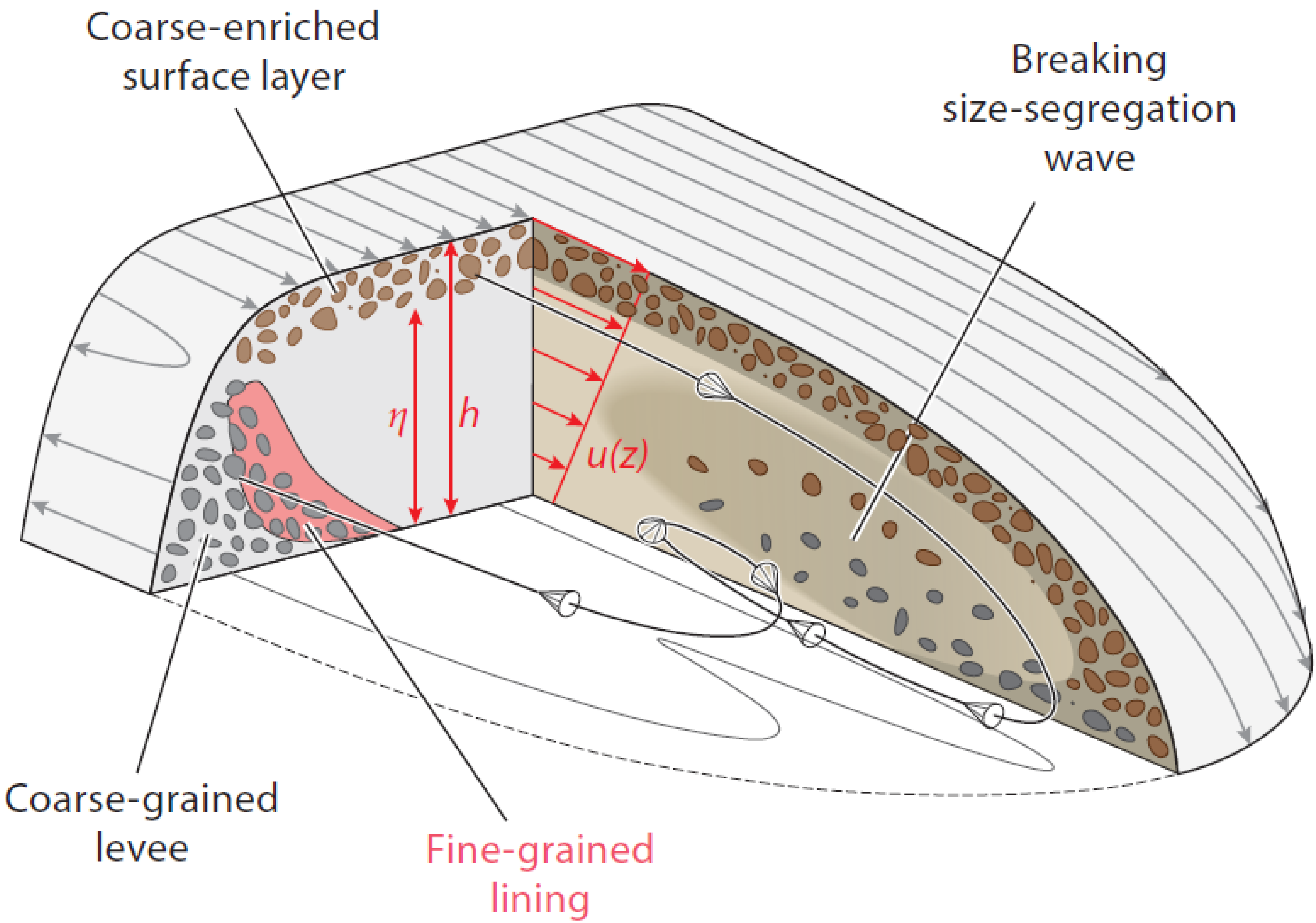
Particle segregation and rheology of dense granular flows

Nico Gray

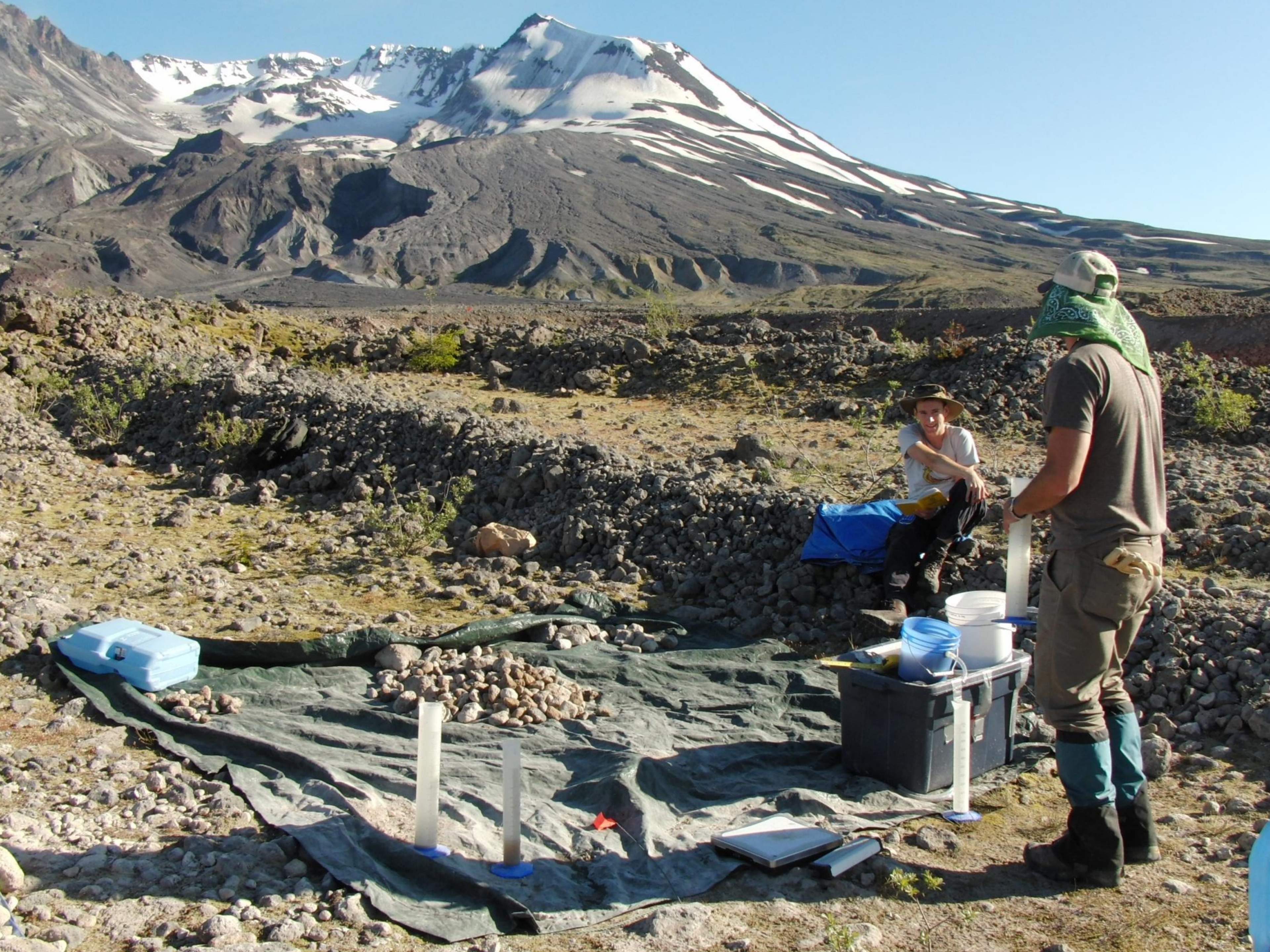
Johnson *et al* (2012) *J. Geophys. Res.* **117**, F01032

MANCHESTER
1824



- larger particles are shouldered to the sides to create levees
- this is an example of a segregation-mobility feedback effect

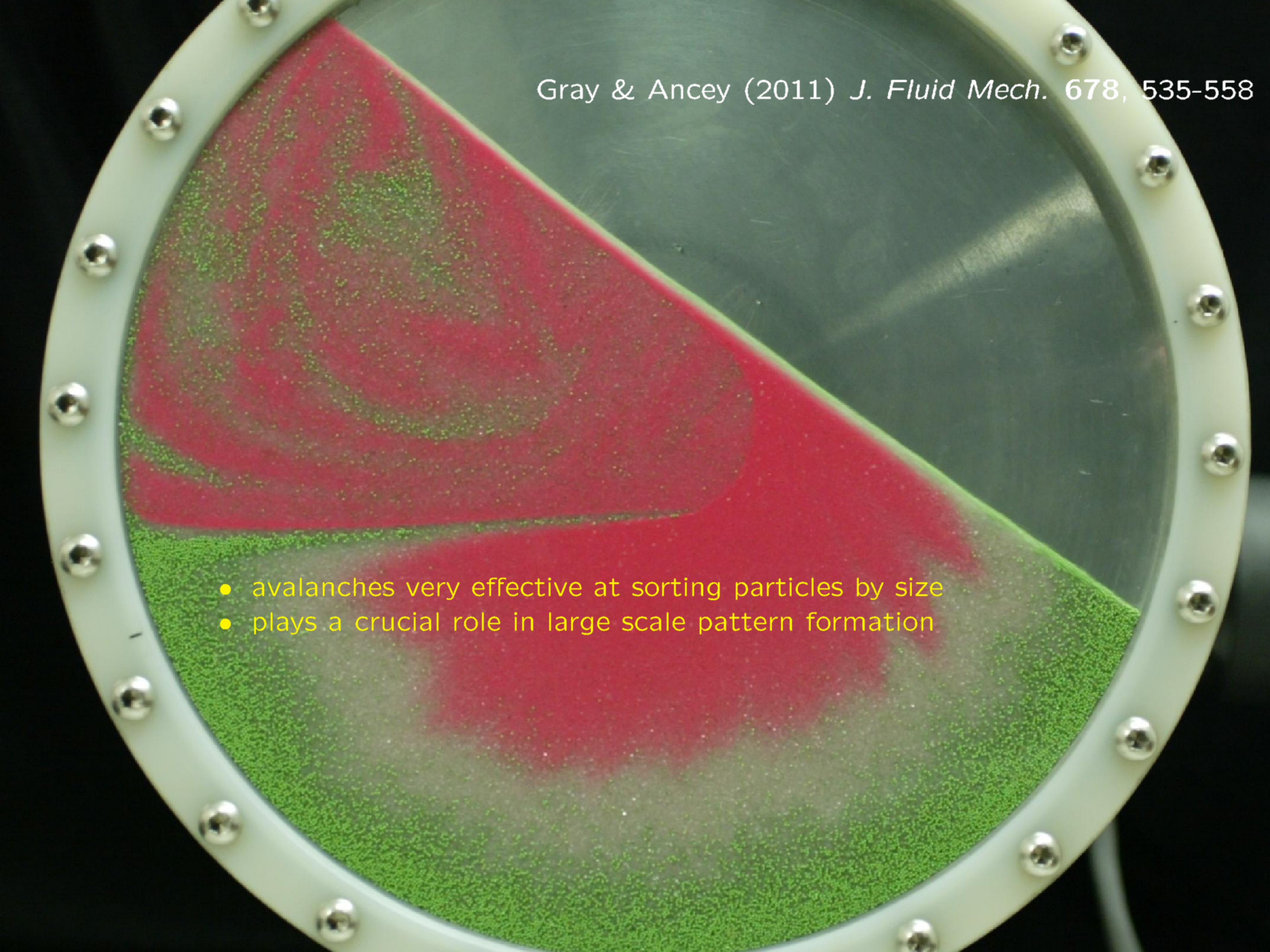
- segregation occurs in many hazardous natural flows
  - debris-flows, pyroclastic flows & snow avalanches
- and leads to spontaneous flow organization and longer run-out

