Permanent shear-band instabilities in dense yield stress materials

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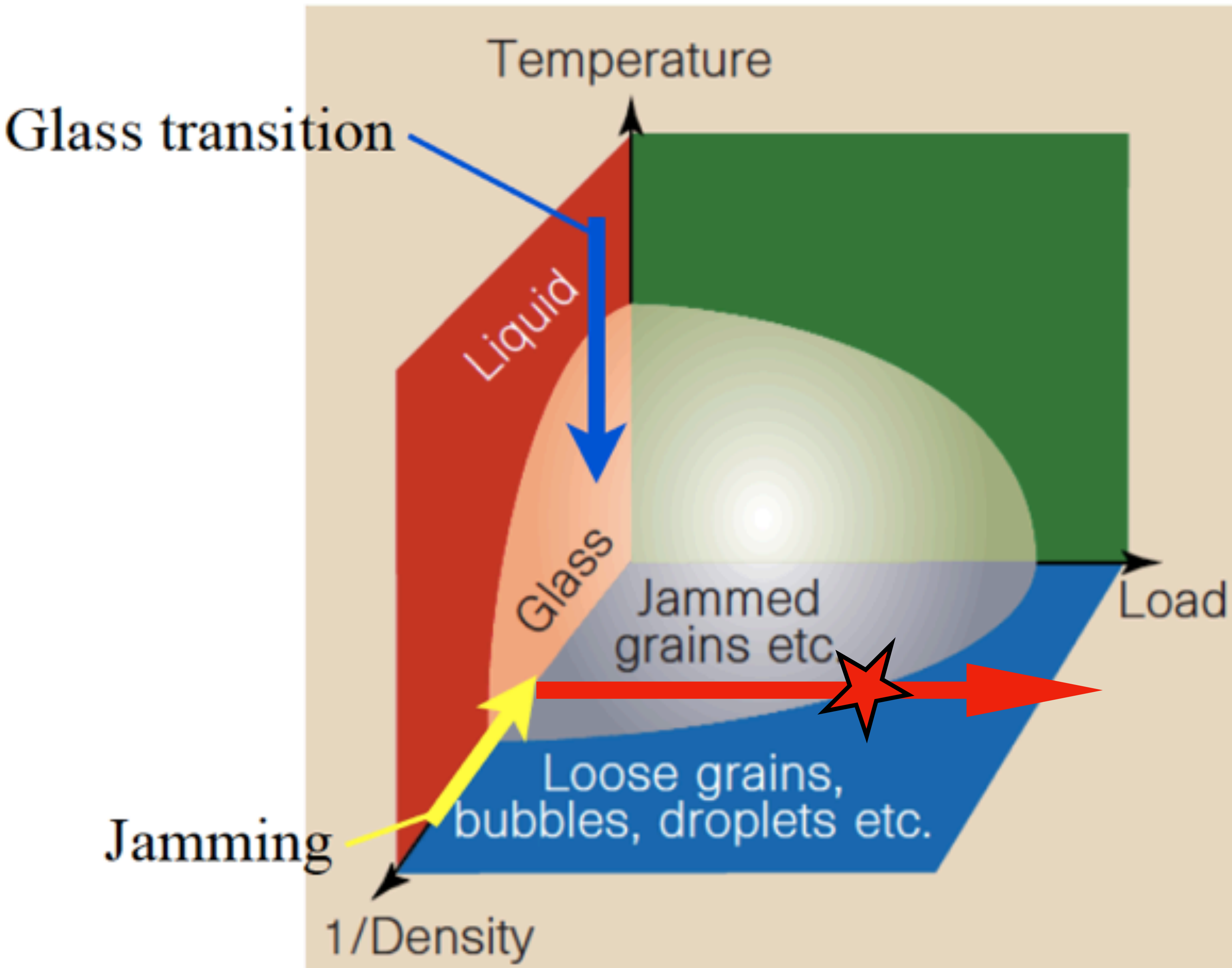
CNRS & University Grenoble Alpes

