Shearing dry granular materials: What is the phase diagram?

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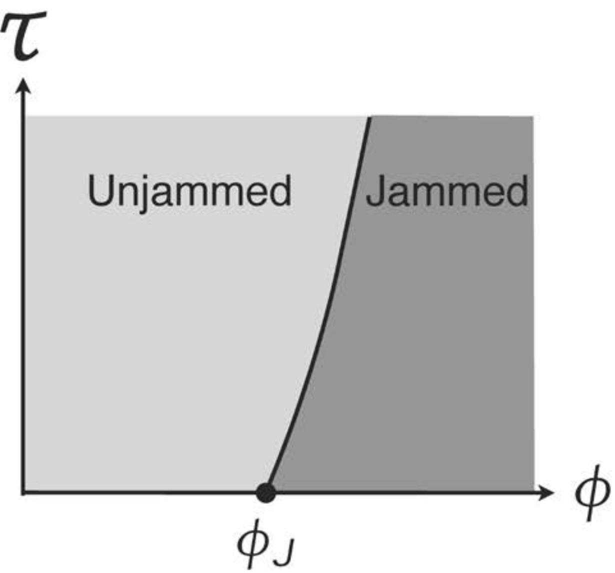
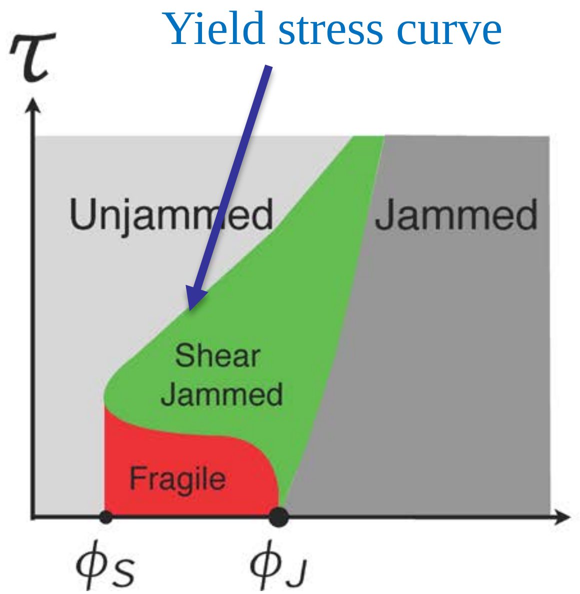
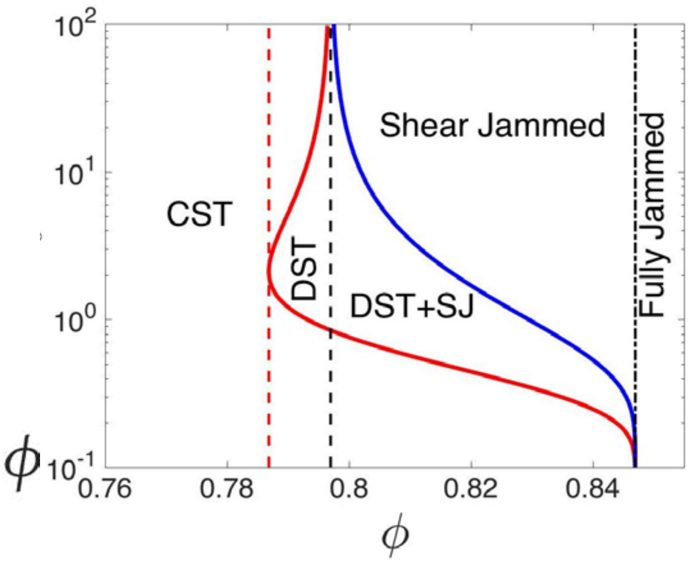
A Tale of Three Figures
Shear stress τ vs. packing fraction φ



Frictionless spheres
static



Frictional spheres
static



Suspensions
dynamic

How do granular materials respond to shear?

- Background
  - Observations from shearing
  - Particles: elastic (soft) and frictional
  - Force networks and protocols
  - Experimental techniques
  - Results from isotropic compression
- Shear jamming—the ‘bottom’ of the jamming phase diagram
- At the Yield Stress Curve—the ‘top’ of the Jamming Phase Diagram

Shear strain applied to granular materials can jam an initially stress-free state. Continued shear drives the system to the yield stress curve

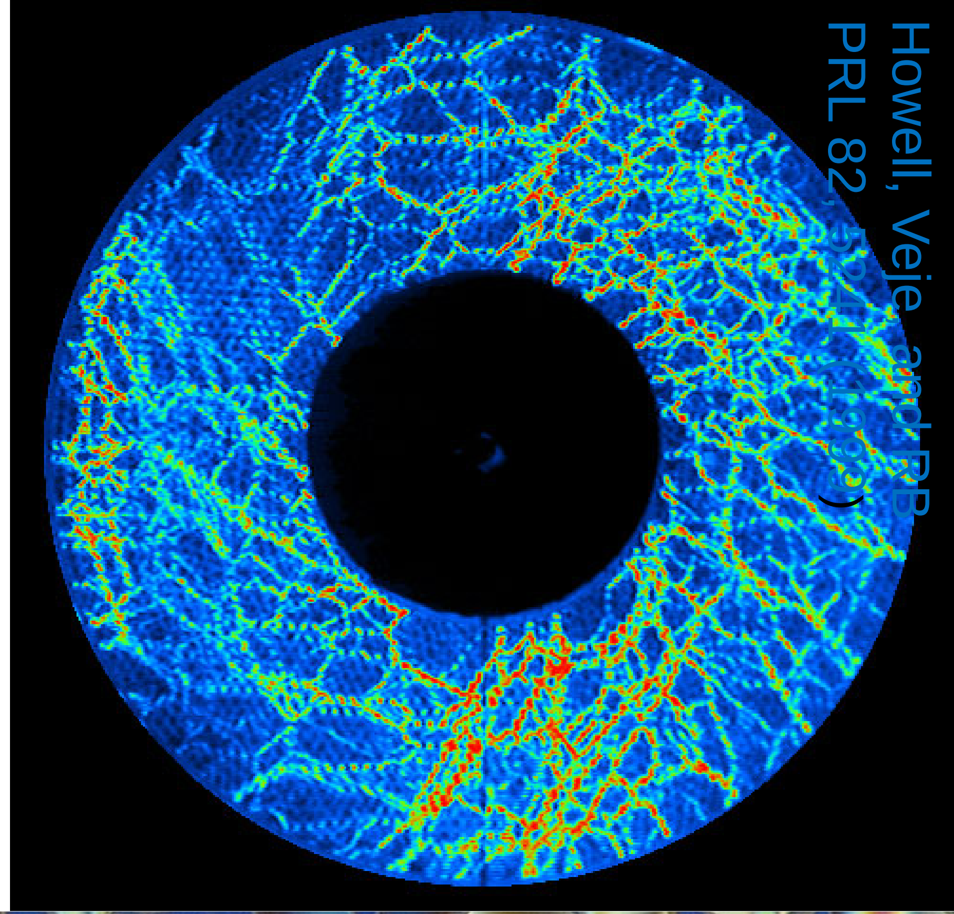
- What is the macroscopic state diagram? Includes fragile, shear jammed and dynamic states at the YSC
- The initial processes leading to shear jamming generate anisotropic networks, called force chains. How should one characterize/distinguish networks?
- What are the statistical properties at the YSC?
- At a yet smaller scale, what processes enable the formation of force chains under shear?
- Do these processes lead to memory? If so, how?
- Experiments to answer these questions require new techniques

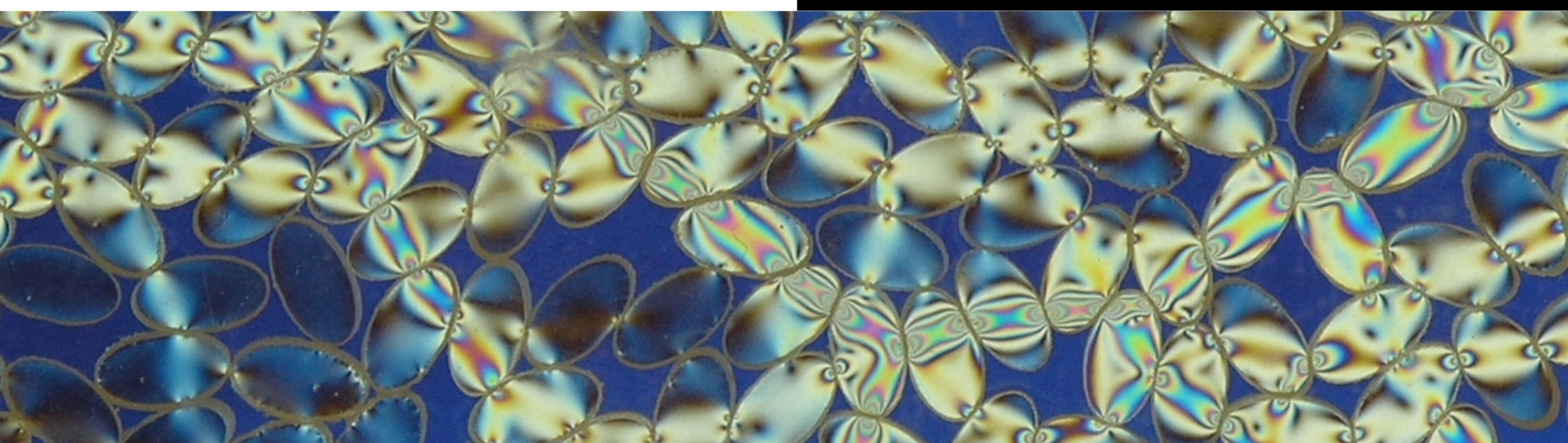
Granular Material:Dense Phases, particularly sheared, frictional

Forces are carried preferentially on force chains (**Networks**) →multiscale phenomena—grains to system

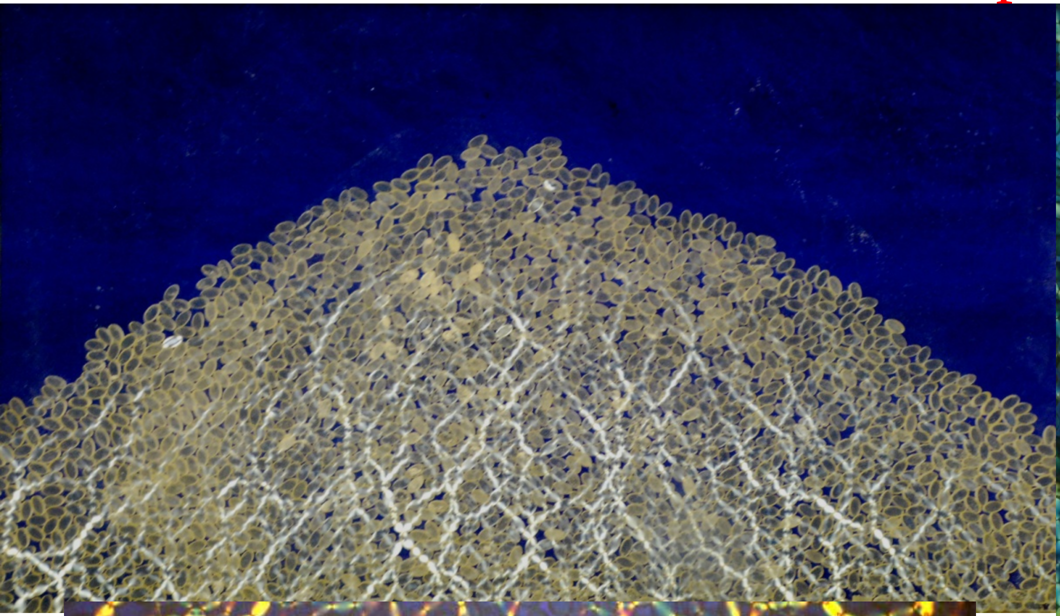
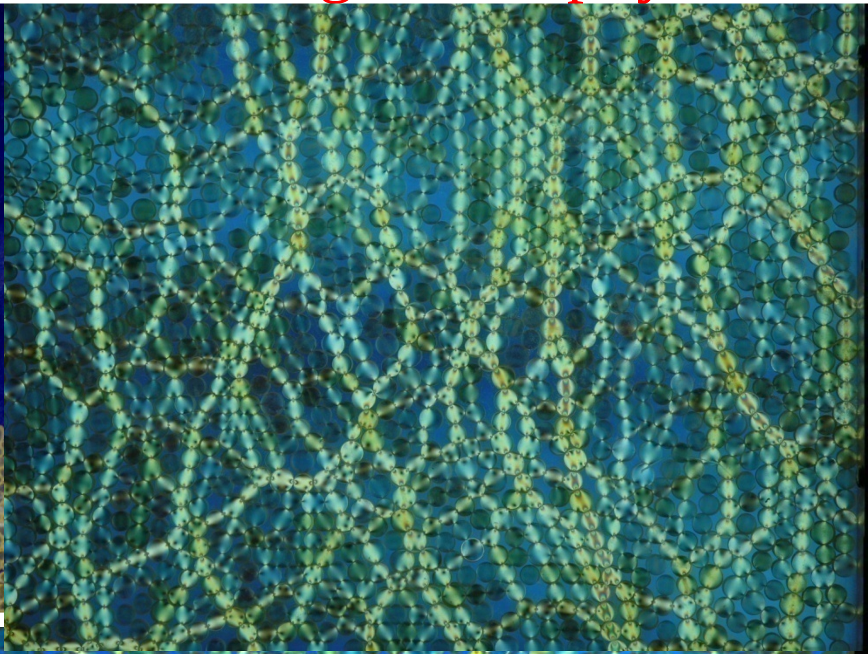
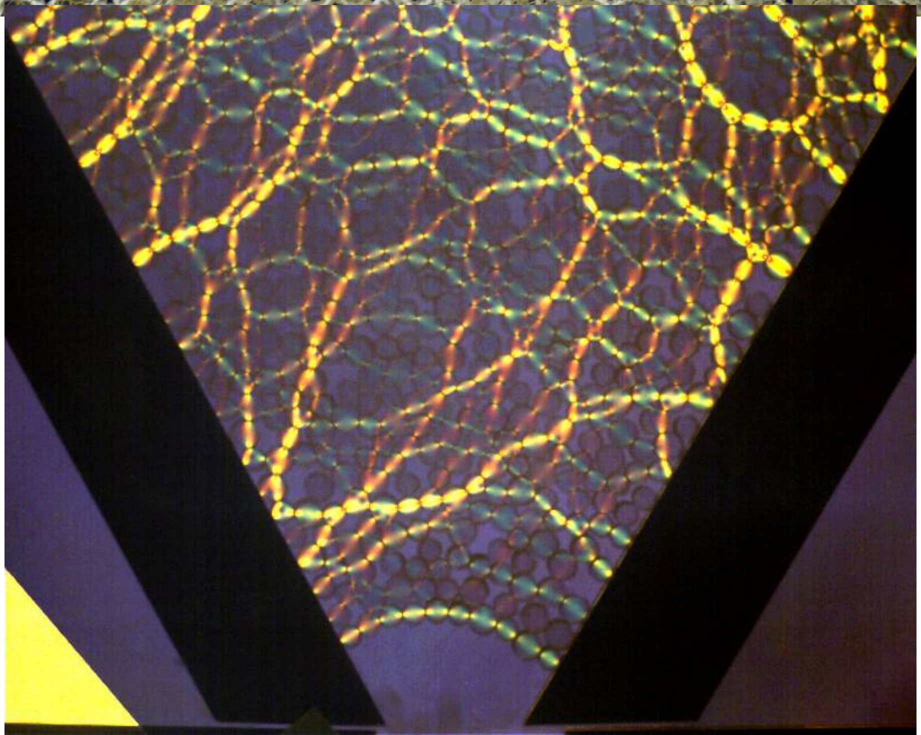
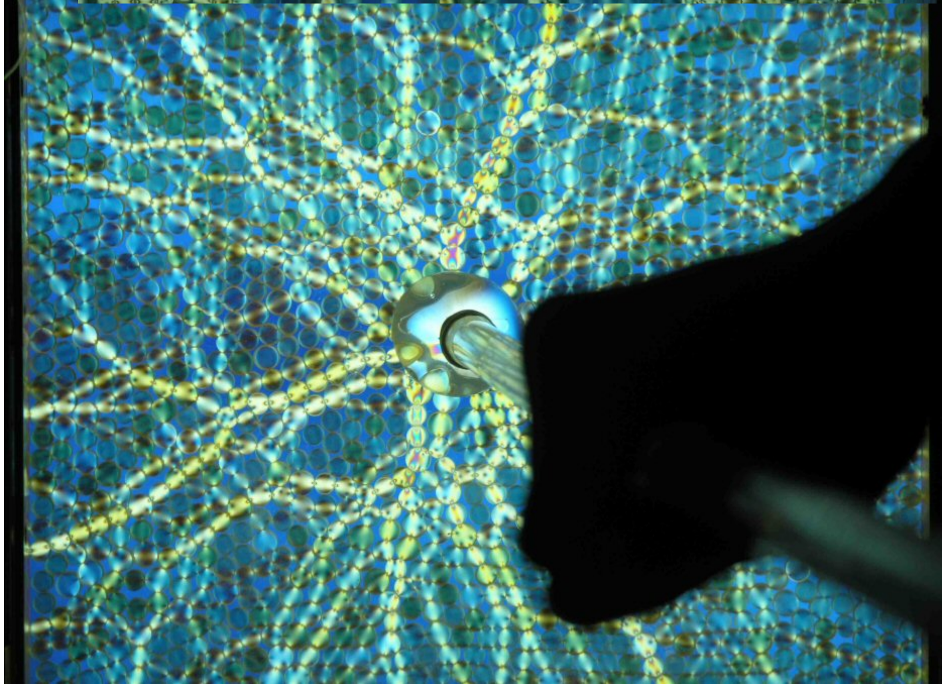
Deformation leads to large spatio-temporal fluctuations

Granular materials jam —fluid ← →solid transition (Howell, P&G1997, PRL 1999)

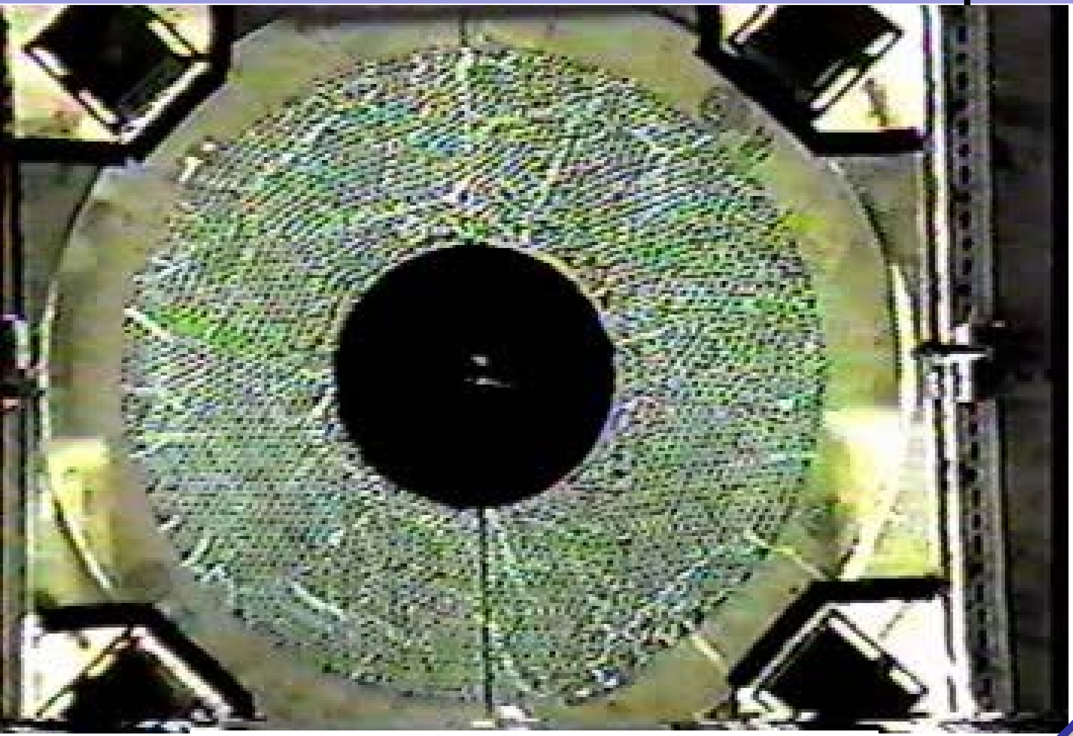


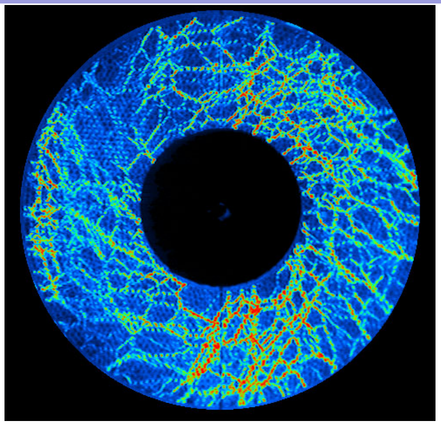


Force networks are an essential part of dense granular physics

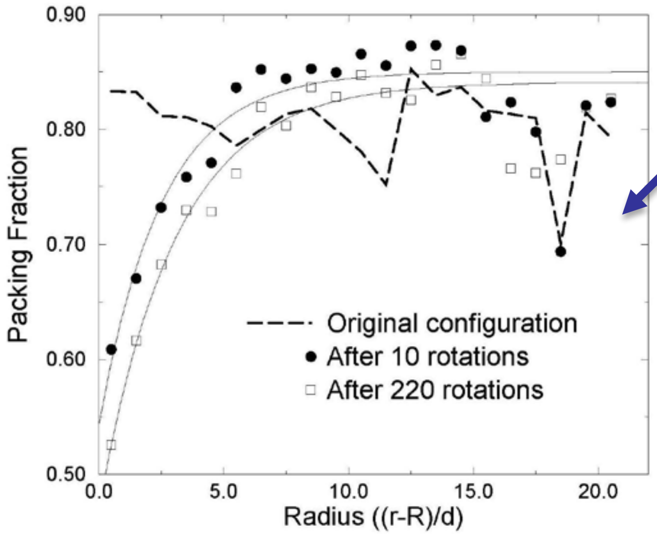





Networks evolve in space-time—are long range and complex *Howell, PRL 1999, Veje PRE 1999*

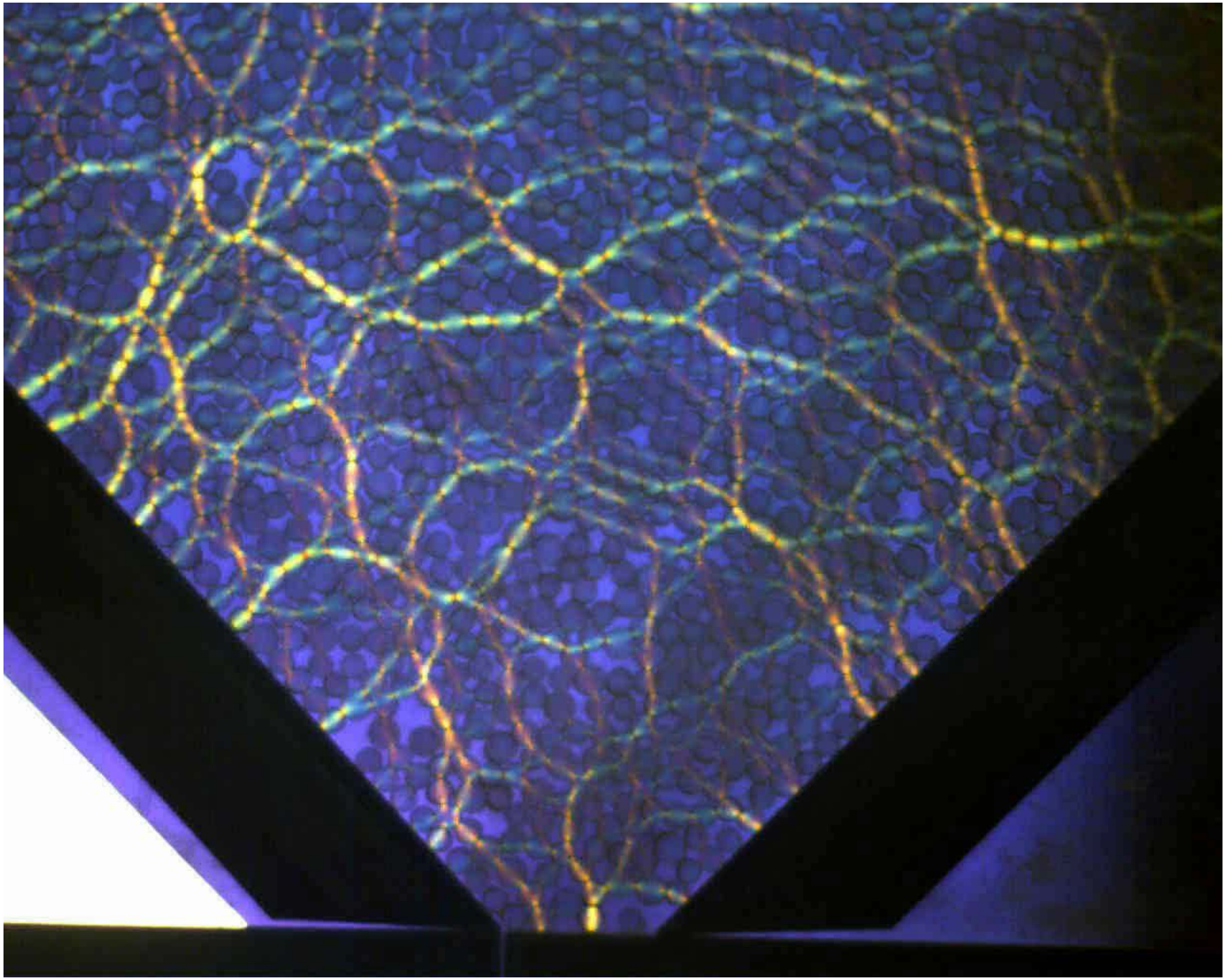




A shear band is a narrow zone of intense shearing strain with reduced local packing fraction—common issue

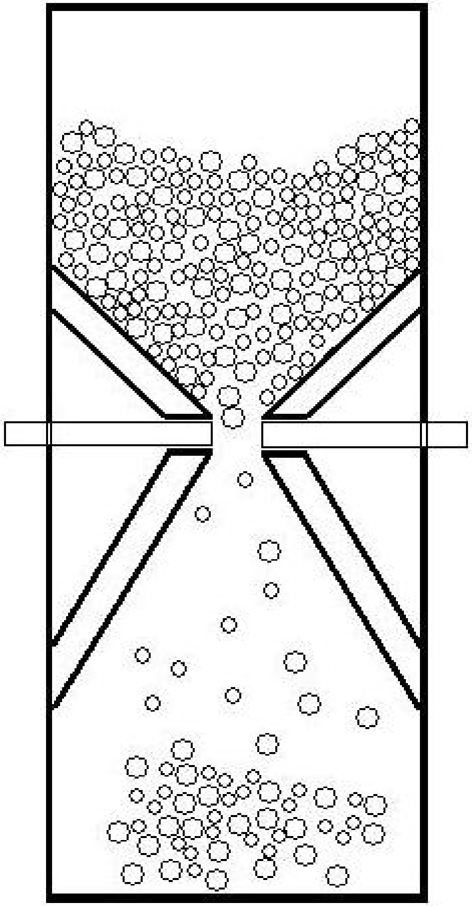


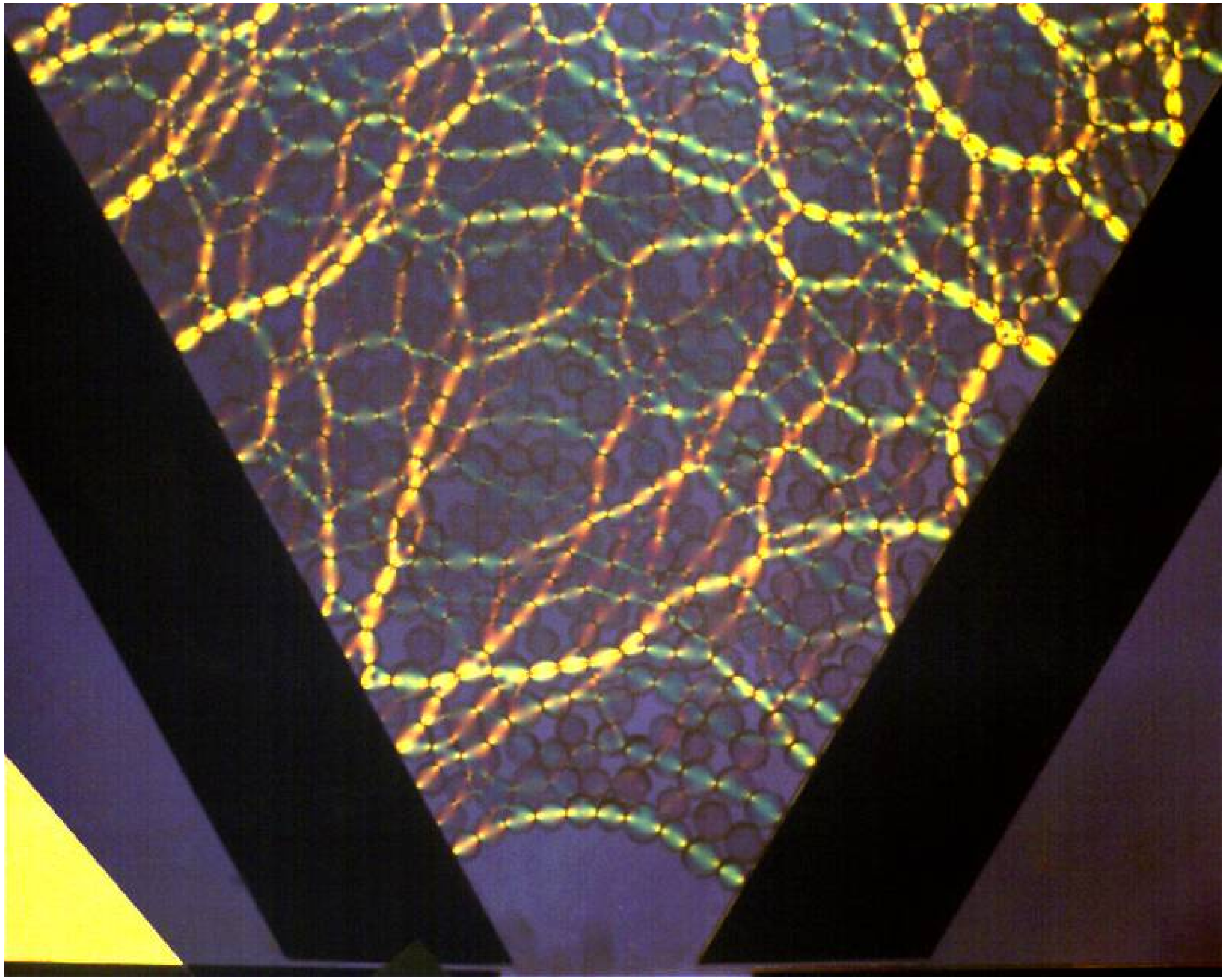
Force networks appear dynamically: formation of force chains arches at outlet leads to clogging (J. Tang & RB-EPL-2016)