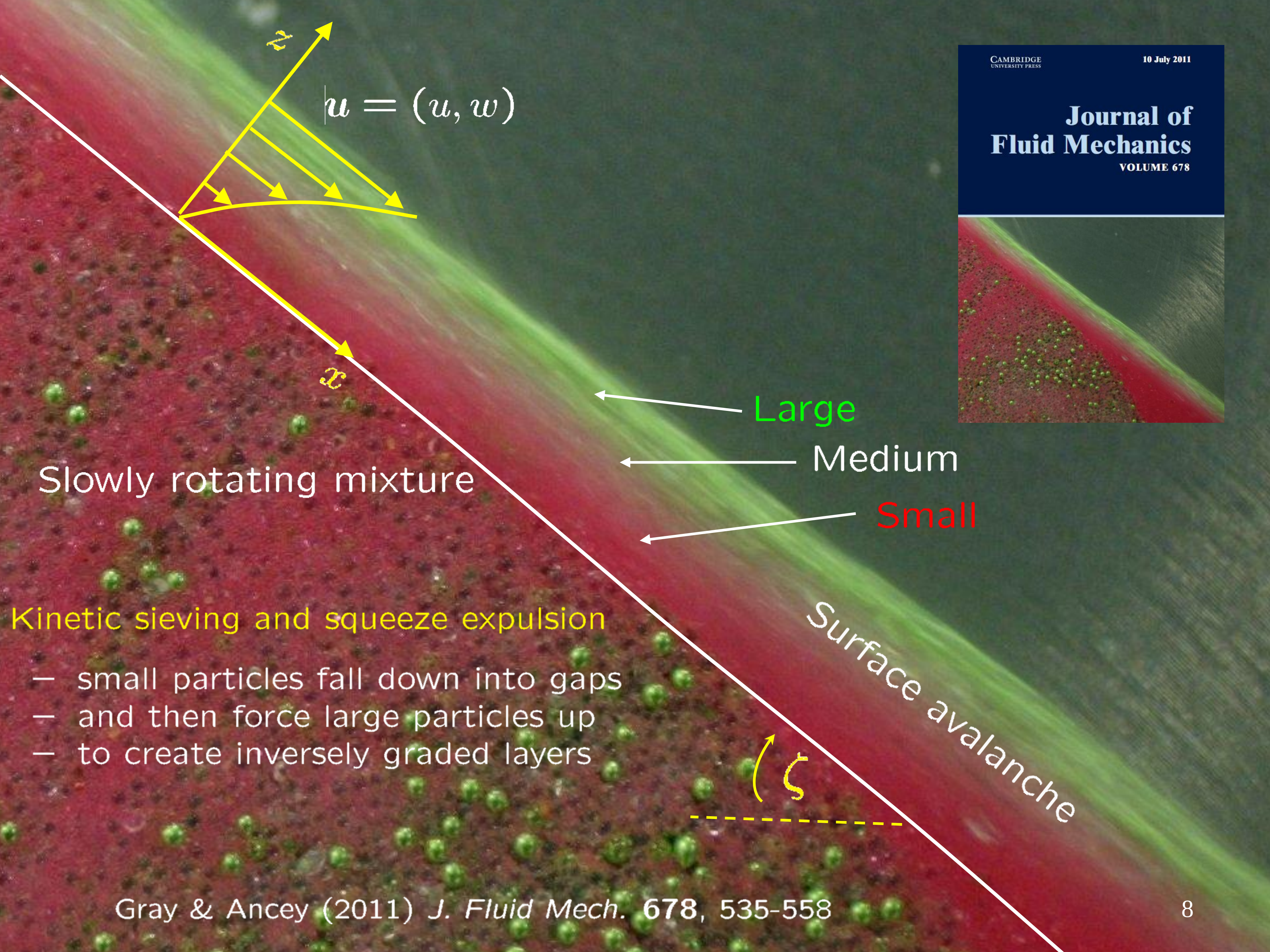


Segregation induced finger formation …

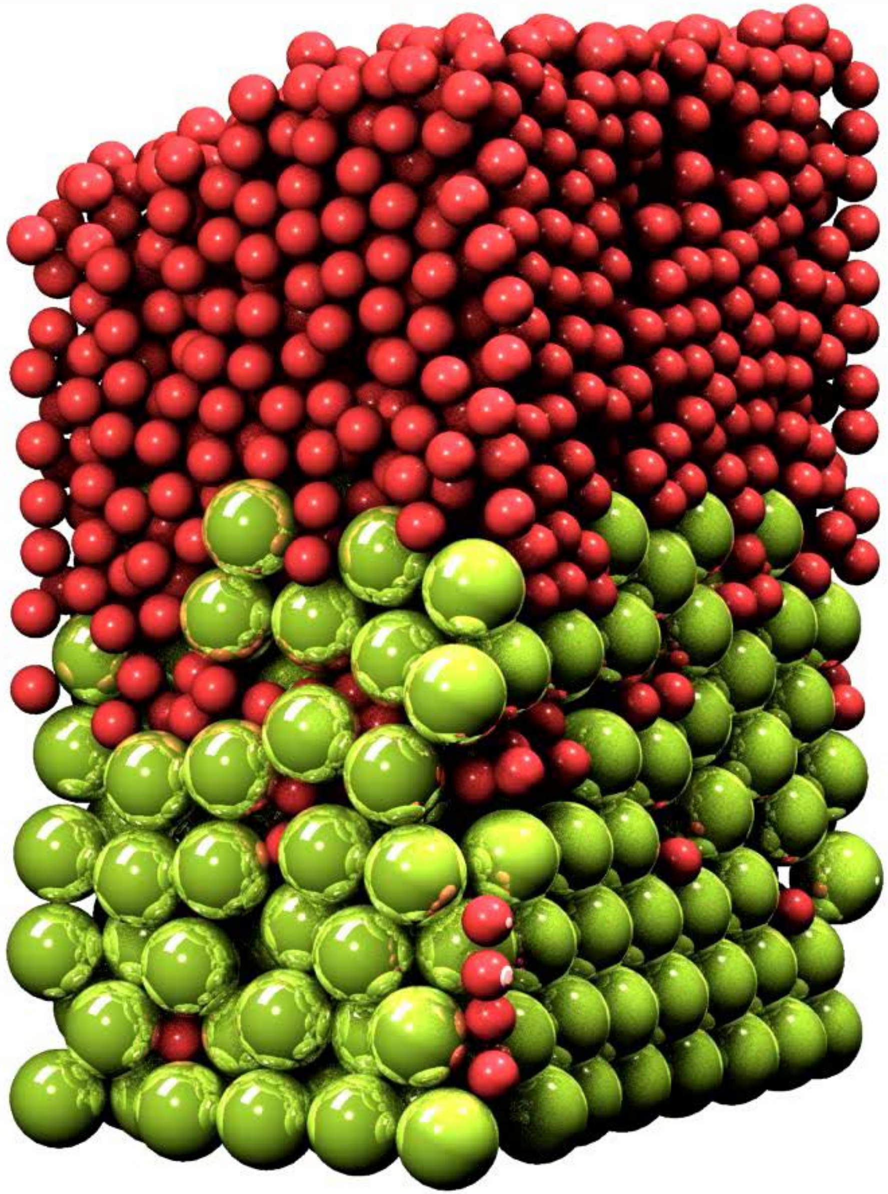
Pouliquen, Delours & Savage (1997), Nature. **386**, 816-817.
Woodhouse *et al.* (2012), J. Fluid Mech. **709**, 543-580.

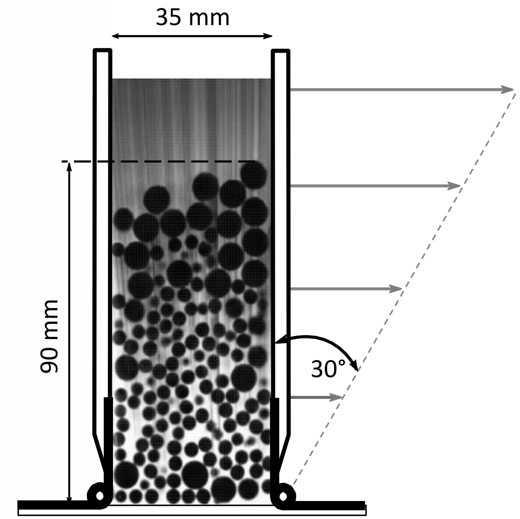
Woodhouse *et al.* (2012), J. Fluid Mech. **709**, 543-580.
Kokelaar *et al* (2014) Earth Planet. Sci. Lett. **385**, 172-180.


Gray & Ancey (2011) *J. Fluid Mech.* **678**, 535-558
• avalanches very effective at sorting particles by size
• plays a crucial role in large scale pattern formation


z
$\boldsymbol{u} = (u, w)$
x
CAMBRIDGE UNIVERSITY PRESS
10 July 2011
Journal of Fluid Mechanics
VOLUME 678
Large
Medium
Small
Slowly rotating mixture
Surface avalanche
ζ
Kinetic sieving and squeeze expulsion
– small particles fall down into gaps
– and then force large particles up
– to create inversely graded layers
Gray & Ancey (2011) *J. Fluid Mech.* **678**, 535-558

Index matched shear box experiments on bi-disperse segregation





van der Vaart, Gajjar, Epely-Chauvin, Andreini, Gray & Ancey (2015) *Phys. Rev Lett.* **114**, 238001

² Bi-disperse segregation equation for small particle concentration Á is

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\phi \boldsymbol{u}) + \frac{\partial}{\partial z}(-S_r F(\phi)) = \frac{\partial}{\partial z}\left(D_r \frac{\partial \phi}{\partial z}\right)$$

² where bulk velocity u, S_r is the segregation rate and D_r is the di®usion

² The segregation °ux F = F(Á) satis¯es the constraints that

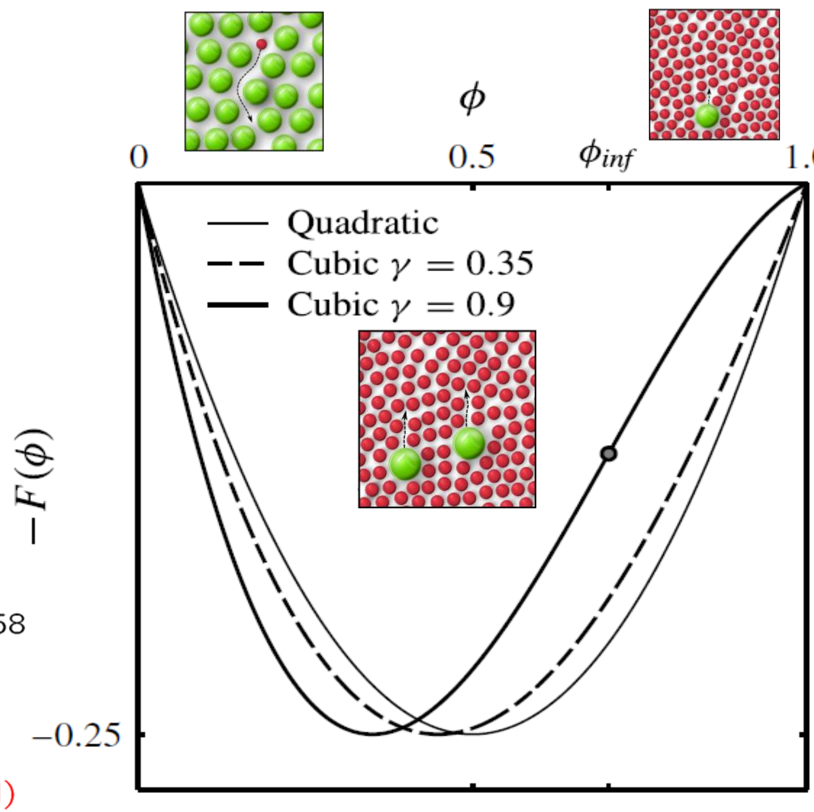
$$\left.\begin{aligned} F &= 0, & \phi &= 0, \\ F &= 0, & \phi &= 1. \end{aligned}\right\}$$

² Gray & Thornton (2005)

$$F(\phi) = \phi(1-\phi)$$

² Gajjar & Gray (2014)

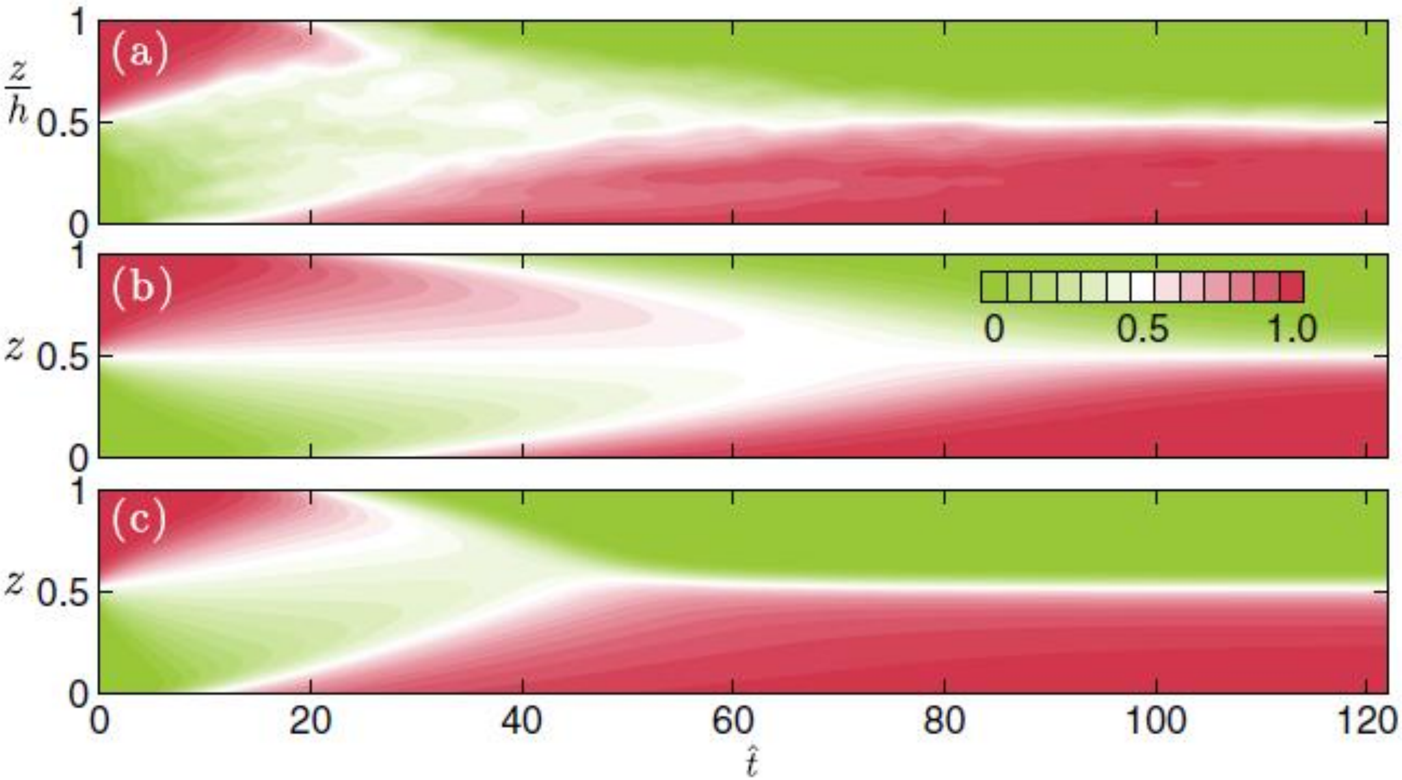
$$F(\phi) = A_\gamma \phi(1-\phi)(1-\gamma\phi),$$



$$\phi^s = \phi, \qquad \phi^l = 1 - \phi$$

Bridgwater, Foo & Stephens (1985), *Powder Technol.* **41**, 147-158
Savage & Lun (1988) *J. Fluid Mech.* **189**, 311-335
Dolgunin & Ukolov (1995) *Powder Technol.* **83**, 95-103
Gray & Thornton (2005) *Proc. Roy. Soc. A.* **461**, 1447-1473.
Gray & Chugunov (2006) *J. Fluid Mech.* **569**, 365-398.
Fan & Hill (2011) *New J. Phys.* **13**, 095009. (fluctuation induced)
Gray & Ancey (2011) *J. Fluid Mech.* **678**, 535-558. (multiple sizes)
Gajjar & Gray (2014) *J. Fluid Mech.* **757**, 297-329. (asymmetry)
Gray & Ancey (2015) *J. Fluid Mech.* **779**, 622-668. (size and density)

