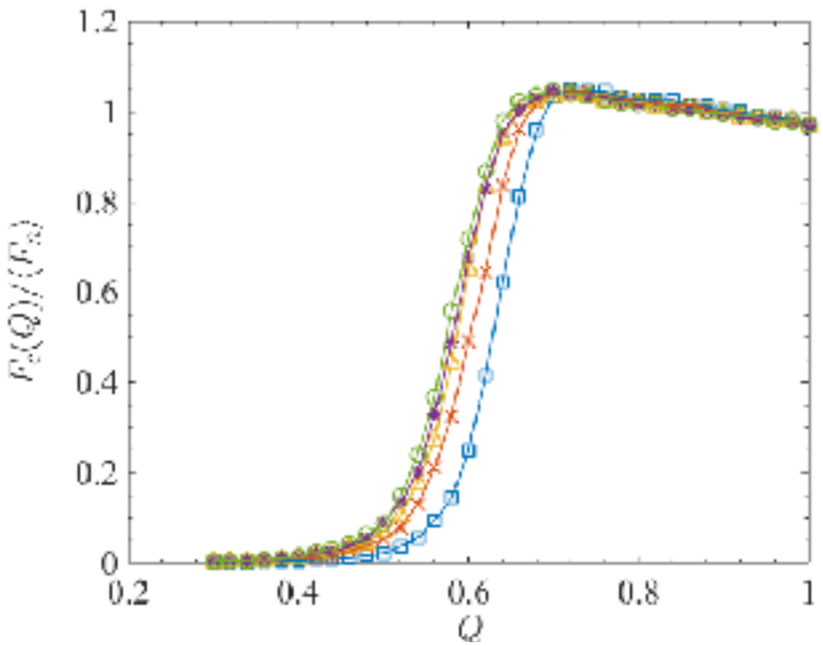
$F_c(Q)$ = Average pair force of the cluster