One frame, showing jam and force chain arch

2D hopper flow

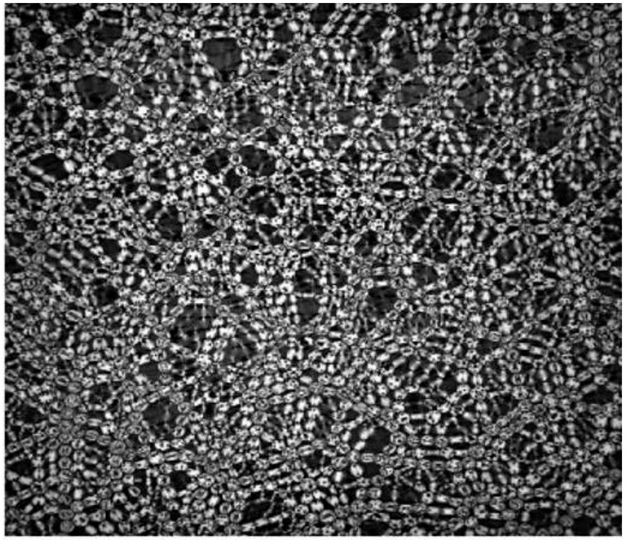




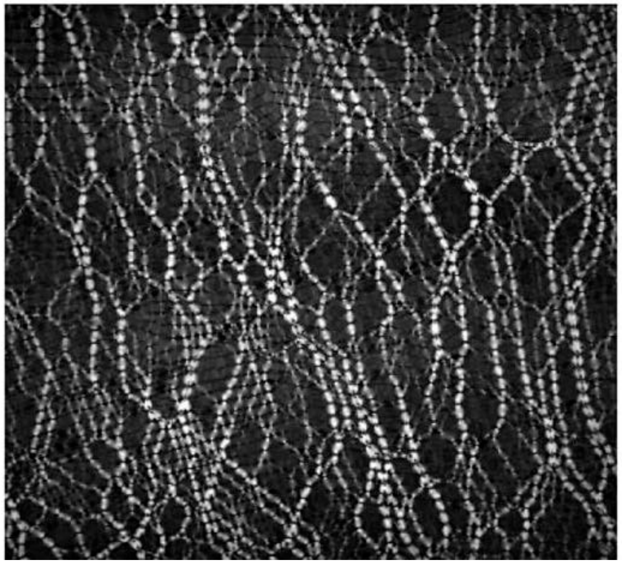
Particle properties for this discussion

- Particles interact when they are in contact—no contact no force
- Particles interact by elastic normal forces and tangential frictional forces
- Normal force, F_n depends on the distance δ by which two particles have been pushed together (overlap)—$F_n \sim \delta^{\alpha}$…
  $\alpha = 1, 3/2$ for Hookean and Hertzian contacts resp.
- Grains typically have friction, coefficient μ…friction forces do not depend on inter-grain positions -> no potential energy—large particle size -> athermal

Relation of force networks to protocols—e.g. compression or shear



Isotropic Compression

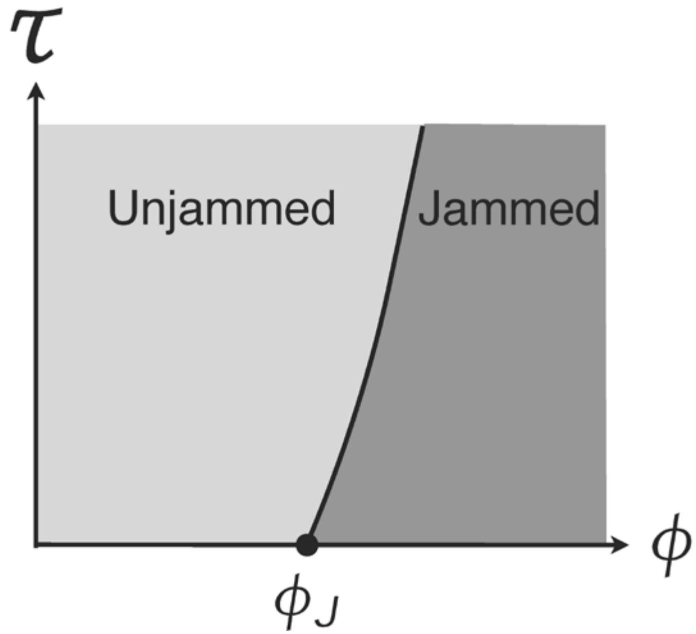


Pure Shear

T. Majmudar and BB, Nature 2005

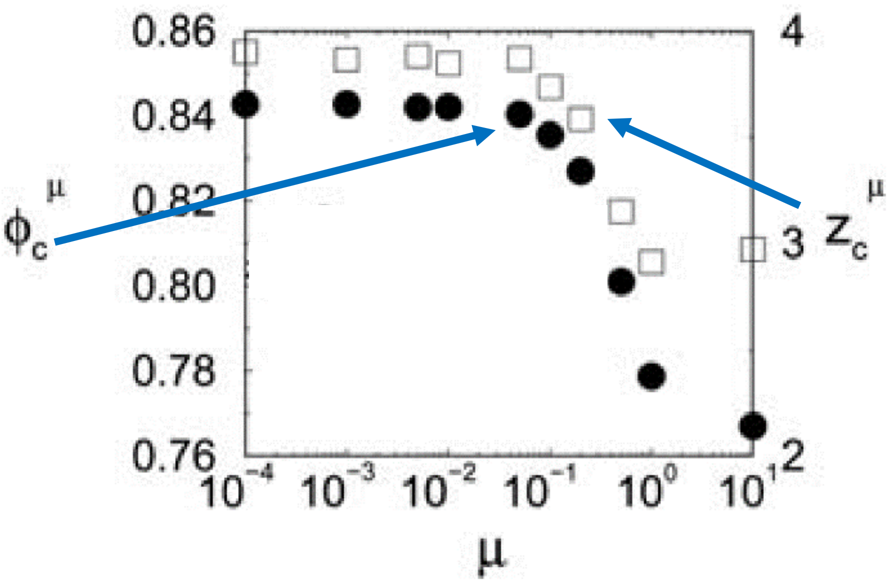
Isotropic jamming of spheres/discs

Schematic of jamming diagram for frictionless spheres



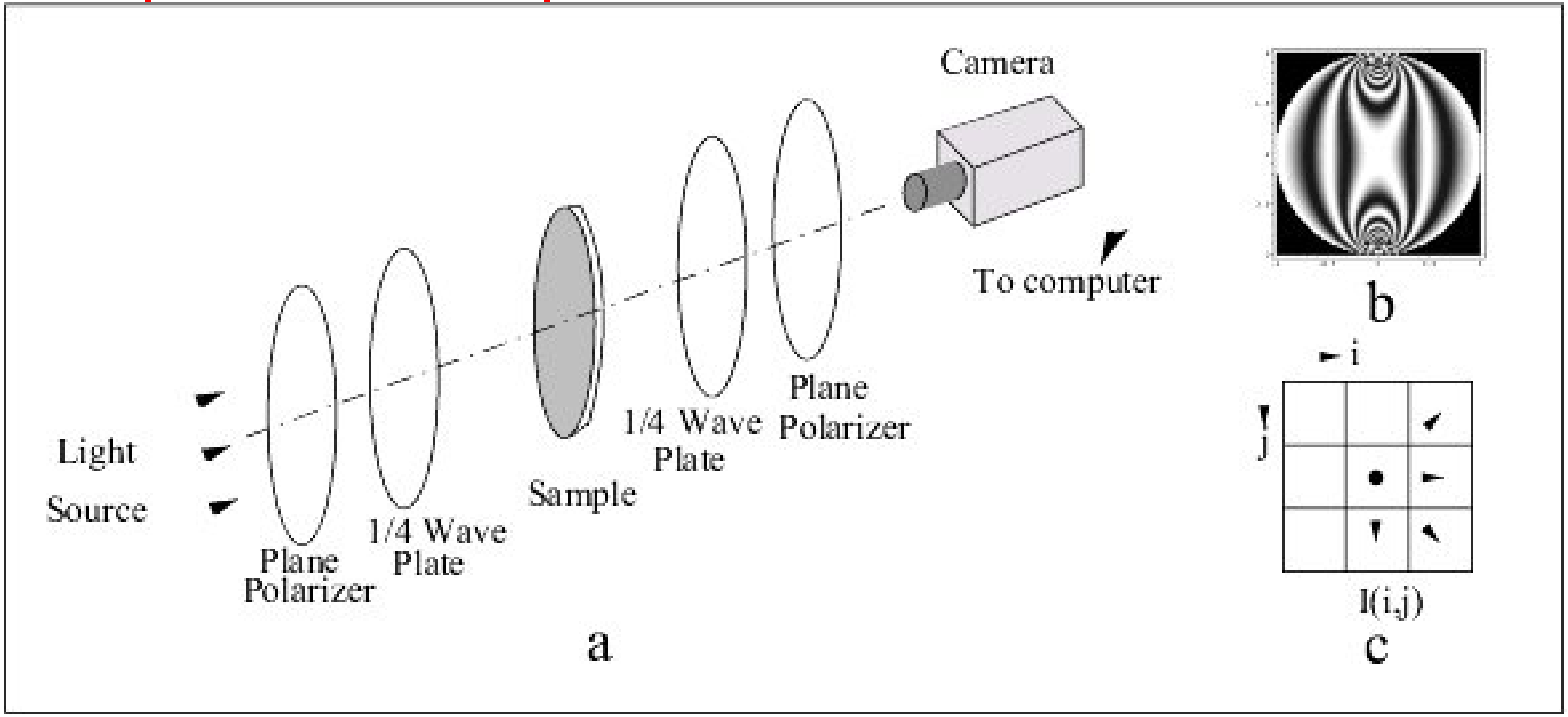
O'Hern et al. PRE 2003

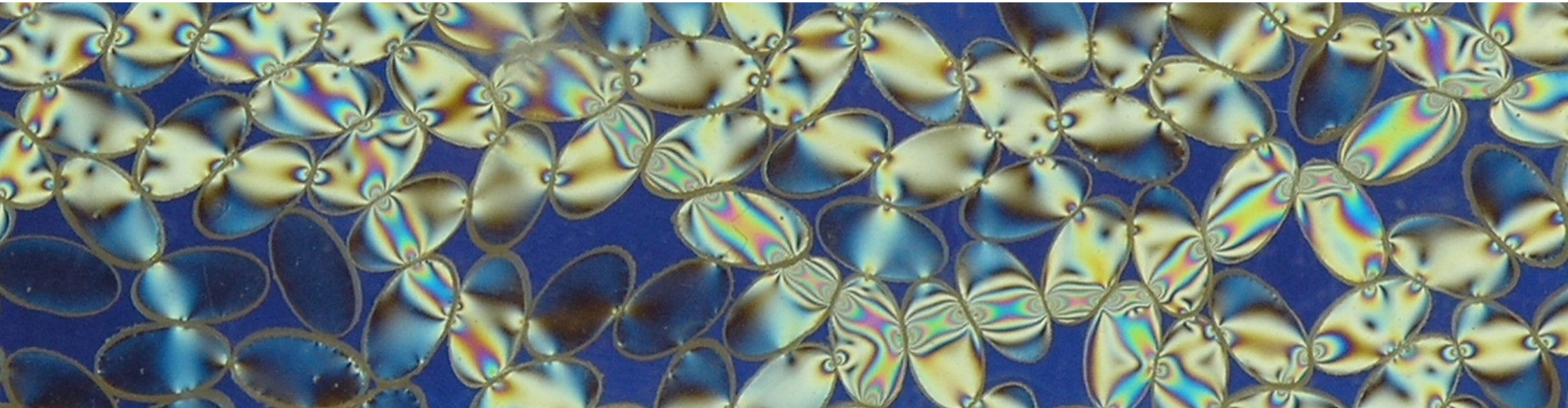
Schematic of jamming diagram for frictional discs



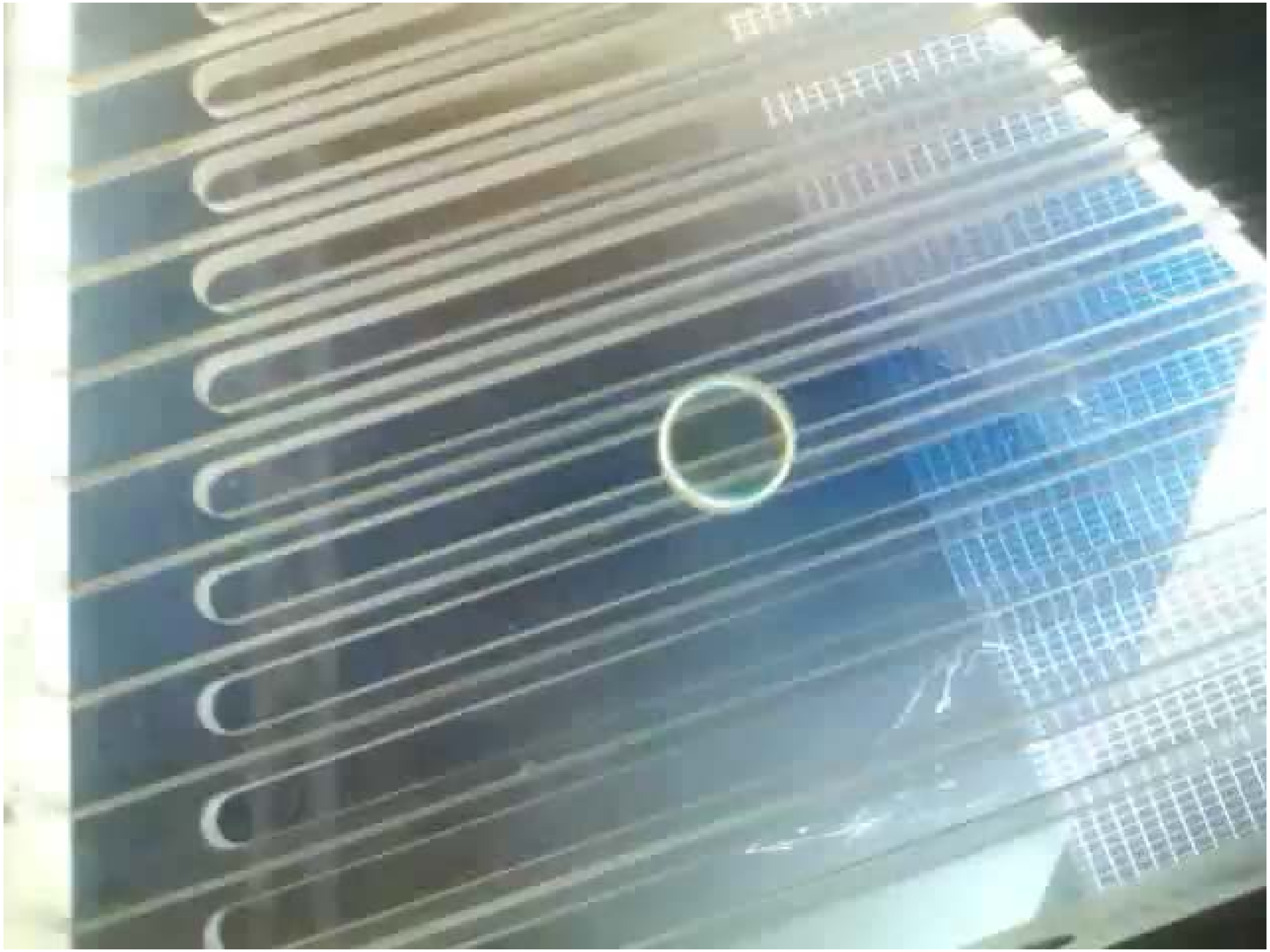
O'Hern et al. PRE 2003

Measuring contact forces by photoelasticity—2D quantiative experiments from smallest scales





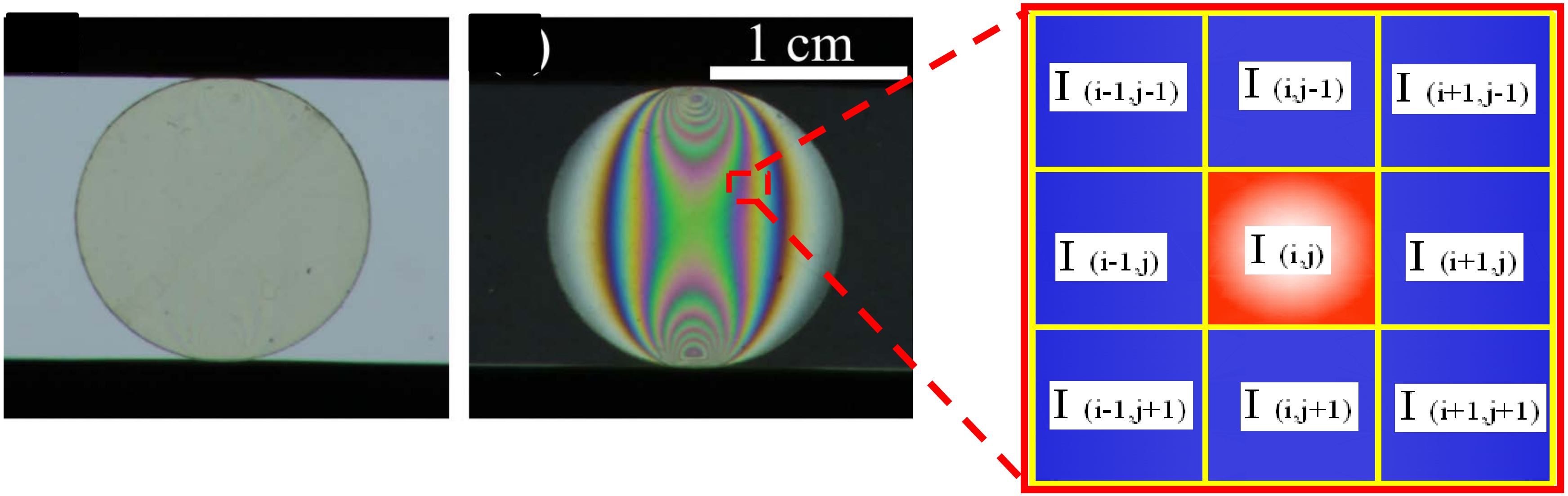
Fun with photoelasticity*



*Joshua Dijksman

Experimental advances allow grain-scale force measurements--I

D. Howell, BB, PRL 1999, PRE 1999



$$\nabla I_{i,j}^2 = \frac{1}{4}\left[\left(\frac{I_{i+1,j} - I_{i-1,j}}{2}\right)^2 + \left(\frac{I_{i,j+1} - I_{i,j-1}}{2}\right)^2 + \left(\frac{I_{i+1,j+1} - 1 - I_{i-1,j-1}}{2}\right)^2 + \left(\frac{I_{i+1,j-1} - I_{i-1,j+1}}{2}\right)^2\right]$$

$$< G^2 > = \frac{1}{N}\sum_{i,j} \nabla I_{i,j}^2$$

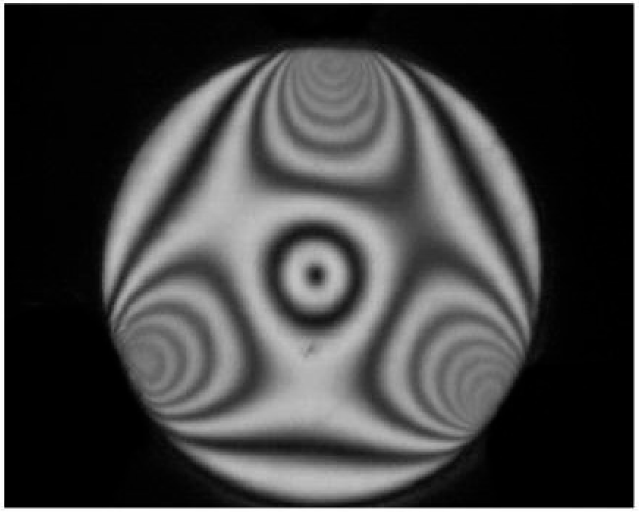
Experimental Advances allow grain-scale contact force measurements--II

T. Majmudar and BB
Nature, 2005

- Contact forces determine exact photoelastic pattern:
- Contact forces → stresses within disk (linear elasticity)
- Planar stresses give pattern:

$$I = I_o \sin^2[(\sigma_2 - \sigma_1)CT/\lambda]$$

c

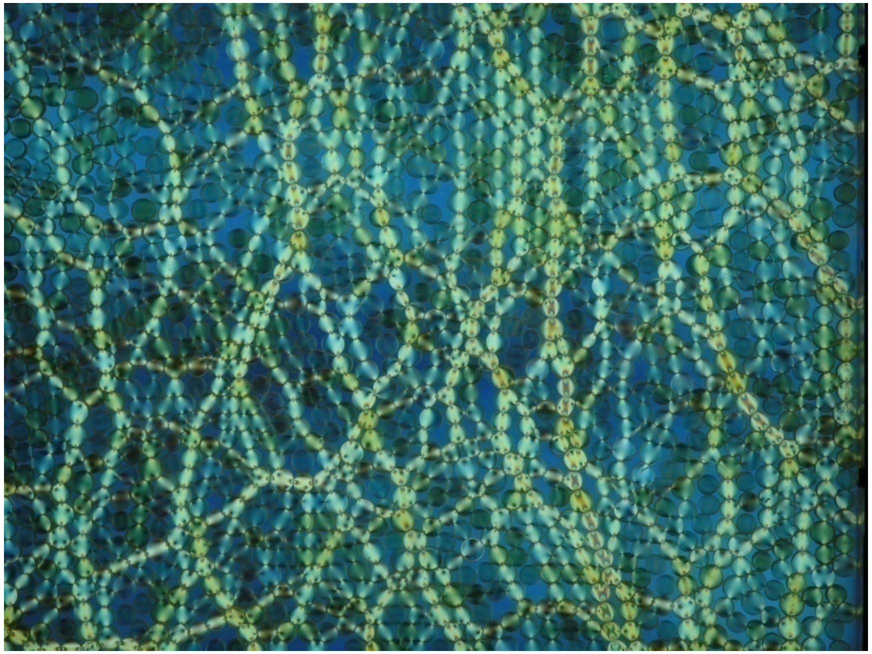


Technique for finding 2D contact forces

- Process images to obtain particle centers and contacts
- Exact solution for stresses (biharmonic equation) has contact forces as parameters
- Make a nonlinear fit to photoelastic pattern using contact forces as fit parameters
- $I = I_o \sin^2[(\sigma_2 - \sigma_1)CT/\lambda]$
- In the previous step, invoke force and torque balance to reduce unknown contact forces
- Newton’s 3d law provides error checking

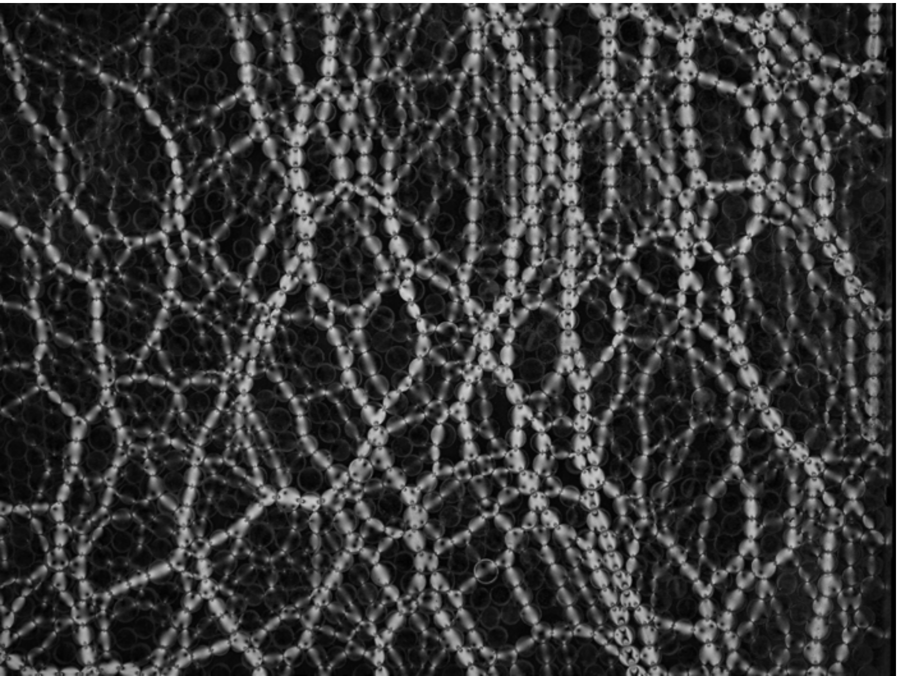
Key new approach: obtain grain contact forces

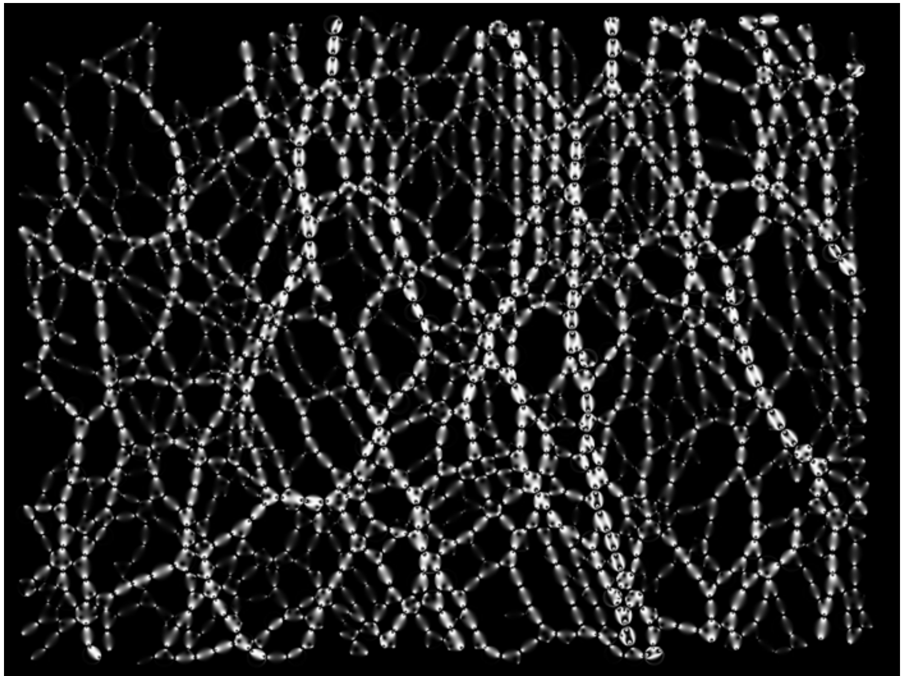
Experiment--raw



Experiment
Color filtered

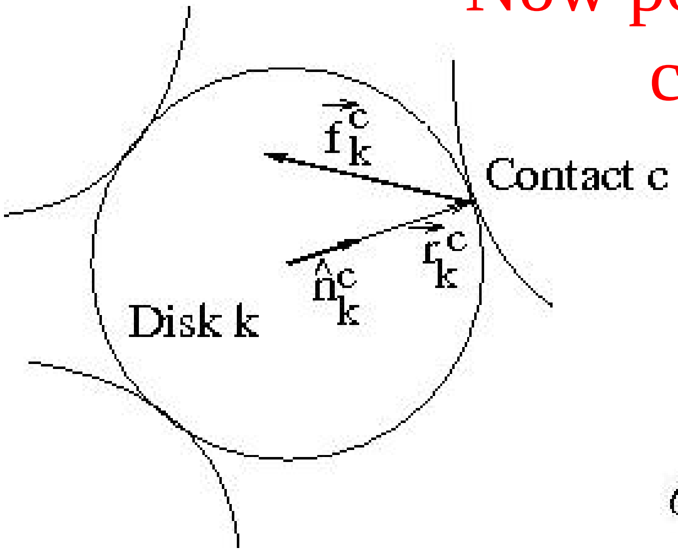
Reconstruction
From force
inverse algorithm





Obtaining stresses and fabric from experimental data

Now possible to obtain direct experimental characterizations at grain scale

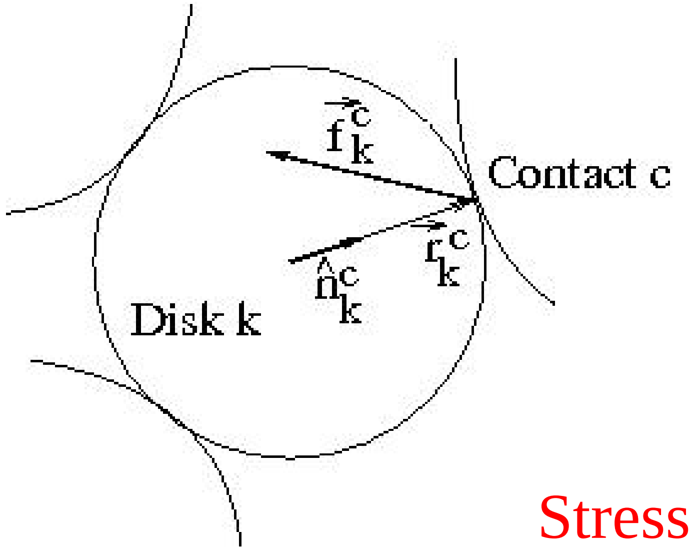


$$\hat{\sigma} = \frac{1}{V} \sum_{i \neq j} \vec{r}_{ij} \otimes \vec{f}_{ij},$$ Stress

$$\hat{R} = \frac{1}{N} \sum_{i \neq j} \frac{\vec{r}_{ij}}{\|\vec{r}_{ij}\|} \otimes \frac{\vec{r}_{ij}}{\|\vec{r}_{ij}\|},$$ Fabric

These quantities can be coarse-grained to produce continuum fields

Stresses, fabric, force moment tensor—2D
evaluate across scales: particles, networks, system



Fabric tensor

$R_{ij} = \Sigma_{k,c}\, n^c_{ik}\, n^c_{jk}$

$Z = \text{trace}[R]$

Stress tensor, force moment tensor

stress: $\sigma_{ij} = (1/A)\, \Sigma_{k,c}\, r^c_{ik}\, f^c_{jk}$

Force moment $\Sigma_{ij} = \Sigma_{k,c}\, r^c_{ik}\, f^c_{jk} = A\, \sigma_{ij}$