KITP, 25 January 2018

Vishwas Vasisht, Magali Le Goff, Romain Mari, Lyderic Bocquet, Jean-Louis Barrat

Jammed and glassy materials

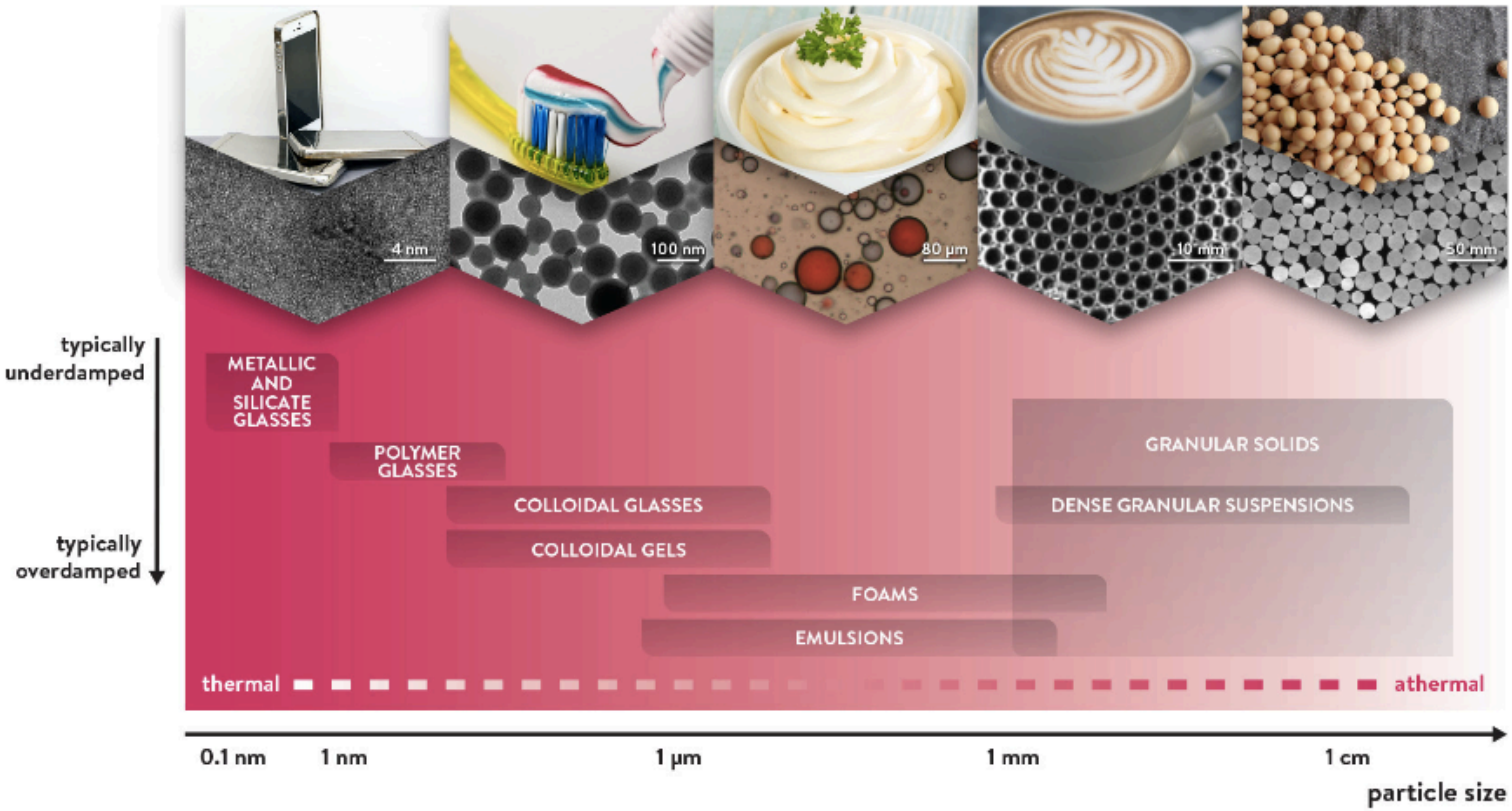


A.J.Liu & S.R.Nagel, 1998

Yielding transition

Elastic, plastic and viscous properties

Examples of yield stress materials



Nicolas, Ferrero, KM, Barrat, arXiv: 1708.09194 (2017)

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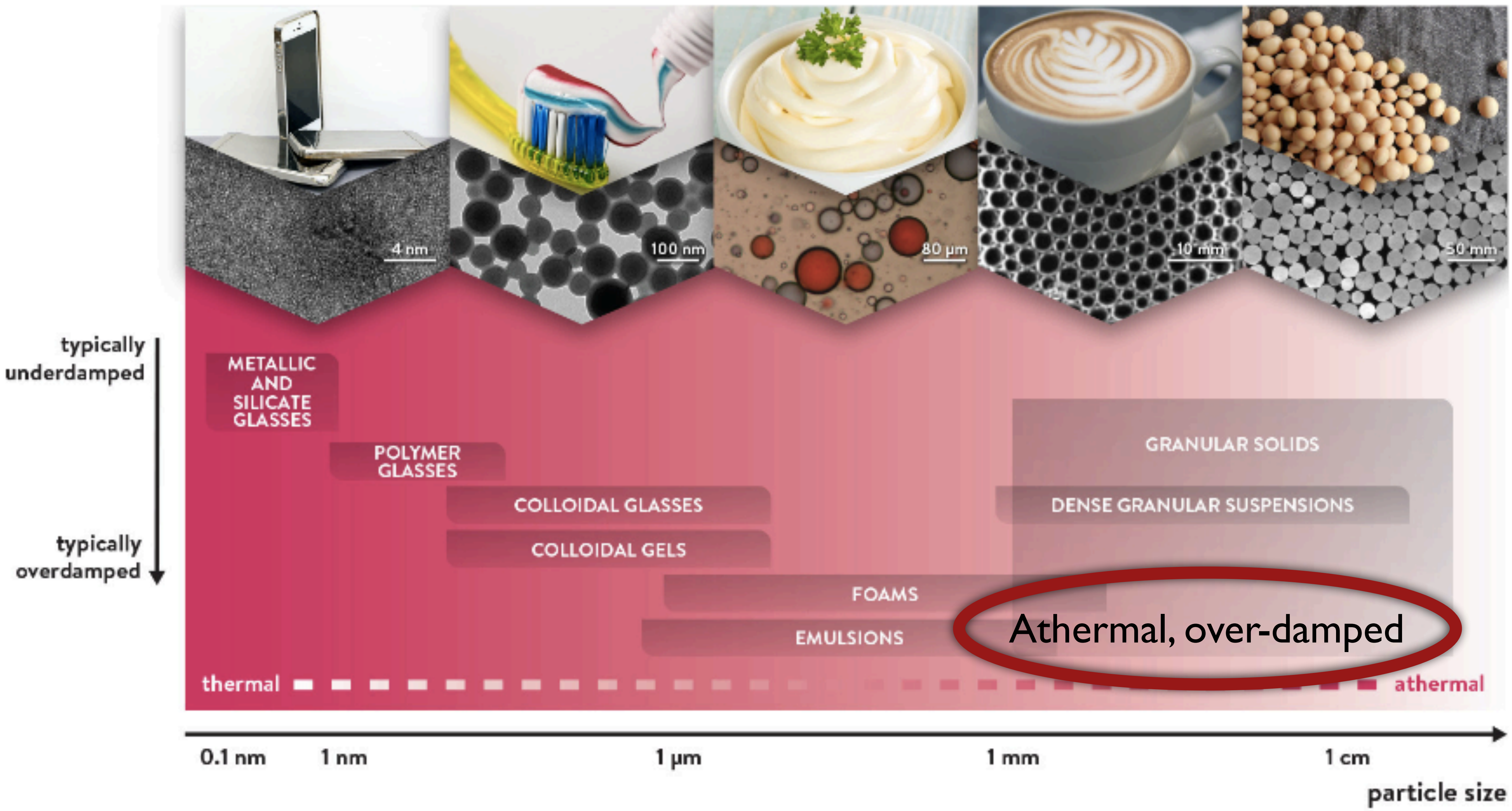