Cubic flux with γ=0.89 captures slower rise of individual large particles, as well as their enhanced collective motion



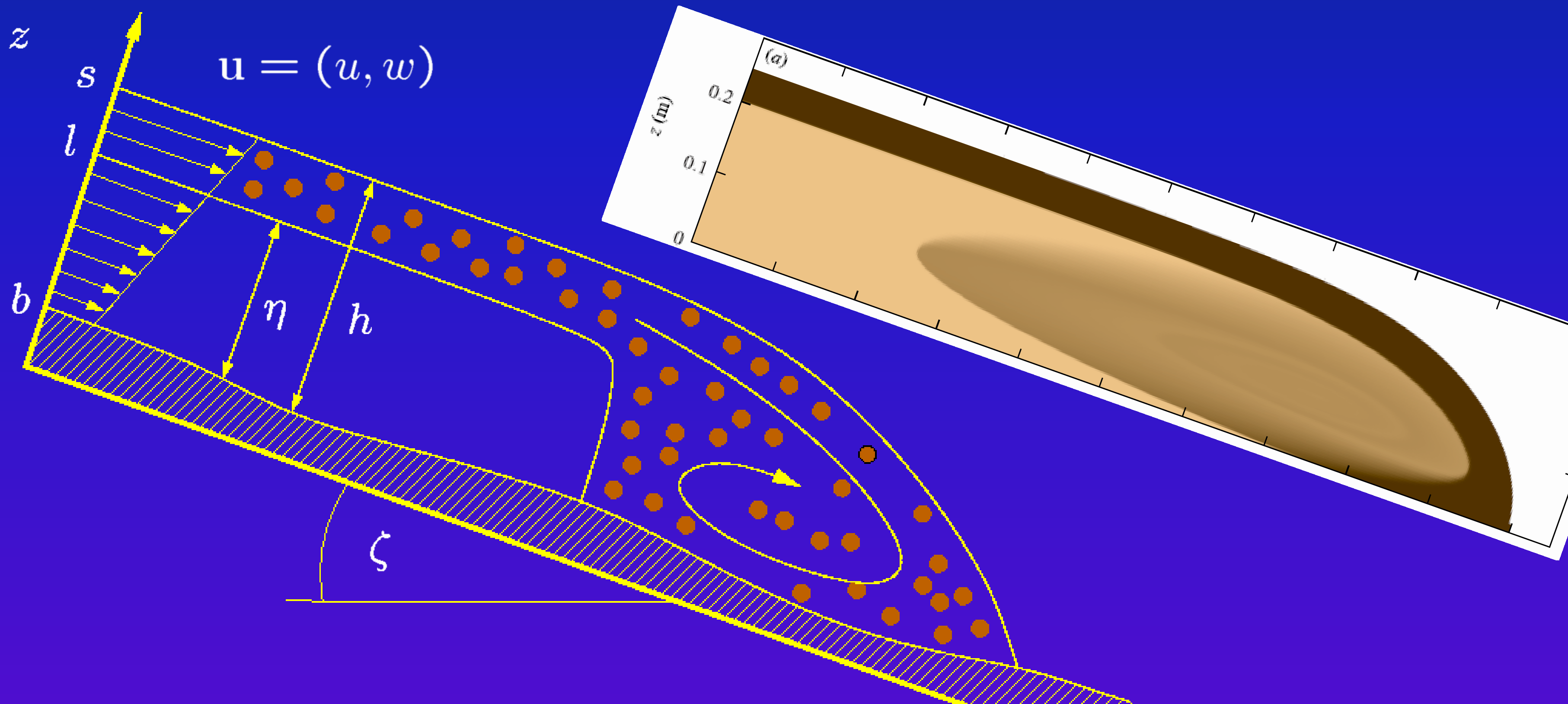
Experiment

$F = \phi(1-\phi)$

$F(\phi) = A_\gamma \phi(1-\phi)(1-\gamma\phi),$

Gollick & Daniels (2009) *Phys. Rev E* **80**, 042301.
van der Vaart,Gajjar, Epely-Chauvin, Andreini, Gray & Ancey (2015) *Phys. Rev Lett.* **114**, 238001
Gajjar & Gray (2014) *J. Fluid Mech.* **757**, 297-329.
Gray (2018) *Annual Review Fluid Mech.* **50**, 407-433.

Transport and accumulation of large particles



- large particles segregate to the surface
- where the velocity is greatest and
- are transported to the flow front where they are
- over run and recirculated by particle size segregation

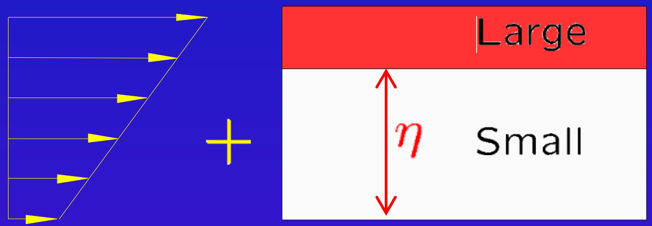
Johnson *et al* (2012) *J.Geophys. Res.* **117**, F01032

A depth averaged theory for particle size segregation

- Integrating the segregation-remixing equation w.r.t z
- subject to the no flux and kinematic boundary conditions gives

$$\frac{\partial}{\partial t}(h\bar{\phi}) + \frac{\partial}{\partial x}(h\overline{\phi u}) = 0$$

- where the integrals evaluated assuming



i.e. linear velocity with basal slip and sharp segregation

$$h\bar{\phi} = \int_b^s \phi^s \, dz = \eta$$

$$h\overline{\phi u} = \int_b^s \phi^s u \, dz = \eta\bar{u} - (1-\alpha)\bar{u}\eta\left(1 - \frac{\eta}{h}\right)$$

- This yields the large particle transport equation

$$\frac{\partial \eta}{\partial t} + \frac{\partial}{\partial x}(\eta\bar{u}) - \frac{\partial}{\partial x}\left((1-\alpha)\bar{u}\eta\left(1-\frac{\eta}{h}\right)\right) = 0.$$

- for the evolution of the inversely graded shock interface η.

Gray & Kokelaar (2010) *J. Fluid Mech.* **652**, 105–137

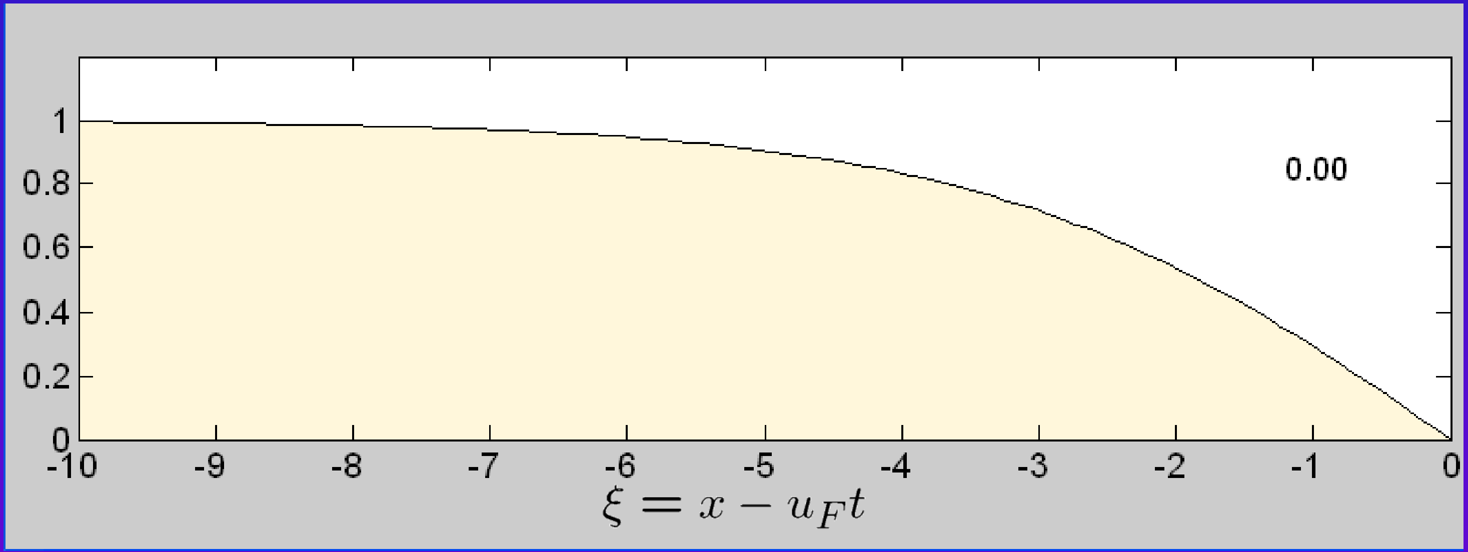
- Using $\eta = h\bar{\phi}$ this can also be rewritten as

$$\frac{\partial}{\partial t}(h\bar{\phi}) + \frac{\partial}{\partial x}(h\bar{\phi}\bar{u}) - \frac{\partial}{\partial x}\left((1-\alpha)h\bar{u}\bar{\phi}\left(1-\bar{\phi}\right)\right) = 0.$$

- Remarkably similar to the segregation equation ...

$$\frac{\partial \phi}{\partial t} + \frac{\partial}{\partial x}(\phi u) + \frac{\partial}{\partial z}(\phi w) - S_{ls}\frac{\partial}{\partial z}\Big(\phi(1-\phi)\Big) = \frac{\partial}{\partial z}\left(D_r \frac{\partial \phi}{\partial z}\right)$$

- Large grains transported forwards to form bouldery flow front



- more RESISTIVE larger particles $\Rightarrow$ feedback on bulk flow

Gray & Kokelaar (2010), J. Fluid Mech. **652**, 105-137.

Inviscid avalanche model for segregation-mobility induced fingers

- For avalanche thickness h, small particle thickness η and depth-averaged velocity $\overline{\boldsymbol{u}}$ the 2D coupled model is

$$\begin{aligned}
\frac{\partial h}{\partial t} + \mathrm{div}(h\overline{\boldsymbol{u}}) &= 0, \\
\frac{\partial \eta}{\partial t} + \mathrm{div}\left(\eta\overline{\boldsymbol{u}} - (1-\alpha)\eta\left(1-\frac{\eta}{h}\right)\overline{\boldsymbol{u}}\right) &= 0, \\
\frac{\partial}{\partial t}(h\bar{u}) + \mathrm{div}(h\overline{\boldsymbol{u}}\otimes\overline{\boldsymbol{u}}) + \mathrm{grad}\left(\frac{1}{2}gh^2\cos\zeta\right) &= hg\boldsymbol{S},
\end{aligned}$$