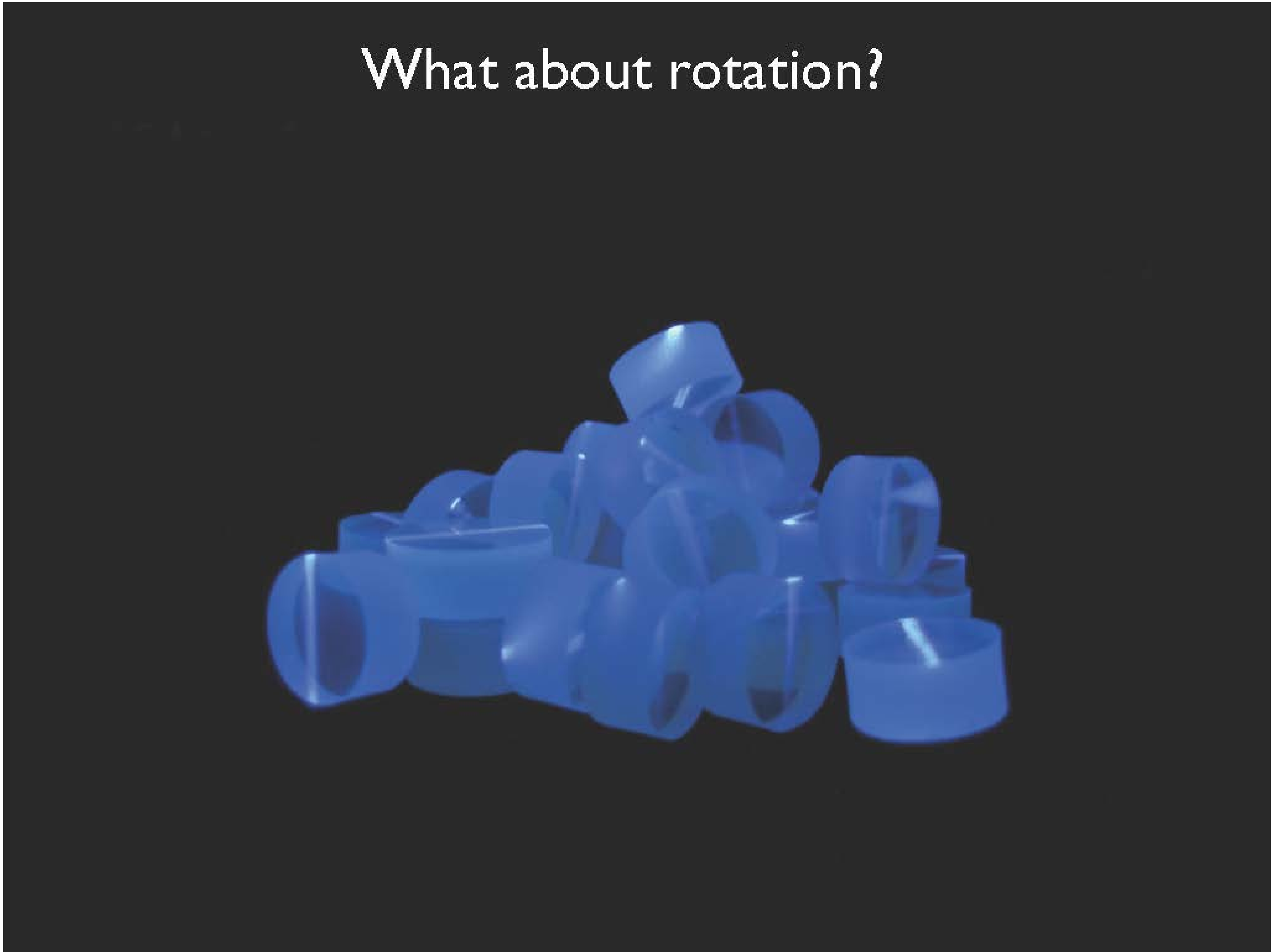
A is particle/system area

Pressure, P and shear stress

$P = \text{Tr}\,(\sigma)/2 = (\sigma_2 + \sigma_1)/2$

$:\tau = (\sigma_2 - \sigma_1)/2$

Displacements and rotations of grains



Track Particle: Forces/Displacements/Rotations

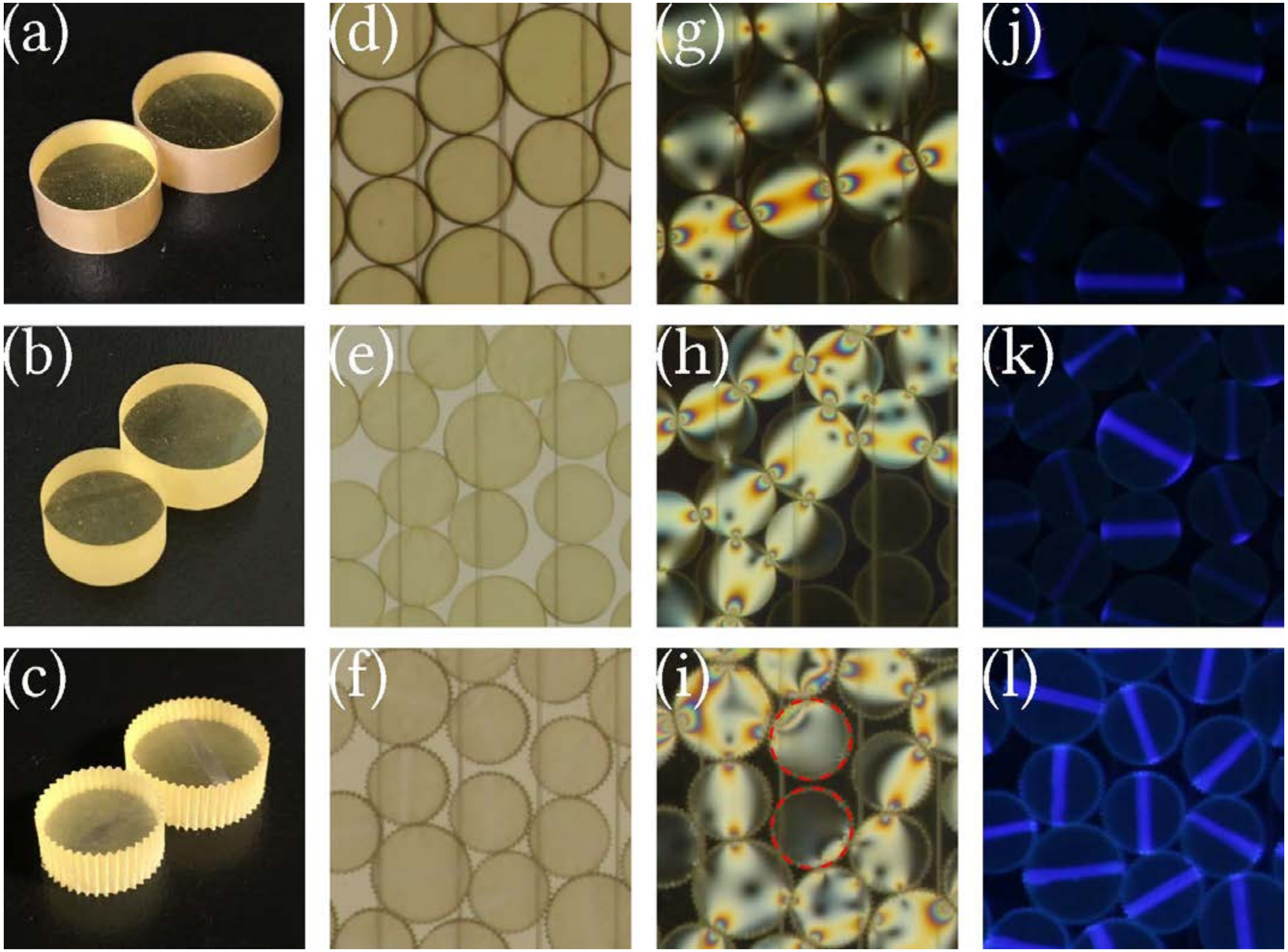
Following a small strain step we track particle displacements

$\mu = 0.15$

$\mu = 0.65$

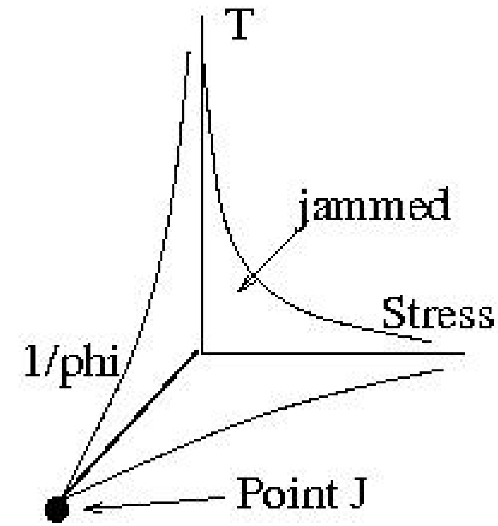
$\mu >> 1$

Under UV light bars Allow rotational tracking

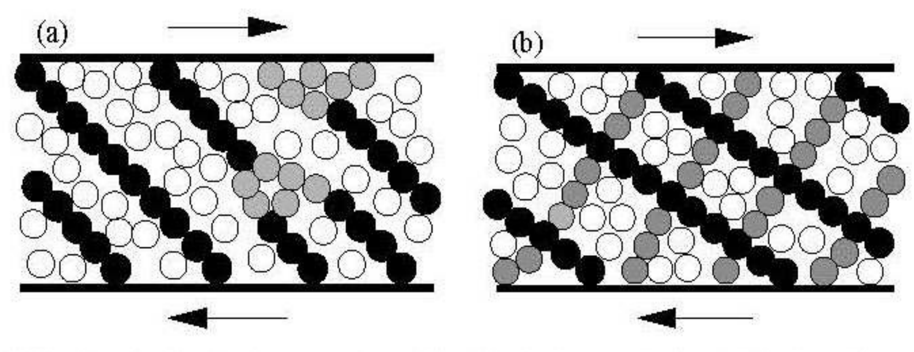


Majmudar and BB Nature, 2005; Majmudar et al. PRL 2007; Zhang et al. Gran.Matt2010; Bi, Zhang, Chacraborty, BB, Nature 2011, Ren et al. PRL 2013, Zheng et al. EPL 2014; Clark et al. PRL 2015; Cox et al. EPL 2016, Barés et al. PRE 2017, Wang et al. 2018

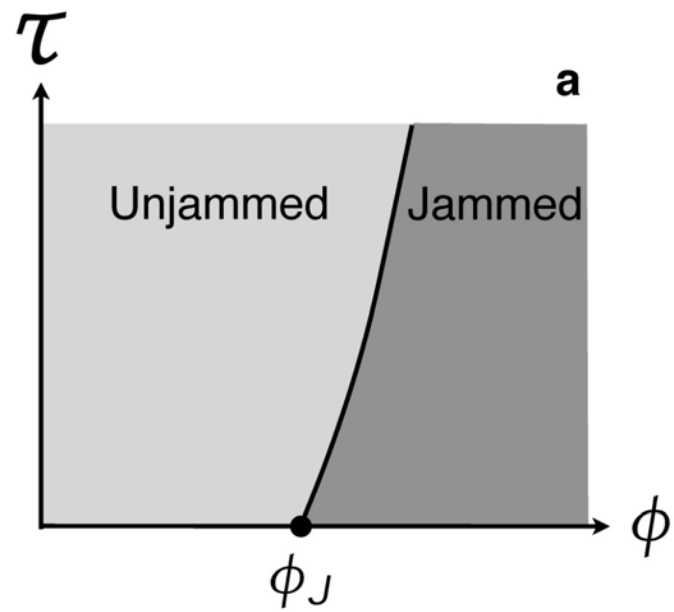
Context: Jamming and Fragility— sheared granular materials



Fragile states: ability to resist strain:
Strong in one direction but weak in reverse



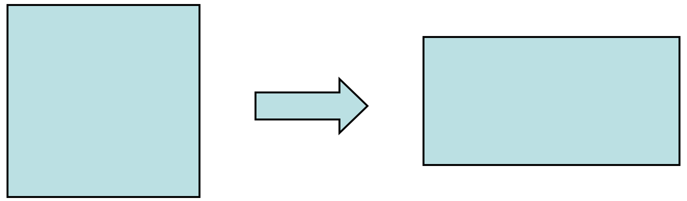
Cates et al. PRL 1998



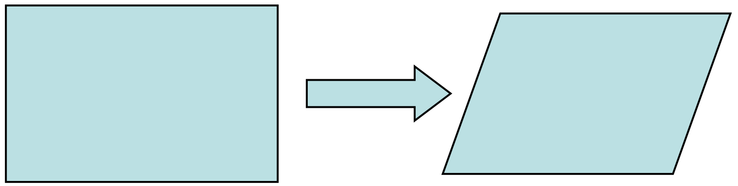
After Liu and Nagel, Nature, 1998, O'Hern et al. PRE 2003

Investigate the response to shear—creation of **stable** anisotropic states

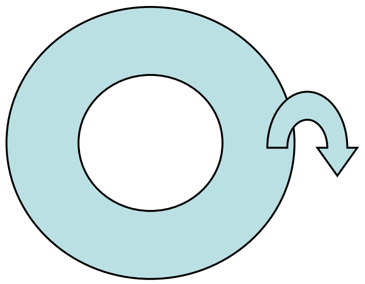
- Example 1: pure shear



- Example 2: simple shear

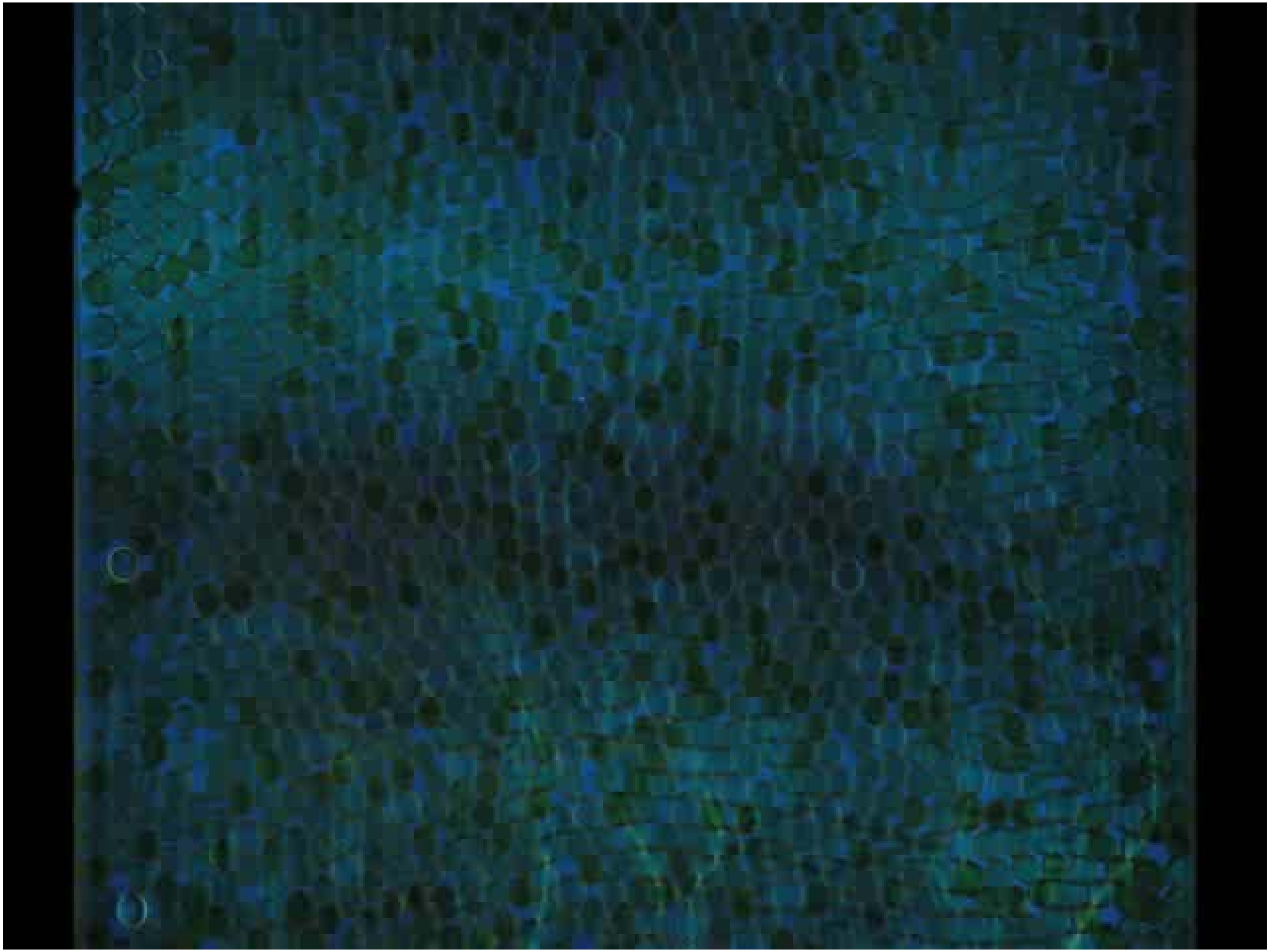


- Example 3: Couette shear

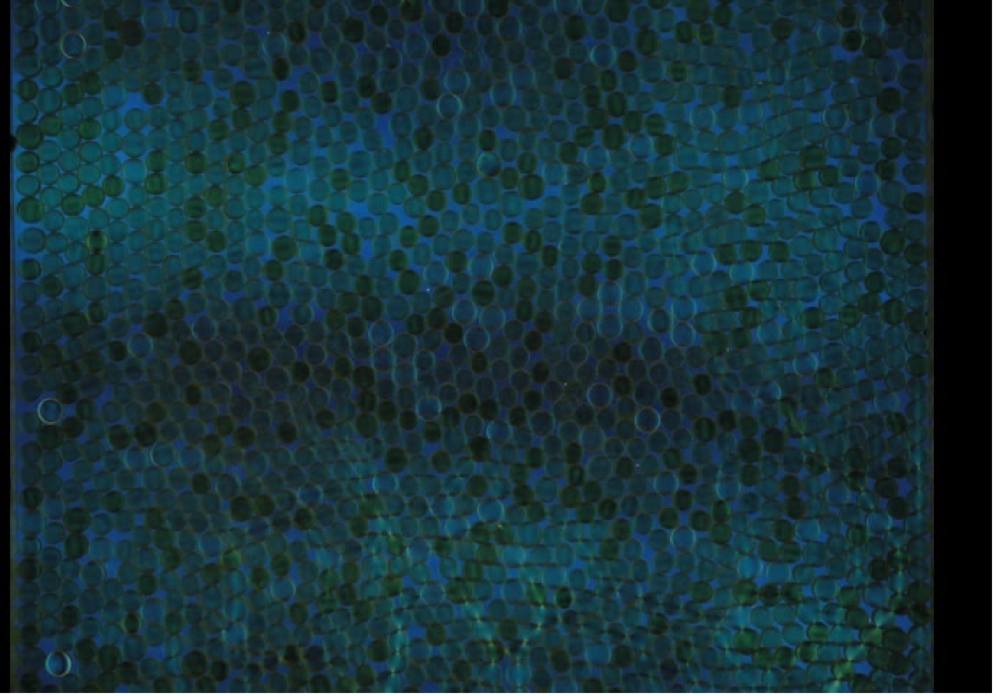


Series of experiments to map out phase diagram

Time-lapse video (one shear cycle) shows force network evolution—Frictional Shear Jamming—



Bi, Zhang, Chakraborty, RPB, Nature, 2011

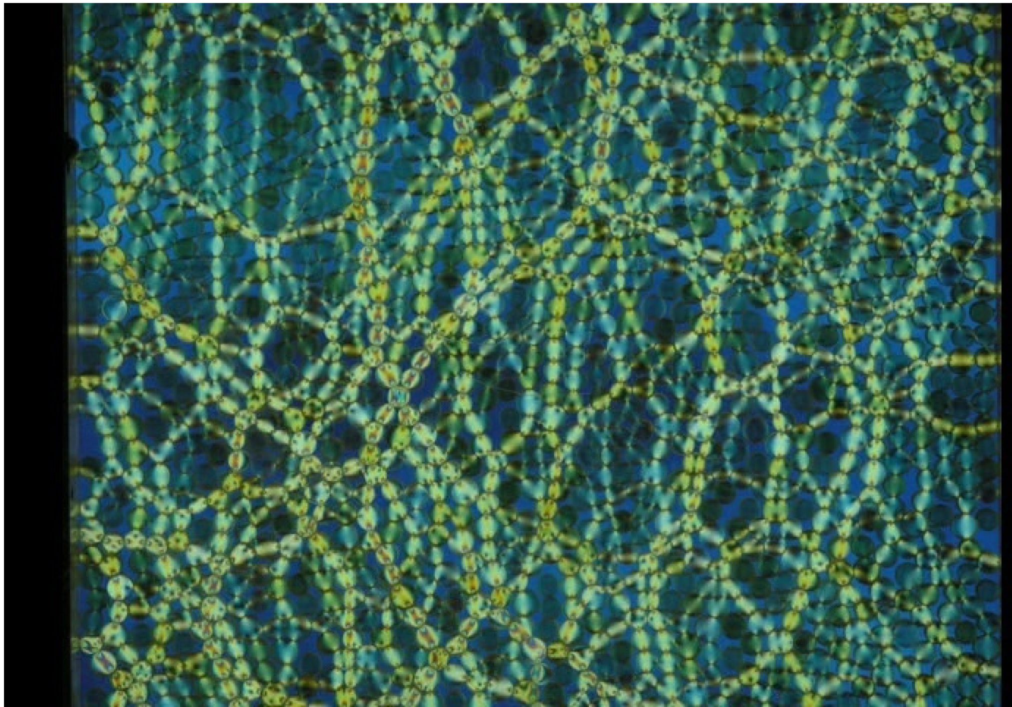


Initial and final states following a shear cycle—no change in area—Density cannot distinguish --but networks can

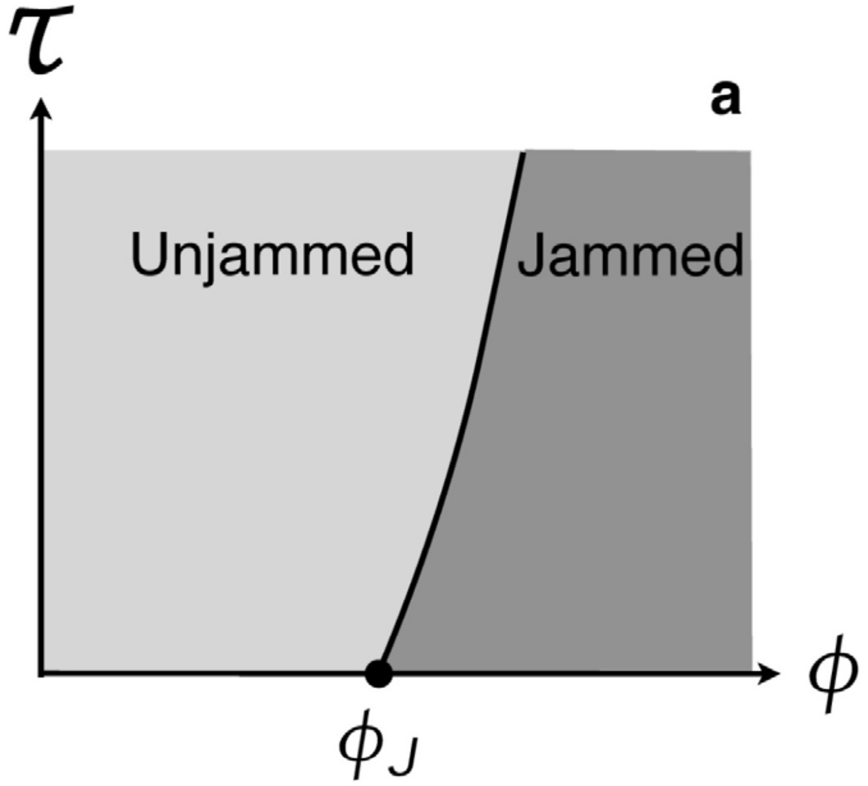
← Initial state, isotropic, no stress

Works between $\varphi_S < \varphi < \varphi_J$

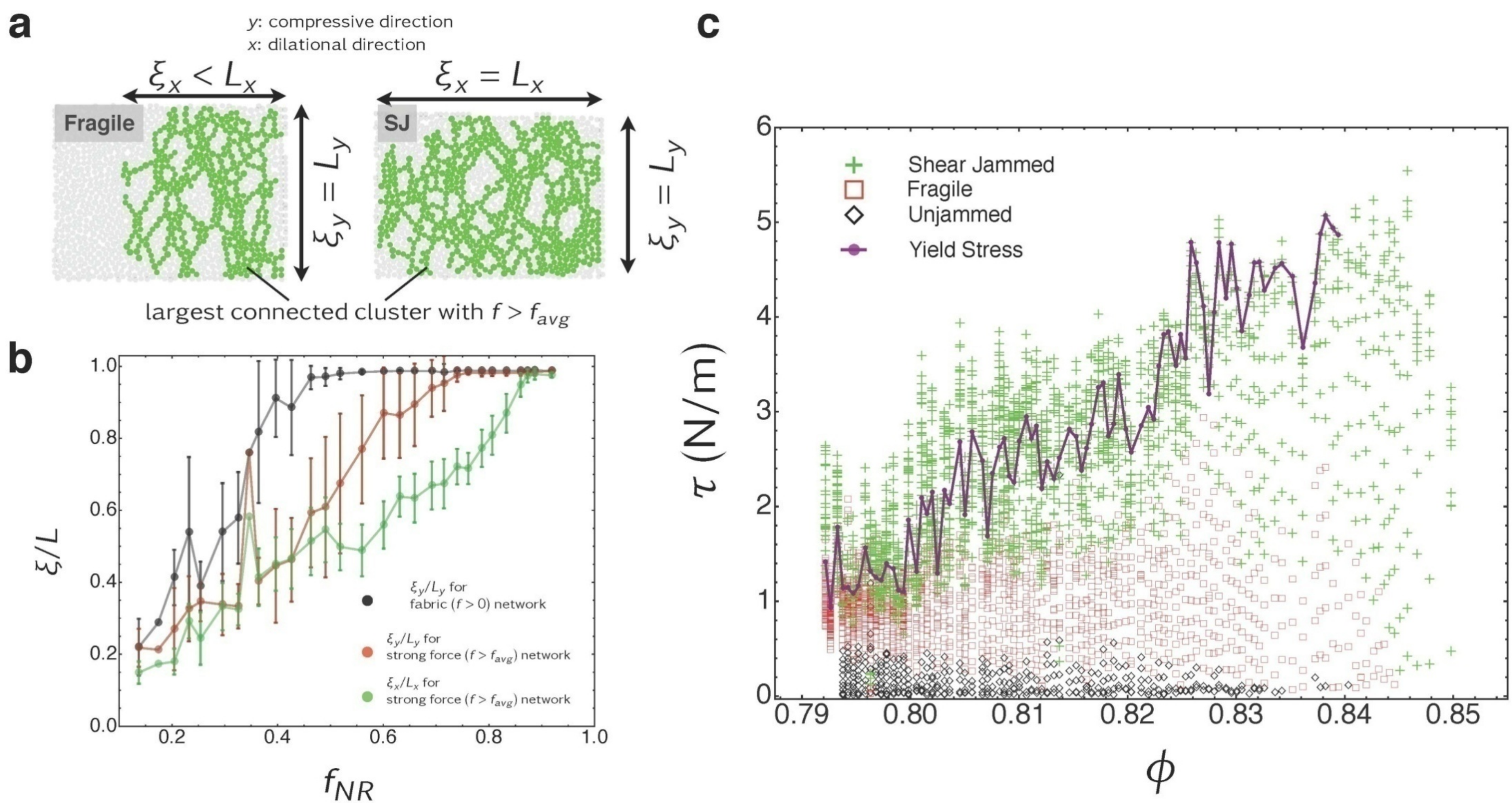
Final state → large stresses jammed



Does not fit frictionless jamming diagram (large-system limit)



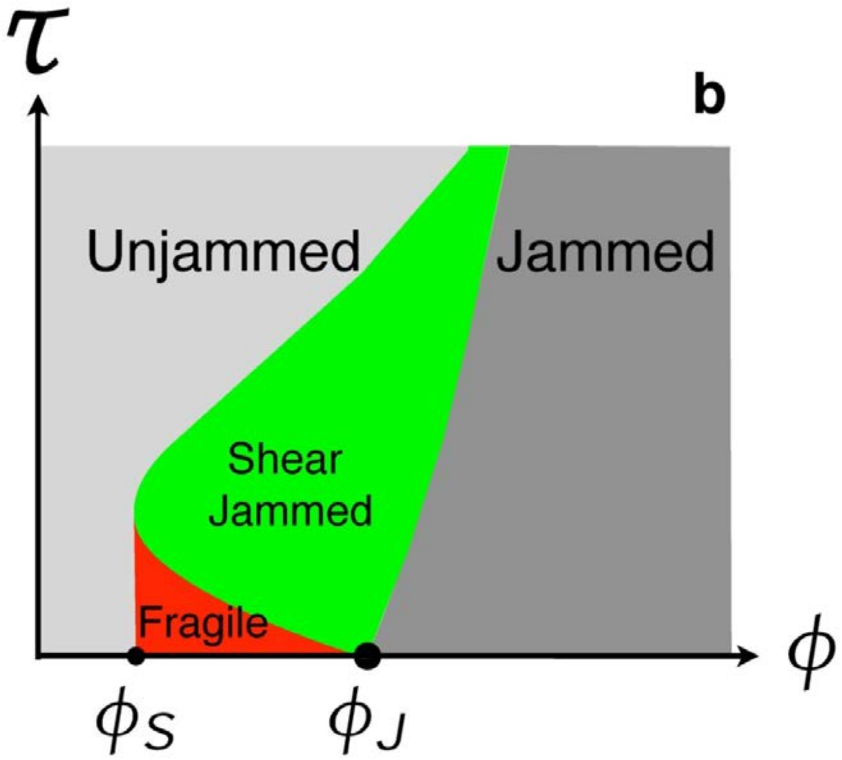
Some special properties of shear jammed states—start with Directional Percolation, Fragile and Shear-Jammed States (Bi et al. Nature, 2011)



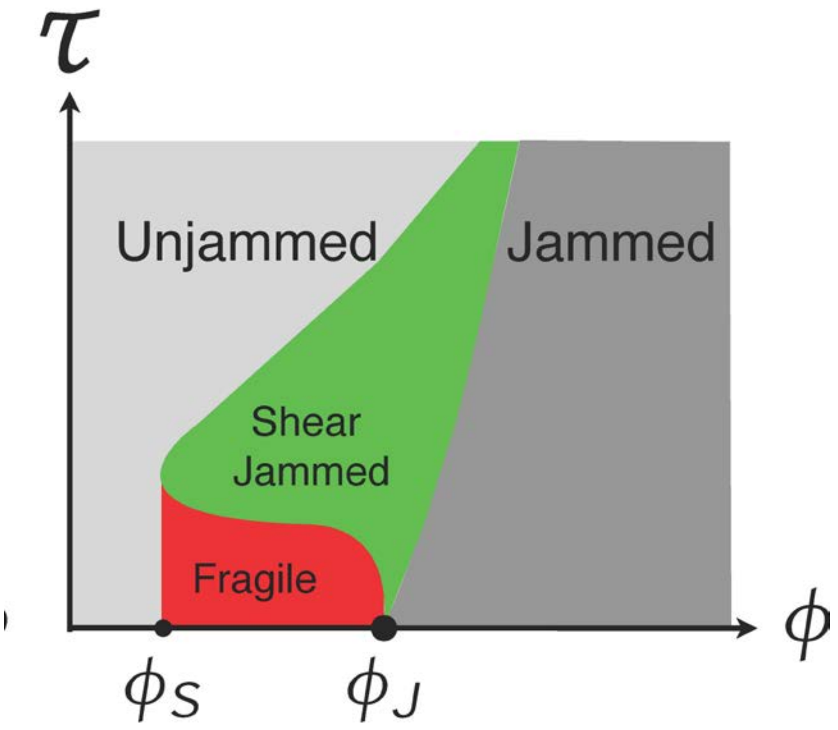
f_{NR} = nonrattler fraction

See Otsuki and Hayakawa, Phys. Rev. E **83,** 051301 (2011)