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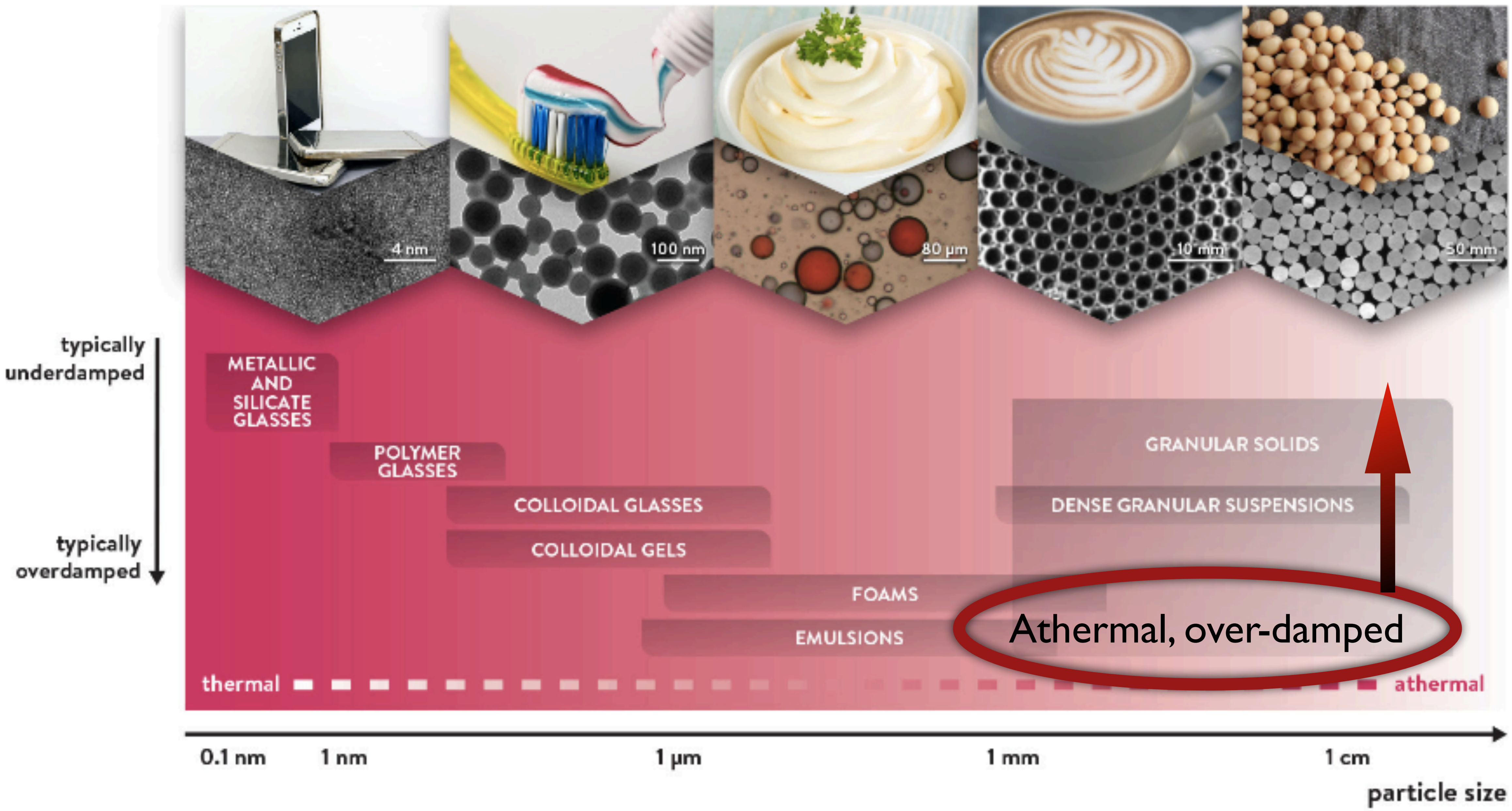
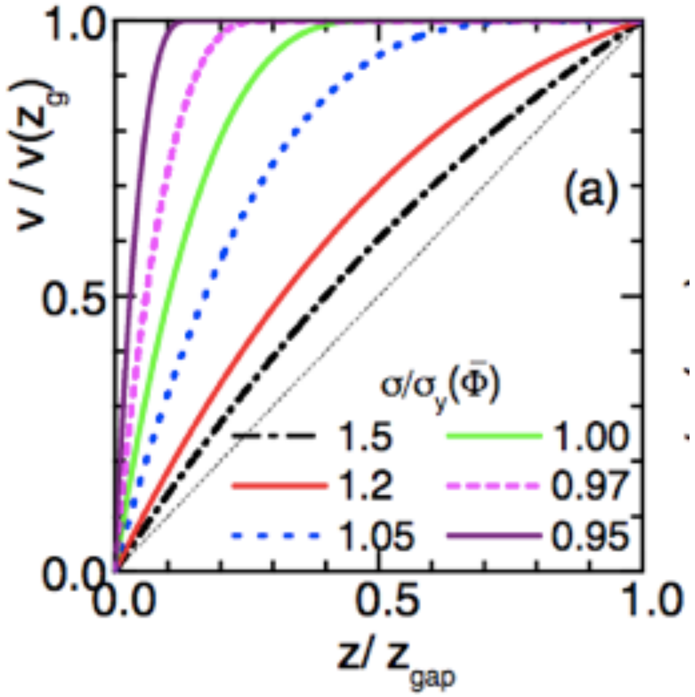


Nicolas, Ferrero, KM, Barrat, arXiv: 1708.09194 (2017)