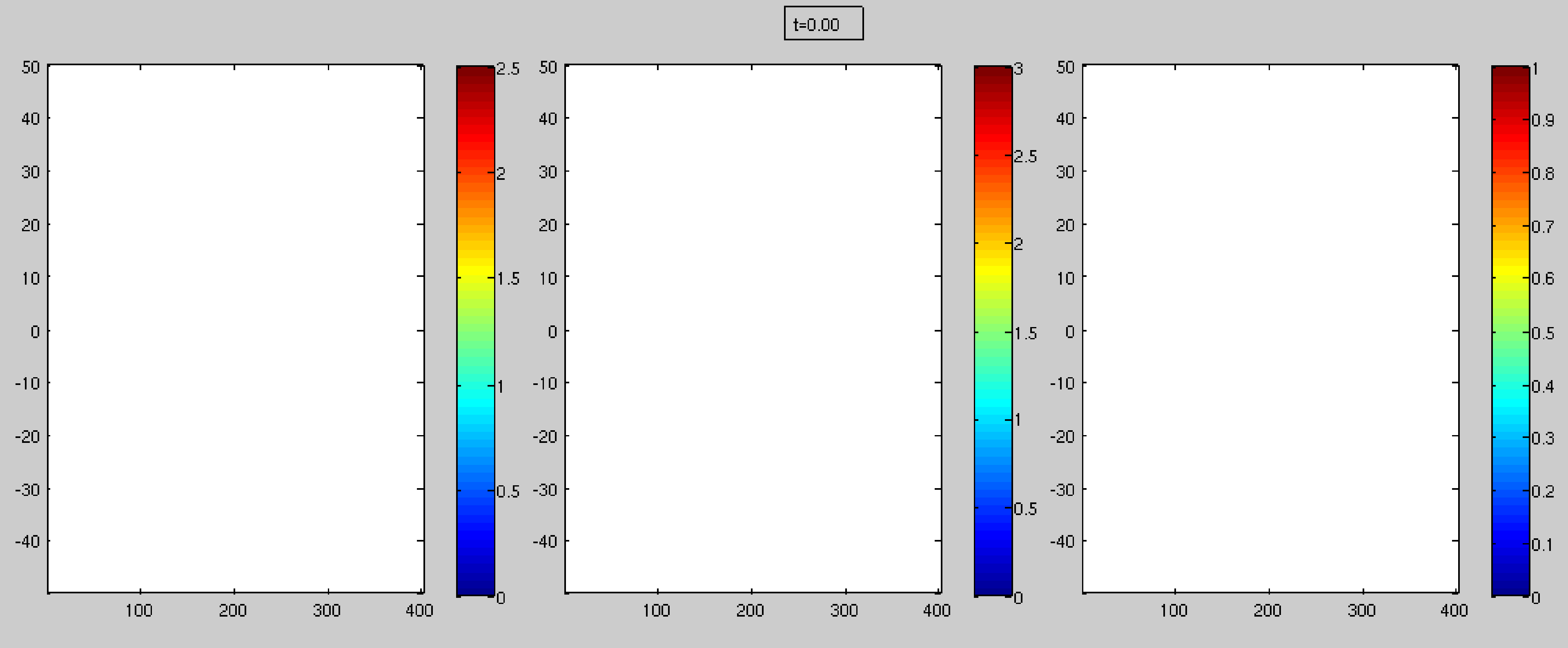
- source terms composed of gravity and basal friction

$$\boldsymbol{S} = \begin{pmatrix} \sin\zeta - \mu(\overline{u}/|\overline{\boldsymbol{u}}|)\cos\zeta, \\ -\mu(\overline{v}/|\overline{\boldsymbol{u}}|)\cos\zeta, \end{pmatrix}$$

- coupling through $\bar{\phi} = \eta/h$ dependent friction coefficient

$$\mu = \left(1-\bar{\phi}\right)\mu^L + \bar{\phi}\mu^S, \qquad \mu^L > \mu^S$$

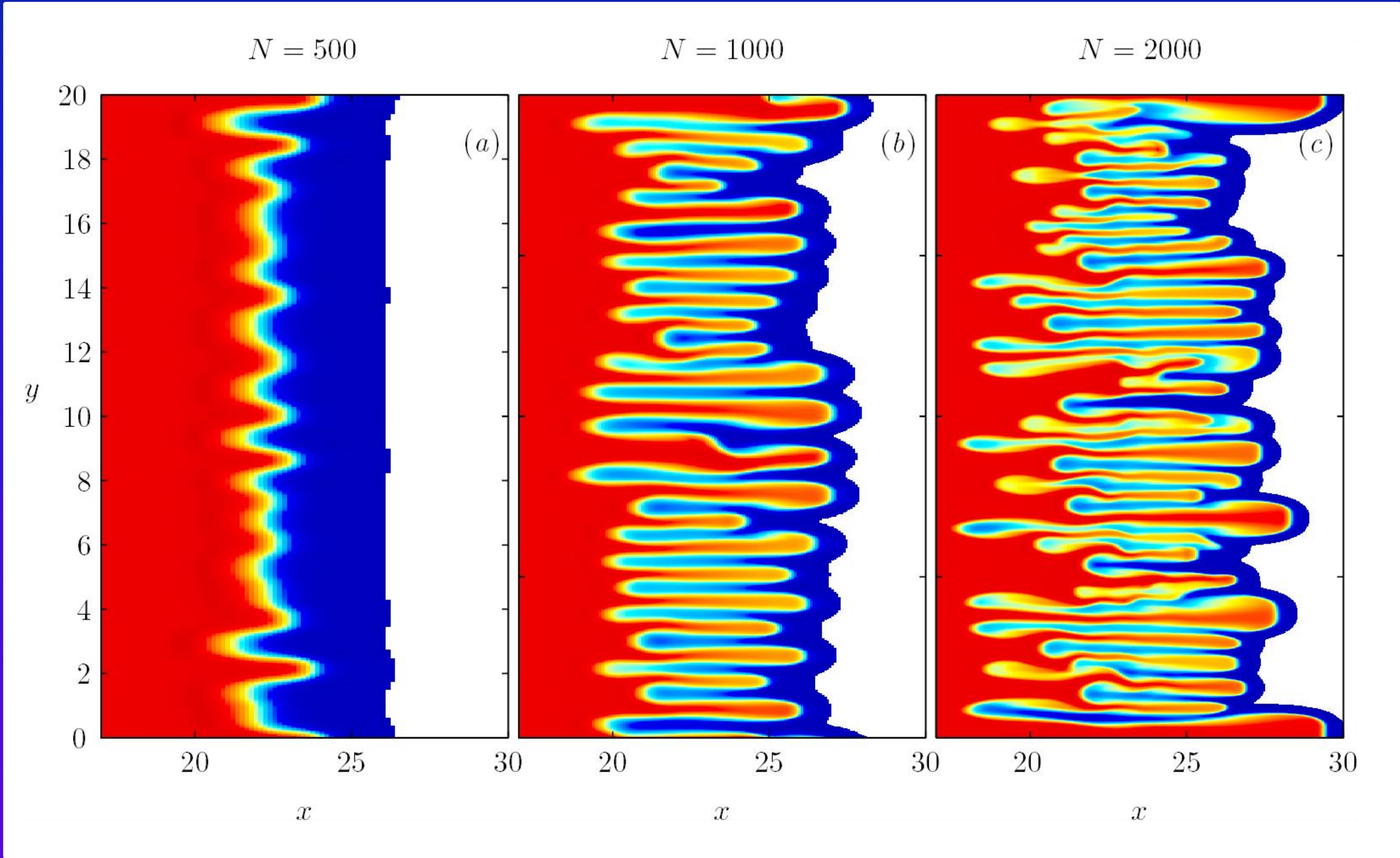
Woodhouse *et al.* (2012), J. Fluid Mech. **709**, 543-580.



h $|\overline{\boldsymbol{u}}|$ $\overline{\phi}$

- The model is hyperbolic
- captures the instability mechanism
- and forms large rich lateral levees, BUT ....

Numerical solutions are grid dependent ...!

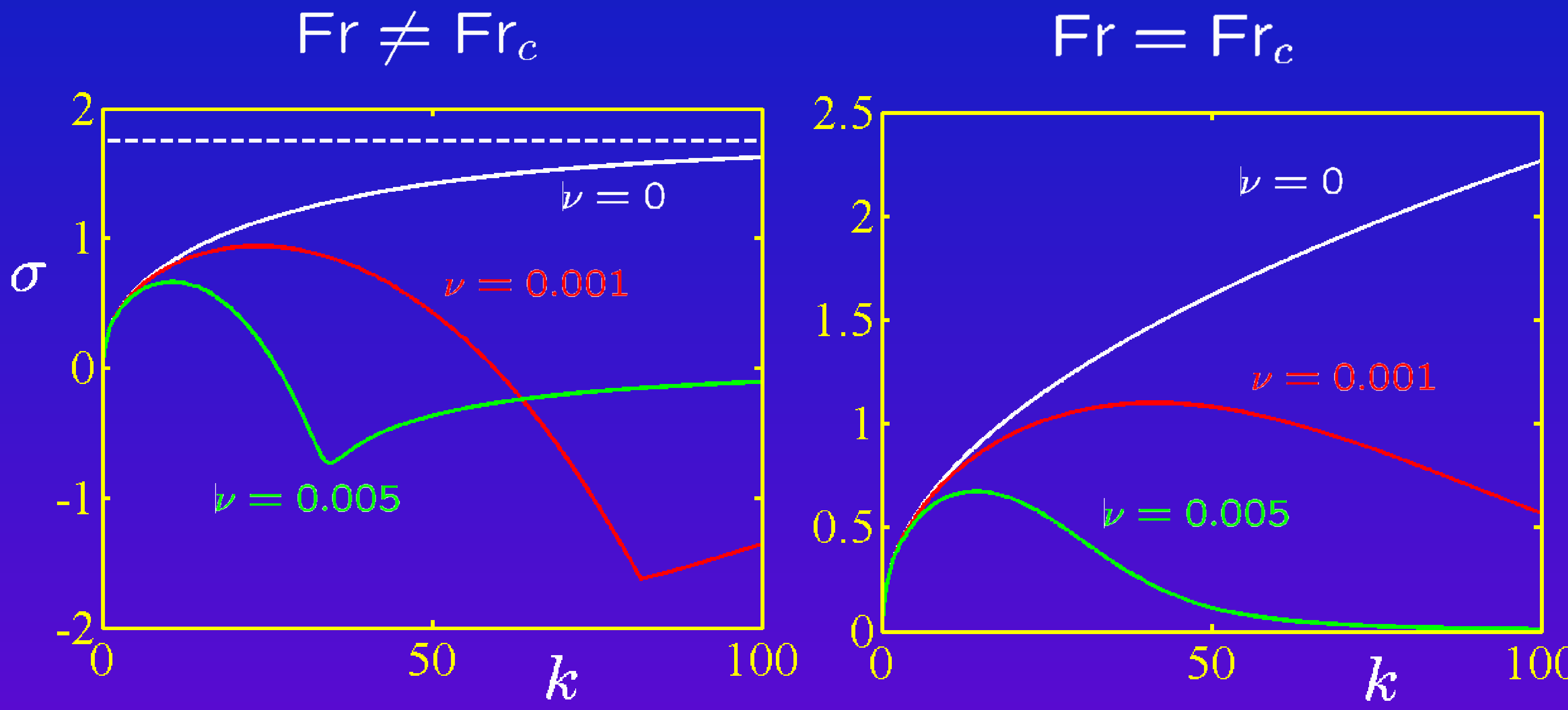


- Numerical viscosity is setting the wavelength

$\Rightarrow$ there is some important missing physics

Woodhouse *et al* (2012), J. Fluid Mech. **709**, 543-580.

- cause of problem is that the equations are ill-posed for one value of the concentration, when $\mathrm{Fr}_c = [(1-\alpha)|2\eta_0 - 1|]^{-1}$, i.e. linear instability growth rate tends to infinity



- introducing a physicaly based viscosity can solve the problem, BUT what is that physics?

A two-dimensional fully coupled model including rheology

- Adding a two-dimensional depth-averaged $\mu(I)$-rheology implies a system of conservation laws of the form

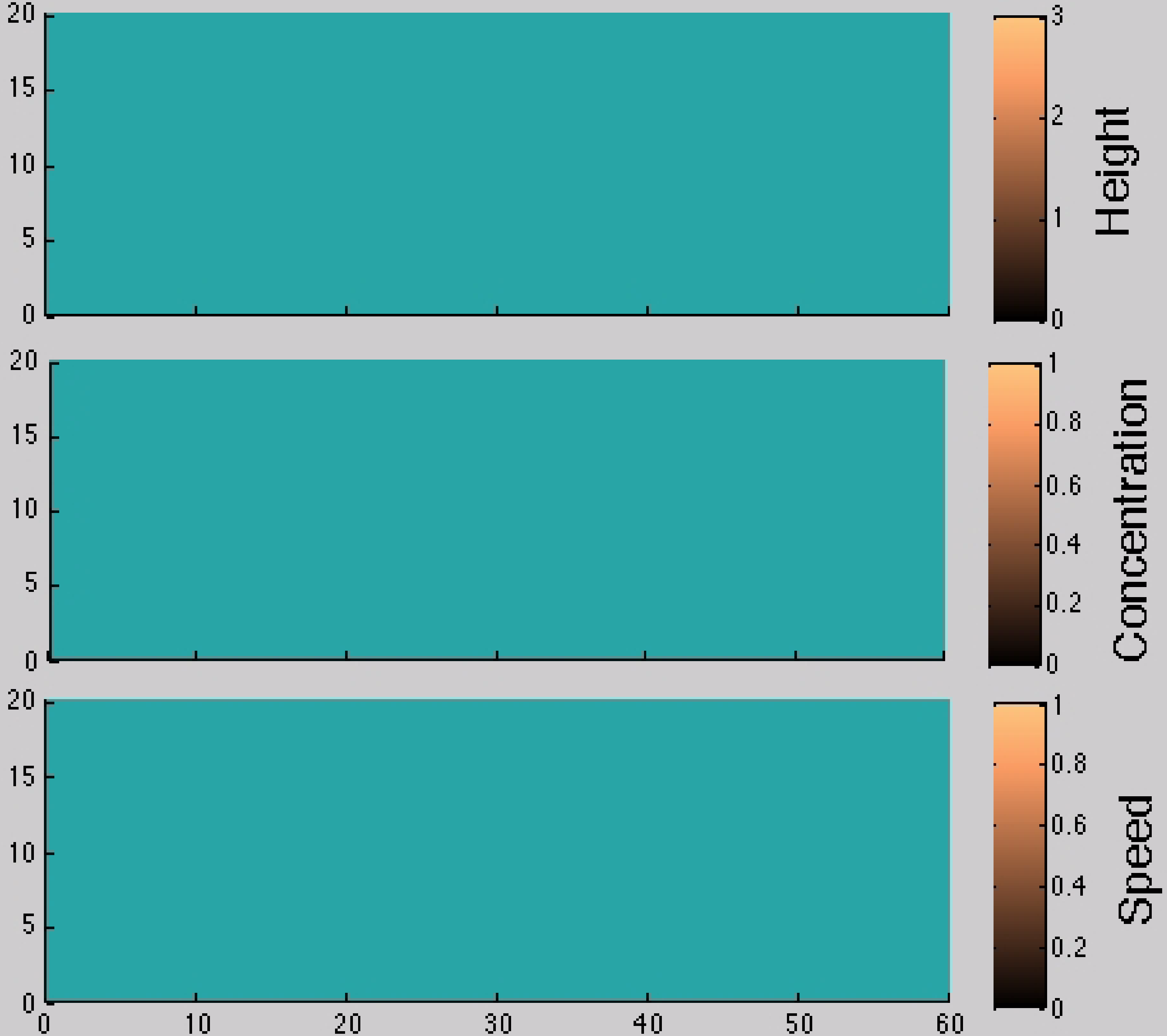
$$
\begin{aligned}
\frac{\partial h}{\partial t} + \mathrm{div}(h\bar{\boldsymbol{u}}) &= 0, \\
\frac{\partial \eta}{\partial t} + \mathrm{div}\left(\eta\bar{\boldsymbol{u}} - (1-\alpha)\eta\left(1-\frac{\eta}{h}\right)\bar{\boldsymbol{u}}\right) &= 0, \\
\frac{\partial}{\partial t}(h\bar{u}) + \mathrm{div}(h\bar{\boldsymbol{u}}\otimes\bar{\boldsymbol{u}}) + \mathrm{grad}\left(\frac{1}{2}h^2 g\cos\zeta\right) &= hg\boldsymbol{S} + \mathrm{div}\left(\nu h^{\frac{3}{2}}\overline{\boldsymbol{D}}\right),
\end{aligned}
$$