Jamming diagram for frictional grains



Original sketch, Bi et al. Nature 2011

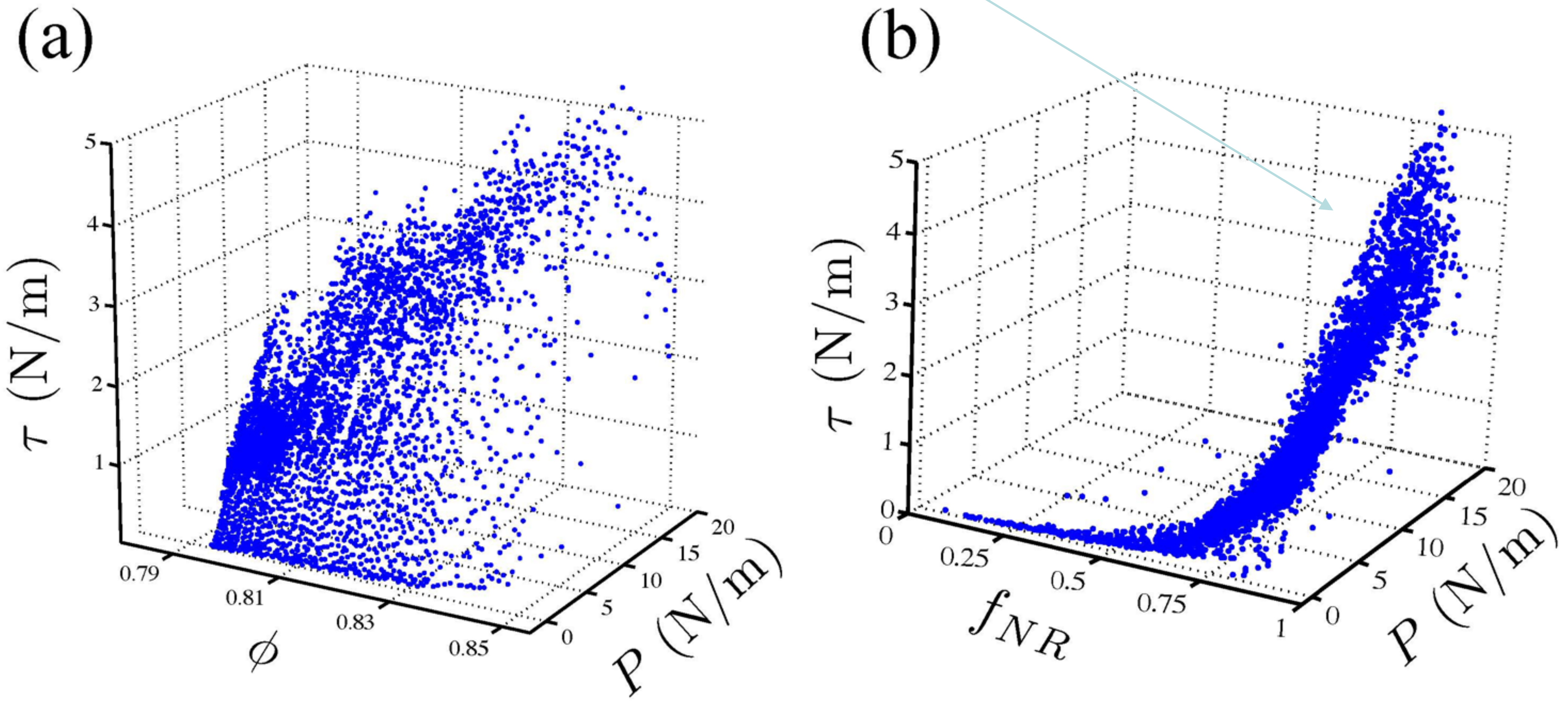


More accurate representation

Other features of shear jamming

Stresses vs. non-rattler fraction f_{NR}

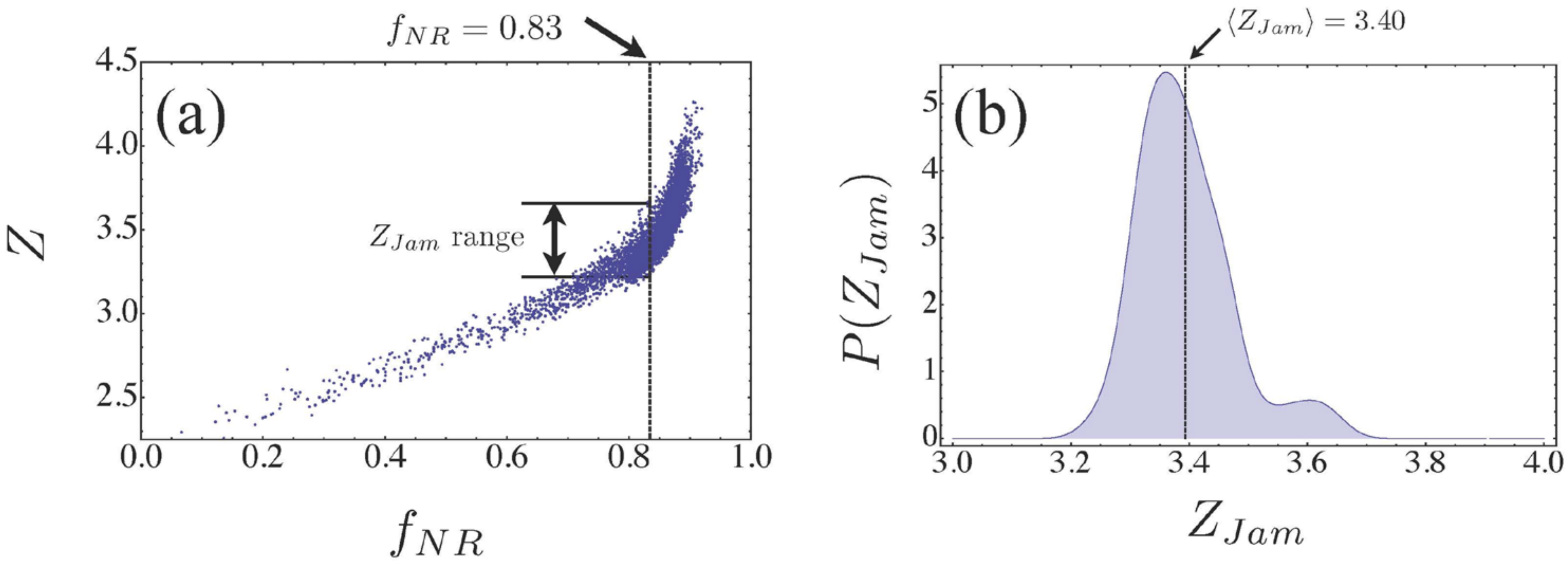
Good collapse of 'classical measures'



f_{NR} = fraction of non-rattlers—a rattler has too few contacts to be mechanically stable

Ditto for contact network properties, e.g. Z

Z is average number of contacts per particle

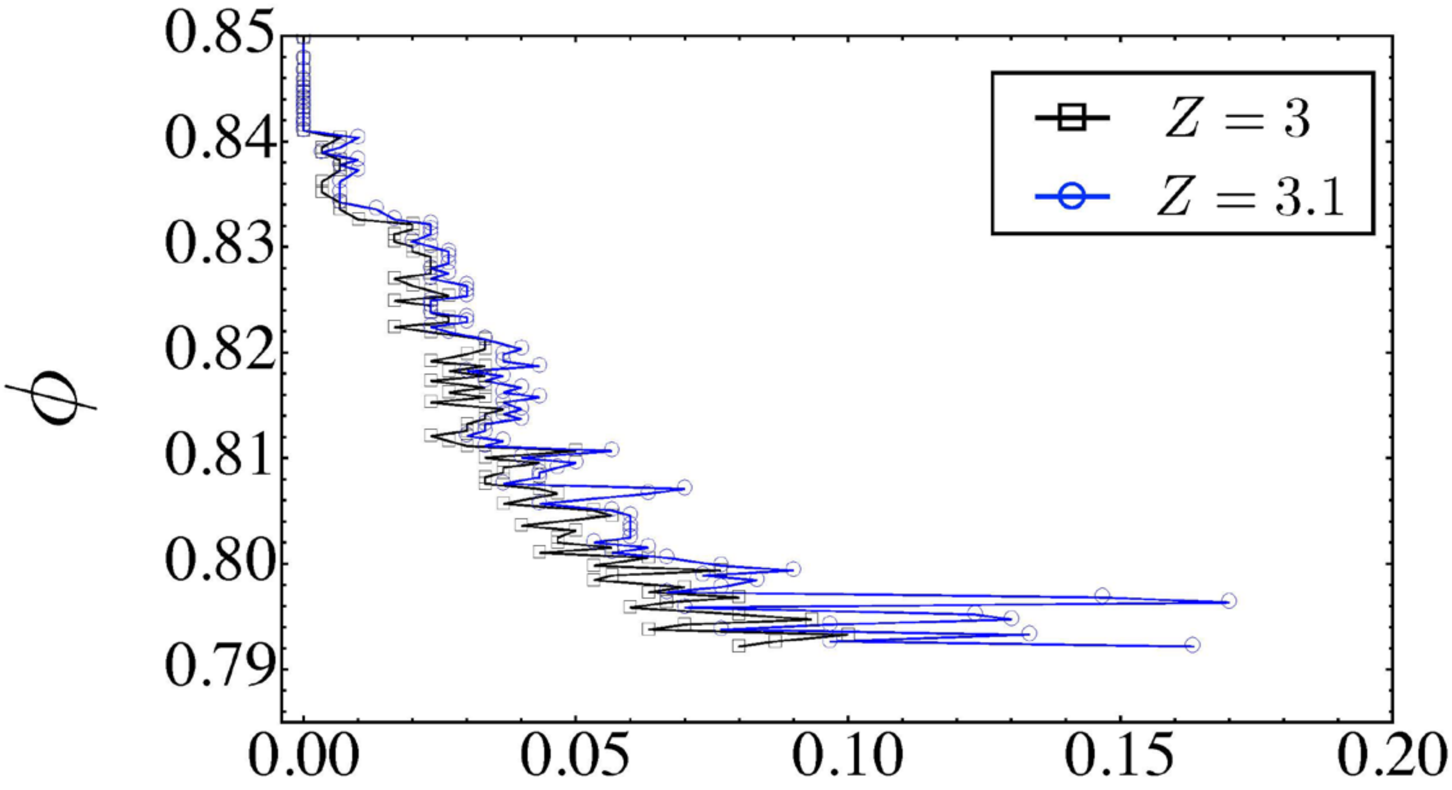


f_{NR} = fraction of non-rattler particles
non-rattlers need at least 2 contact

Range of densities for which shear jamming can be achieved

Random dense

Random loose

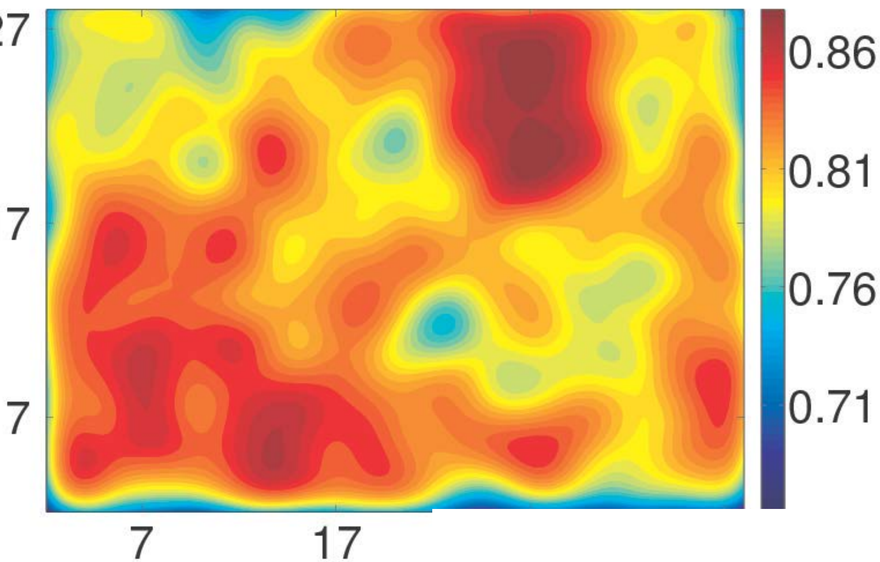


$\gamma_J(\phi) = min\{\gamma(Z \geq Z_{iso}^{\infty}; \phi)\}$

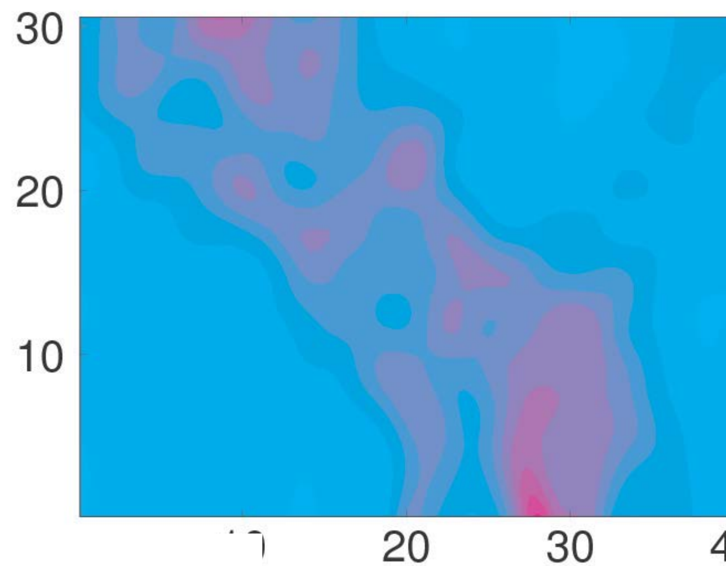
Minimum strain to shear jam

Shear band forms: result of driving soft system from wall, base friction

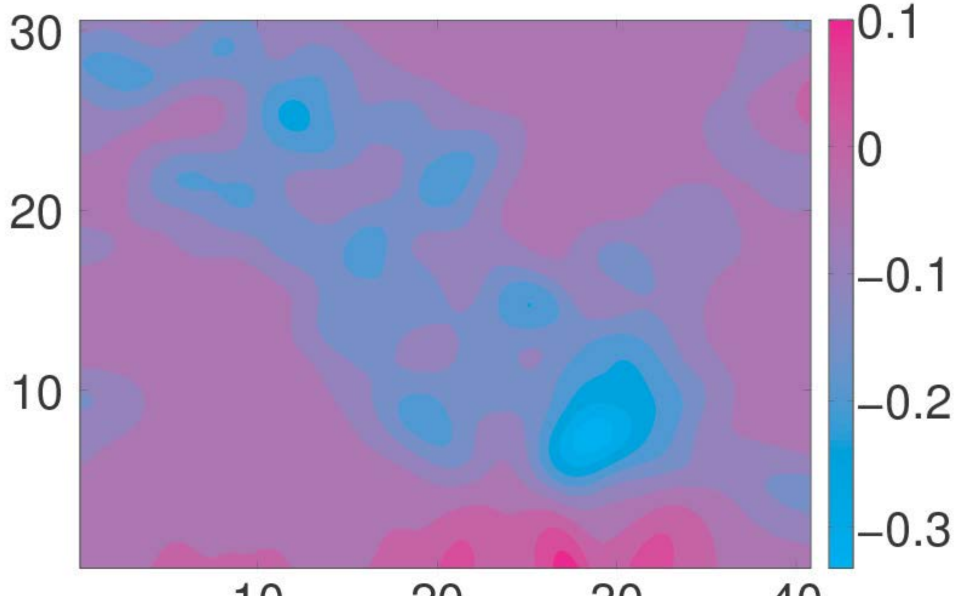
Contour plots of coarse-grained local density and strain components,
at a strain of γ=9.3%}



density



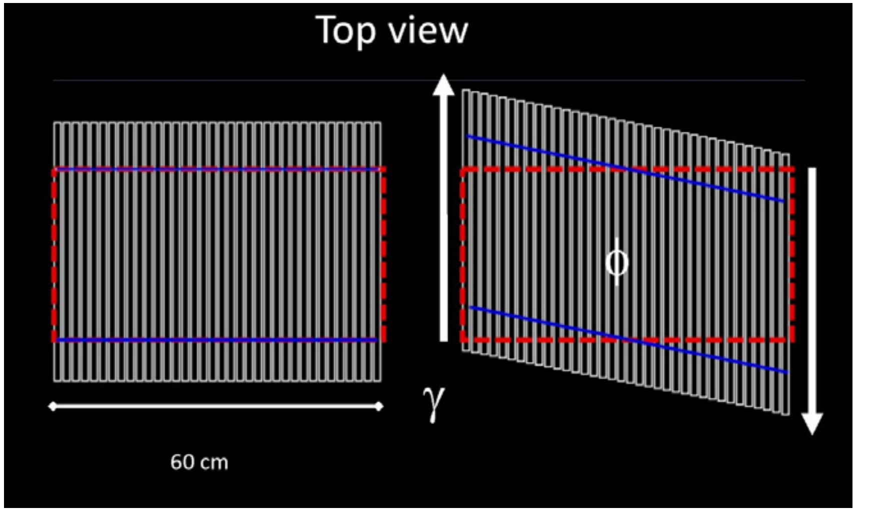
ε_{yy}



ε_{xx}

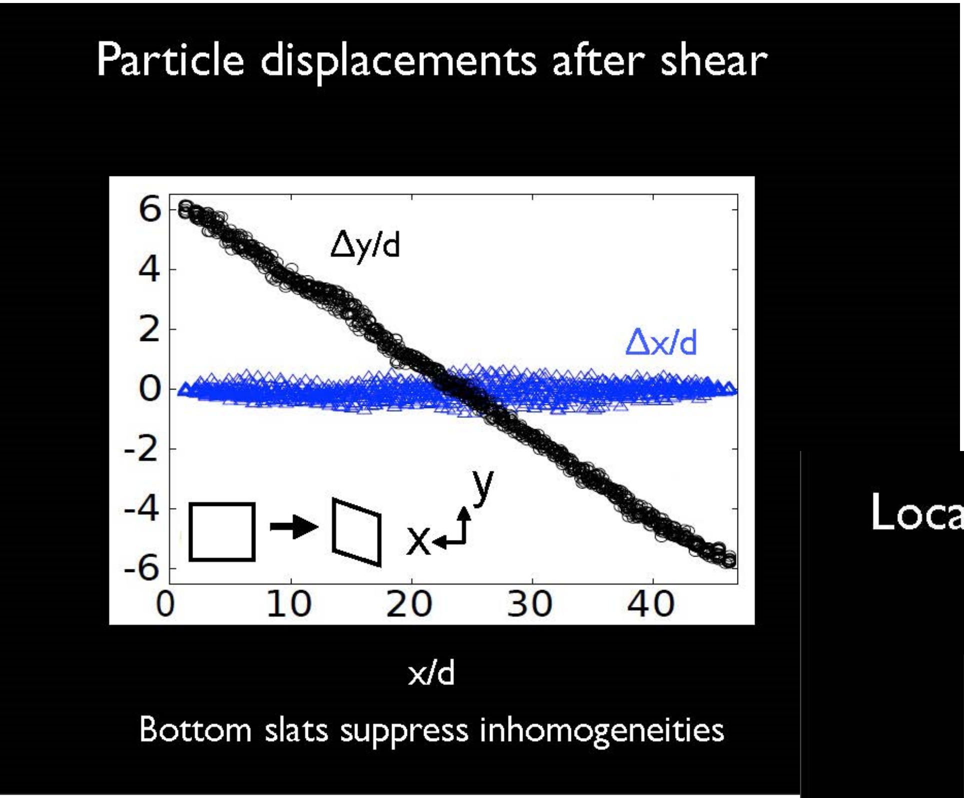
Jie Zhang,
I.Goldhirsch
BB, Supp.Prog.
Theor.Phys2010

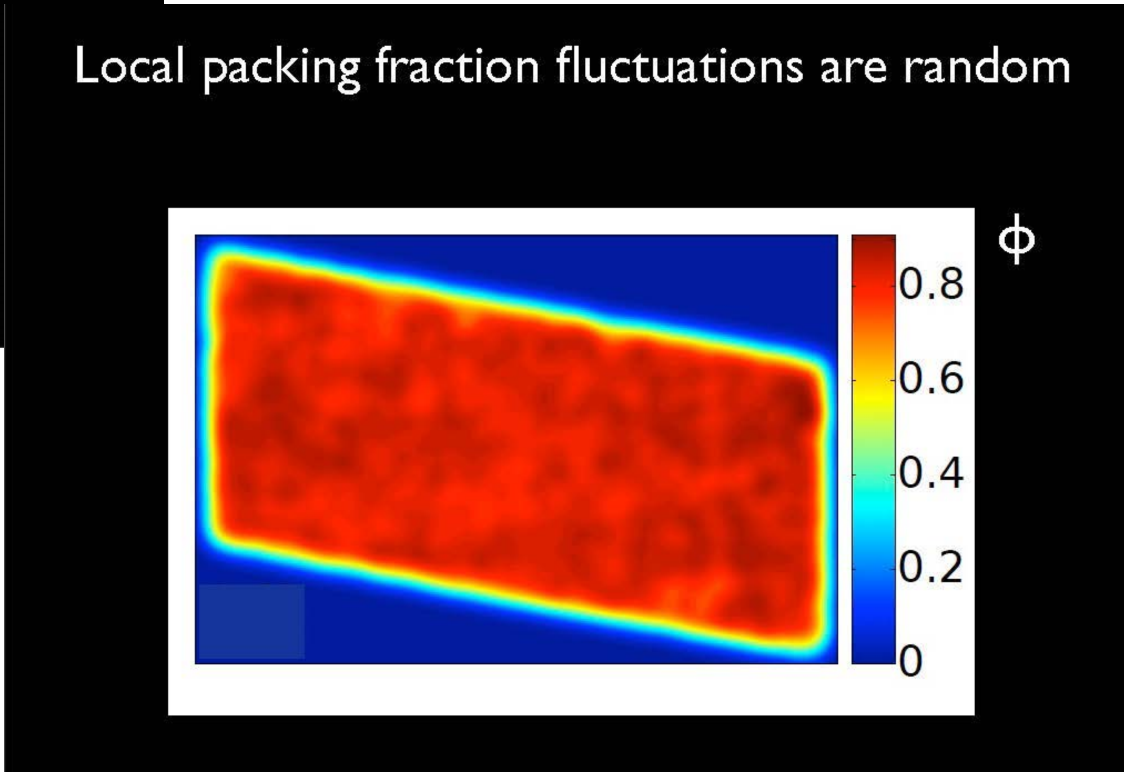
2^{nd} apparatus: uniform simple shear throughout system



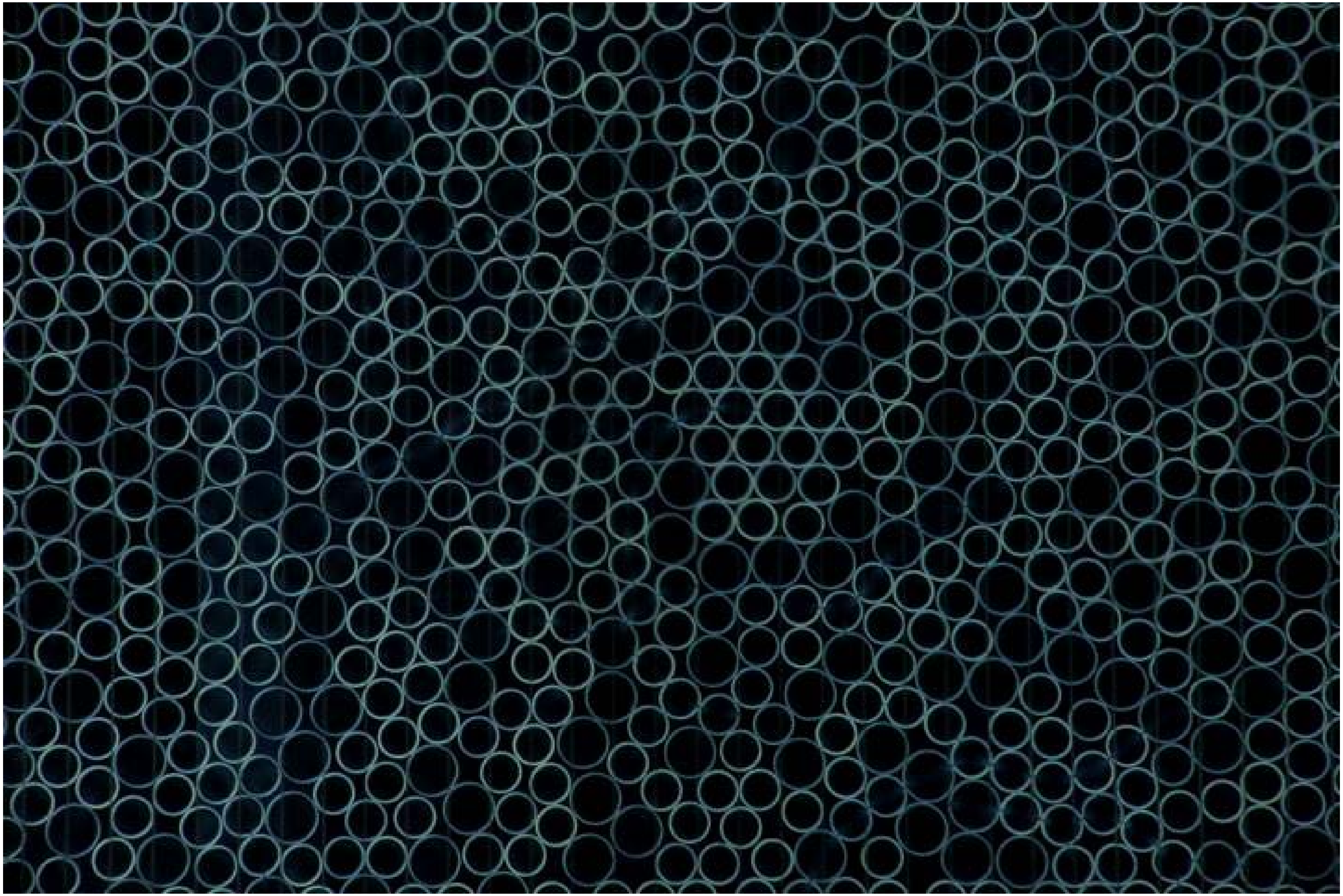
Joshua Dijksman, Jie Ren, Dong Wang
BB, PRL 2013

This new experimental approach supplies uniform shear—max strain ~ γ = 0.5





Shear-Jamming—clean experiment, constant φ—states well characterized

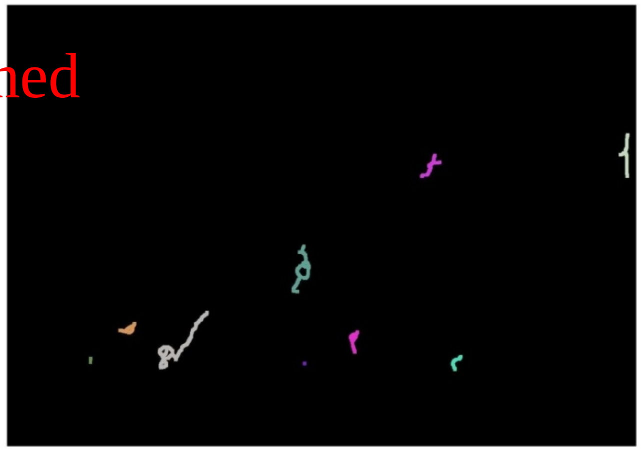


Networks are key to shear jamming

Increasing shear strain—first unidirectional, then all-directional percolation of strong force network (e.g. Cates et al. PRL 1998)

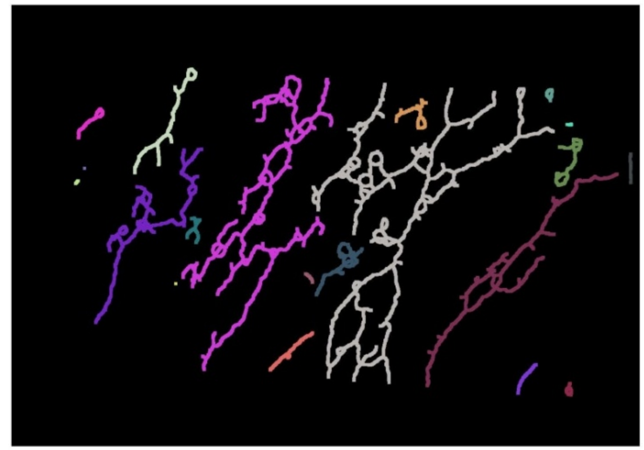
The force chains look differently at different stages of linear shear:

Unjammed not fragile



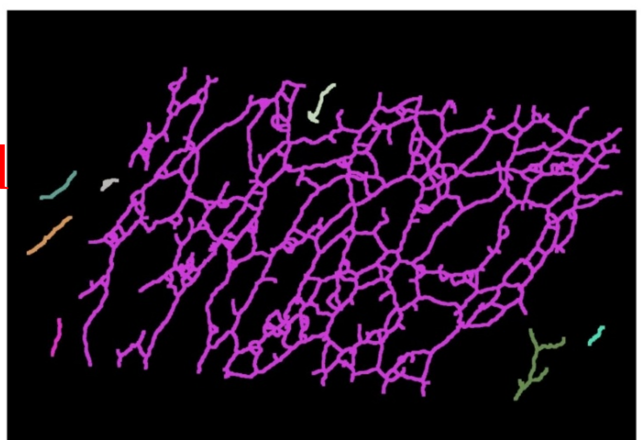
1. minimal force, unjammed

Fragile



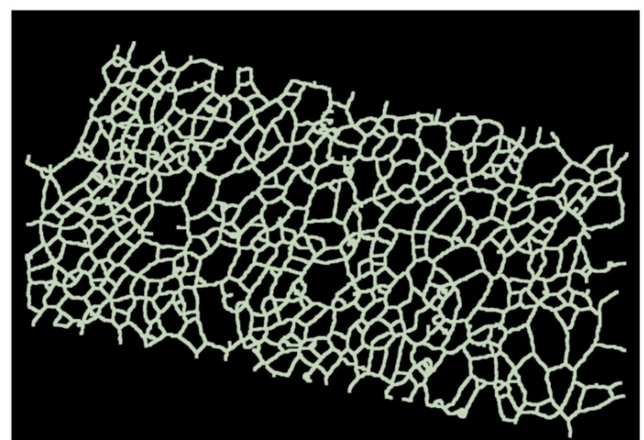
2. more force, multiple clusters; fragile

