ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Friction law and hysteresis in granular materials

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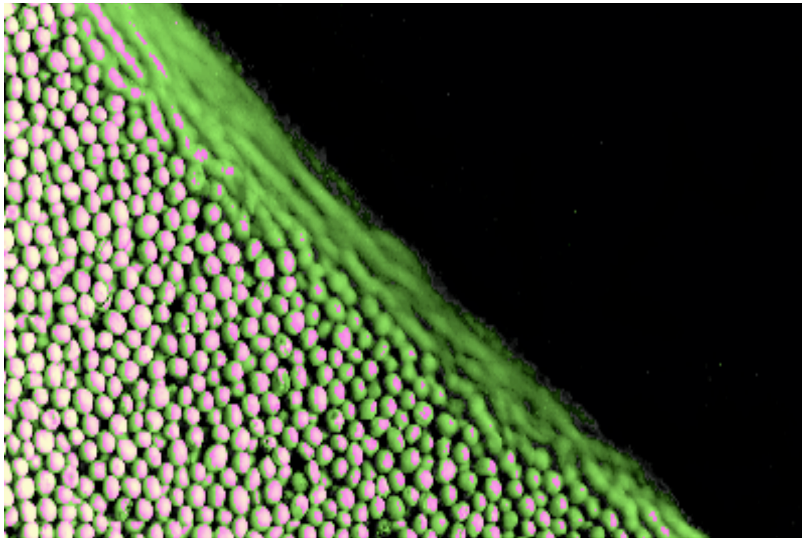
Granular materials (T=*0*, hard)

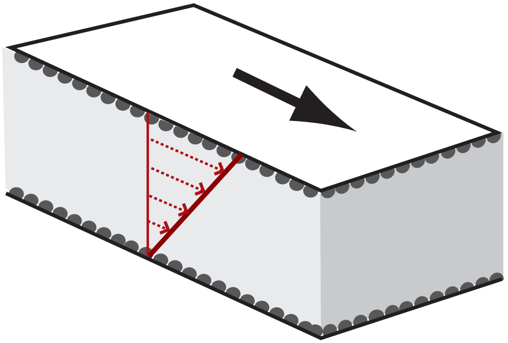
Andreotti, Forterre, Pouliquen



Poppy seeds, Chicago group

- Solid and Liquid phases separated by a jamming transition
- Mesoscopic scale: collective effects important
- Local scale: interaction law between grains complicated
- Rich interplay between these two scales (e.g. shear thickening)

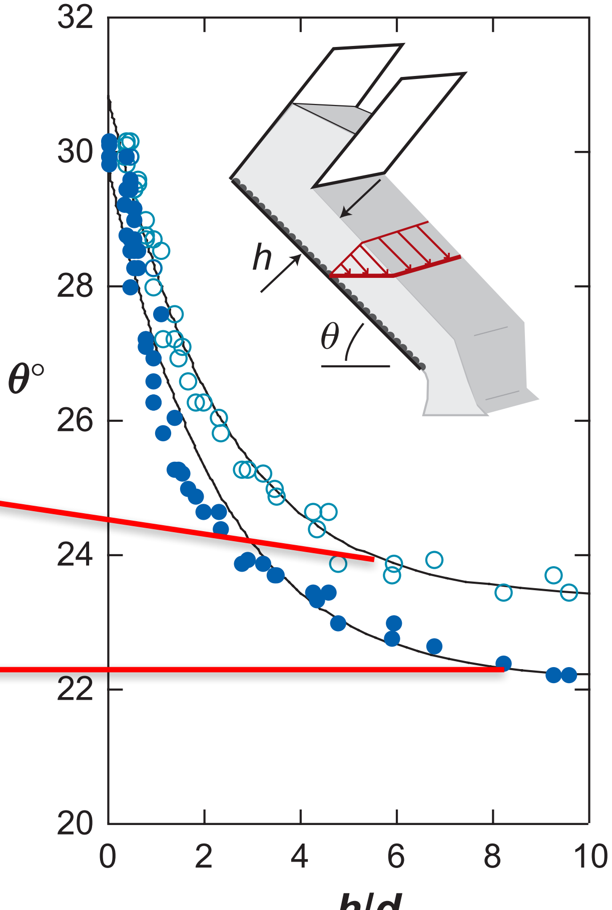
Hysteresis at the jamming transition



minimal stress σ_c needed for steady flow

with $\sigma_c = \mu_c$ p

μ_c = tan(θ_c)