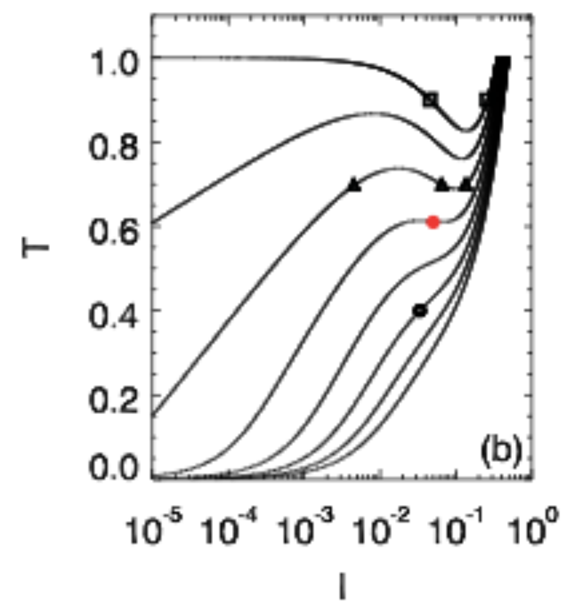
Origins for permanent shear banding

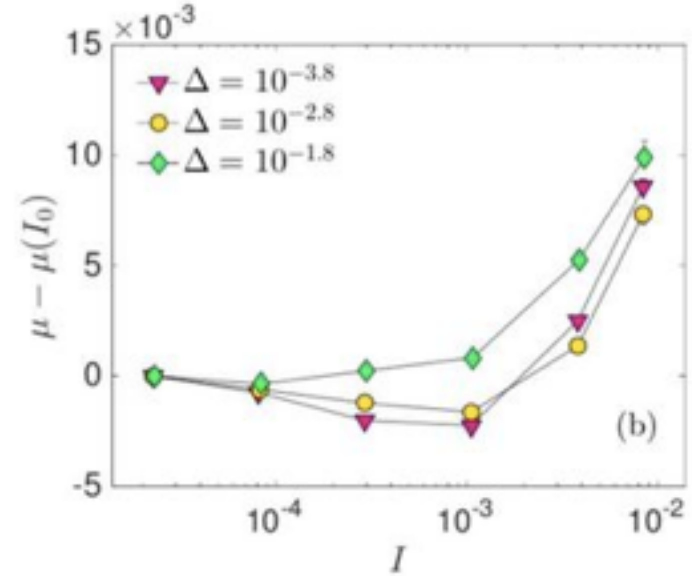
Shear-concentration coupling



Besseling et al., PRL 2010

Self-generated mechanical noise lubrificates contacts in frictional systems



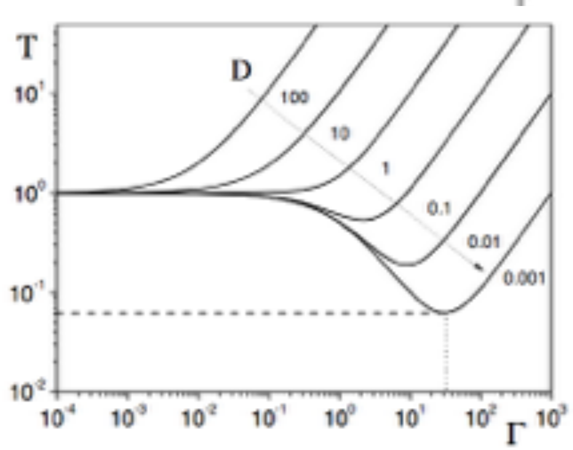


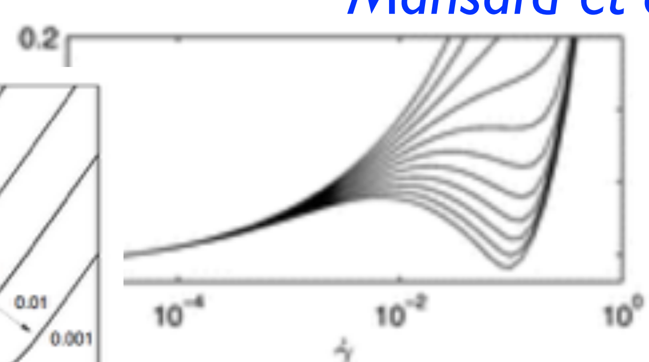
Wortel, Dauchot & Van Hecke PRL 2016

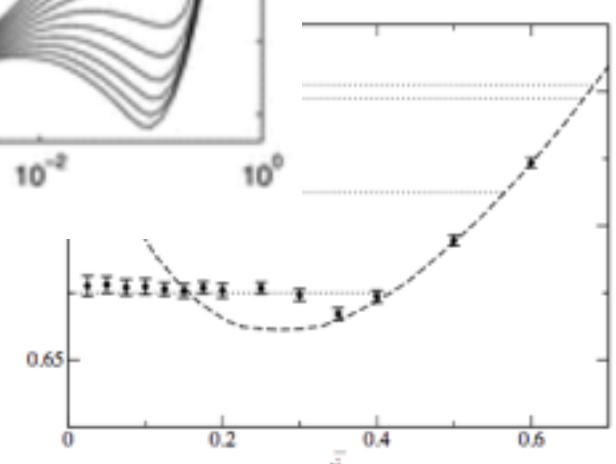
Di Giuli, Wyart PNAS 2017

Local shear weakening

Mansard et al, Soft Matter 2011



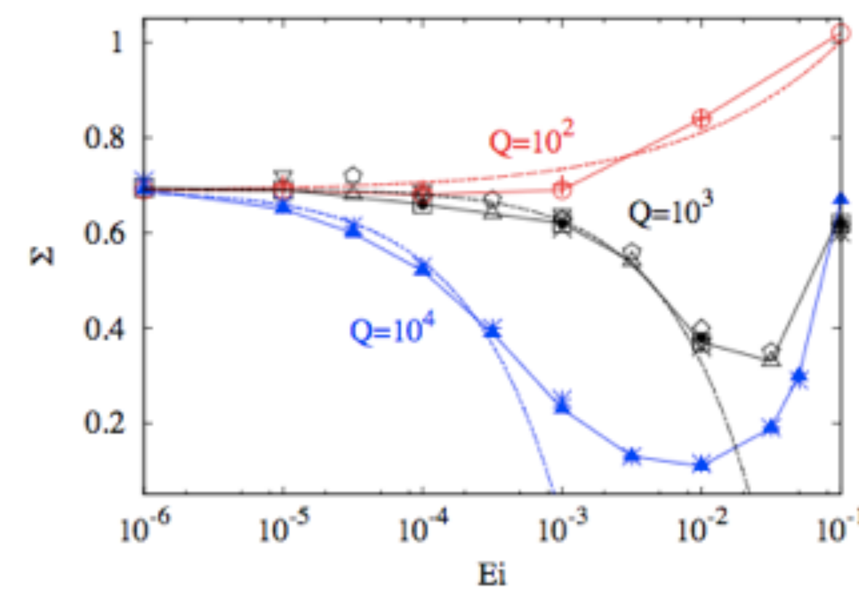




Coussot & Ovarlez, EPJE 2010

Fielding, Cates, Sollich, Soft Matter 2009