- where the two-dimensional strain-rate tensor is

$$
\overline{\boldsymbol{D}} = \frac{1}{2}\left(\nabla\bar{\boldsymbol{u}} + \nabla\bar{\boldsymbol{u}}^T\right)
$$

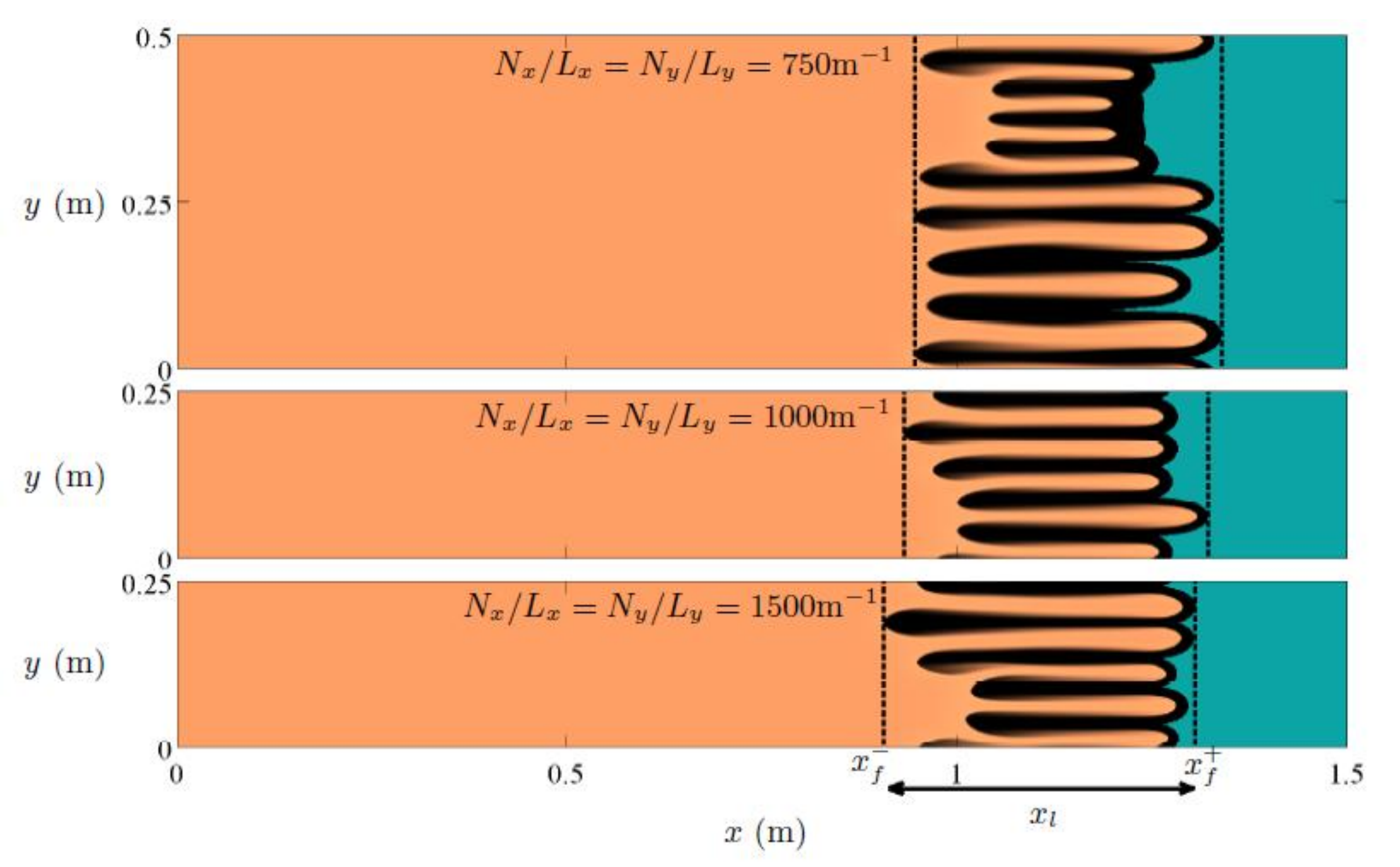
- the coefficient ν in the viscosity $\nu h^{1/2}/2$ is determined from the friction law

Baker, Barker & Gray (2016) *J. Fluid Mech.* **787**, 367-395.
Baker, Johnson & Gray *J. Fluid Mech.* **809**, 168–212.



20
15
10
5
0
Height
3
2
1
0
Concentration
1
0.8
0.6
0.4
0.2
0
Speed
1
0.8
0.6
0.4
0.2
0
0
10
20
30
40
50
60

The depth-averaged mu(I)-rheology is crucial for grid resolved fingers



- We have grid convergence!
- However, still can't bring levees fully to rest
- We don't understand the viscosity of mixtures of grain sizes

The μ(I)-rheology for liquid-like granular flows

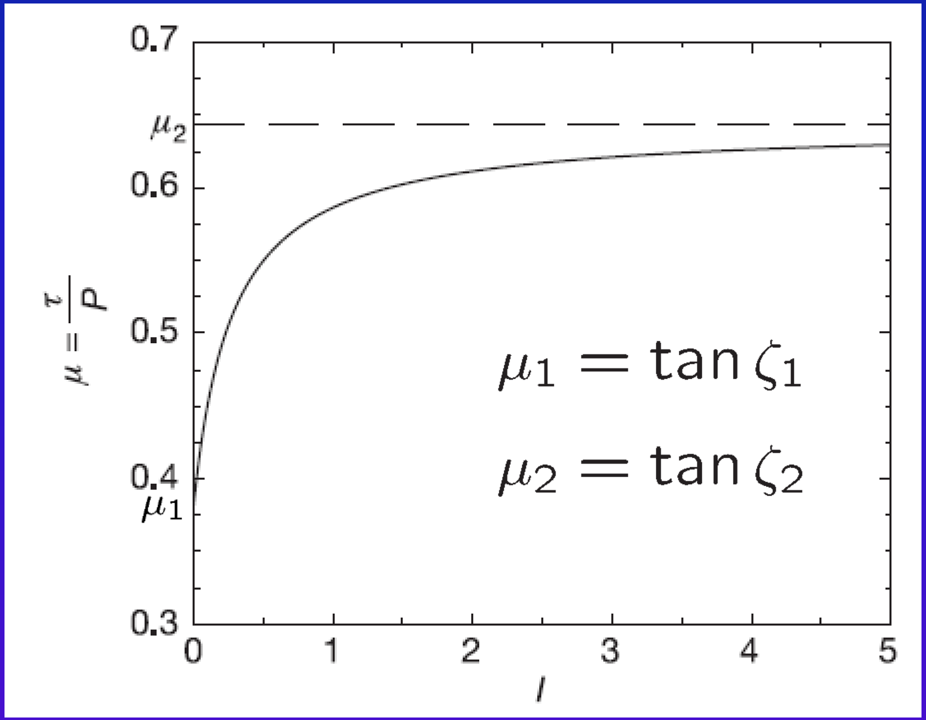
- GDR MIDI (2004) and Jop *et al.* (2006): proposed constitutive law

$$\boldsymbol{\tau} = \mu(I)p\frac{\boldsymbol{D}}{||\boldsymbol{D}||}$$

- where 2nd invariant

$$||\boldsymbol{D}|| = \sqrt{\frac{1}{2}\mathrm{tr}\boldsymbol{D}^2}$$

- If $\mu =$ const this reduces to Drucker-Prager



- BUT, friction μ is a function of the inertial number I

$$\mu(I) = \mu_1 + \frac{\mu_2 - \mu_1}{I_0/I + 1}, \qquad I = \frac{2||\boldsymbol{D}||d}{\sqrt{p/\rho^*}},$$

- where d is the particle diameter and ρ^* is the intrinsic density.

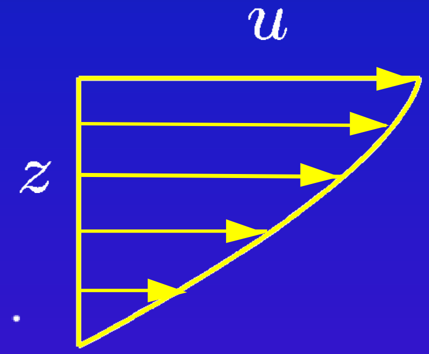
The depth-averaged μ(I)-rheology for granular flows

Gray & Edwards (2014) *J. Fluid Mech.* **755**, 503-534.

- Steady-uniform flow has constant inertial number, a litho-static pressure and Bagnold velocity

$$I = I_\zeta, \qquad p = \rho g(h-z)\cos\zeta.$$

$$u = \frac{2I_\zeta}{3d}\sqrt{\Phi g\cos\zeta}\left(h^{3/2}-(h-z)^{3/2}\right).$$



- Depth-averaging the inviscid avalanche equations emerge naturally at first order with basal friction law

$$\mu_b(h,\mathrm{Fr}) = \mu_1 + \frac{\mu_2-\mu_1}{\beta h/(L\mathrm{Fr})+1}, \qquad \mathrm{Fr} > \beta,$$

- This is just Pouliquen & Forterre's (2002) law, where

$$\mathrm{Fr} = \frac{|\bar{\boldsymbol{u}}|}{\sqrt{gh\cos\zeta}}$$

- Add in small in-plane deviatoric stress correction τ_{xx}
- evaulate using the steady-uniform Bagnold solution

$$\tau_{xx} = \mu(I)p\frac{D_{xx}}{||\boldsymbol{D}||} = 2\rho g \sin\zeta \left(h^{1/2}(h-z)^{1/2} - (h-z)\right)\frac{\partial h}{\partial x}.$$

- formal depth-integration gives

$$h\bar{\tau}_{xx} = \frac{1}{3}\rho g \sin\zeta\, h^2 \frac{\partial h}{\partial x}.$$

- Use depth-averaged Bagnold velocity $\bar{u} = \frac{2I_\zeta}{5d}\sqrt{\Phi g \cos\zeta}\, h^{3/2}$ to convert to viscous-like term

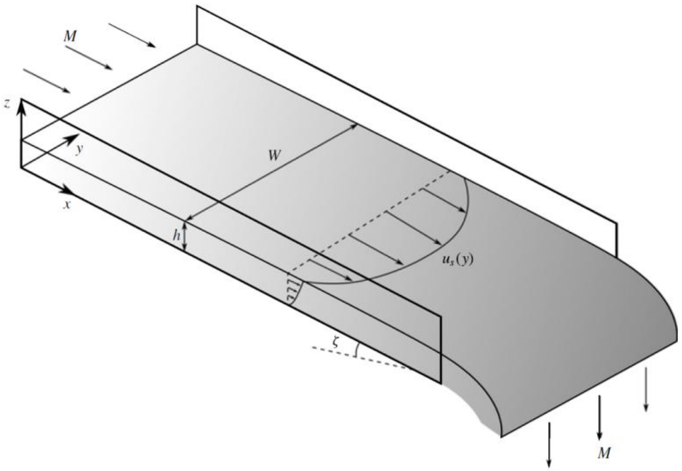
$$h\bar{\tau}_{xx} = \rho\nu h^{3/2}\frac{\partial \bar{u}}{\partial x}$$

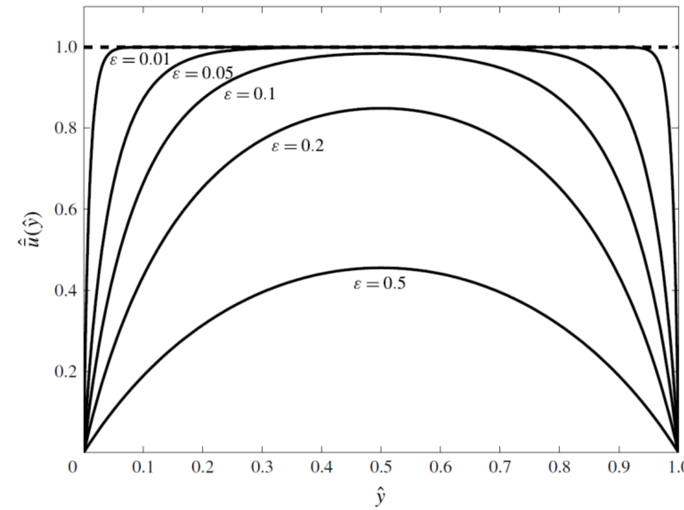
- where angle dependent coefficient ν is determined