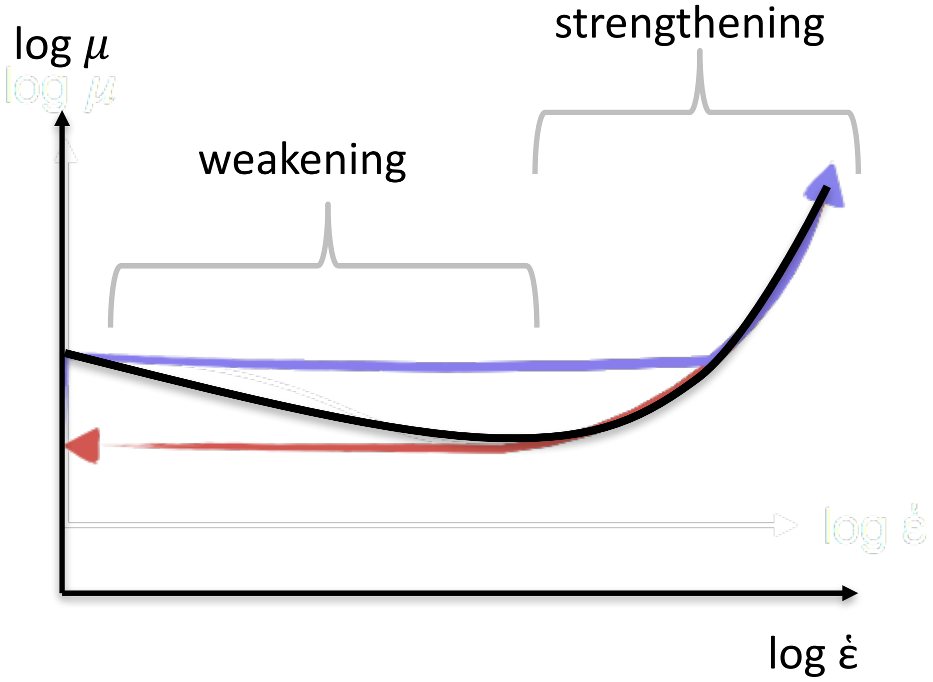
θ_{start}

θ_c

Glass beads, sand:

$$\frac{\mu_{start} - \mu_c}{\mu_c} \sim 10\%$$

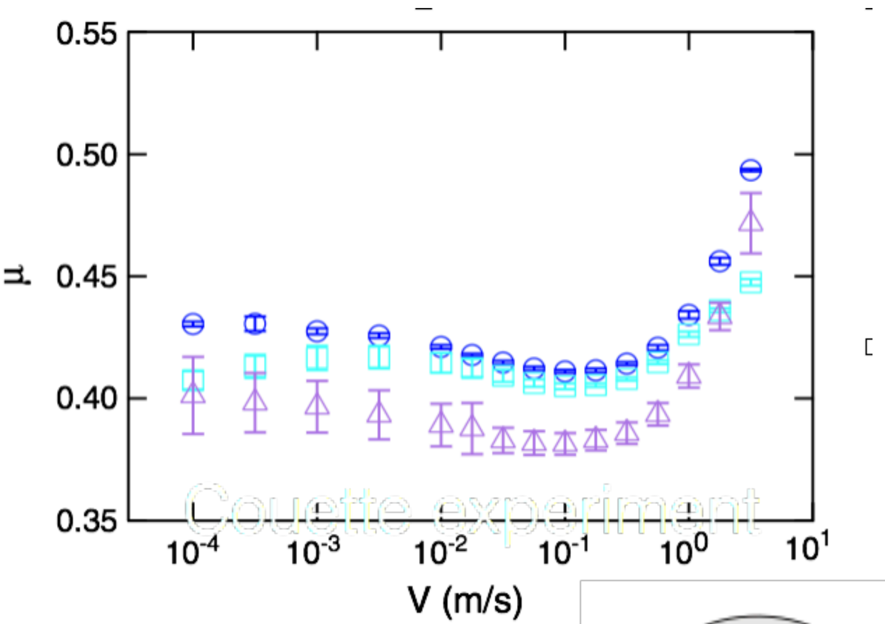
Hysteresis and Velocity weakening



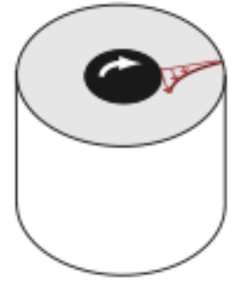
- Sudden starts

but what are the microscopic controls?

- Tricks needed to stabilize against shear band
- Microscopic theory?



Couette experiment



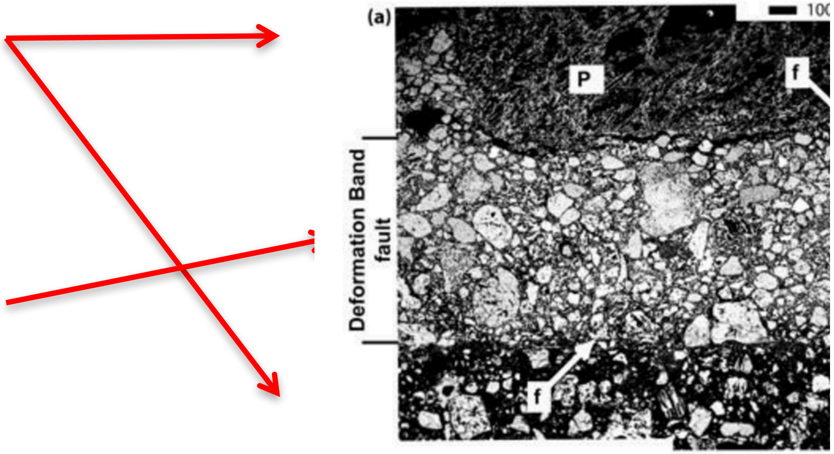
Van Hecke, Dauchot 2010, 2016
Kuwano, Hatano 2013,