Inertia



Nicolas, Barrat & Rottler, PRL 2016

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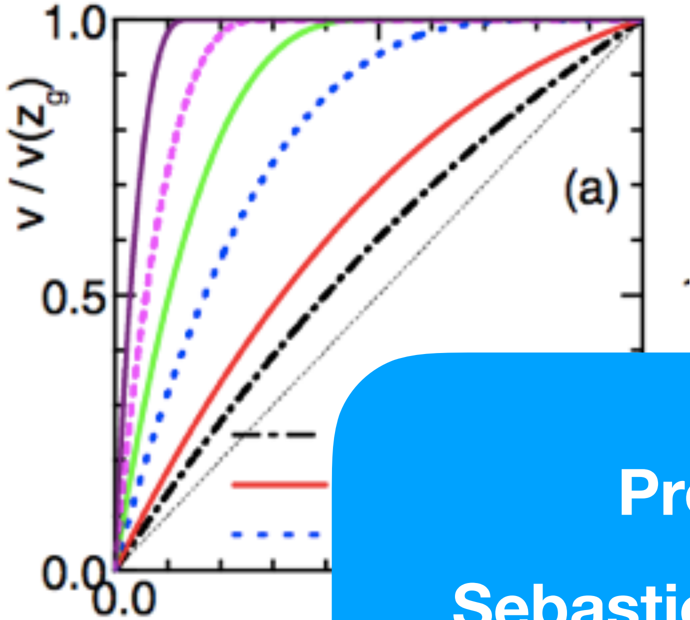
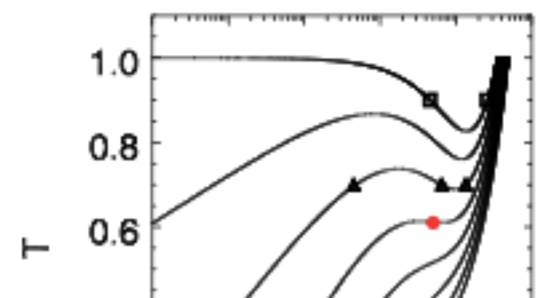
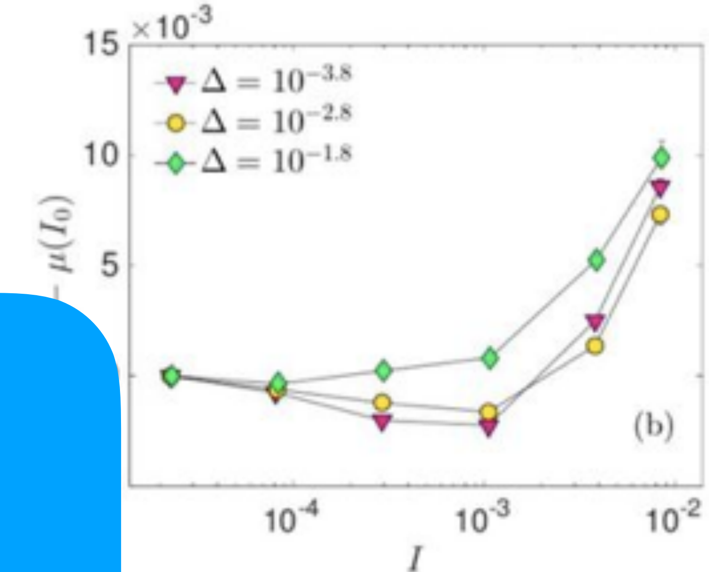


Bess

Self-generated mechanical noise lubrificates contacts in frictional systems





Di Giuli, Wyart PNAS 2017

Program talks of last week by

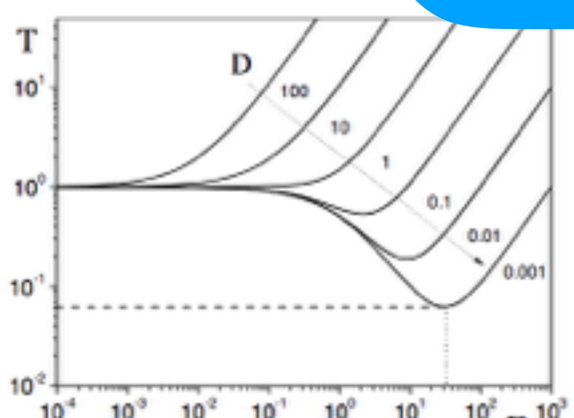
Sebastien Manneville and Peter Olmsted

Reviews by

Olmsted, Dhont, Fielding, Divoux…

Local shear w

Inertia



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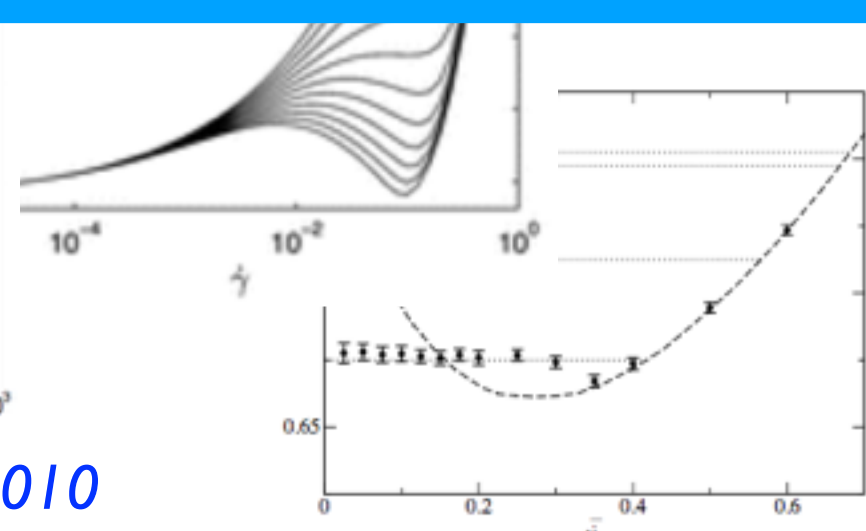