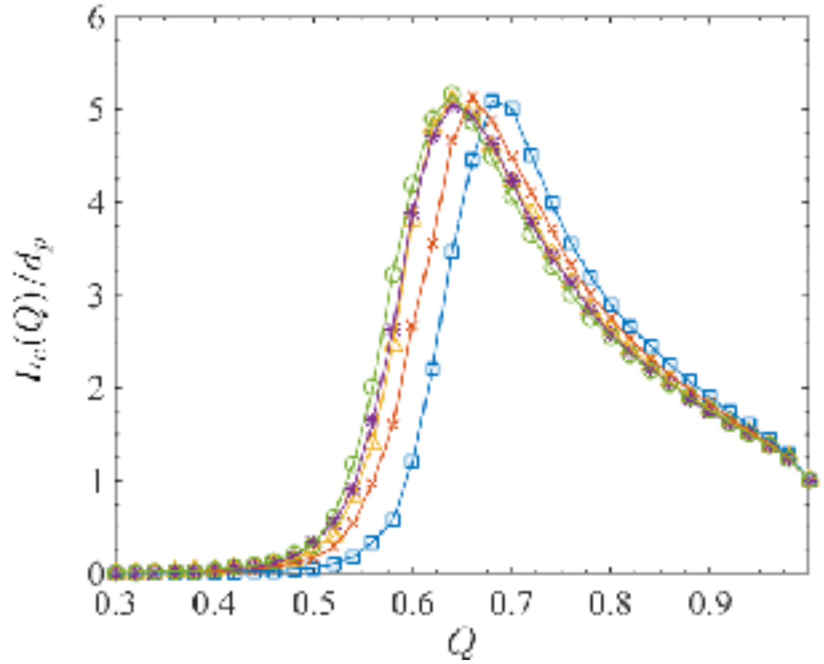
$L_c(Q)$ = Average size of the clusters

Note that percolating clusters are neglected

Effect of friction







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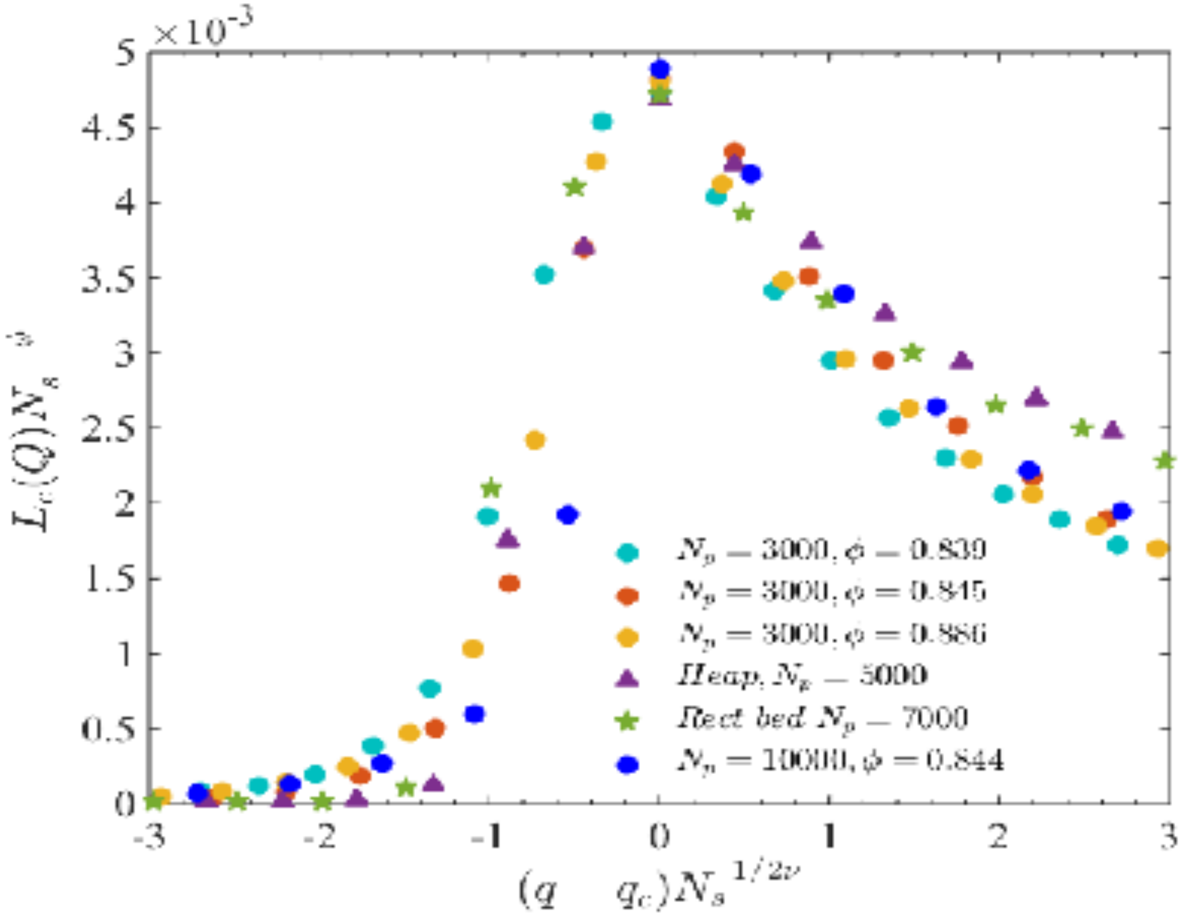
Finite size scaling