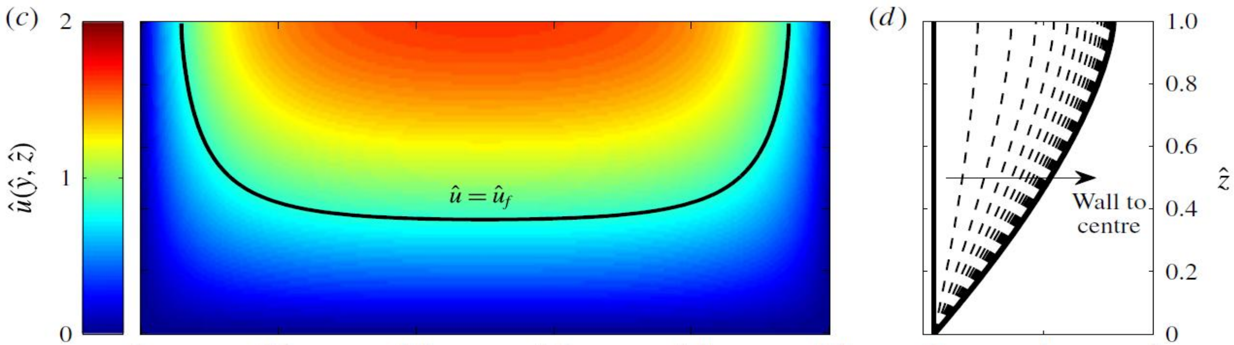
$$\nu = \frac{2L\sqrt{g}}{9\beta}\frac{\sin\zeta}{\sqrt{\cos\zeta}}\left(\frac{\mu_2 - \tan\zeta}{\tan\zeta - \mu_1}\right).$$

Gray & Edwards (2014) *J. Fluid Mech.* **755**, 503-534.

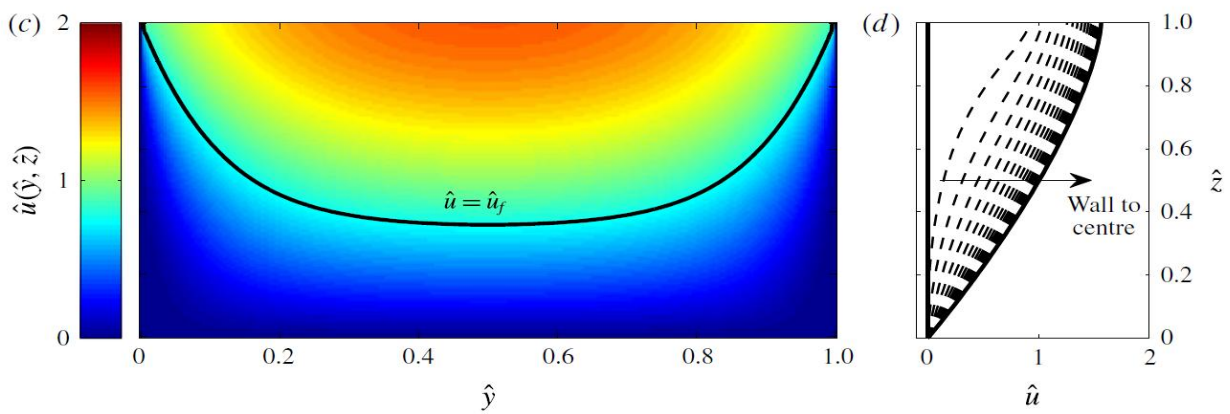
(a) cross-slope velocity profiles







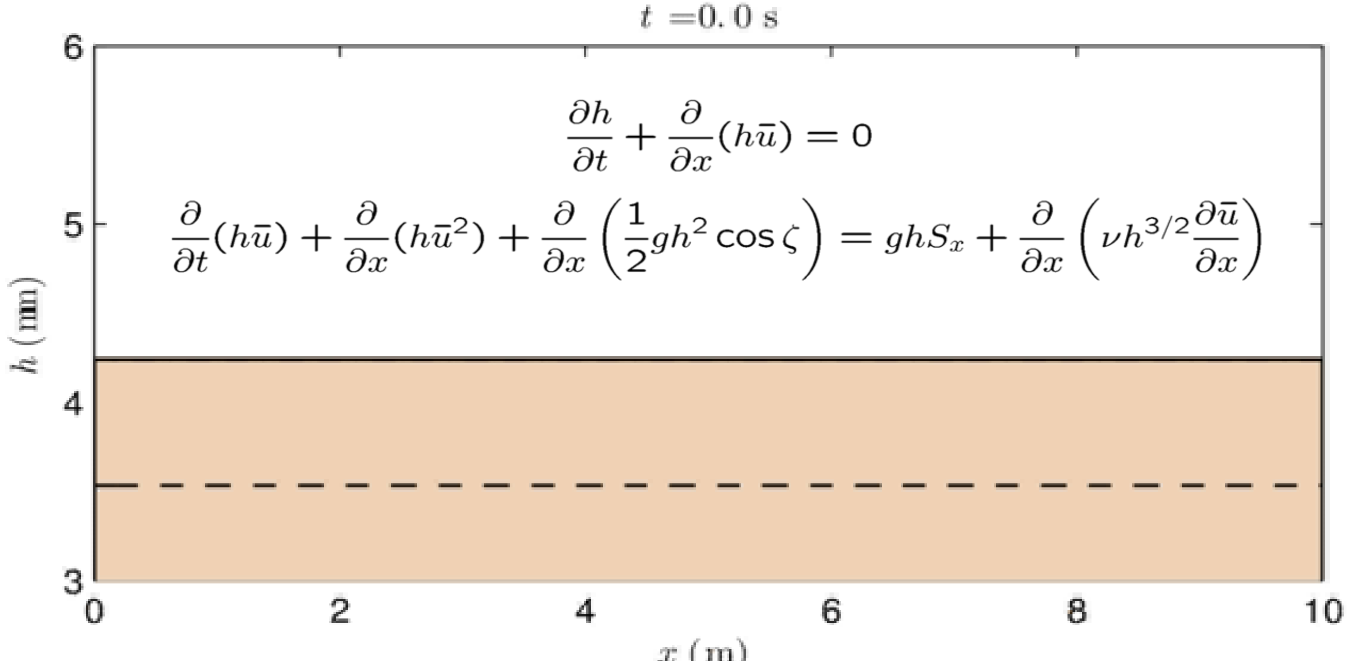
Reconstructed with Bagnold velocity profile

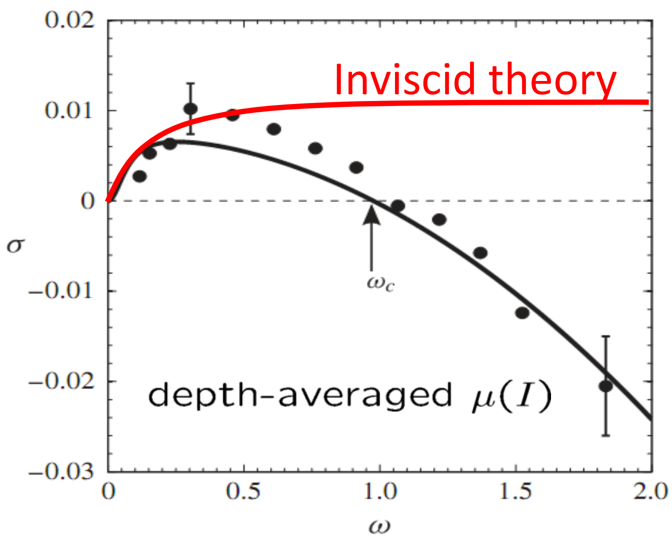


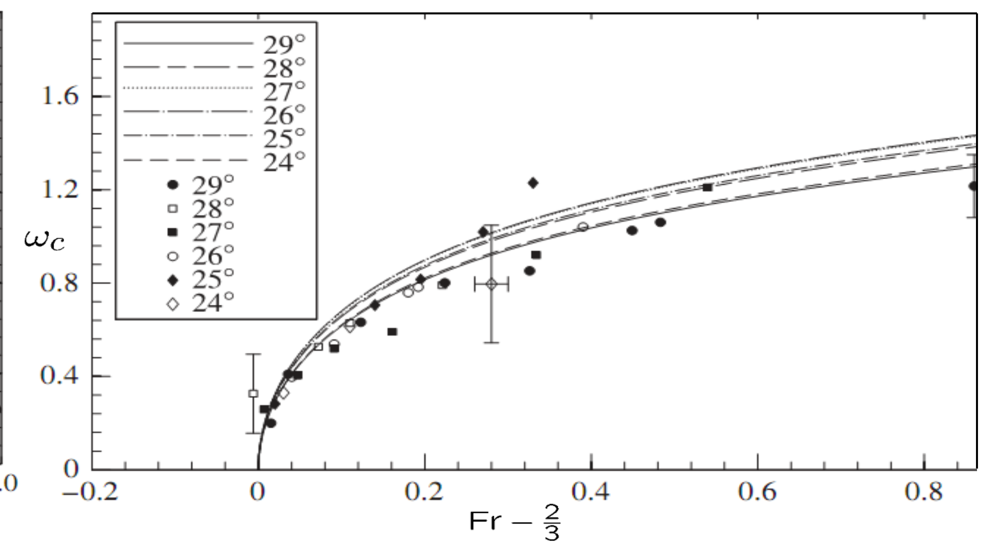
Steady 2D-solutions of full $\mu(I)$-rheology

Baker, Barker & Gray (2016) *J. Fluid Mech.* **787**, 367-395.