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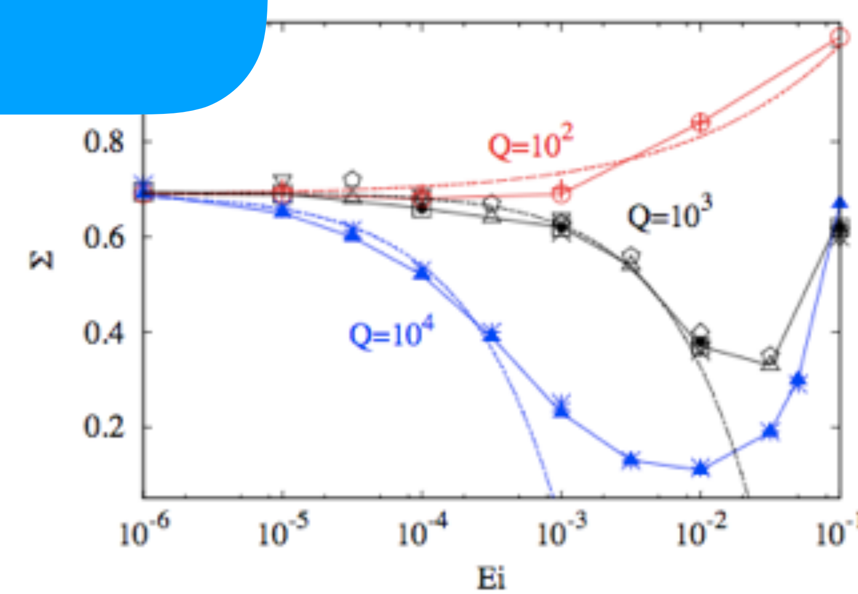