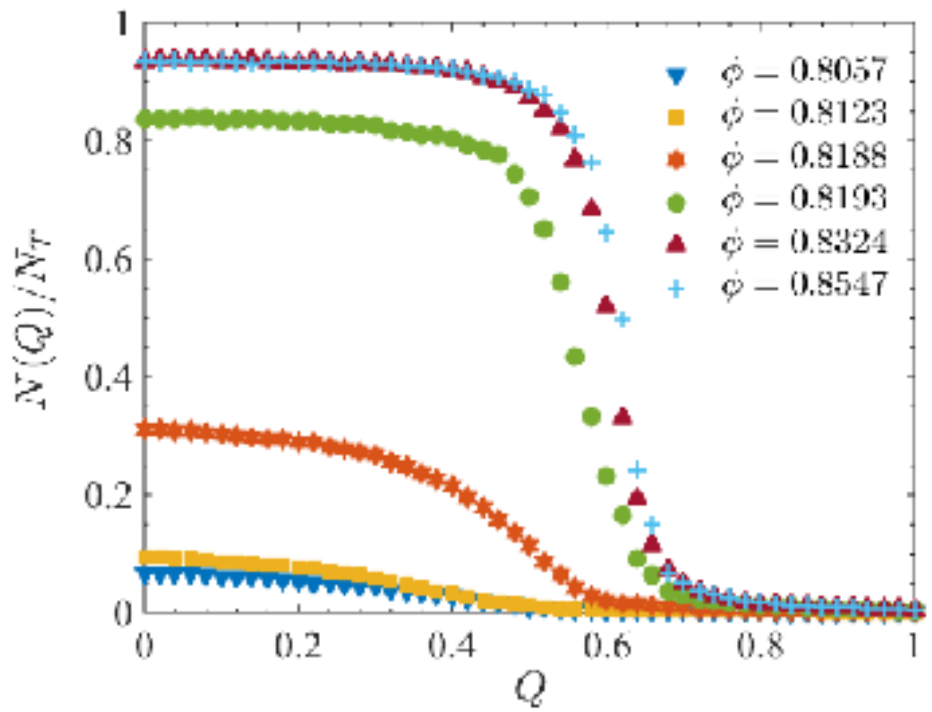


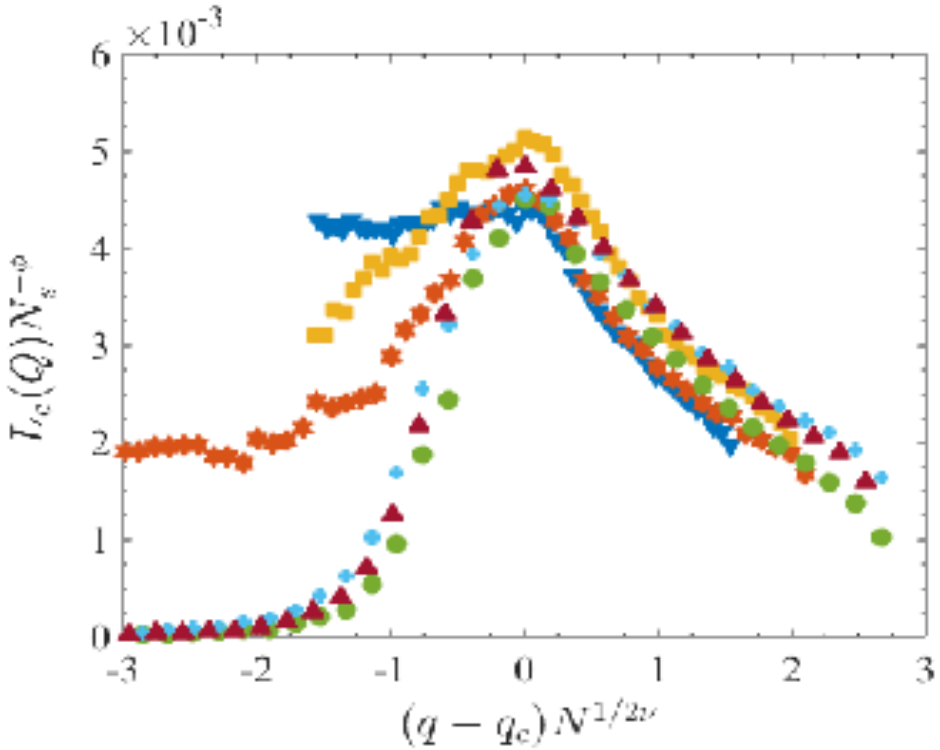
Scaled radius of gyration of clusters. Only static configurations are shown.

Here N_s is the total number of force bearing contacts in the system.

The value of the exponents are $\varphi = 0.29$ and $\nu = 1.52$.

Finite size scaling (Sheared systems)





Scaled radius of gyration of clusters. Only 2D plane shear configurations are shown.

Here N_s is the total number of force bearing contacts in the system

The value of the exponents are $\varphi = 0.275$ and $\nu = 1.8$