Importance of hysteresis in particulate materials

- Hysteresis: once it starts, flow is fast
- Landslides
- Earthquakes

rock

Granular matter



Zooming in a fault

Fault that velocity weaken prone to earthquake

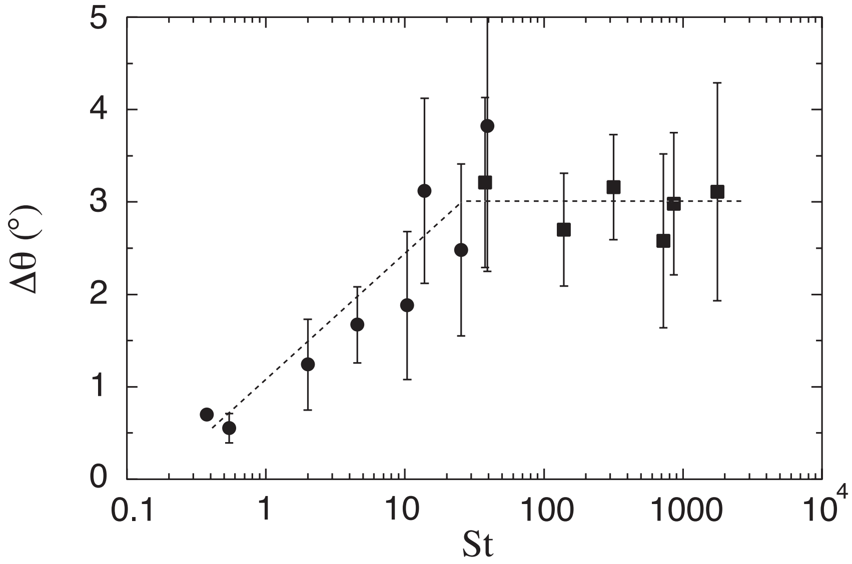
$$\frac{\mu_{start} - \mu_c}{\mu_c} \sim 10$$

Observations to explain in granular matter

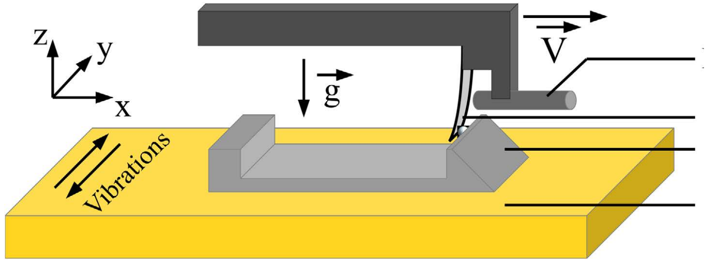
- Inertia favors hysteresis

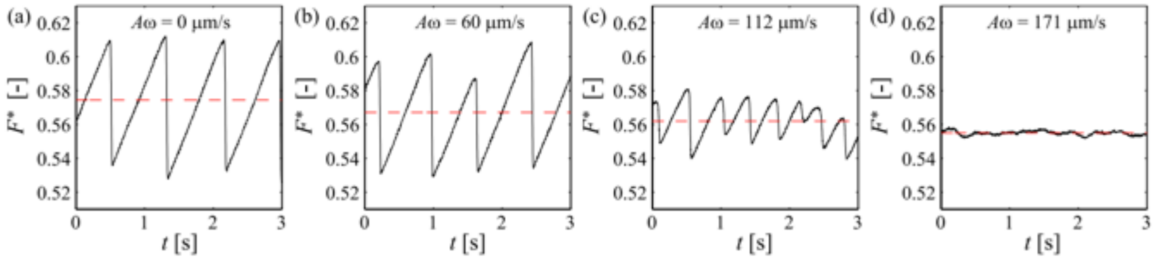
- Friction appears necessary
  Peyneau roux 08

- Acoustic vibrations can eliminate hysteresis



Dupont et al, 2003





Lastakowski et al 2015, Johnson and Jia 05

Dimensional analysis and constitutive relations

GDR Midi, Roux, Radjai, Lemaitre, Pouliquen, Forterre,…

- Δ = deformation at contacts / diameter ~ 10^{-5}

- Rigid limit: if limit Δ -> 0 not singular, dimensional analysis implies one dimensionless number controls flow

$$\boxed{\mathcal{I} = \dot{\epsilon} D \sqrt{\rho/p}}$$

$\dot{\epsilon} : strain \quad rate$

$D : particle \ diameter$

$\rho : mass \ density$

- $\mathcal{I} < 0.1$ dense flows. Most numerical studies report $\mathcal{I} > 10^{-3}$

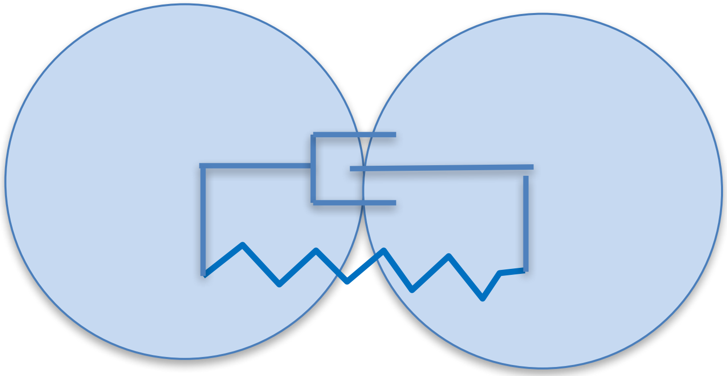
(show data)

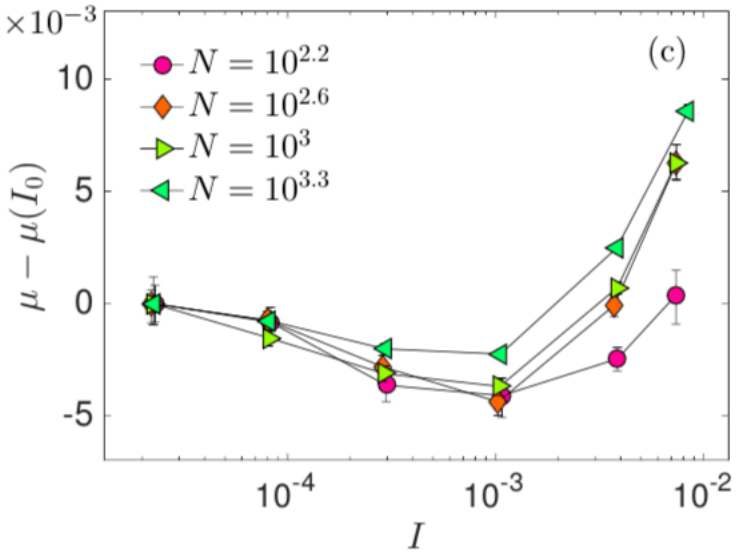
Velocity strengthening:

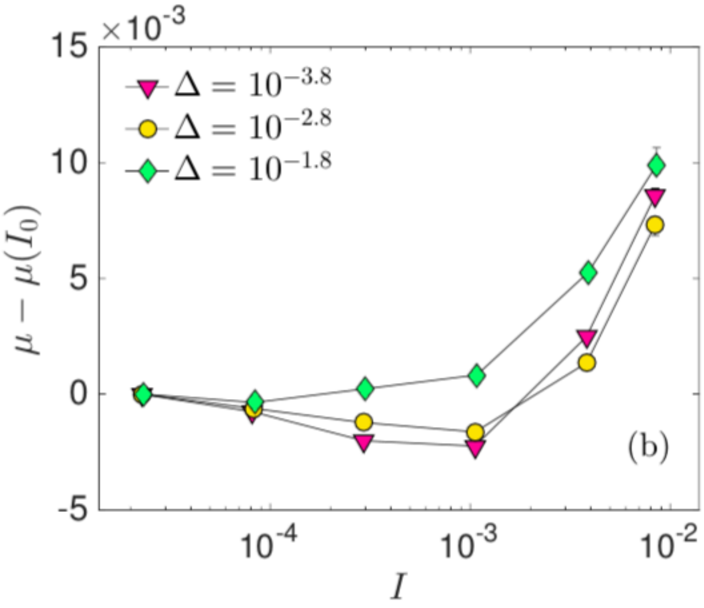
$$\mu \approx \mu_c + c_0 \mathcal{I}$$

Simulations

- Static and dynamic friction coefficient identical μ_f
- Restitution coefficient e
- Small N to avoid shear bands

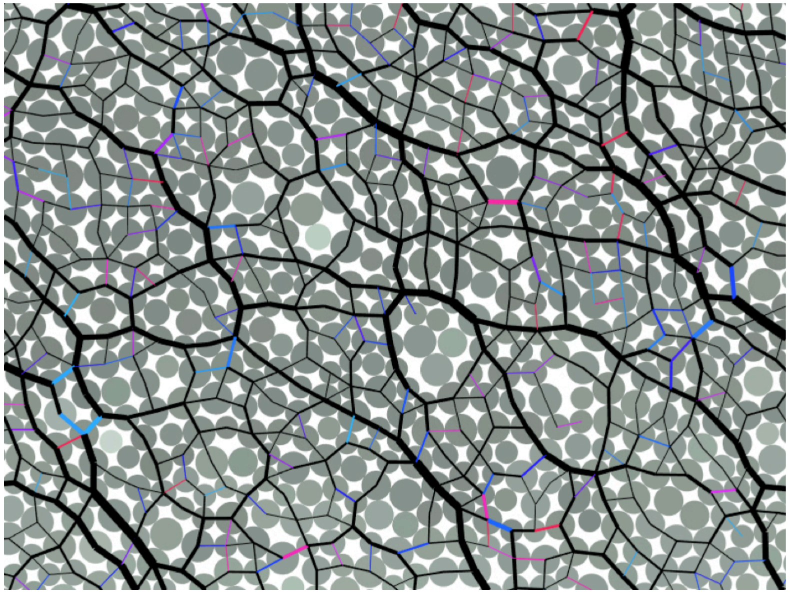






- Effects disappear as Δ increases (or friction removed)

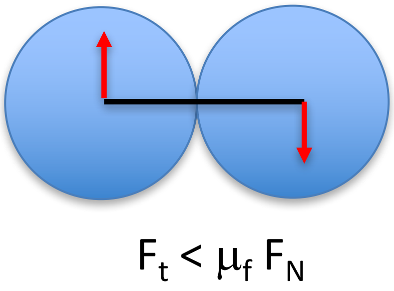
Two ideas



- Near jamming, dense

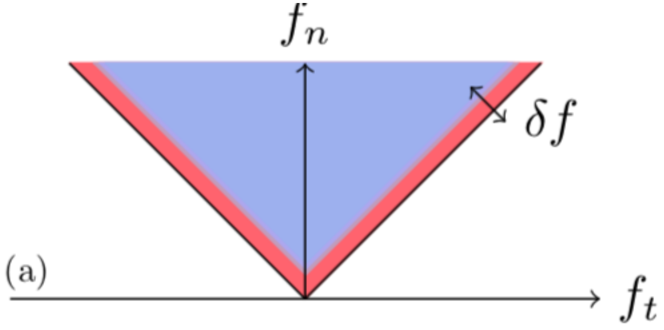
Network of contacts

1/ Collisions induce elastic vibration of the network.
These vibrations make some contacts slide. *Melosh 80's*



2/ soft spots in granular flows (~ dislocation in metals)?
Contact near the coulomb cone.

Macroscopic friction and fraction of sliding contacts



- Gedanken experiment:

Add vibrations to sample.