Shear Jammed



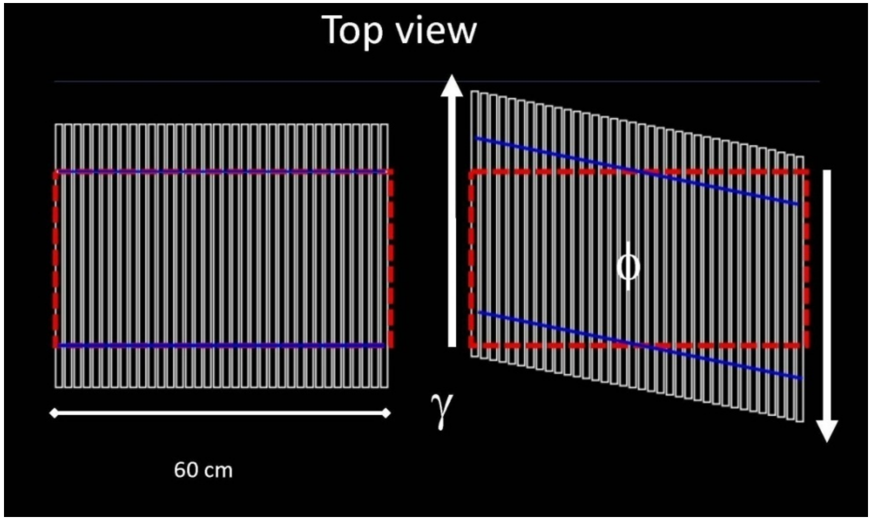
3. percolating cluster, onset of jamming

Evolves towards more isotropic



4. one large cluster, jammed

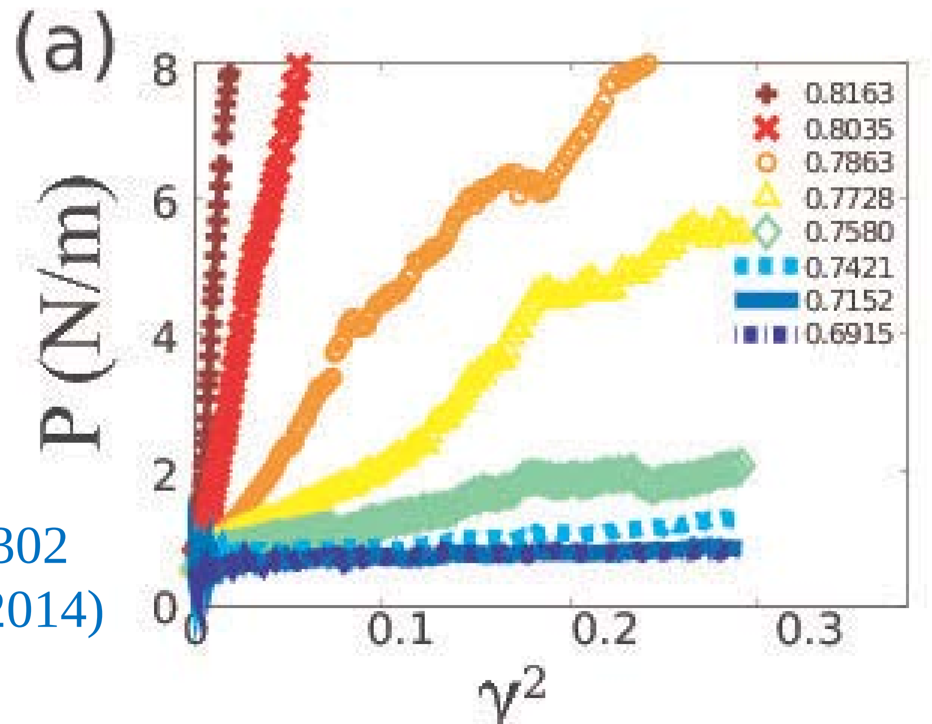
Nonlinear stress vs. strain below φ_J



We introduce Reynolds coefficient

$$P \approx R\gamma^2$$

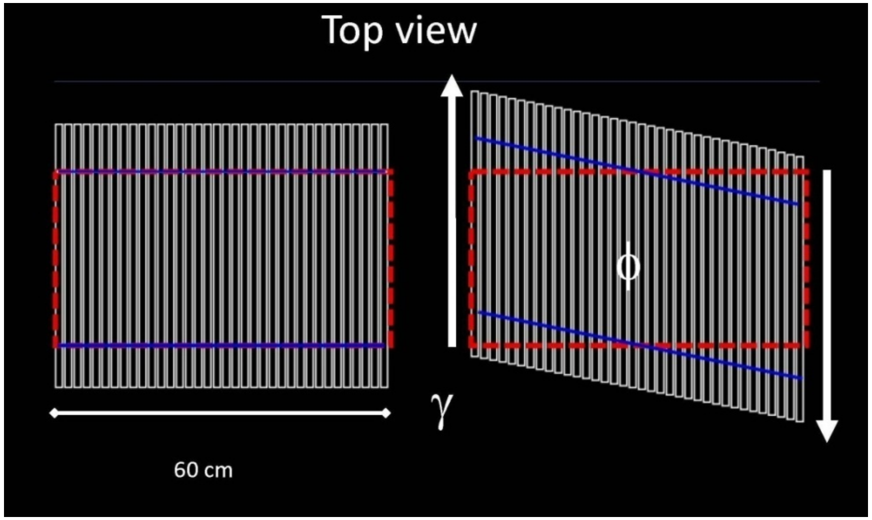
Manifestation of Reynolds dilatancy in fixed volume
Define Reynolds coefficient, R



Ren et al.PRL 110, 018302 (2013), Hu et al. EPL (2014)

Tighe, Gran. Matter (2014)

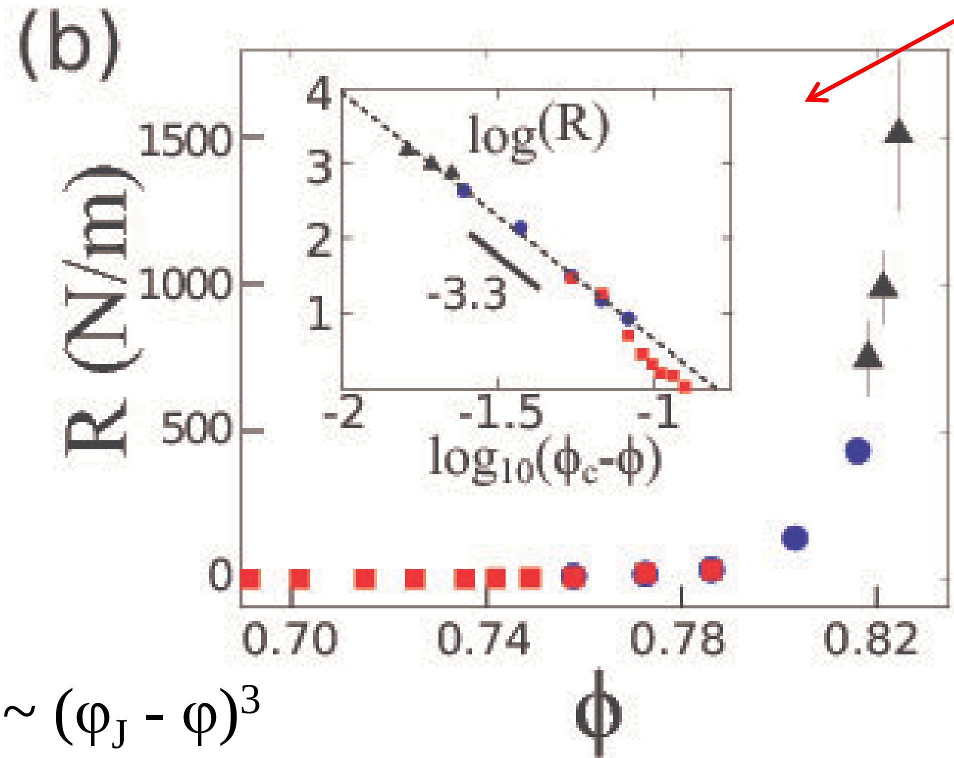
Shear jamming dynamics below φ_J

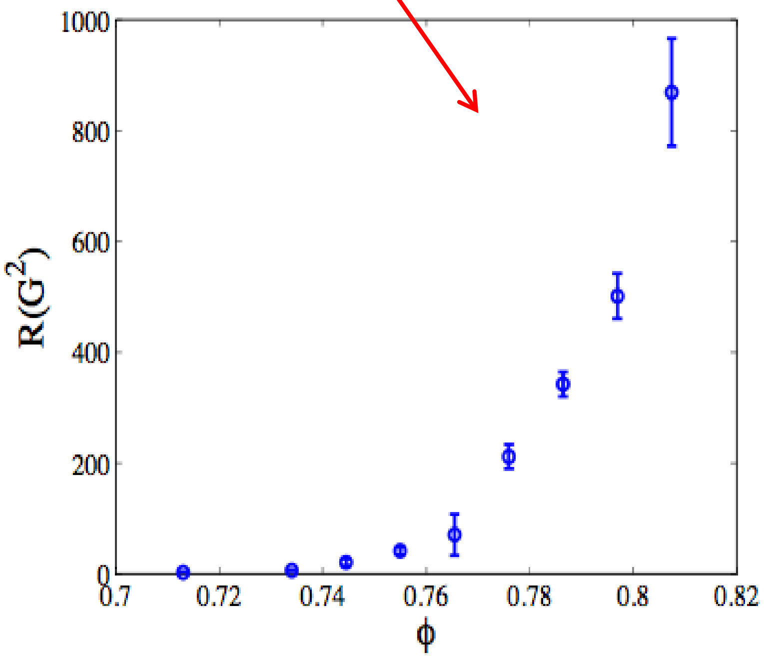


Introduce Reynolds coefficient, R

$$P \approx R\gamma^2$$

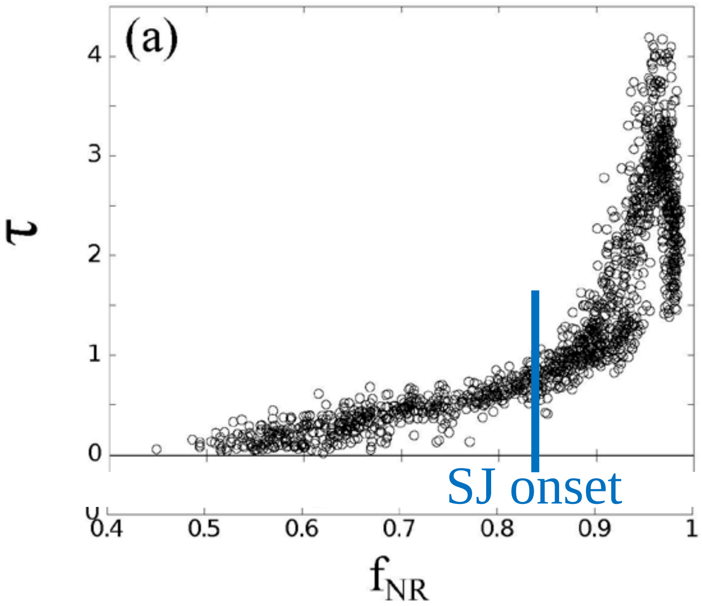
$\mu = 0.65 \quad \mu >> 1$



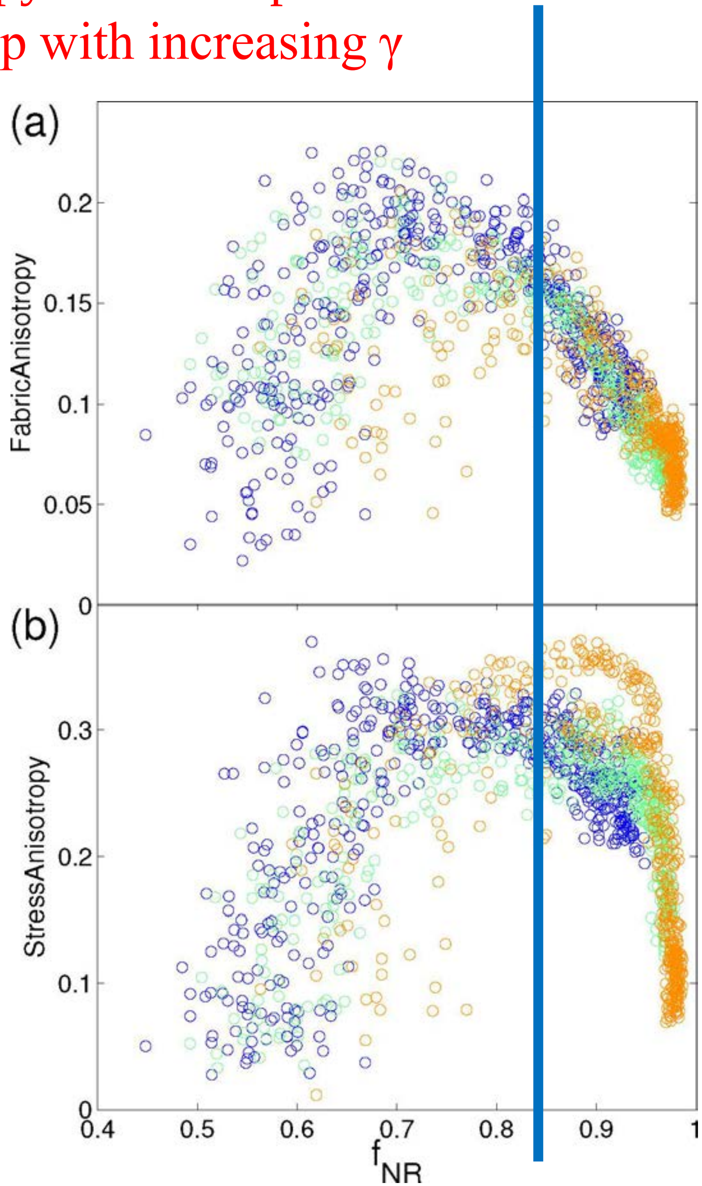


$R \sim (\varphi_J - \varphi)^3$

Stress and fabric anisotropy



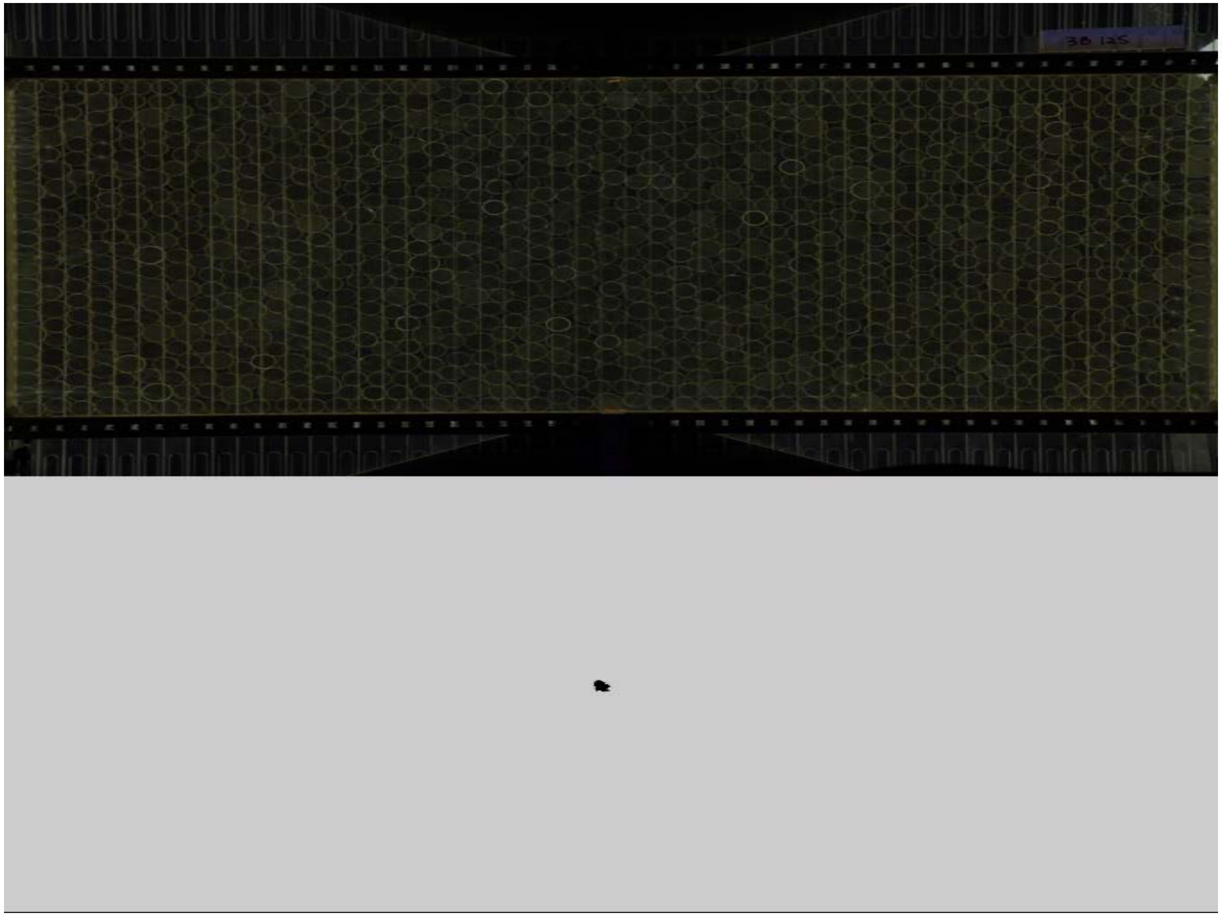
Anisotropy measures peak
Then drop with increasing γ

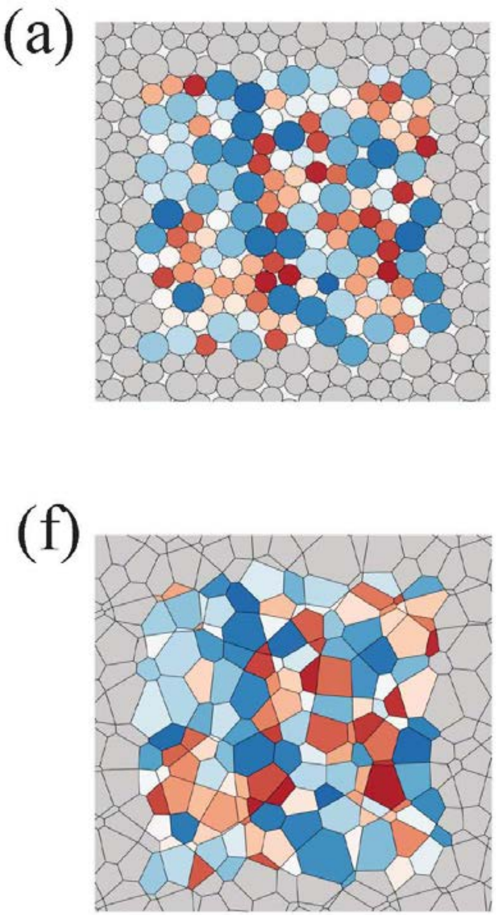


$SA = (\sigma_2 - \sigma_1)/(\sigma_2 + \sigma_1) = \tau/P$

FA defined similarly

Ordering in a space of force tiles



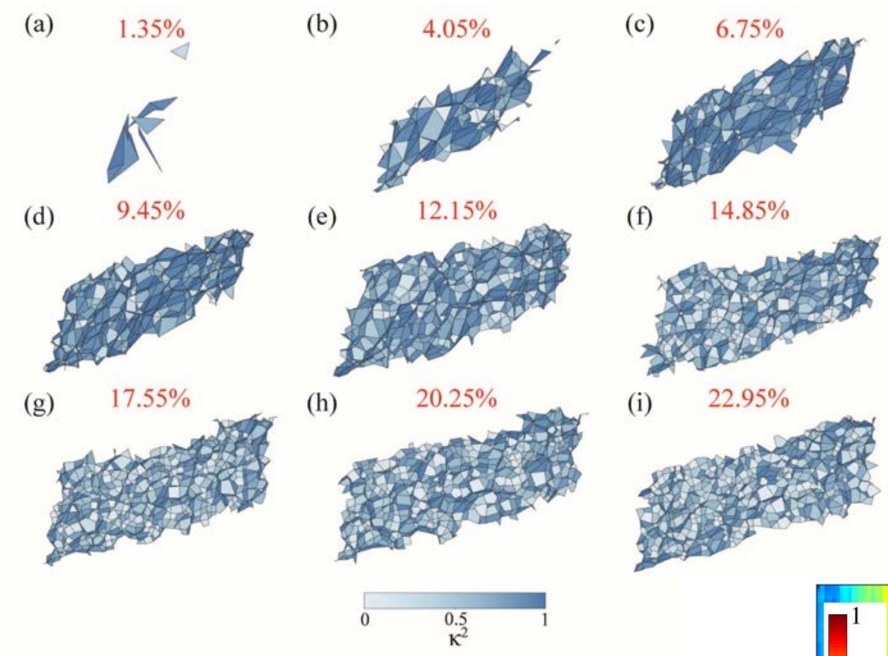


1) For one tile: align contact forces for particle i head to tail—force balance -> closed polygon

2) Repeat for all particles—contacting particles share common edge

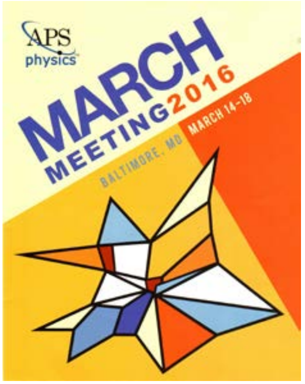
3) Polygons are space-filling in a space of forces

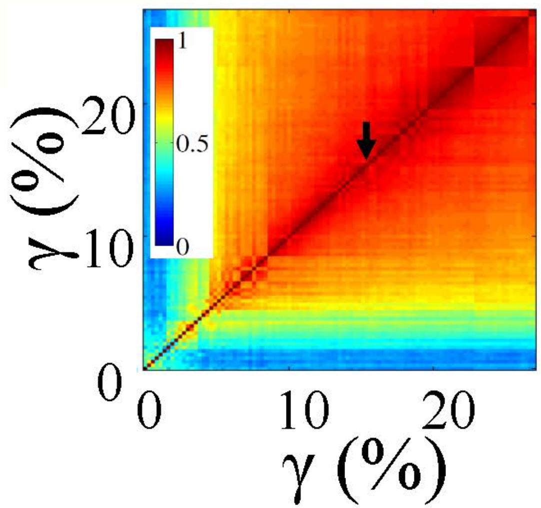
Ordering in a space of force tiles

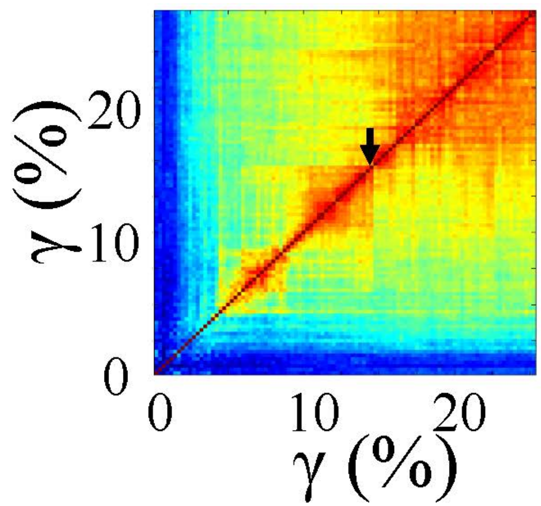


Evolution of force tiles under shear

Overlaps: position space—no ordering
force tile space shows ordering at
$\gamma \approx 12\%$

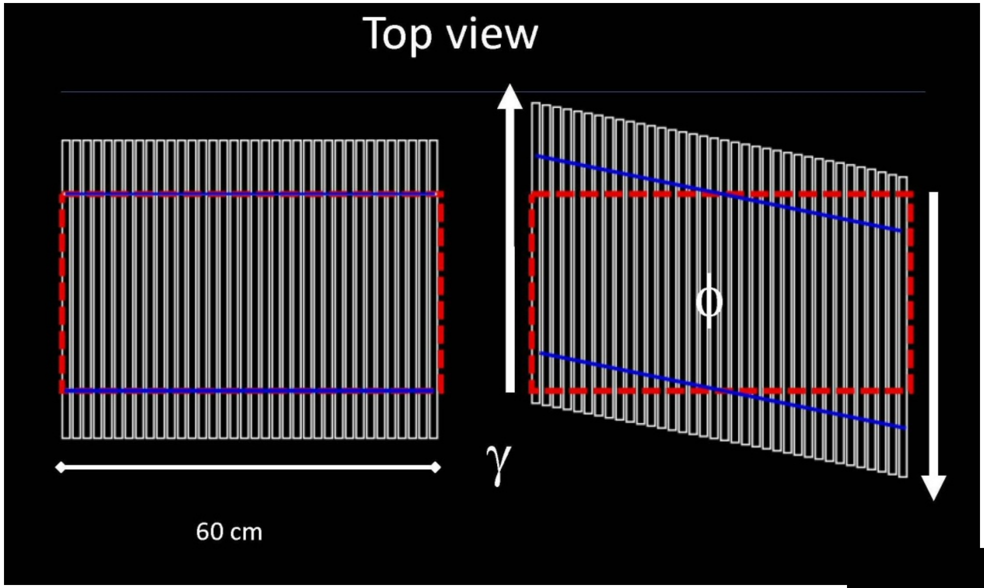






S.Sarkar, D.Bi, J. Zhang, J.Ren, BB,
B.Chakraborty, PRE 2016

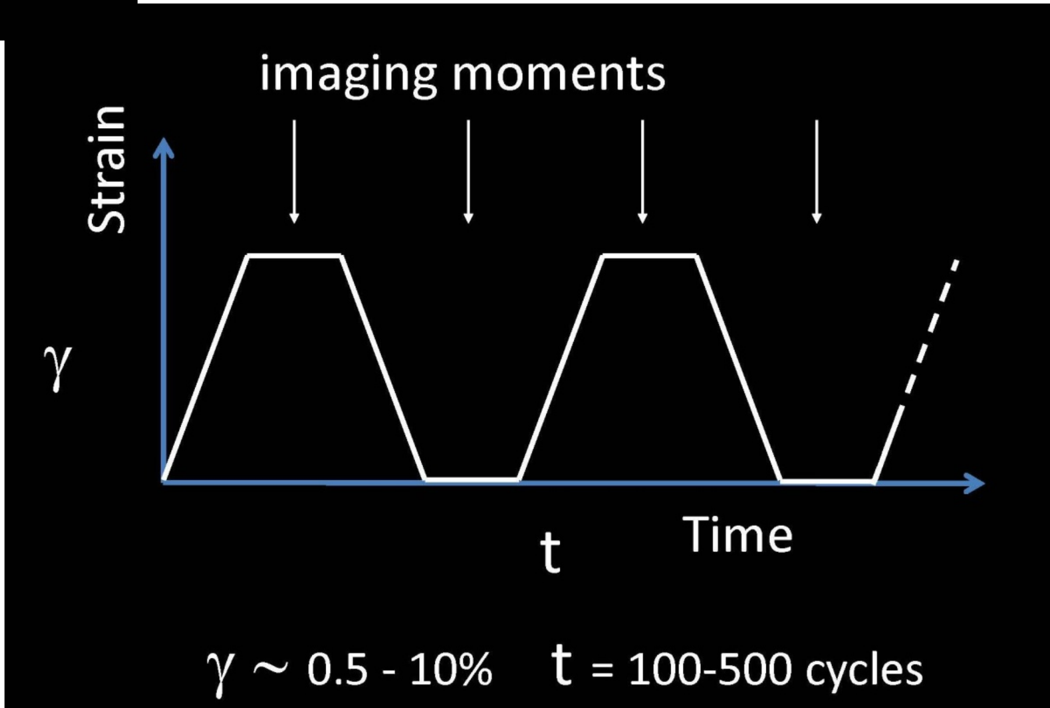
Memory forms and evolves under cyclic shear



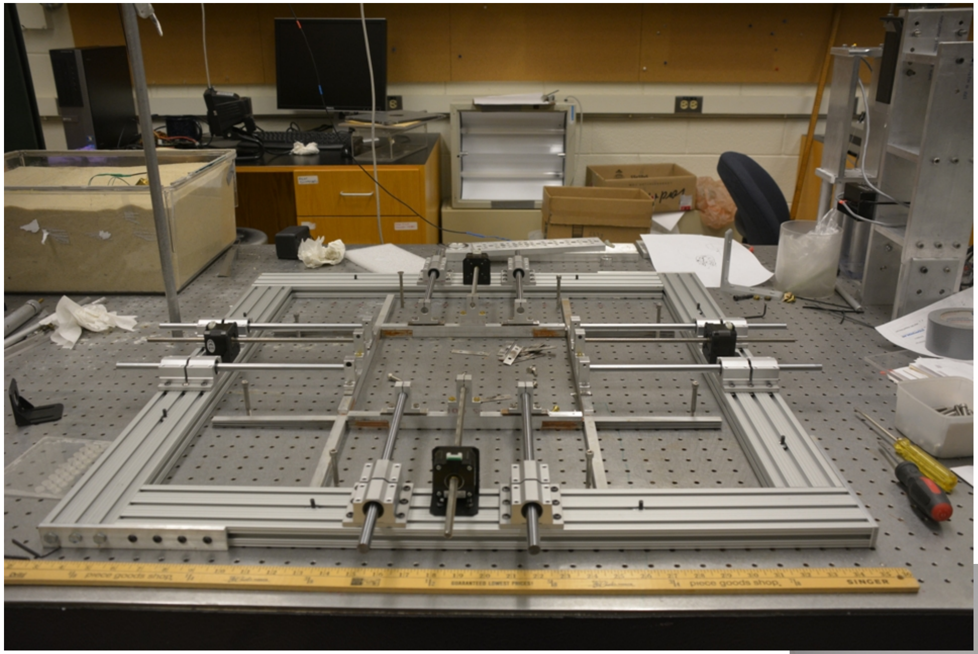
Granular analogue of dense suspension experiment