(b) Matches roll-wave cut-off frequency without any fitting parameters

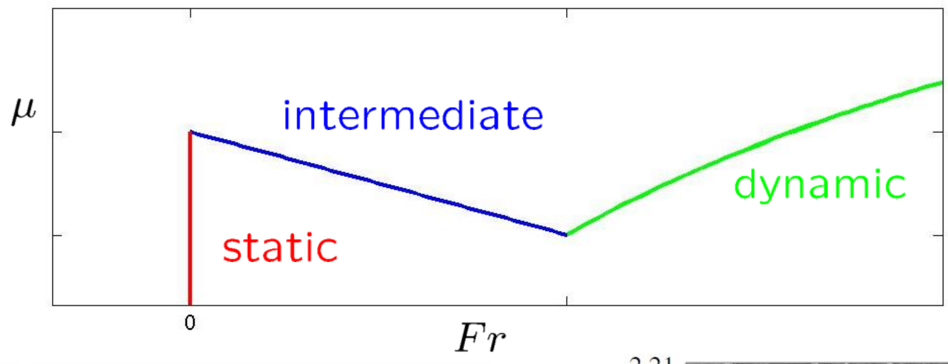


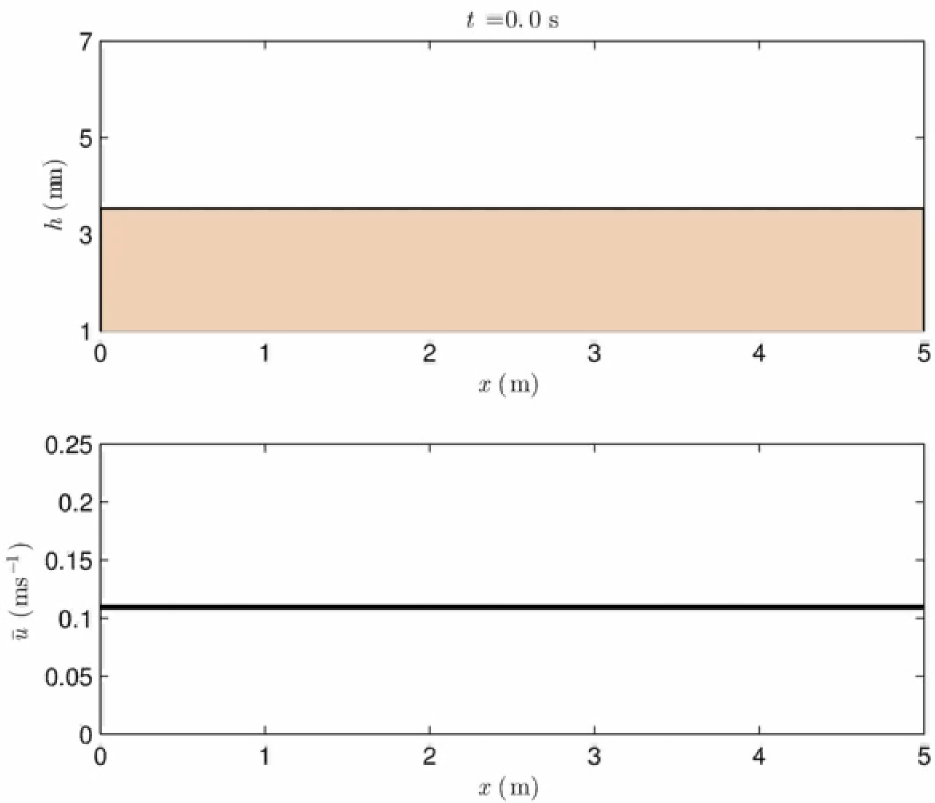


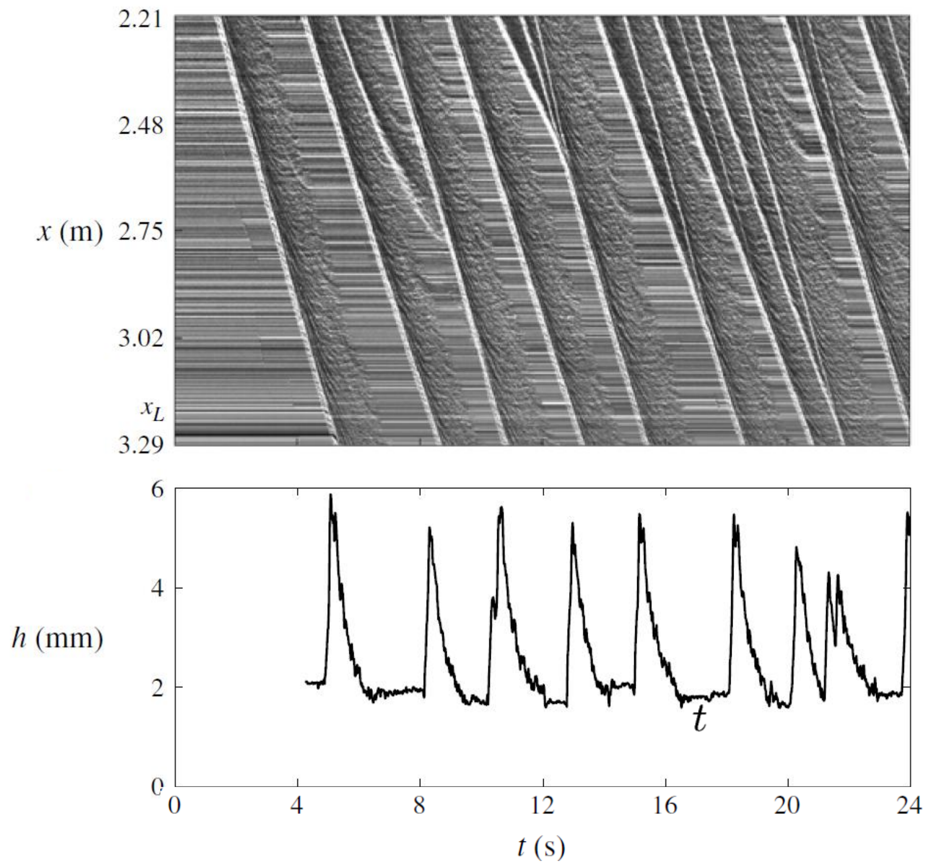


Gray & Edwards (2014) *J. Fluid Mech.* **755**, 503-534.
Pouliquen & Forterre (2002) *J. Fluid Mech.* **453**, 133-151.

(c) Frictional hysteresis leads to erosion-deposition waves

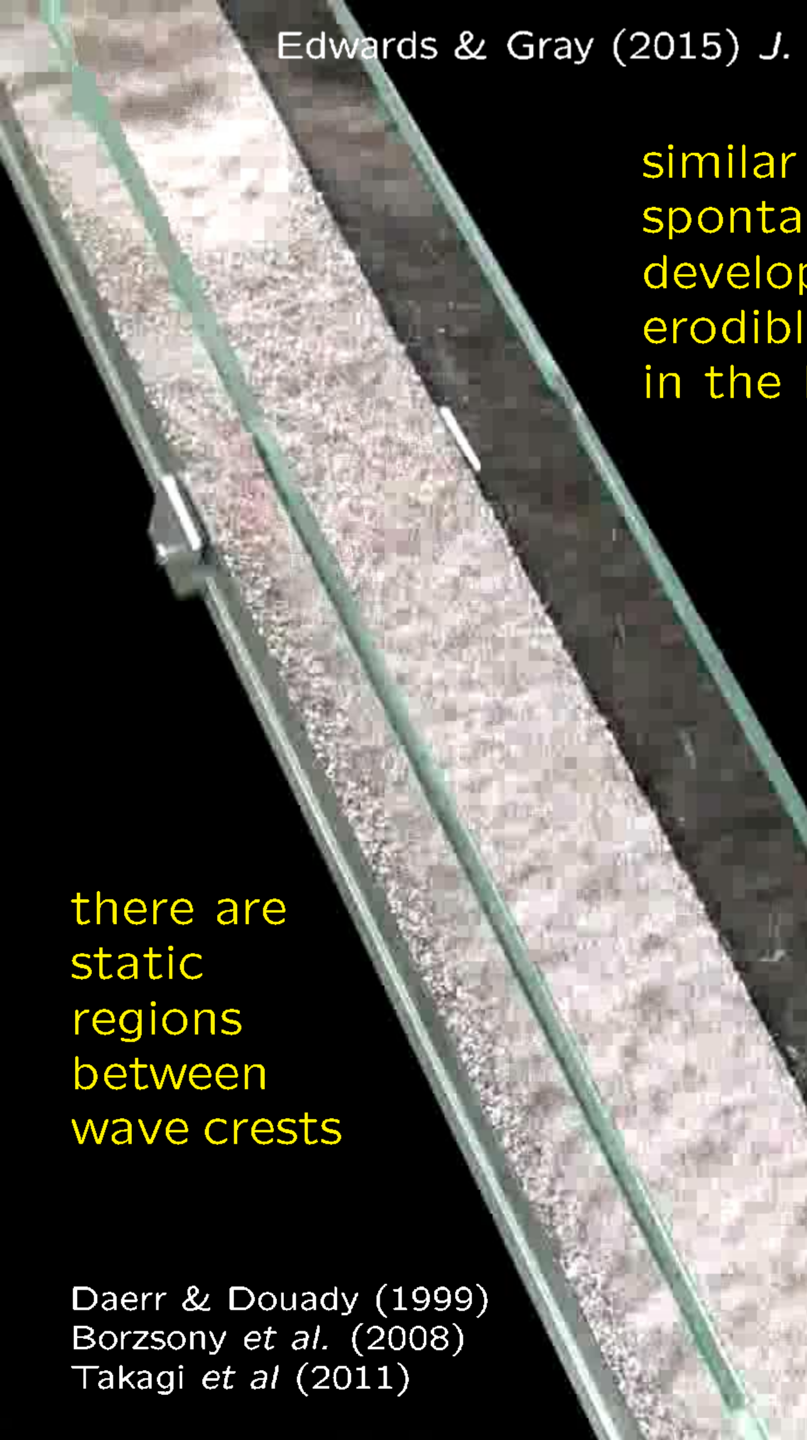






Edwards & Gray (2015) *J. Fluid Mech.* **762**, 35-67.
Pouliquen & Forterre (2002) *J. Fluid Mech.* **453**, 133-151.

Edwards & Gray (2015) *J. Fluid Mech.* **762**, 35–67.



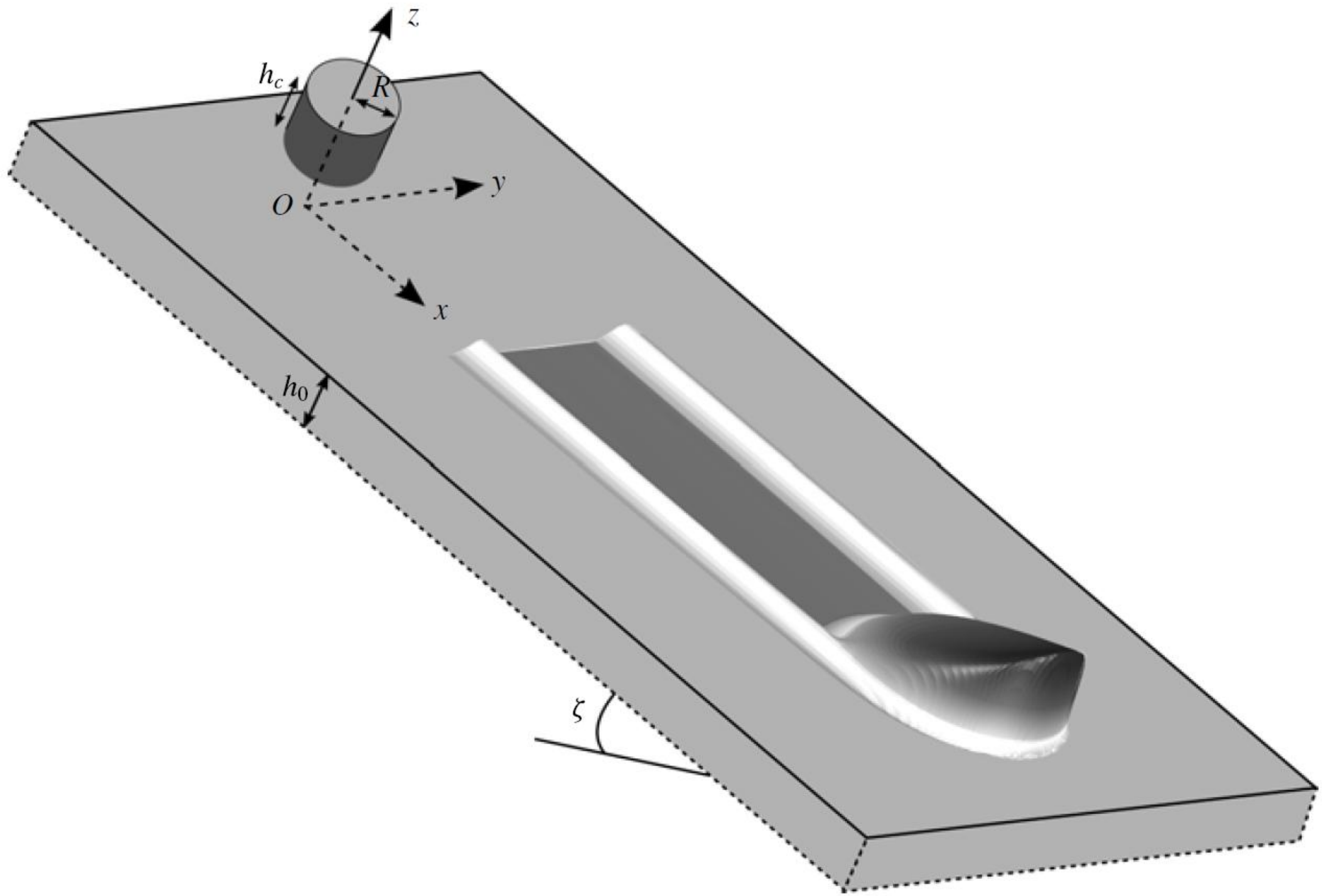
similar waves spontaneously develop on erodible beds in the lab

there are static regions between wave crests

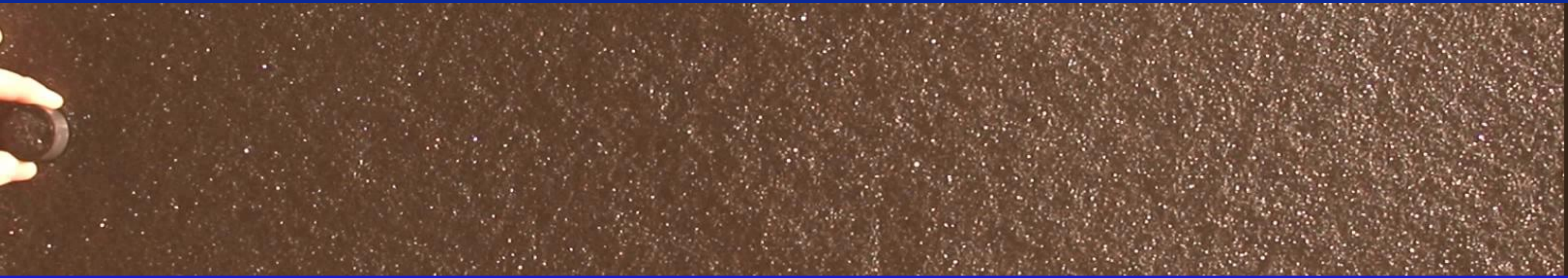
Daerr & Douady (1999)
Borzsony *et al.* (2008)
Takagi *et al* (2011)

Erosion-deposition waves in debris flows

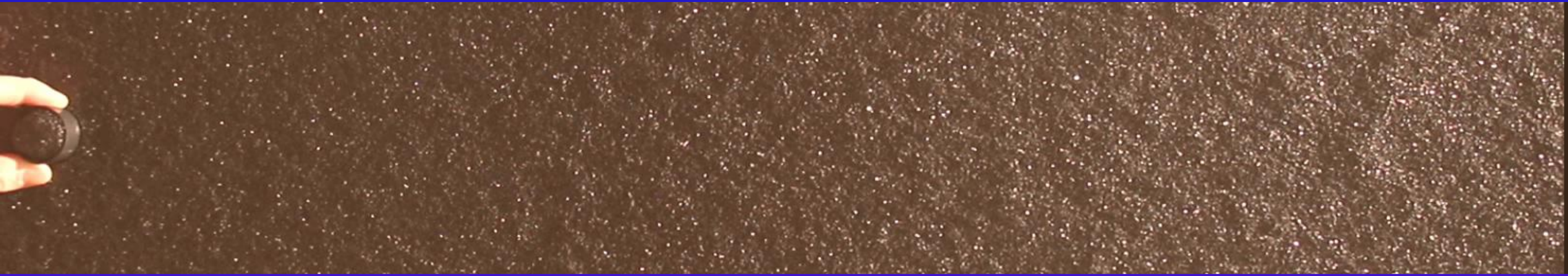
- 15^{th} Oct 2000 an unintentional release of 150 000 m^3 water led to a debris flow in Fully Switzerland that had regular surges



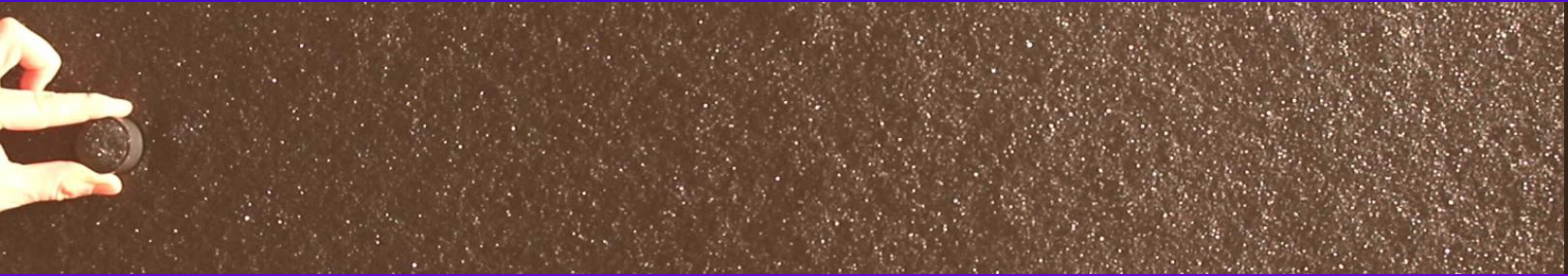
Edwards *et al.* (2017) Formation of levees, troughs and elevated channels by avalanches on erodible slopes, *J. Fluid Mech.* **823**, 278–315.