- Fraction χ of contacts near Coulomb

Cone will slide in average

- Flow threshold will decrease $\tilde{\mu}_c = \mu_c(1 - g(\chi))$

With $g(0) = 0$

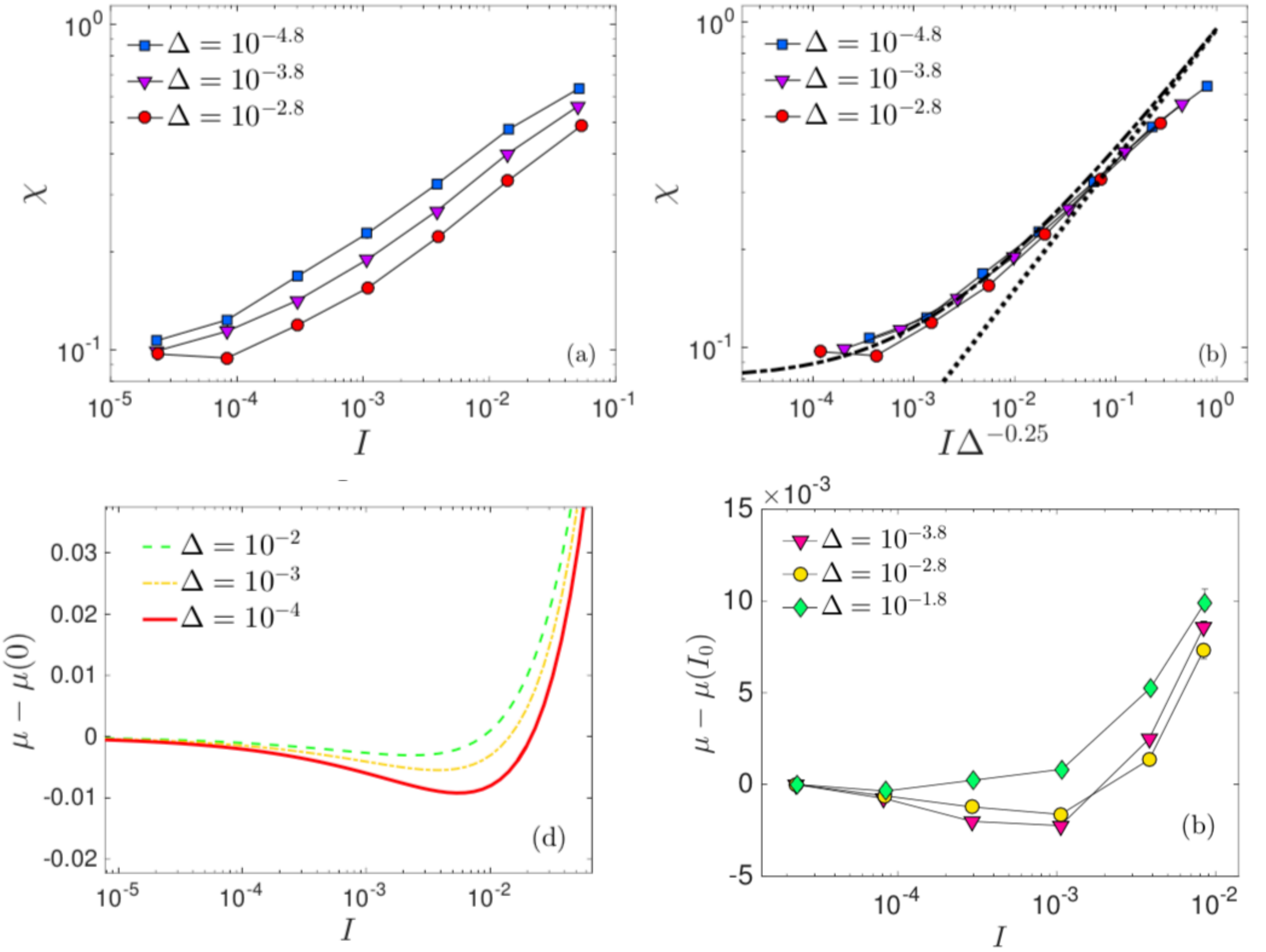
- Same if noise is endogenous, leading to:

$$\boxed{\mu(\mathcal{I}) = \mu_c(1 - g(\chi)) + c_0\mathcal{I}}$$

- Below: use and test model with $g(\chi) = b\chi$

  (b interaction dependent)