Example below is asymmetric shear

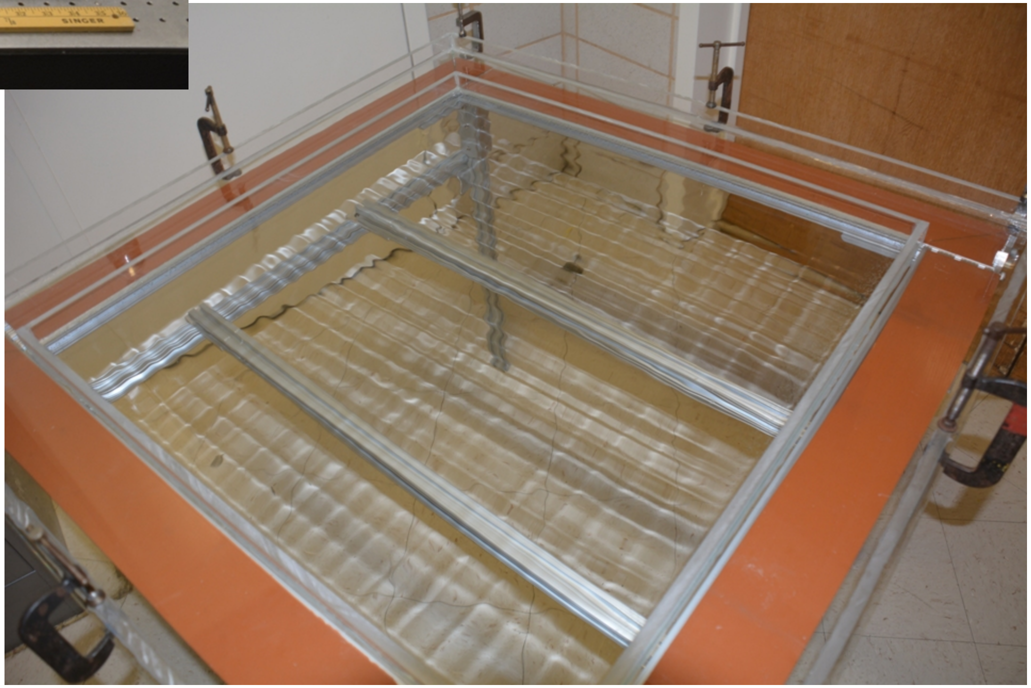
Also: symmetric cyclic shear



Very new experiments



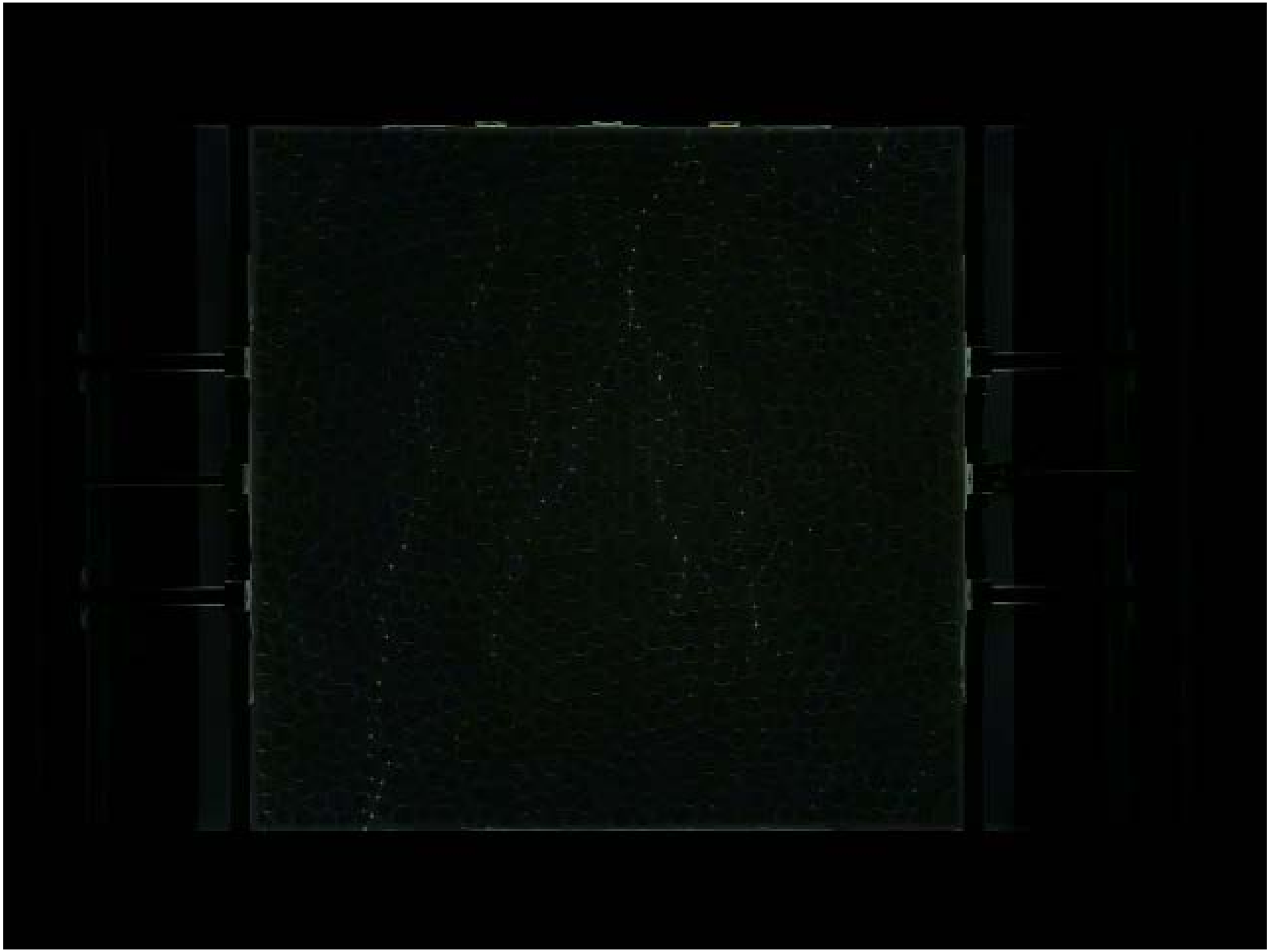
Special biaxial apparatus:
particles float,
four walls move independently



Hu Zheng, Dong Wang
Meimei Wang, David Chen

See also Zheng et al. EPL 2014

How important is friction with the base?
Remove it by floating grains—Pure Shear



Hu Zheng, Dong Wang, Cacey Stevens, David Chen

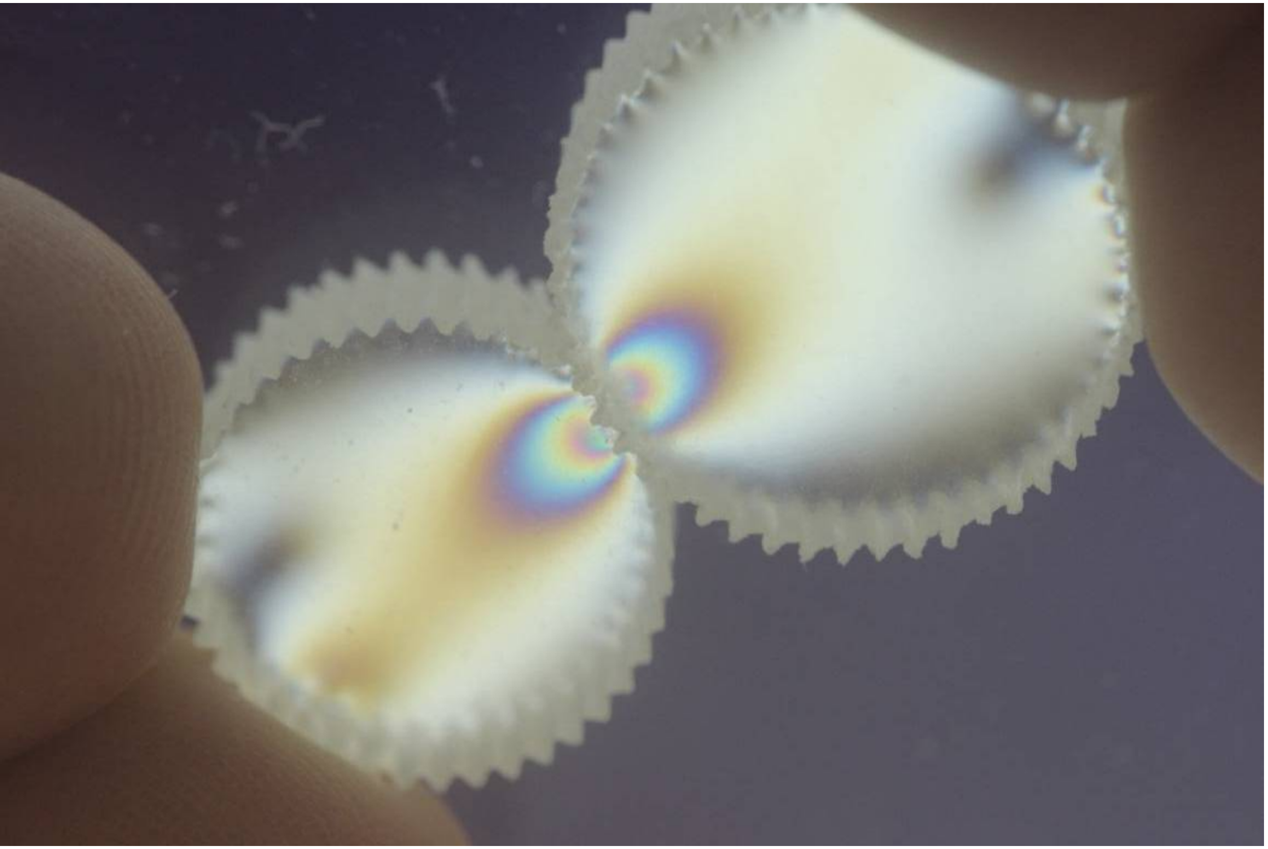
Alternative protocol: compress to just above jamming, then shear (floating grains)

Cumulated strain

Hu Zheng, Cacey Stevens-Bester, Dong Wang, David Chen

Changing friction: higher (lower) μ gives lower (higher) φ_S

$\mu >> 1$



Make gear particles with very high μ

Wrap particles with Teflon for low μ

Compare effect of friction (Dong Wang, Jie Ren, Jonathan Barés, BB)

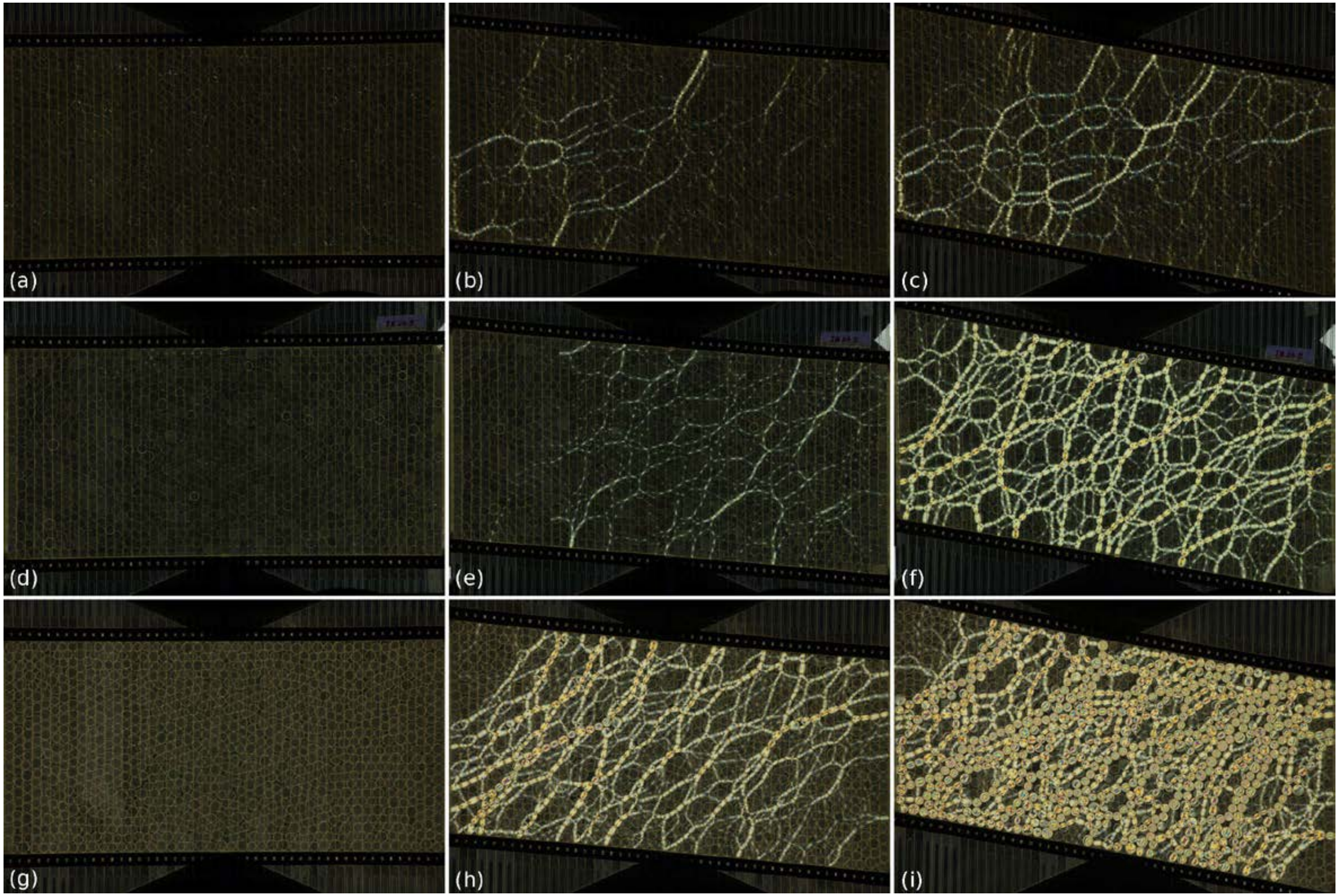
$\mu >> 1$ $\mu = 0.65$ $\mu = 0.15$

Effect of friction (Dong Wang, Jie Ren, Jonathan Barés, BB)

$\mu = 0.15$

$\mu = 0.65$

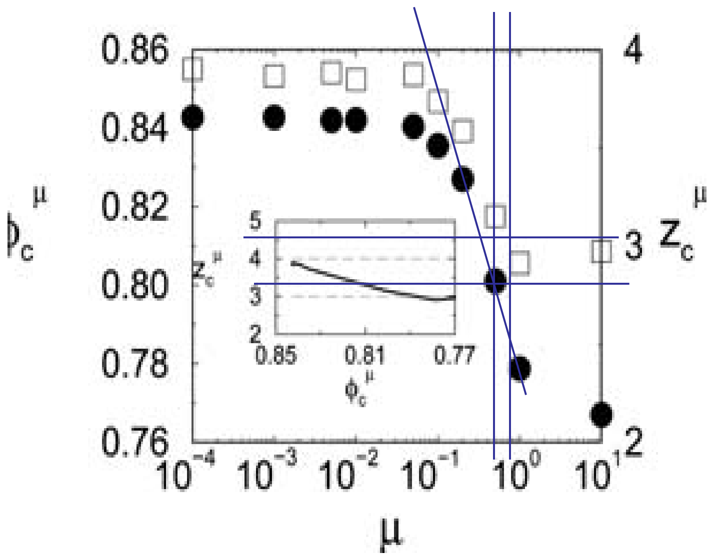
$\mu >> 1$



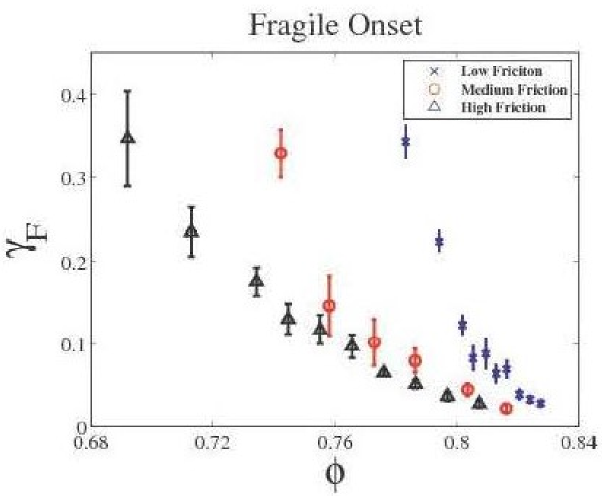
Increasing strain, γ →

Higher (lower) μ gives lower (higher) φ_S

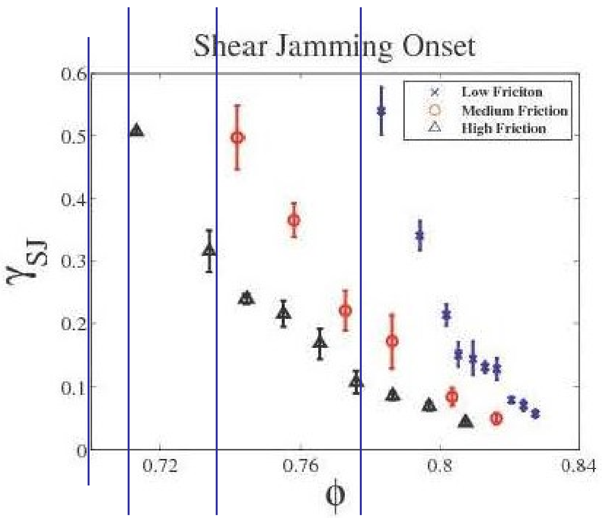
Simulations by Silbert Soft Matter 2010-isotropic jamming



For $\mu >> 1$, $\varphi_S \approx 0.70$
For $\mu = 0.65$, $\varphi \approx 0.74$
For $\mu = 0.15$, $\varphi \approx 0.78$



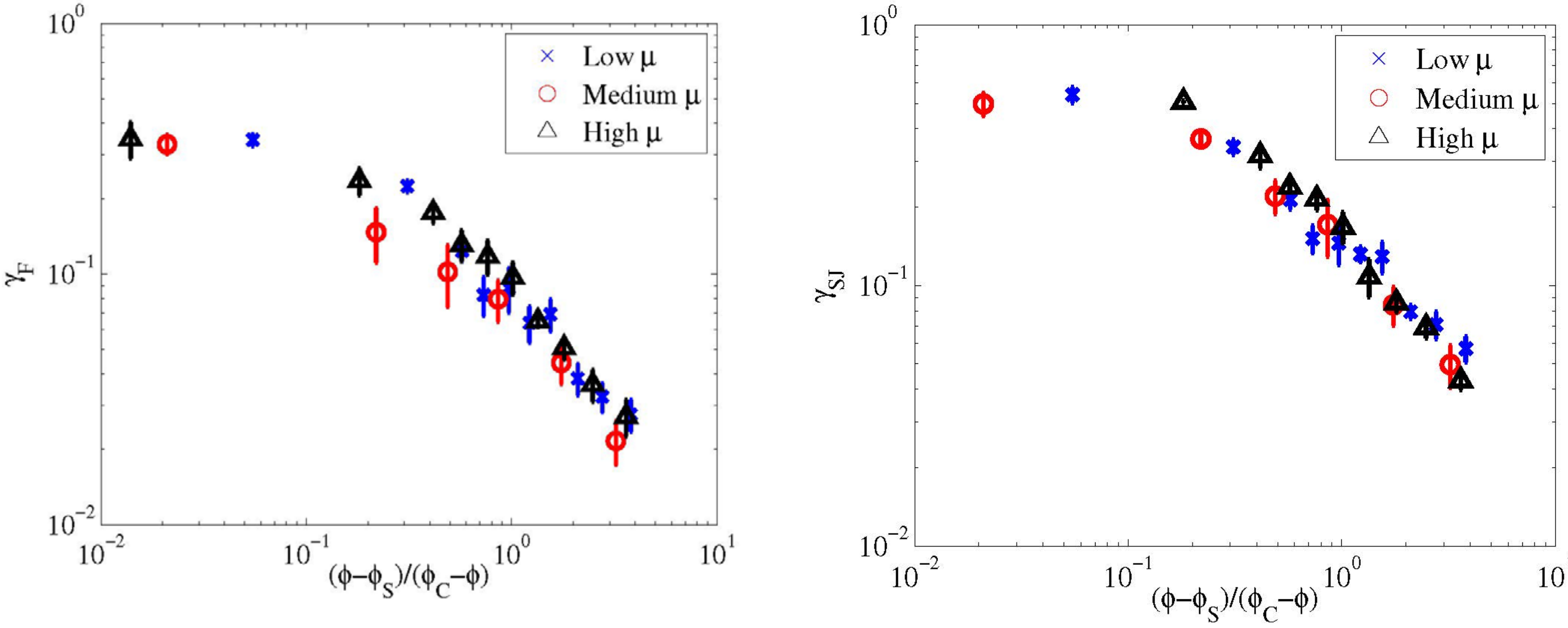
Fragile onset



Shear jamming onset

Jonathan Barés, Dong Wang

Large strains—shear jamming limit, and Yield Stress Curve (YSC)



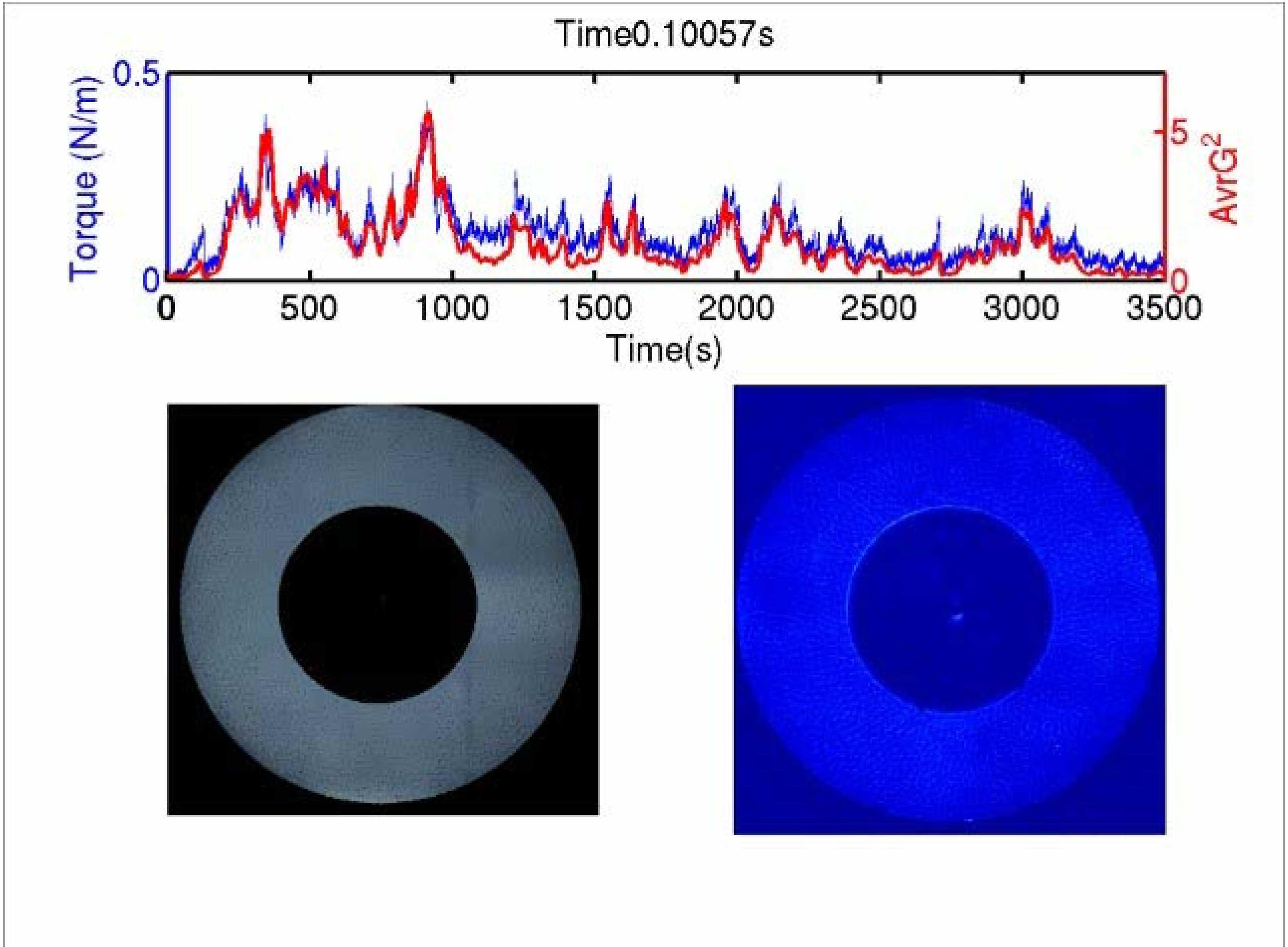
Exponents ≈ 0.8

Large strains—shear jamming limit, and Yield Stress Curve (YSC) Work in Progress

Understanding origin of networks—removing base friction

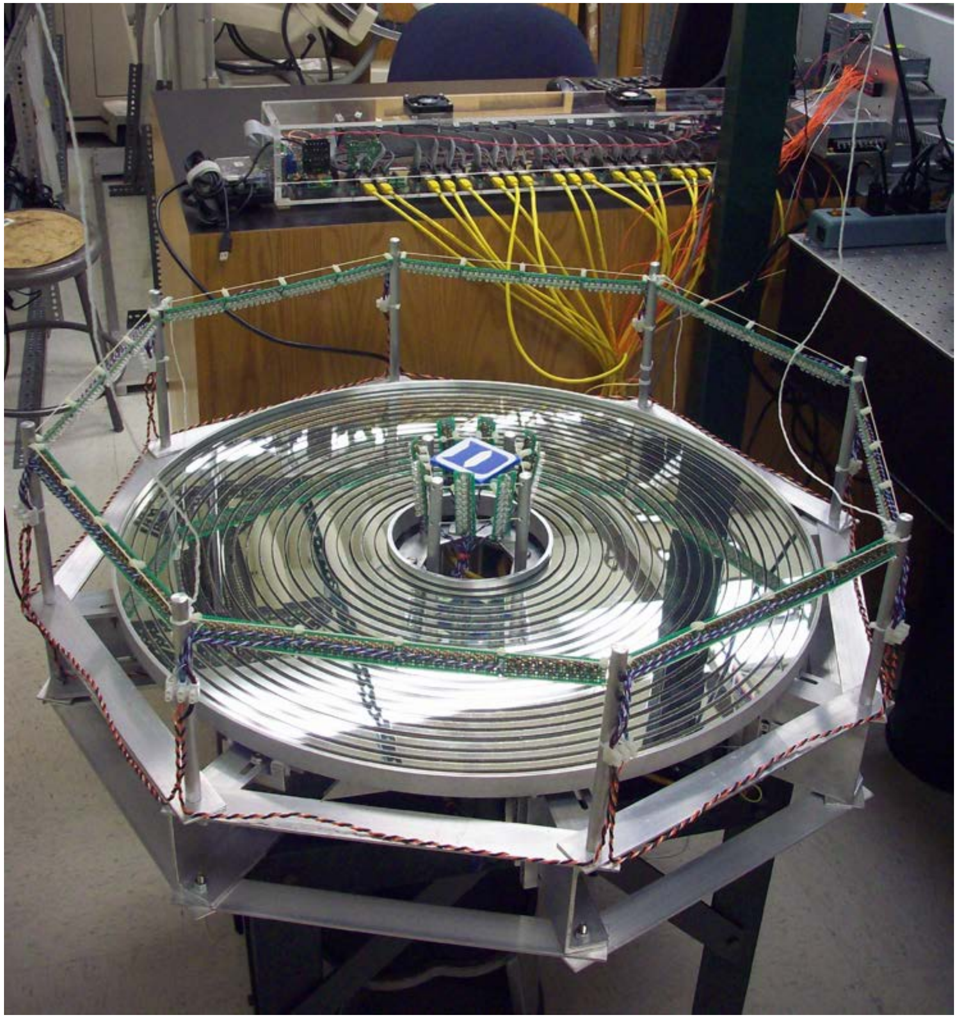
Particles float

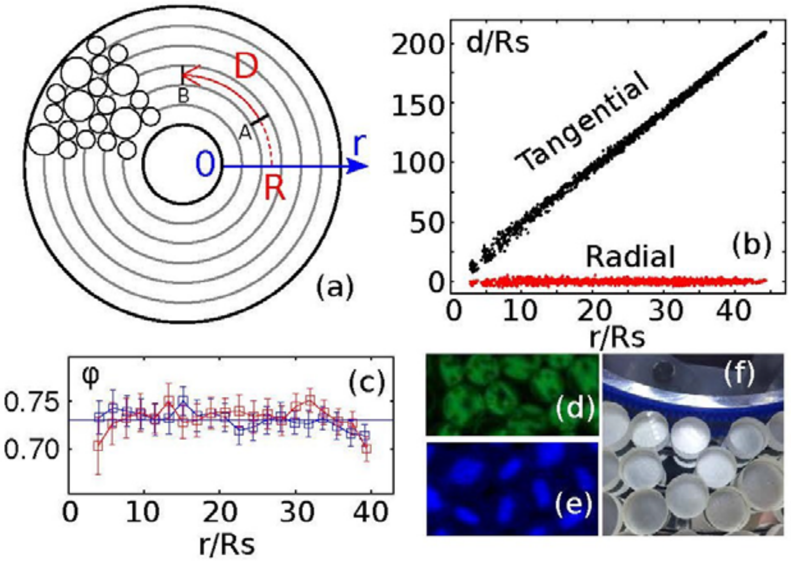
Understanding origin of networks—removing base friction



Shear induces net outflow of grains from inner shearing surface

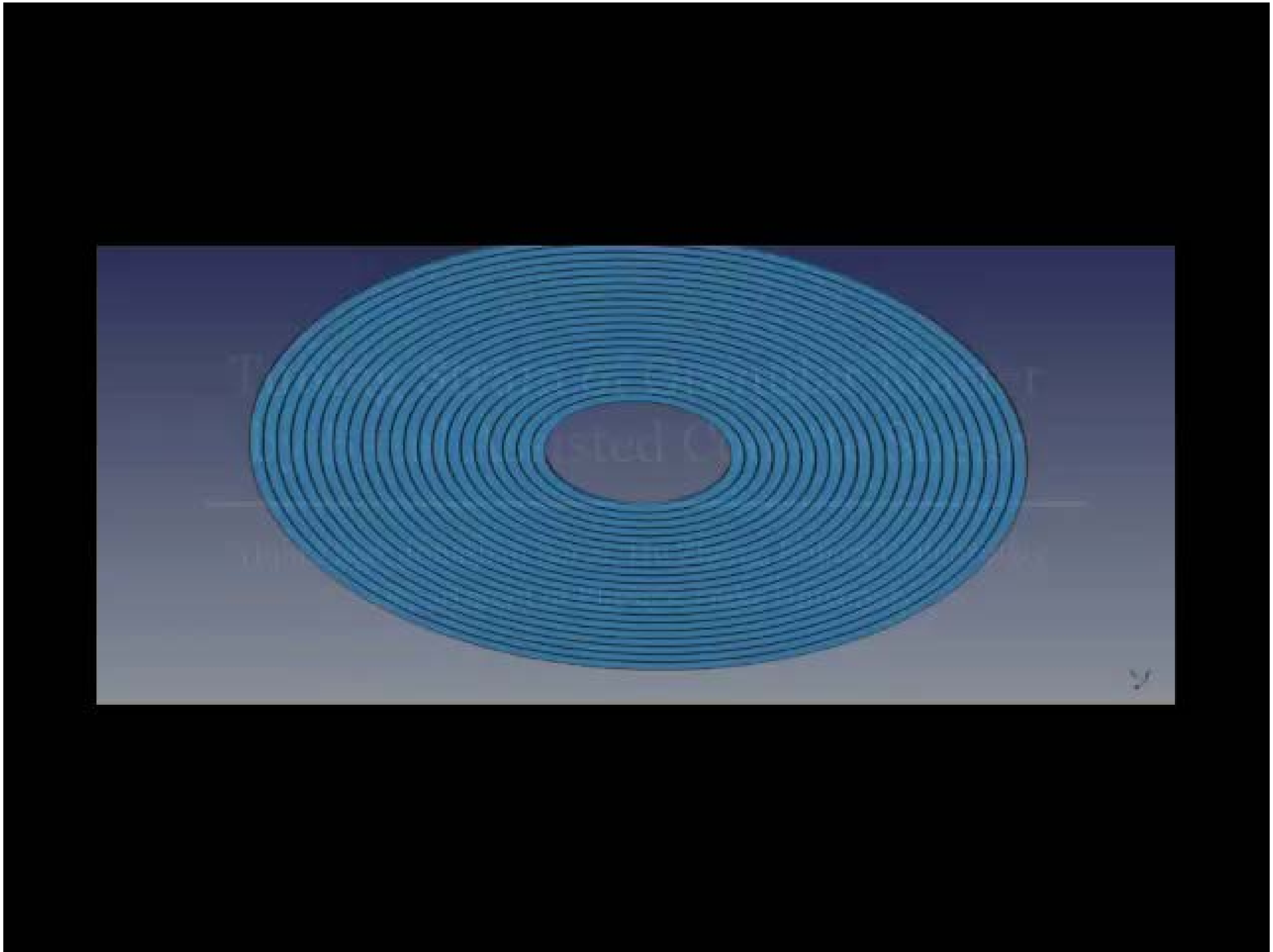
Achieving unlimited shear strain without shear banding-- Co-axial version of simple shear experiment