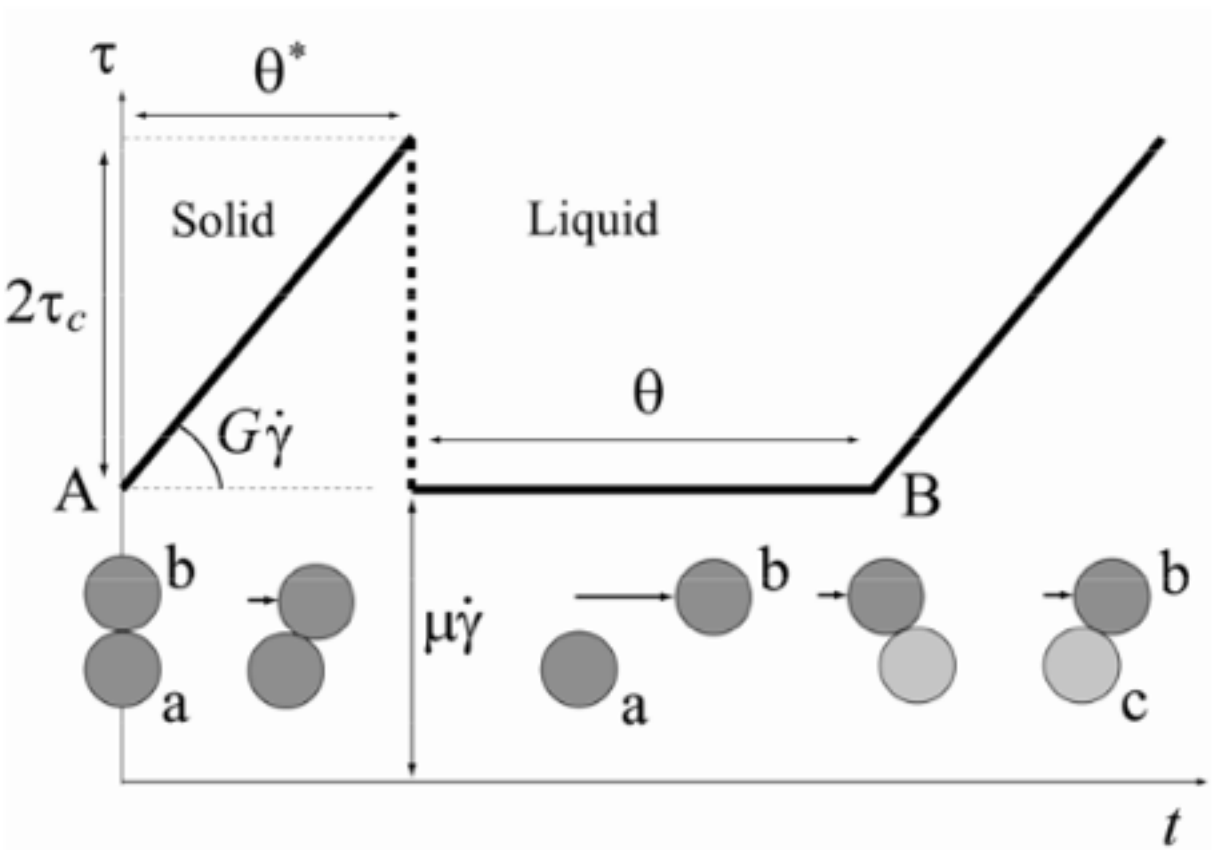
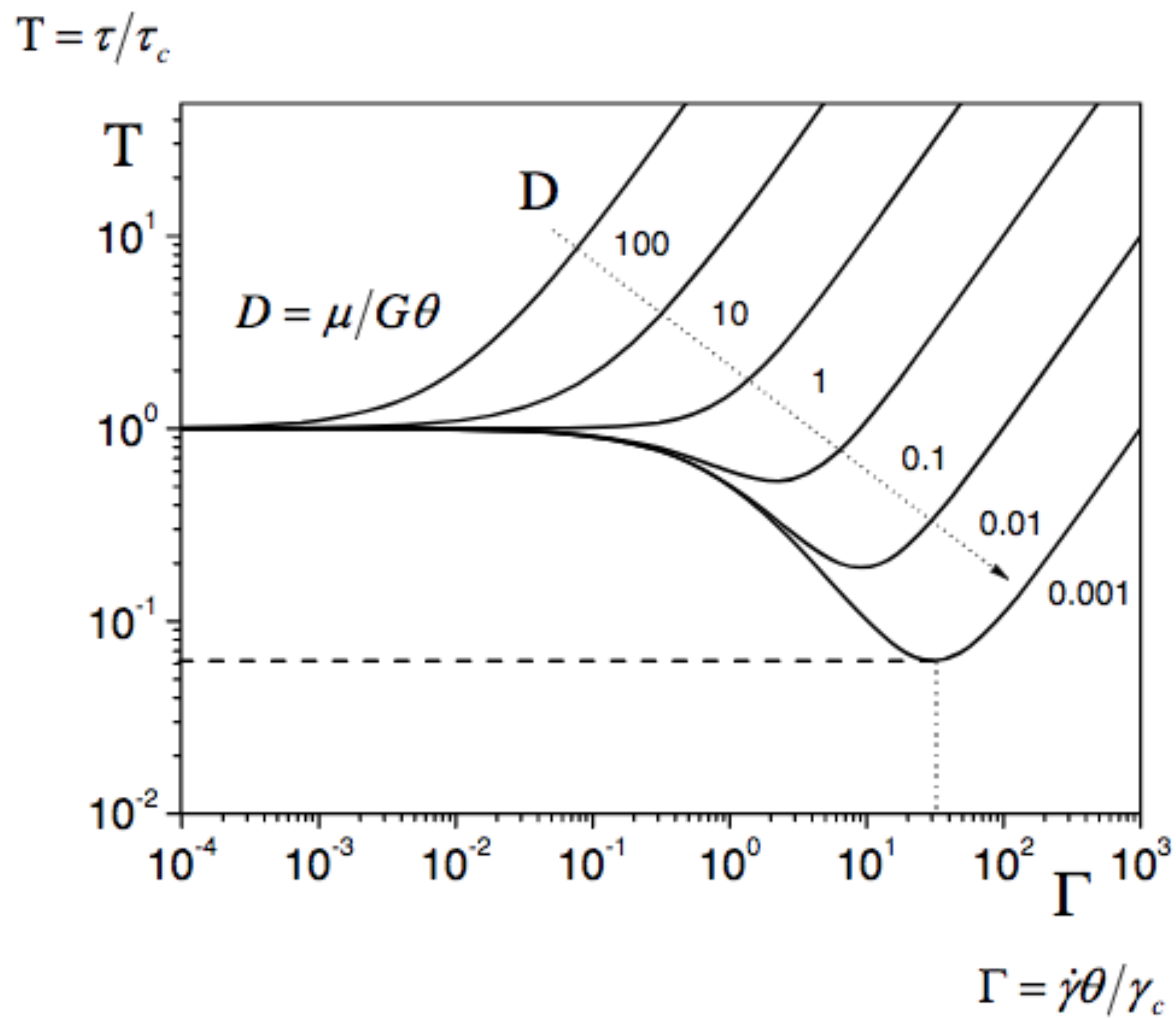