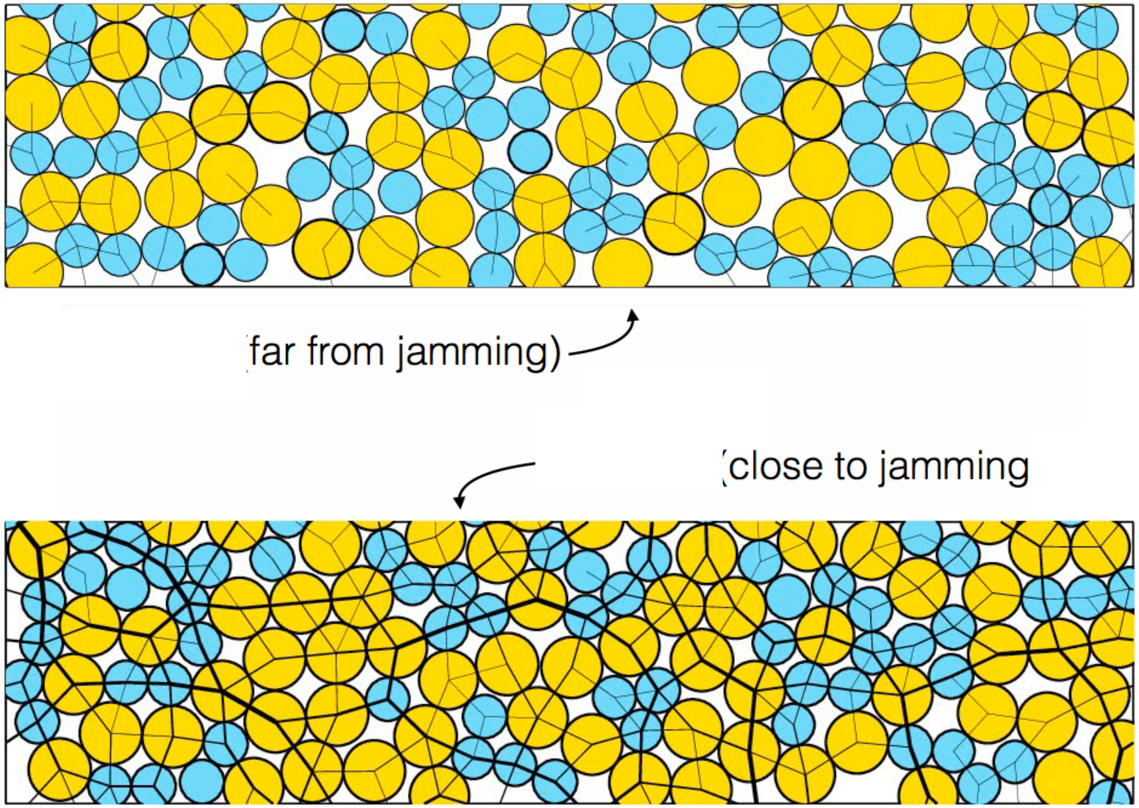
Macroscopic friction and fraction of sliding contacts



- Phenomenological Model agrees well with data
- Limit Δ -> 0 singular for χ! $\chi(\mathcal{I},\Delta) \sim \chi(\mathcal{I}\Delta^{-0.25})$??

Microscopic observables in granular flows

Simple model of frictionless suspensions



Lerner et al, PNAS 2012

- Velocity fluctuations $\mathcal{L} = \frac{V_r}{\dot{\epsilon} D}$
- Strain scale ϵ_v

Extracting Key microscopic properties

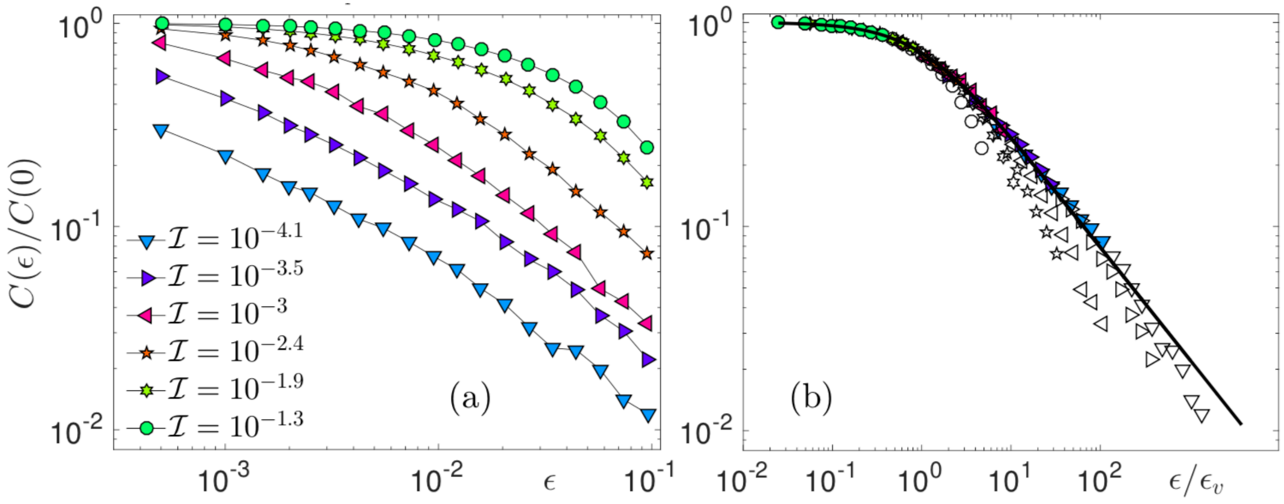
DeGiuli et al, PRE 2015

$$\boxed{C(\epsilon) = \langle \vec{V}_r(0) \cdot \vec{V}_r(\epsilon) \rangle}$$

- Captures velocity fluctuations:

$C(0) = \langle \vec{V}_r^2(0) \rangle \sim \mathcal{L}^2$ appears to diverge near jamming

- Captures ϵ_v (where memory is lost, function drops off)



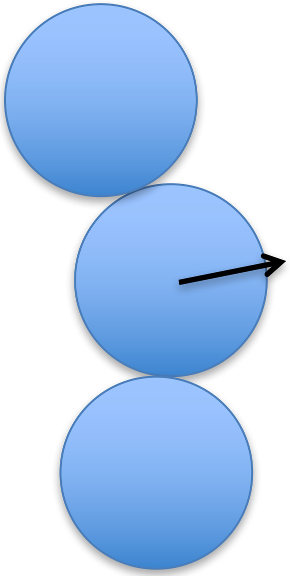
$$\boxed{\epsilon_v \sim \mathcal{I}}$$

Menon Durian 97
DeGiuli et al, PRE 2016

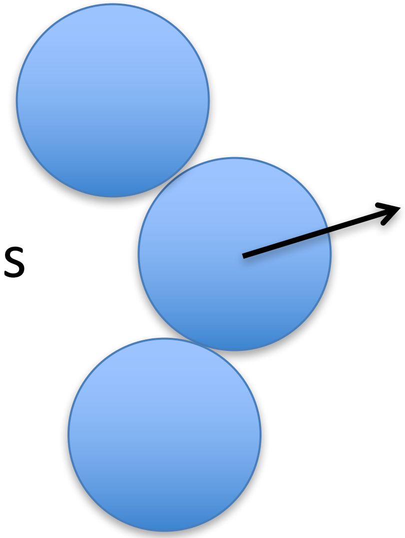
Idea behind $\epsilon_v \sim \mathcal{I}$

DeGiuli et al, PRE 2016

- Network almost force balance



Net force F << contact forces



F grows in between collisions

- Newton equation: velocity increases exponentially.