Growth

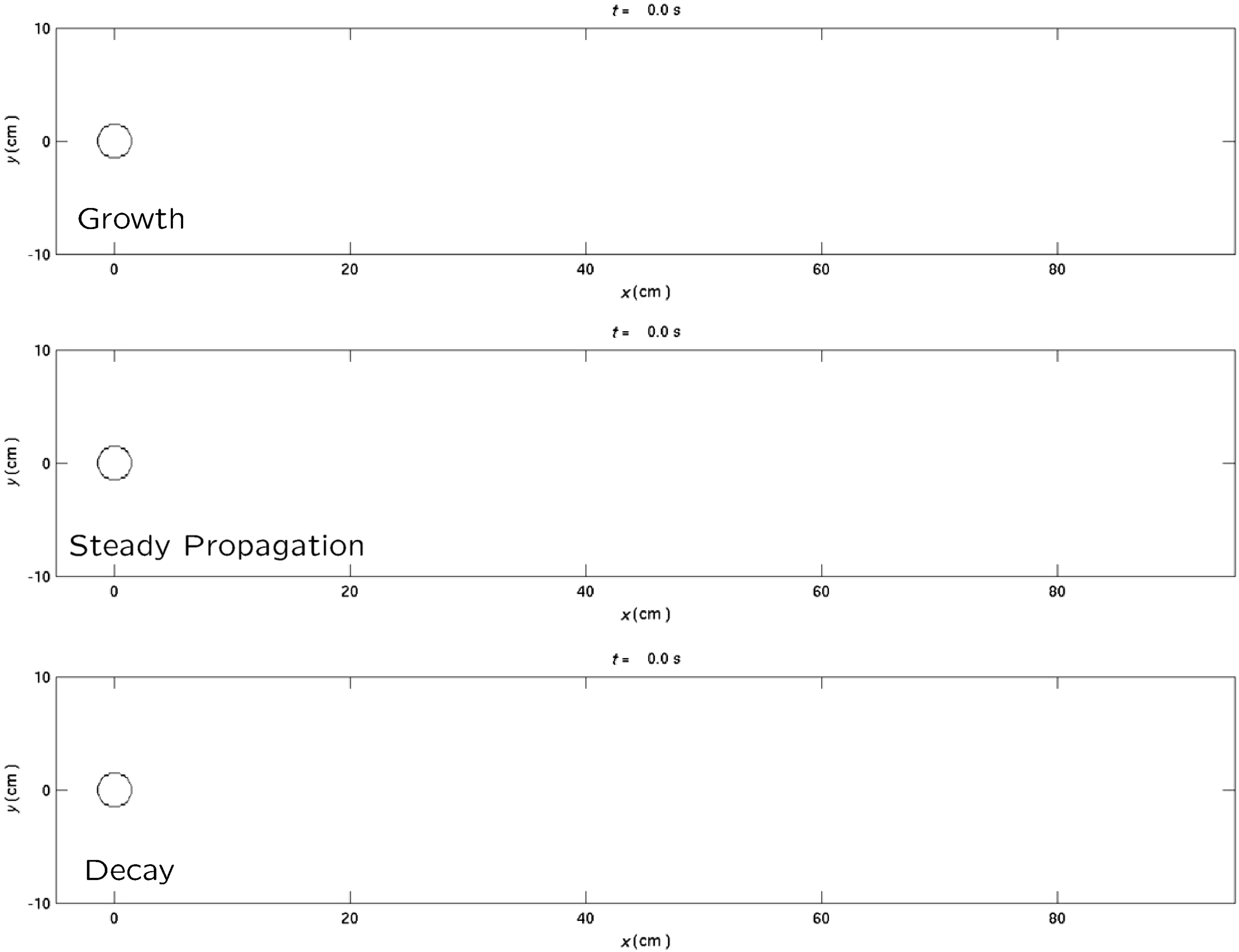


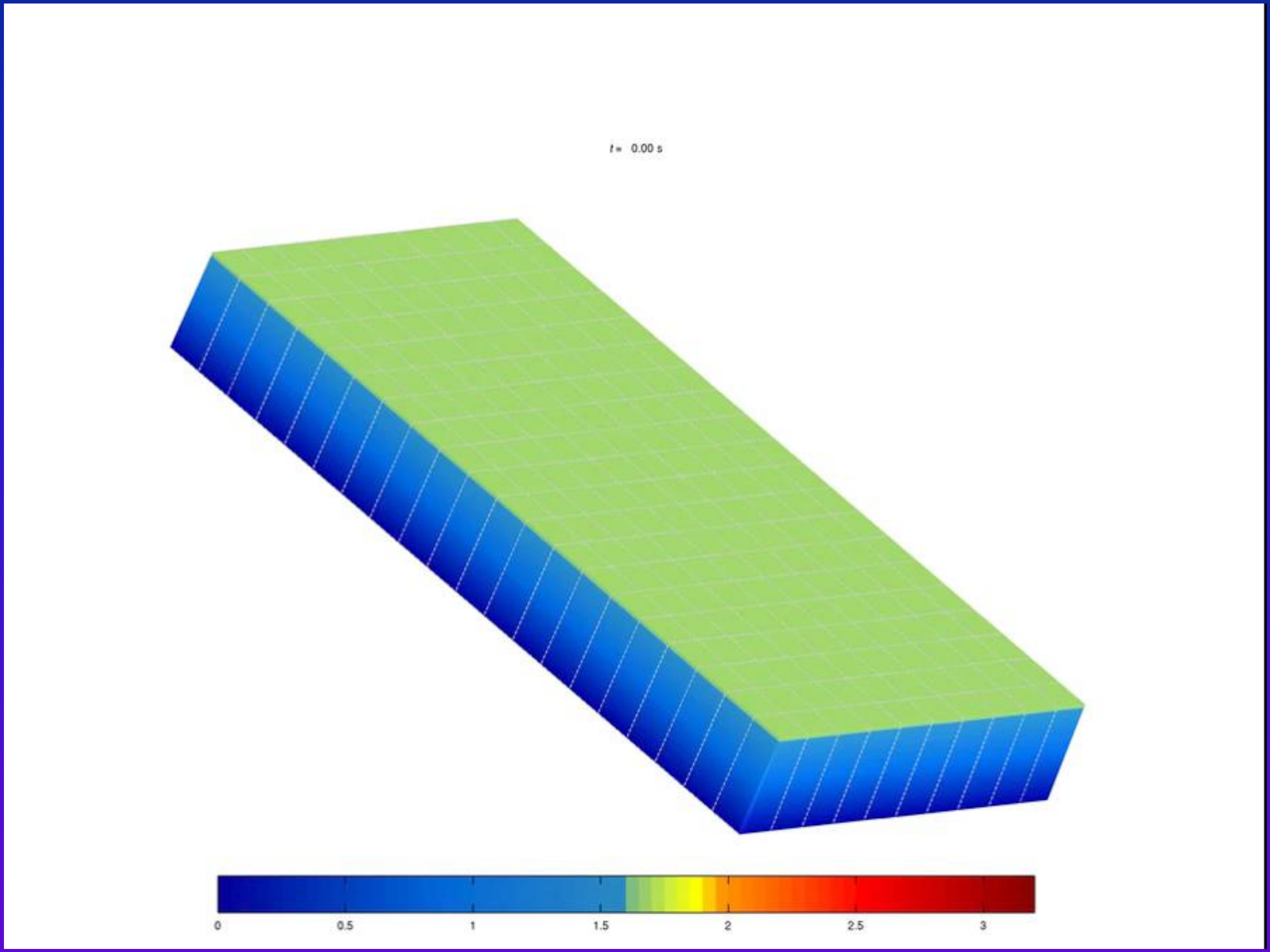
Steady Propagation



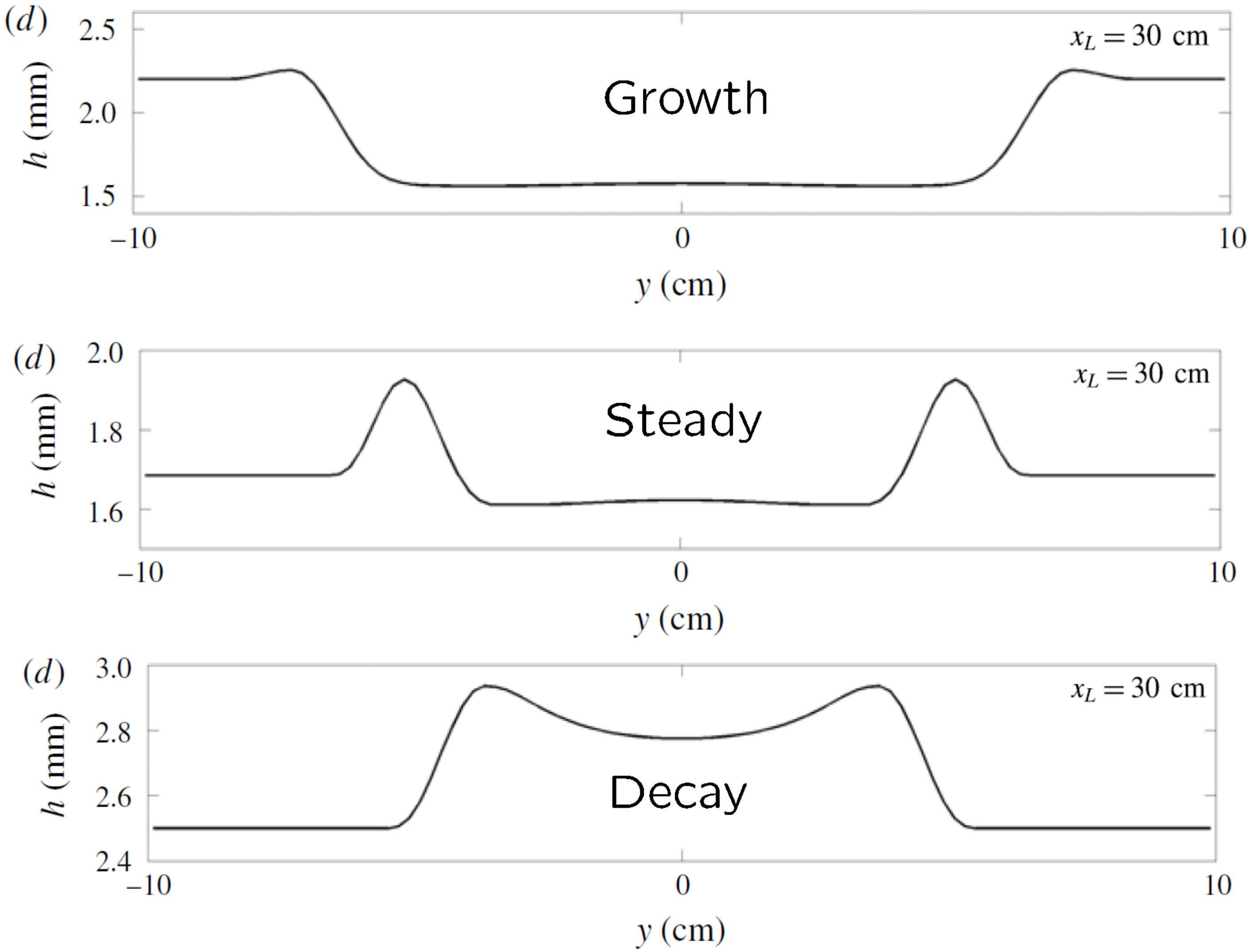
Decay

Edwards *et al.* (2017) Formation of levees, troughs and elevated channels by avalanches on erodible slopes, *J. Fluid Mech.* **823**, 278–315.


t = 0.0 s
10
0
-10
y (cm)
Growth
0
20
40
60
80
x (cm)
t = 0.0 s
Steady Propagation
t = 0.0 s
Decay



Edwards *et al.* (2017) Formation of levees, troughs and elevated channels by avalanches on erodible slopes, *J. Fluid Mech.* **823**, 278–315.



Edwards *et al.* (2017) Formation of levees, troughs and elevated channels by avalanches on erodible slopes, *J. Fluid Mech.* **823**, 278–315.

PHYSICS TODAY Identifying avalanche tipping points BY RACHEL BERKOWITZ