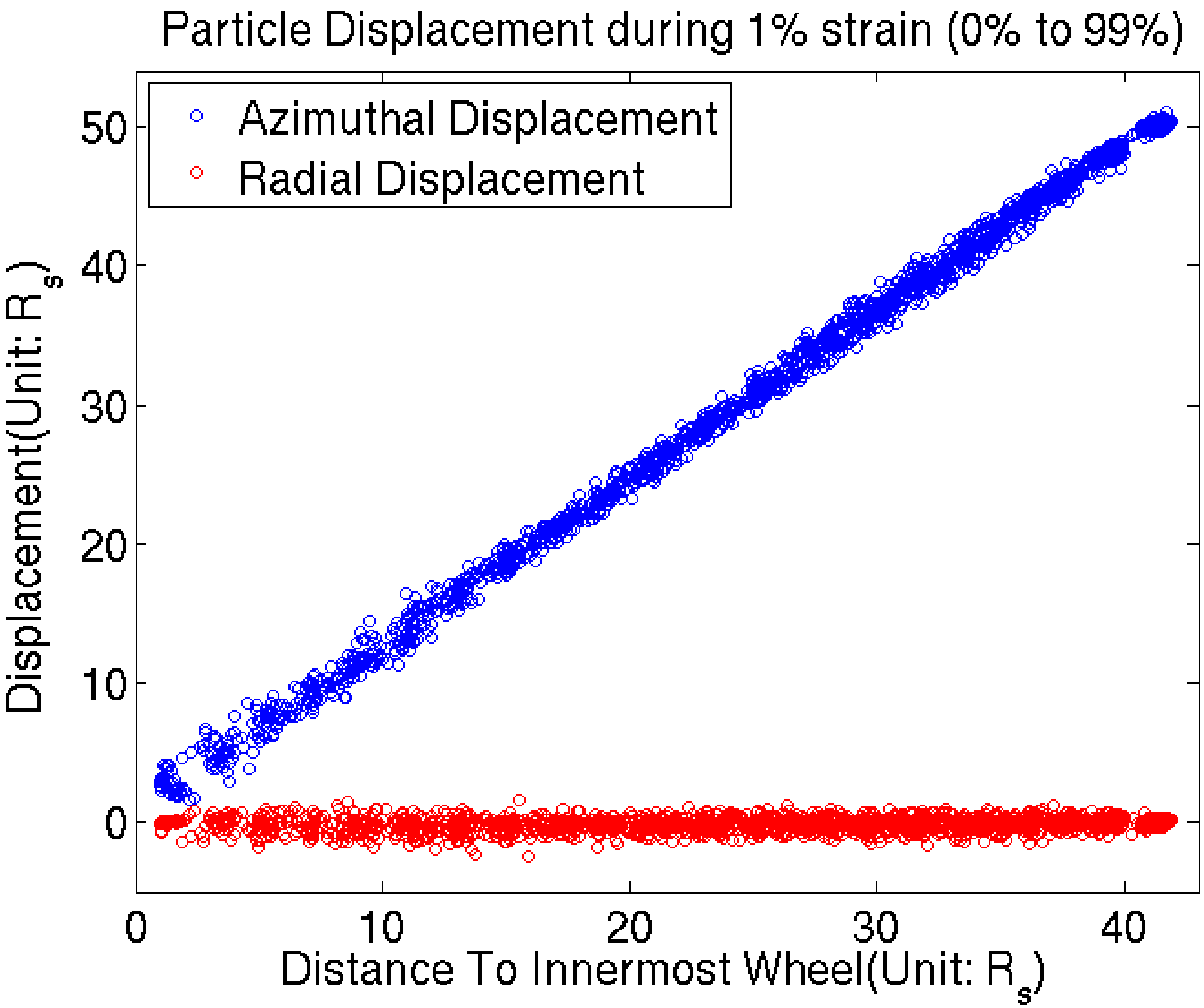


Yiqiu Zhou, Jonathan Barés

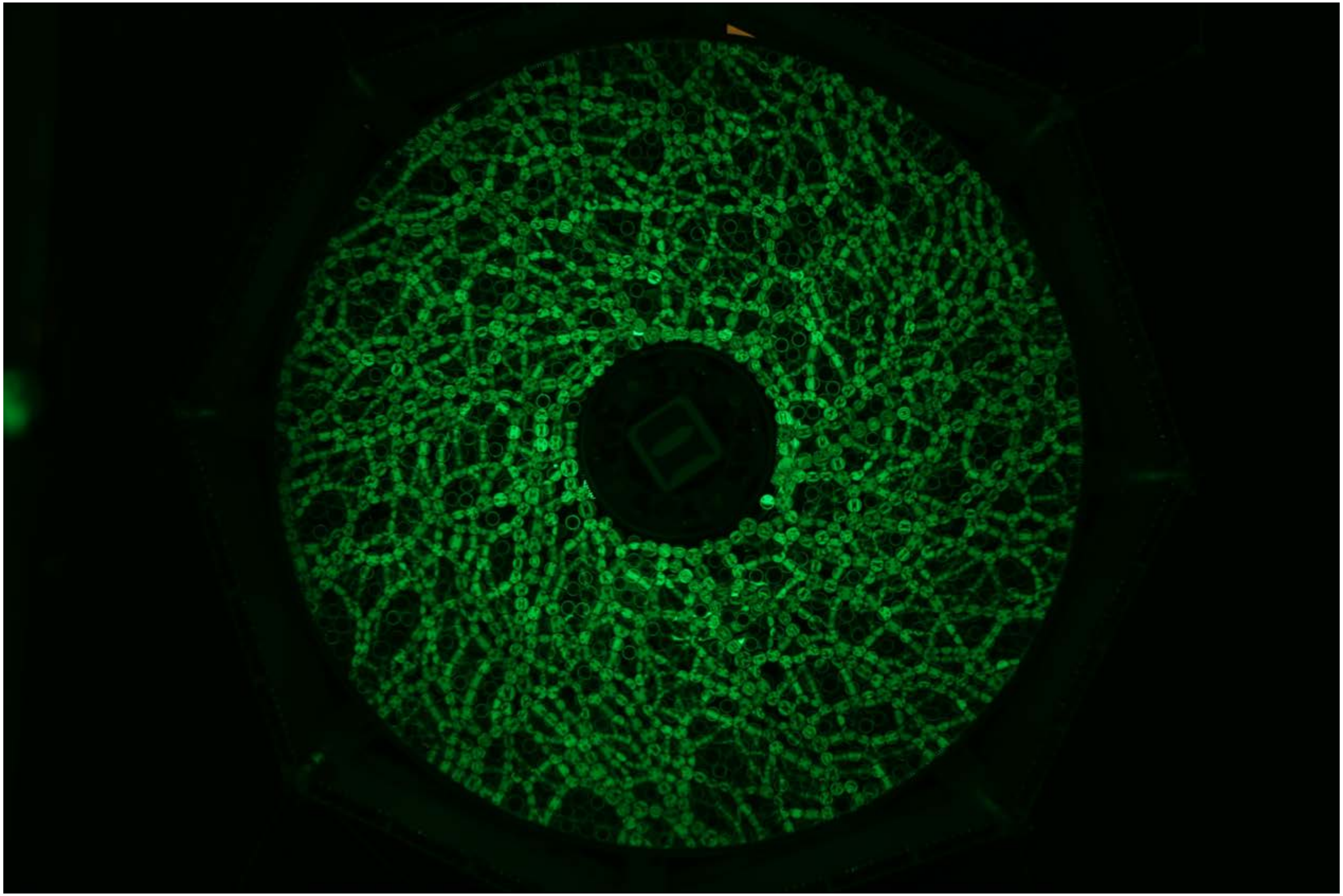
Individual rings are driven independently achieves broad range of shear profiles



Achieving unlimited shear strain
Controlled linear profile



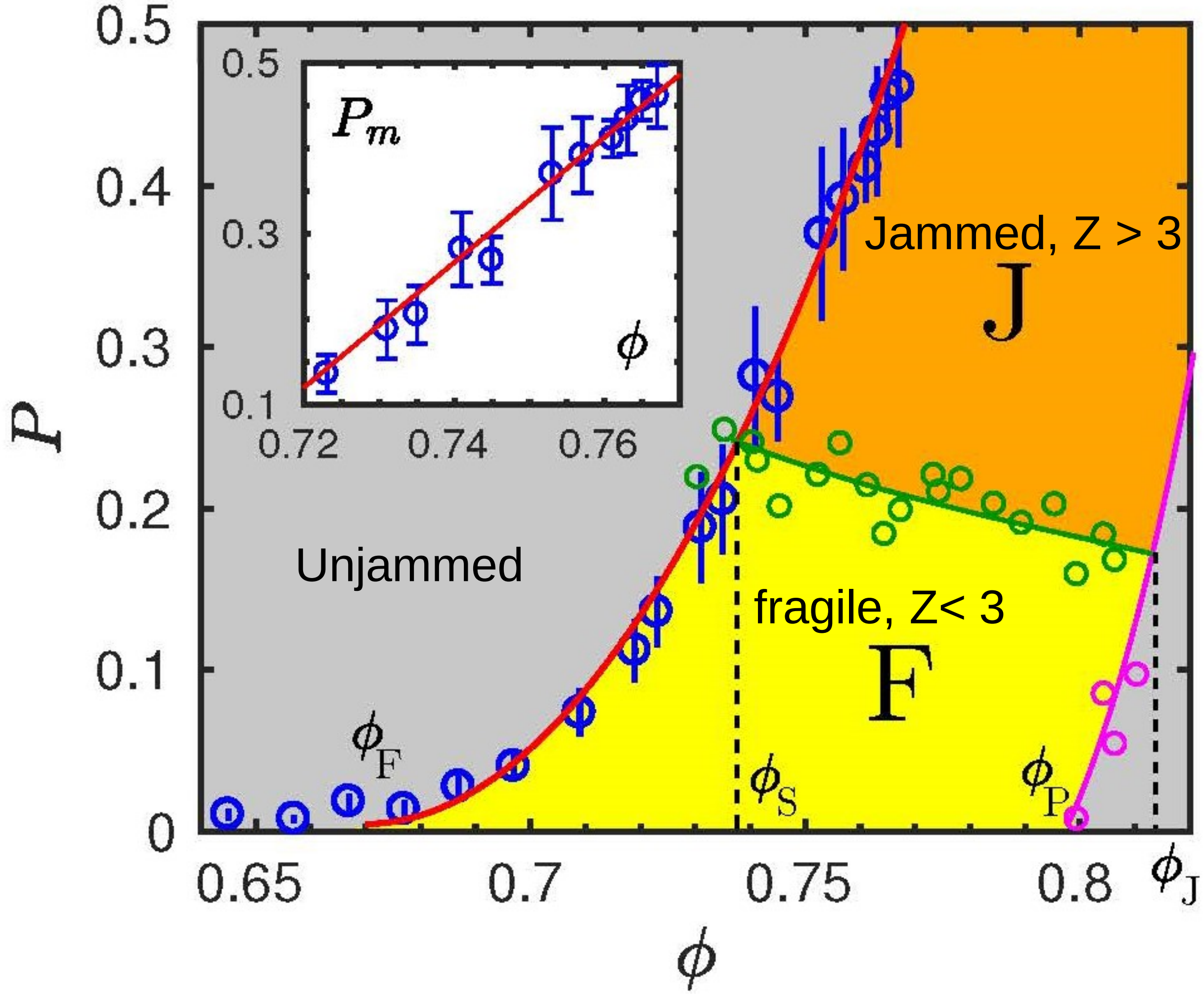
Achieving unlimited shear strain
Force network bend around the core



Achieving unlimited shear strain

Force network bends around the core

Pressure at yield, ring Couette experiment: μ = 0.6

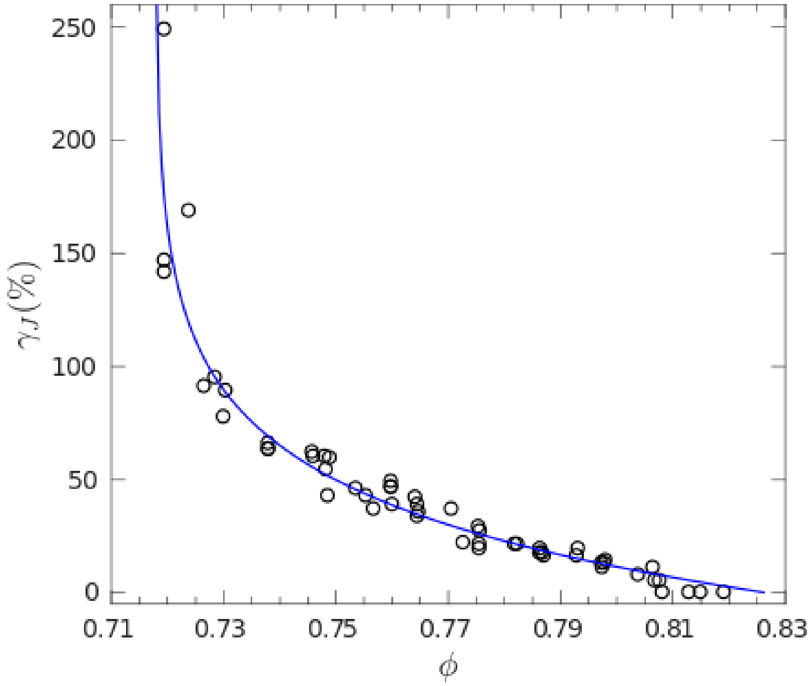


$\gamma_J(\varphi)$ = Strain to achieve jamming

- Control parameter: What is the strain needed to jam a system when $\phi \in [\phi_S, \phi_J]$?

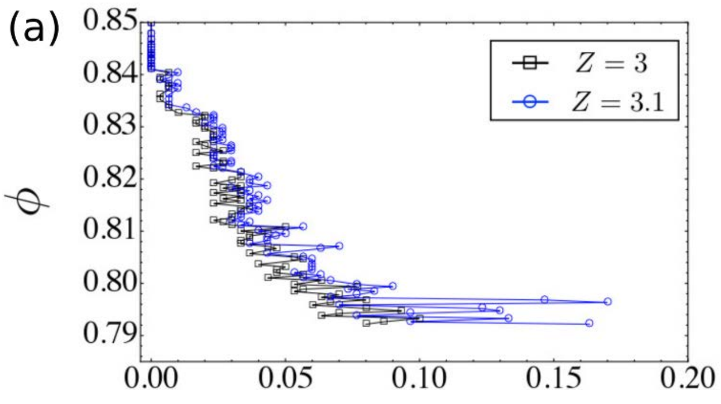
$$\gamma_J = -\gamma_C ln(\frac{\phi - \phi_S}{\phi_J - \phi_S})$$

N. Kumar et al. Granular Matter, 2016



$$\Rightarrow \phi_J = 0.827 \pm 0.004$$

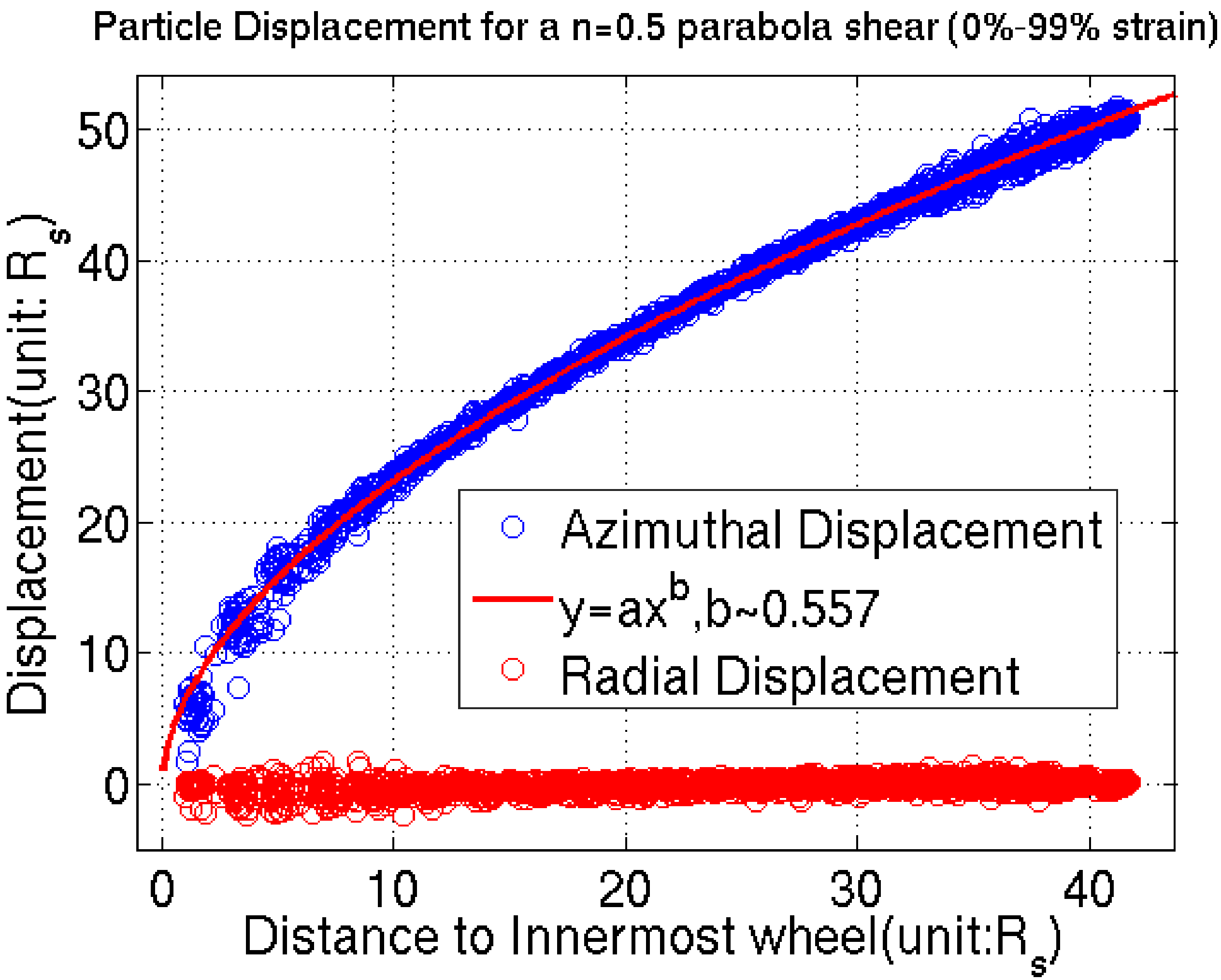
- Note: $\phi_J - \phi_S \approx 0.11$: big separation!



D. Bi et al. Nature, 2011

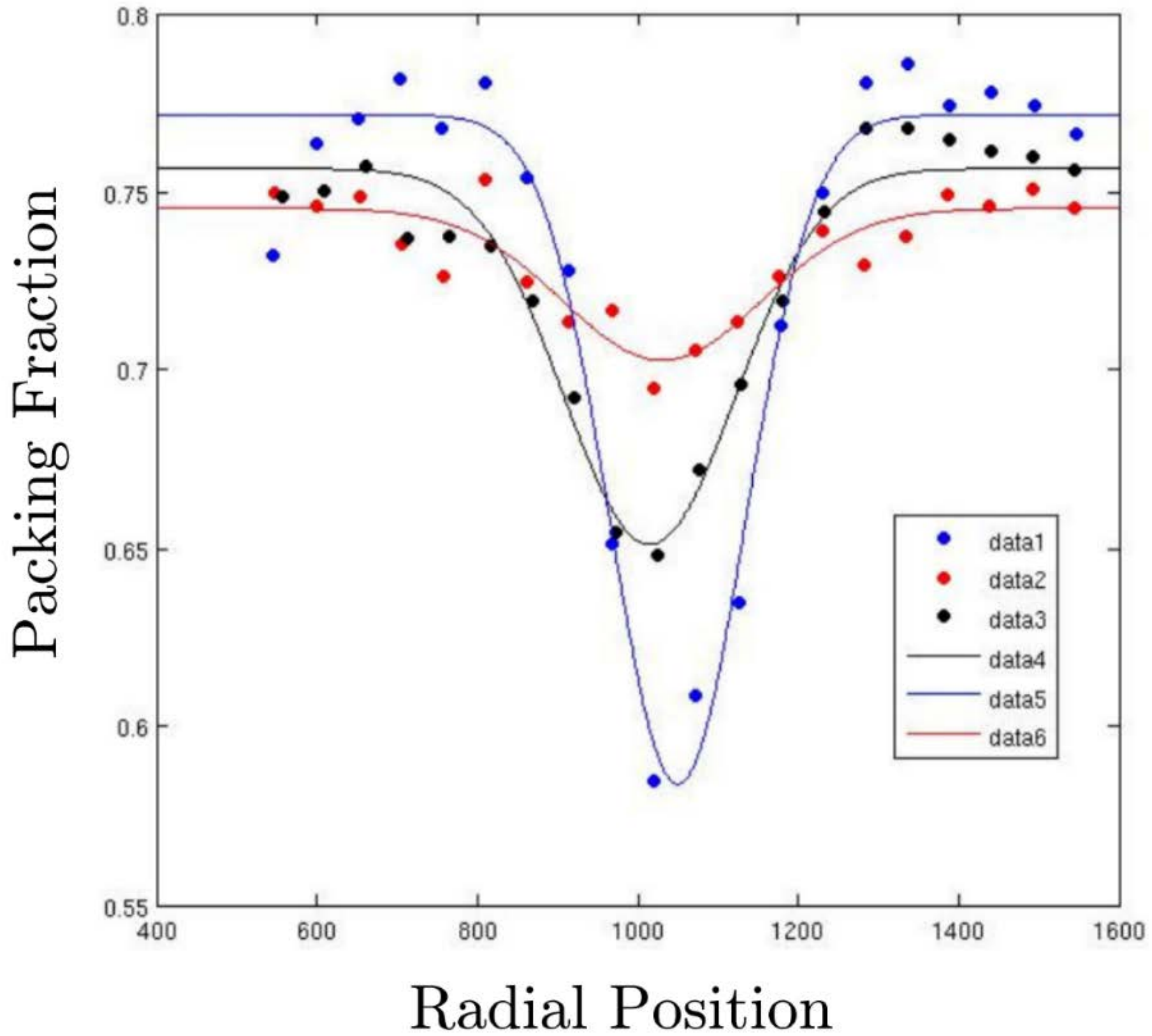
$$\gamma_J(\phi) = min\{\gamma(Z \geq Z^{\infty}_{iso}; \phi)\}$$

Achieving unlimited shear strain
Other profiles: here square-root

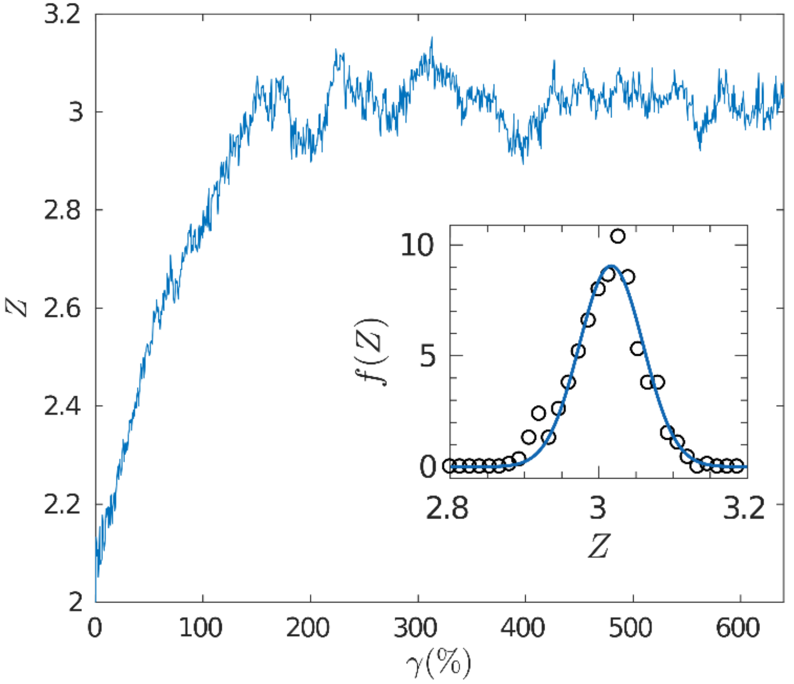


Achieving unlimited shear strain

Controlled placement and depth of shear band



Find the value of φ_S then move on to YSC



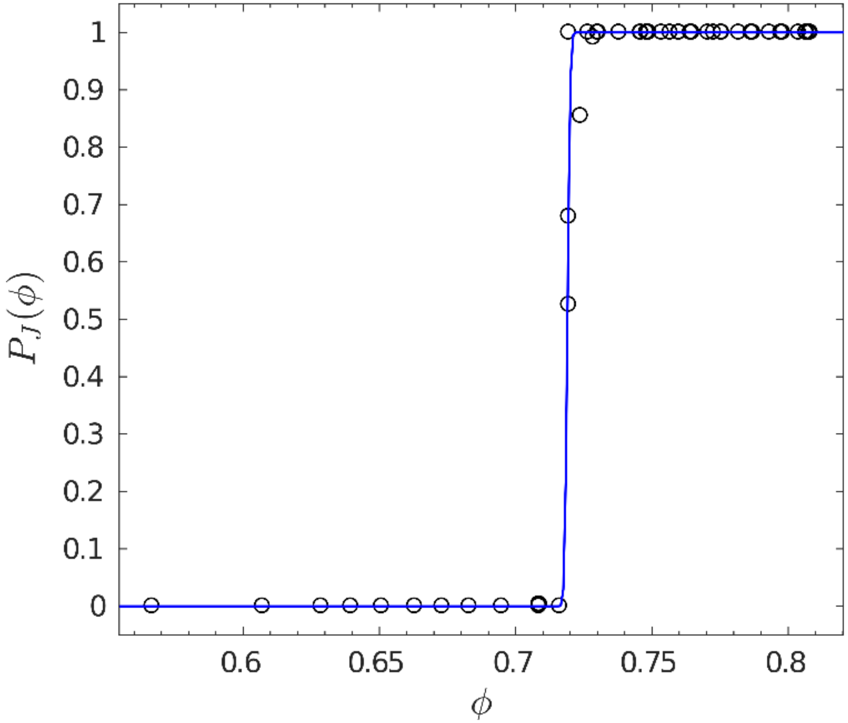
1. Typical behavior of Z: rapid growing before steady state fluctuation.
2. The steady state fluctuations of Z follow a gaussian-like distribution
3. Jamming Probability: $P_J(\phi) = P(Z \geq Z_{iso}^{\infty}; \phi)$

- Fit $P_J(\phi)$ as error function to get ϕ_S:

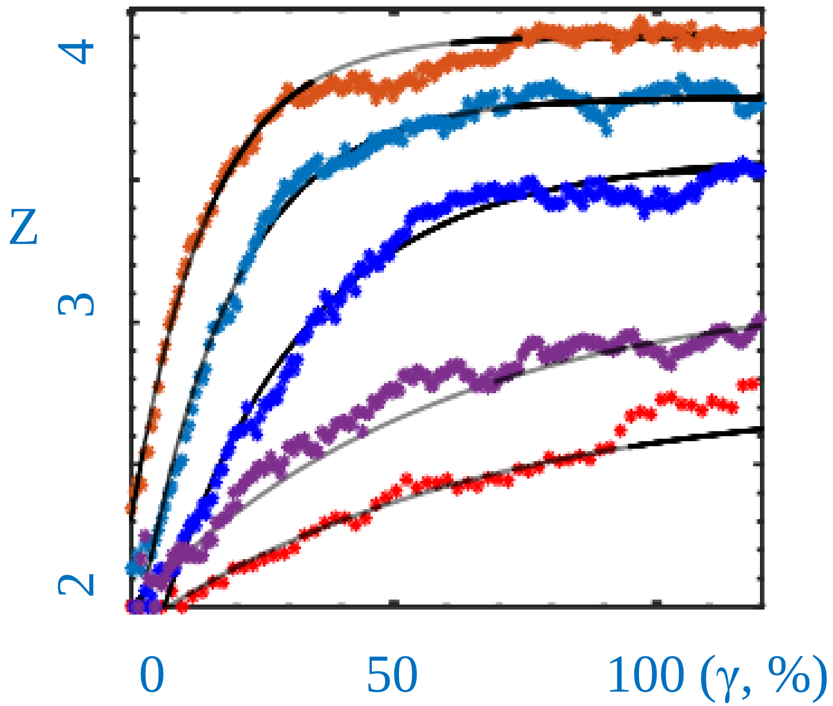
$$P_J(\phi) = \frac{1}{2} erf\big(a * (\phi - \phi_S)\big) + \frac{1}{2}$$

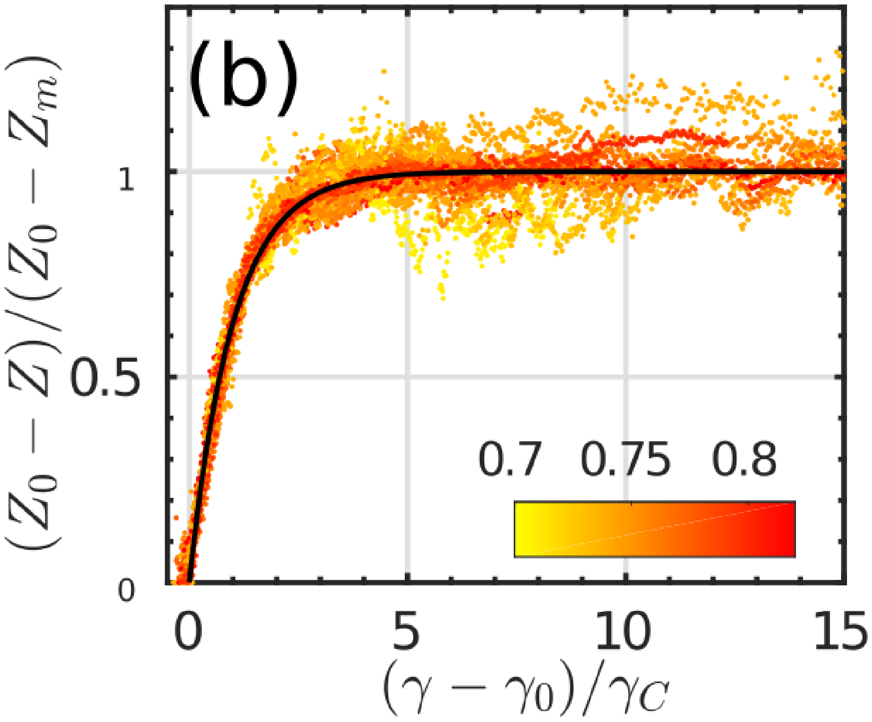
$$\Rightarrow \phi_S = 0.72 \pm 0.01$$

- This value is much smaller than any reported value: 77.8% [D. Howell et al, Couette, 1998], 78% [D. Bi et al., Biaxial, 2011], 75% [R. Jie et al., Linear, 2013]