Characteristic strain $\mathcal{I}$

Collision must occur on that strain scale in a stationary state.

Energy balance

DeGiuli et al, PRE 2016

$\Omega\sigma\dot{\epsilon}$: power injected = power dissipated $\mathcal{D}_{tot}$

$$\mathcal{D}_{tot} = \mathcal{D}_{col} + \mathcal{D}_{sliding}$$

$\mathcal{D}_{col}$ = N * (collisional rate) * (kinetic energy of particles)

$$\sim N \times (\dot{\epsilon}/\epsilon_v) \times (m\mathcal{L}^2\dot{\epsilon}^2)$$

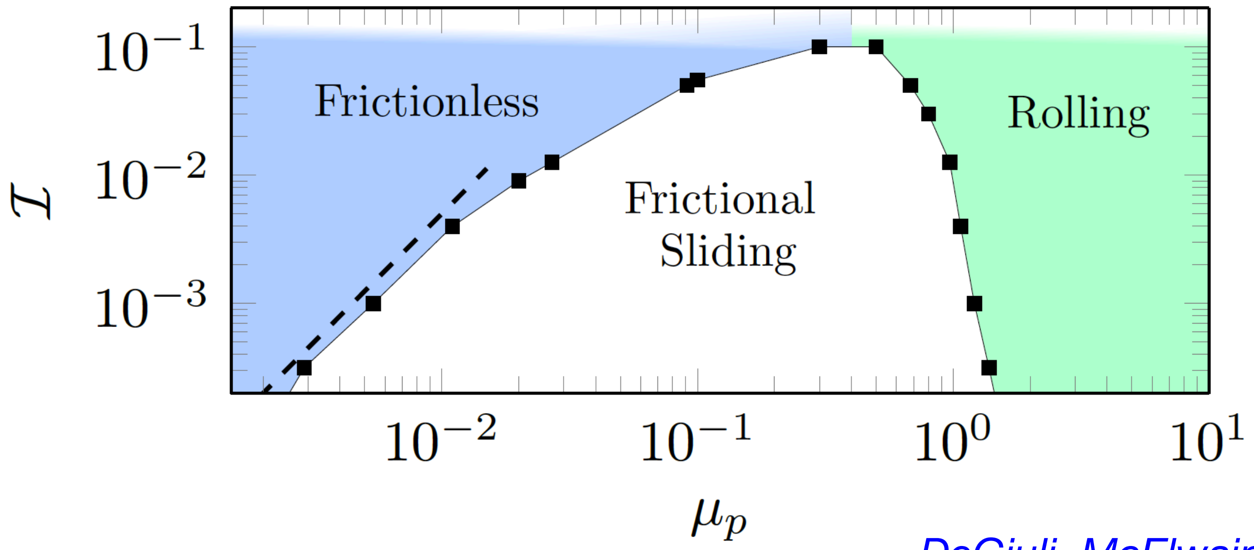
$\mathcal{D}_{sliding}$ = (number of sliding particles)* (transverse force) *(velocity)

$$\sim (N\chi) \times (\mu_f p) \times (\mathcal{L}\dot{\epsilon})$$

χ : Fraction of sliding contact

μ_f : friction coefficient

Sliding dissipation dominates in dense flows

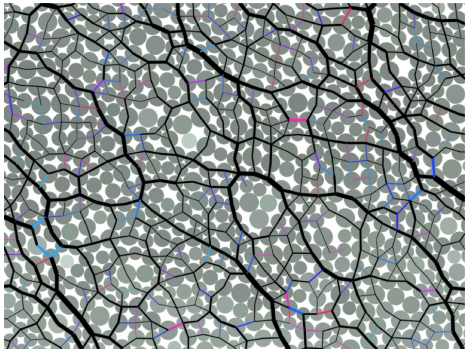


DeGiuli, McElwaine, Wyart PRE 2016

$$\mathcal{D}_{tot} \approx \mathcal{D}_{col} \qquad \boxed{\rightarrow \quad \mu_p \mathcal{L} \chi \sim 1}$$

All quantities $\mathcal{L}, \epsilon_v, \mathcal{D}_{col}$ can be expressed in terms of $\mathcal{I}, \chi$

Estimating the mechanical noise δf



- vibrational energy $\sim \tau \; D_{col}$

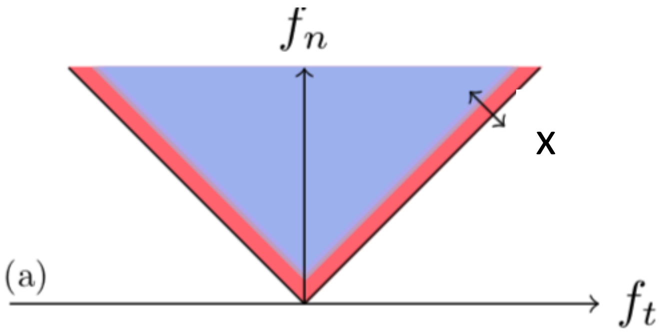
- As to be compared to potential energy $\sim \; \Delta^2$

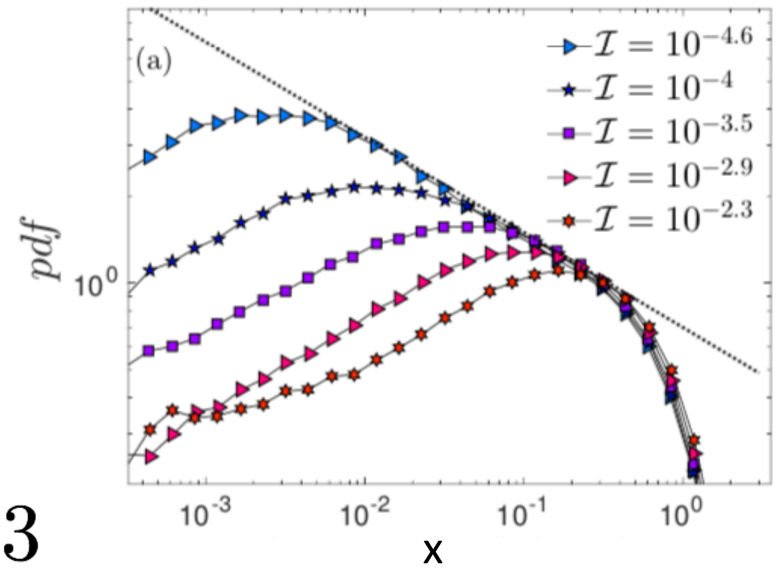
-> explains why rigid limit is singular

- Finally one finds

$$\boxed{\tilde{\delta} f \equiv \frac{\delta f}{p D^{d-1}} \sim (\mathcal{I} \Delta^{-1/4})/\chi}$$

Estimating χ from the mechanical noise





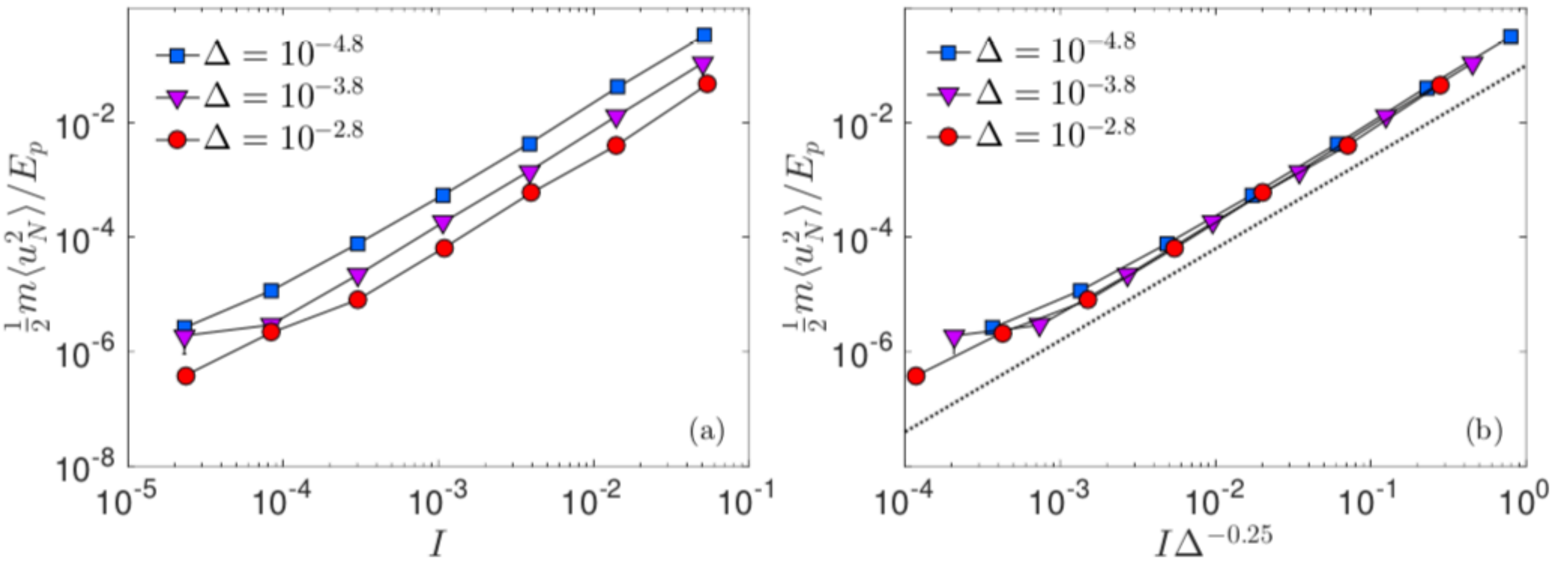
$$P(x) \sim x^{-\theta_s} \quad \theta_s \approx 0.3$$

$$\chi \sim \int_0^{\delta f} P(x) dx$$

$$\begin{array}{rcl} \chi & \sim & (I/\Delta^{1/4})^{\alpha}, \\ \mathcal{L} & \sim & (I/\Delta^{1/4})^{-\alpha}, \\ \mathcal{R} & \sim & (I/\Delta^{1/4})^{\gamma}, \\ \delta f & \sim & (I/\Delta^{1/4})^{\beta} \end{array}$$

Close the problem

Further tests



Conclusion

- Hysteresis emerges as collective effect, even if not present at contact level
- Mechanical noise that lubricate contacts, explains main observations
- Role of Delta testable
- Key role of contacts
- close to the coulomb cone
- Suspensions?
- Earthquakes?