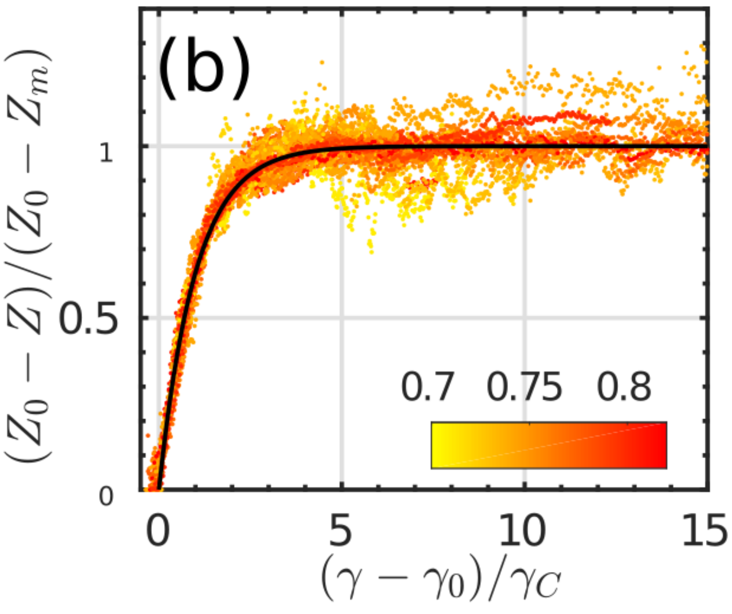


Contact number evolution—
Universal scaling relation

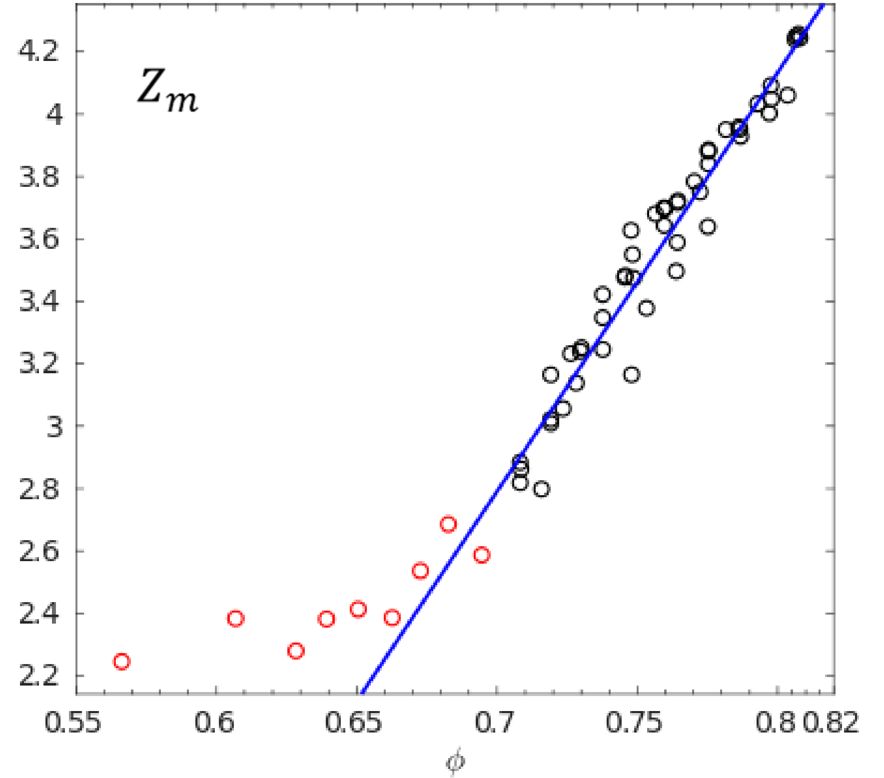




Contact number evolution

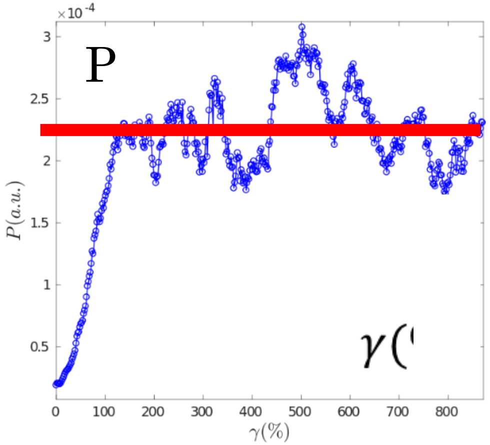


- The growing region of Z can be captured by an exponential fitting.
- $Z = Z_0 + (Z_m - Z_0)e^{-(\gamma-\gamma_0)/\gamma_C}$
  - Z_m, γ_0, γ_C are fit parameters

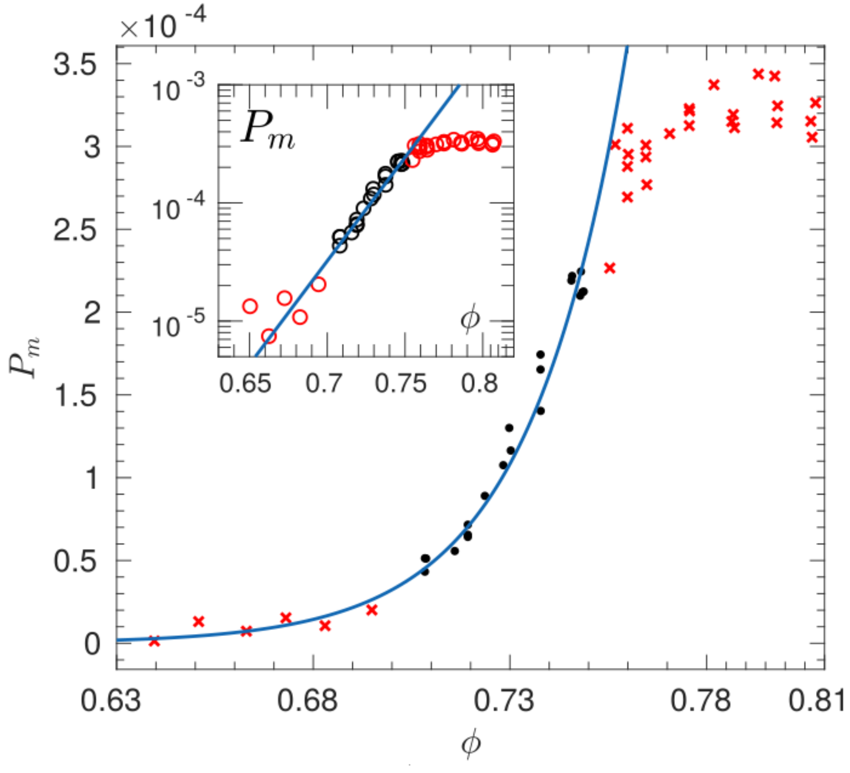


- Rescaling data using fit result gives good collapse for all packing fraction under investigation.
- ⇒ Universal mechanism to generate new contact. [D. Wang et al., under review]
- $Z_m(\phi)$ is linear in region $\phi \in [\phi_S, \phi_J]$
- $Z_m - Z_{iso}^{\infty} \propto (\phi - \phi_S)$

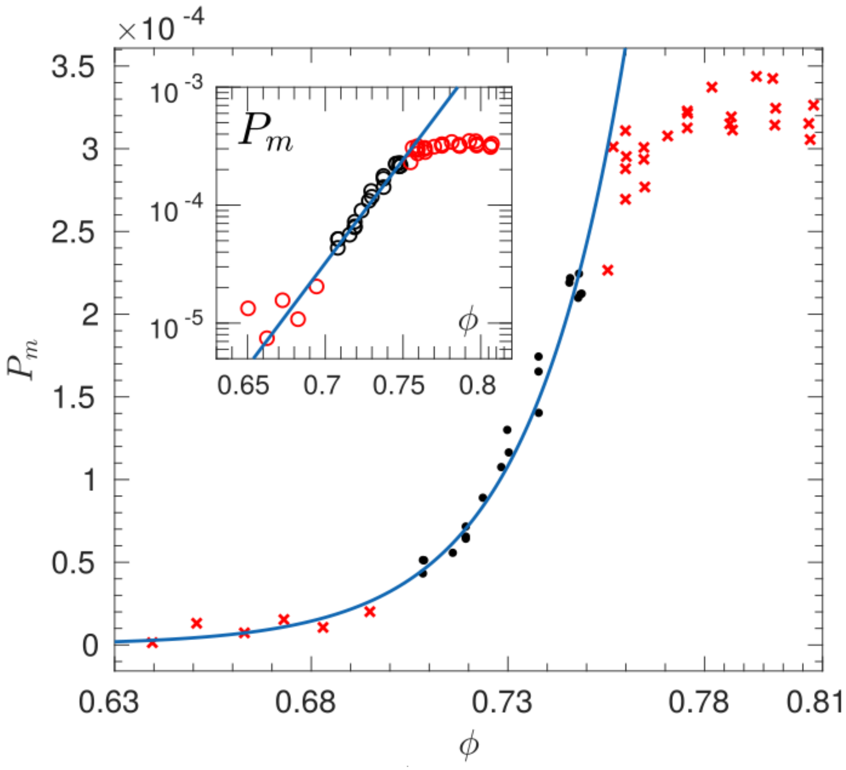
Pressure response at yield surface—use G^2

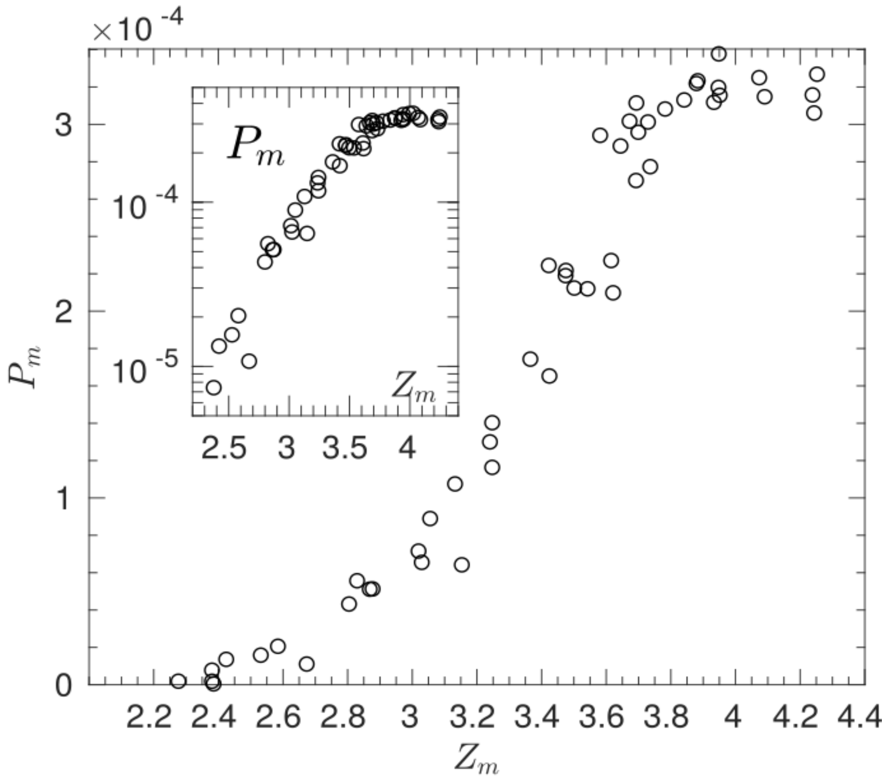


- Fast rising followed by steady state fluctuation.
- P_m: steady state average→ yield stress



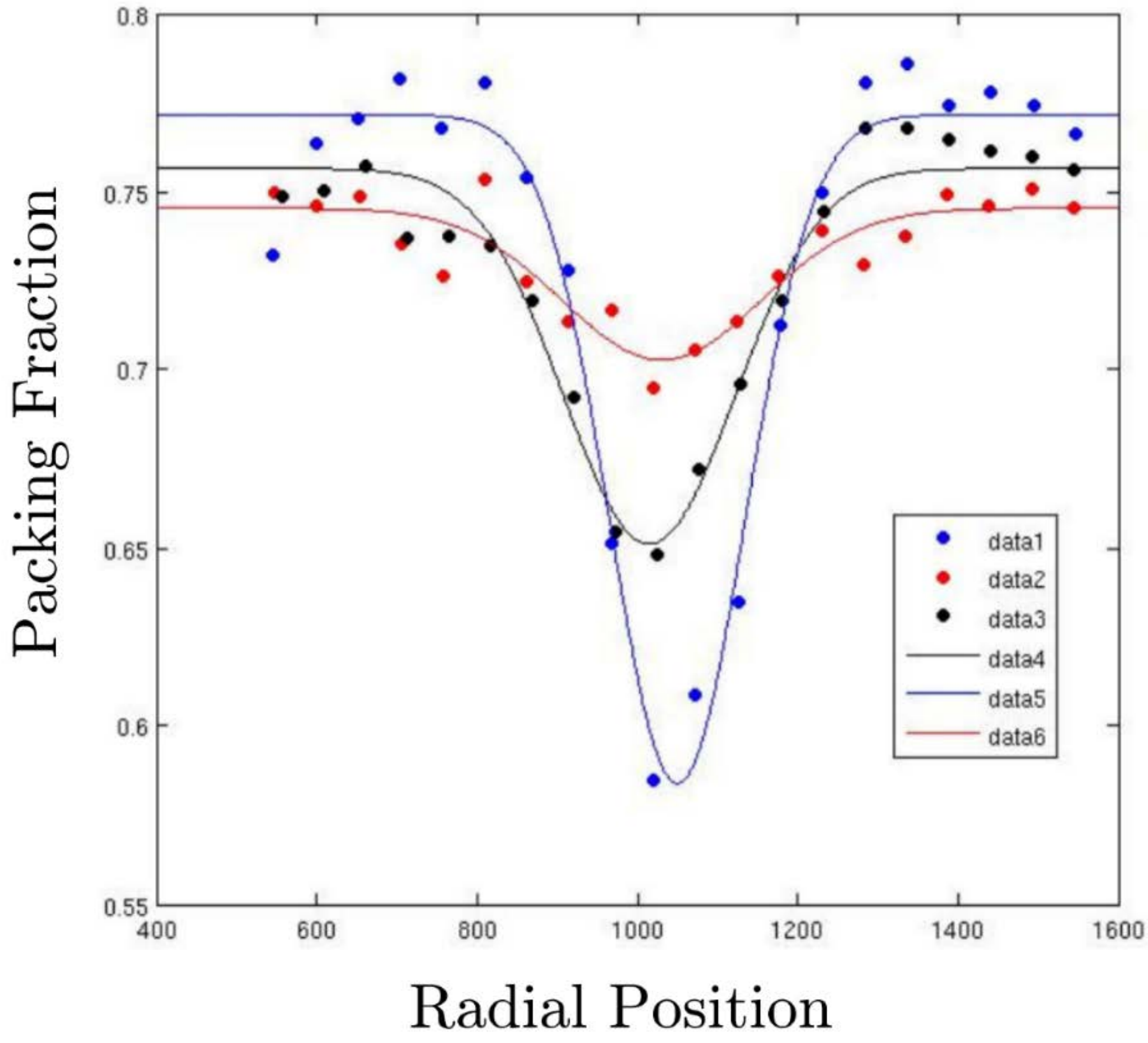
Pressure response—vs. Z



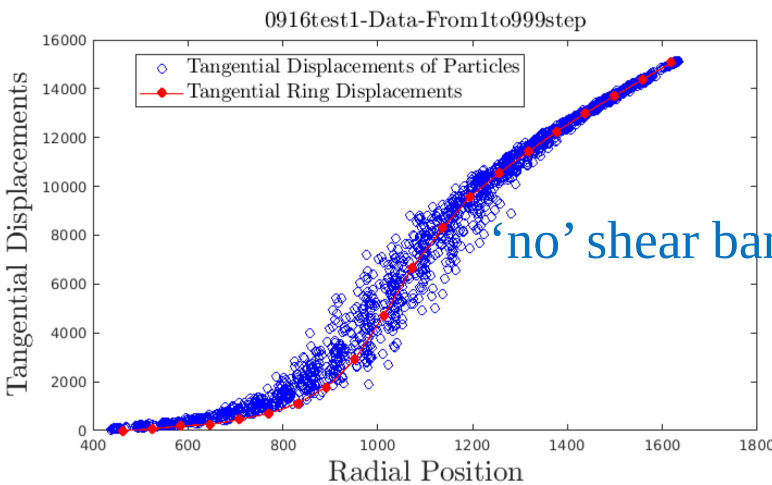


- No critical change at $Z_m = Z_{iso}^{\infty}$
- The saturation of P_m needs further investigation because G^2 measurement is not accurate when the pattern is dense.
- Onset of yield stress $\phi_F < \phi_S$

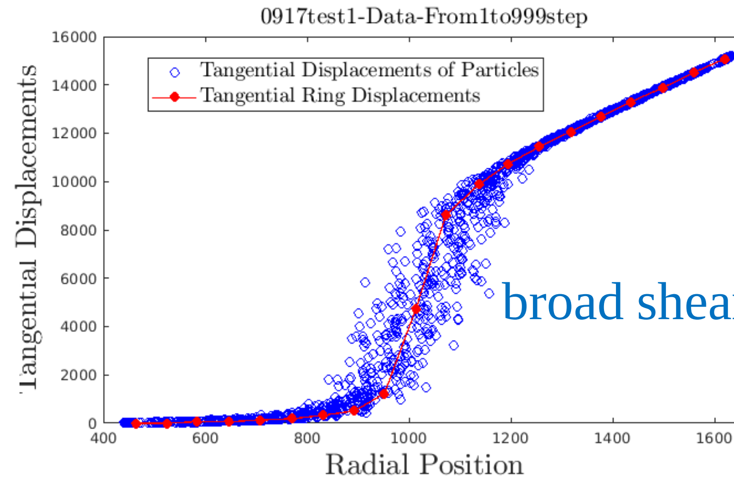
Return to controlled shear band



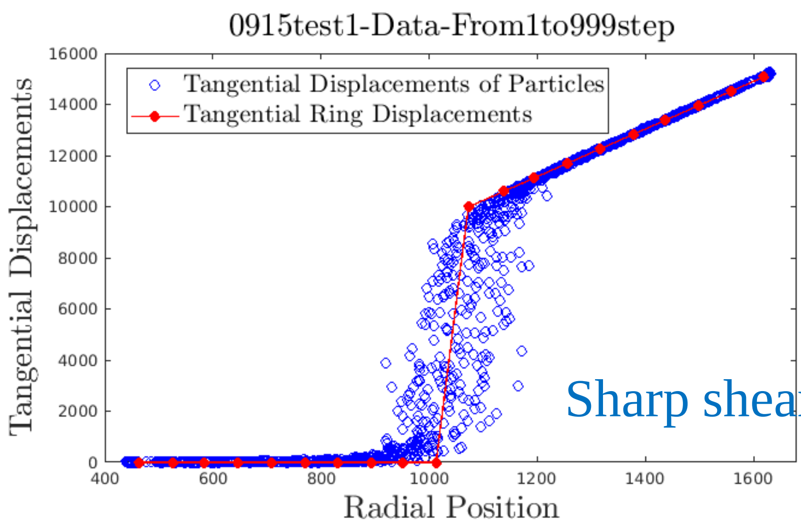
Three different ring protocols to give no, wide, and sharp shear bands—as seen in tangential displacement profiles



'no' shear band



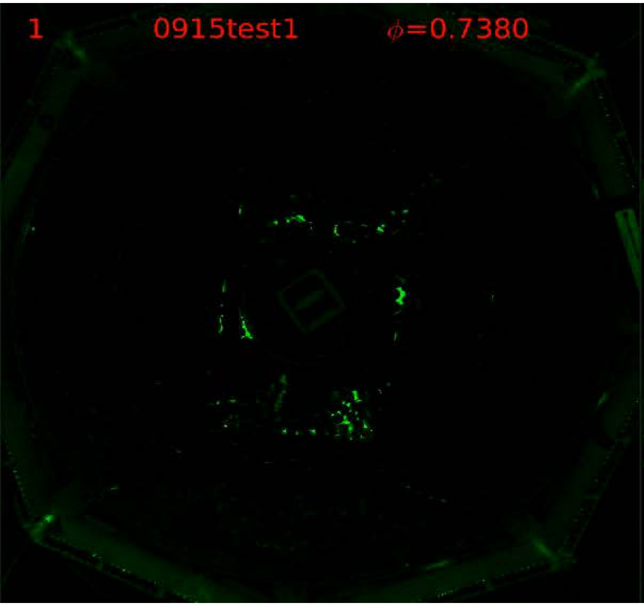
broad shear band



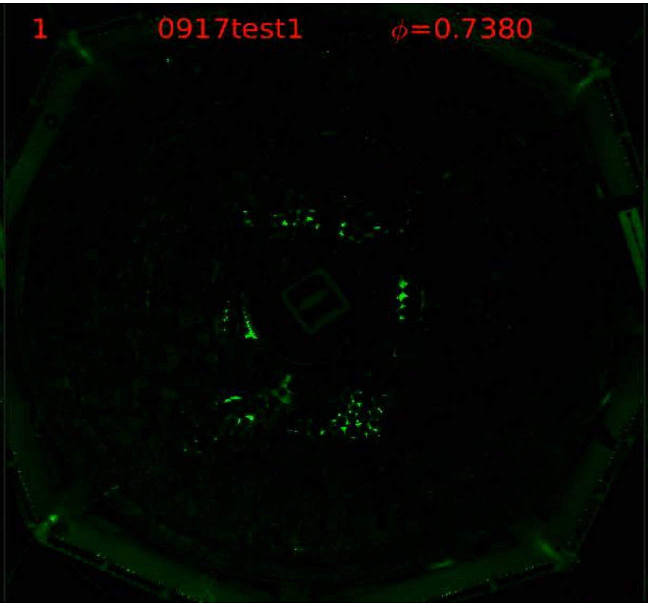
Sharp shear band

Markedly different force responses

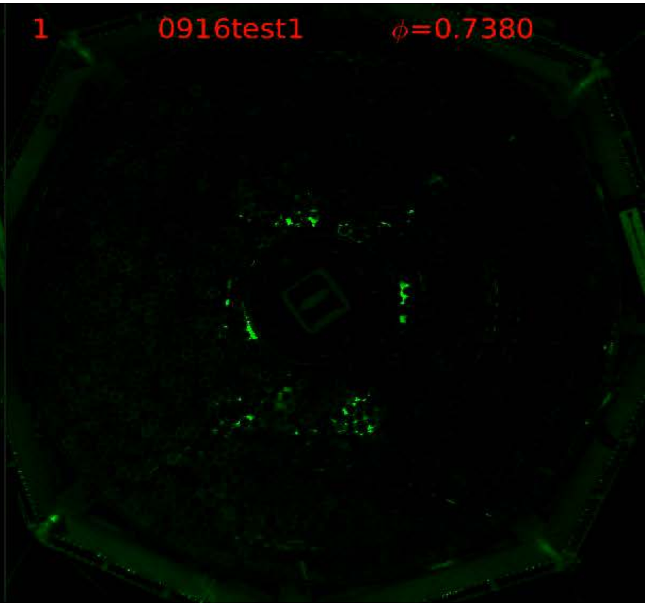
Sharp SB



Broad SB

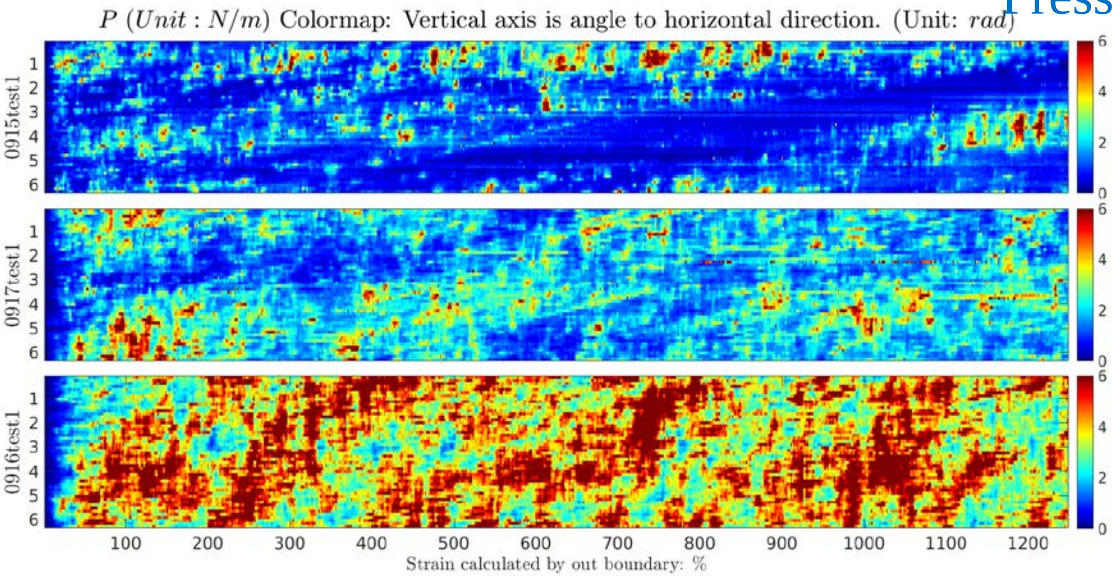


No SB



Pressure and shear stress averaged radially

Pressure, P



Sharp SB

Wide SB

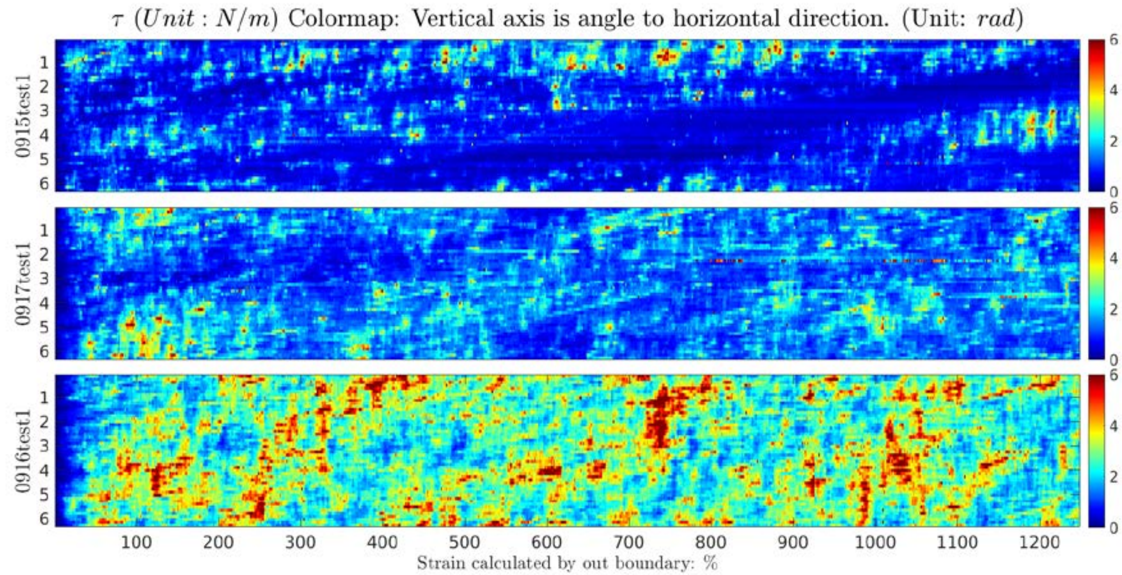
No SB

Shear Stress, τ

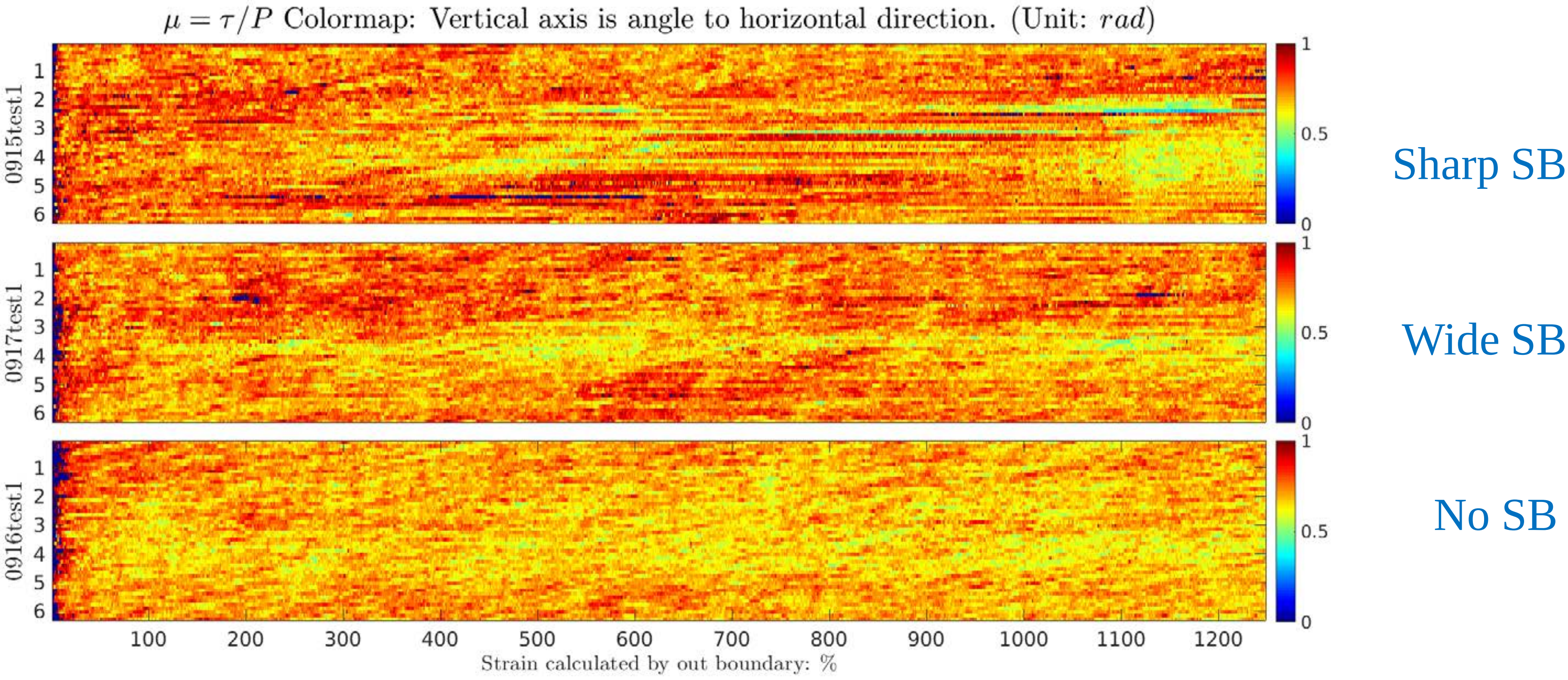
Sharp SB

Wide SB

No SB



Stress ratio, τ/P averaged radially



Overall: non-shearbanding case is more homogeneous, and supports larger stresses

Shear applied to granular materials leads to complex phase diagram with 'Nose'

- Particles with friction jam under shear-- 'bottom of diagram': What is connection with dynamic states?
- States occur at lower φ than isotropic protocols: How does this relate to rlp?
- Understood as ordering in force-tile space: How universal is this?
- Networks structures control process—how do they form? What is a minimal characterization?
- What happens in 3D?