Nicolas, Barrat & Rottler, PRL 2016

Permanent shear bands due to local shear weakening and elasticity

Physical origin of shear-banding in jammed systems



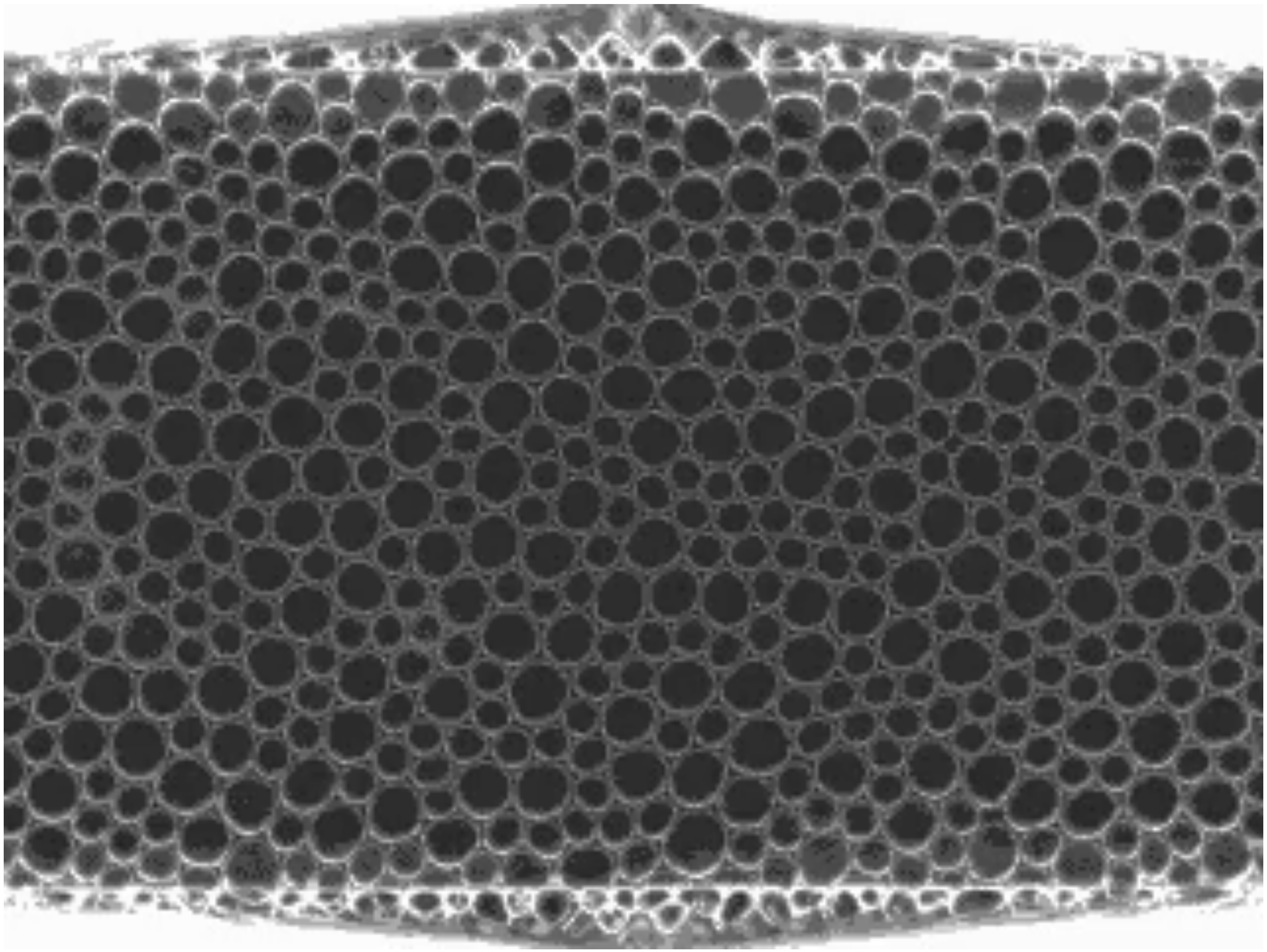


Important parameter:
local restructuring time

Coussot & Ovarlez, EPJE 2010

Dense yield stress materials

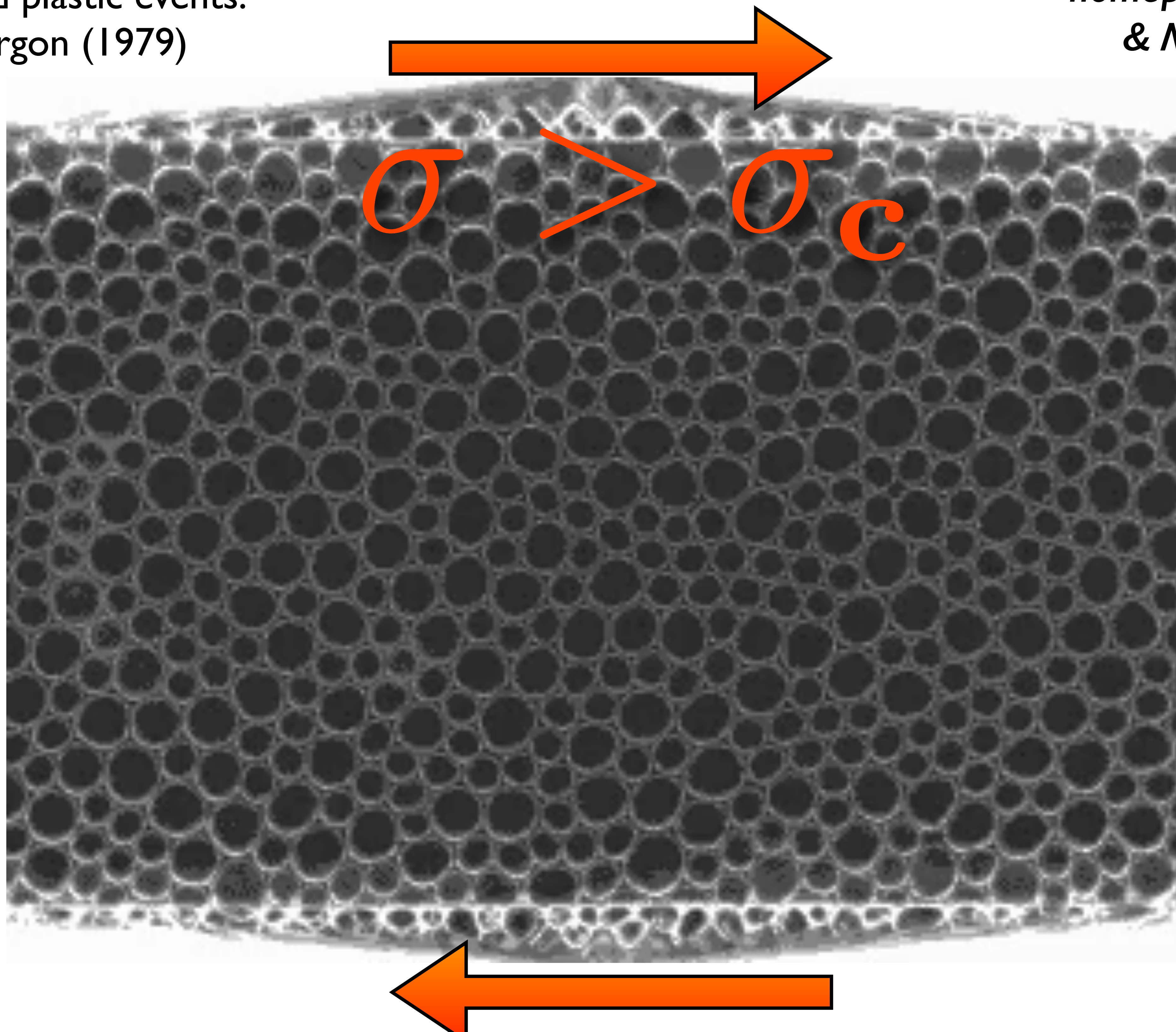
homepage: G. Katgert,
& M. van Hecke



Dense yield stress materials

Localized plastic events:
Ali Argon (1979)

homepage: G. Katgert, & M. van Hecke



The Eshelby problem

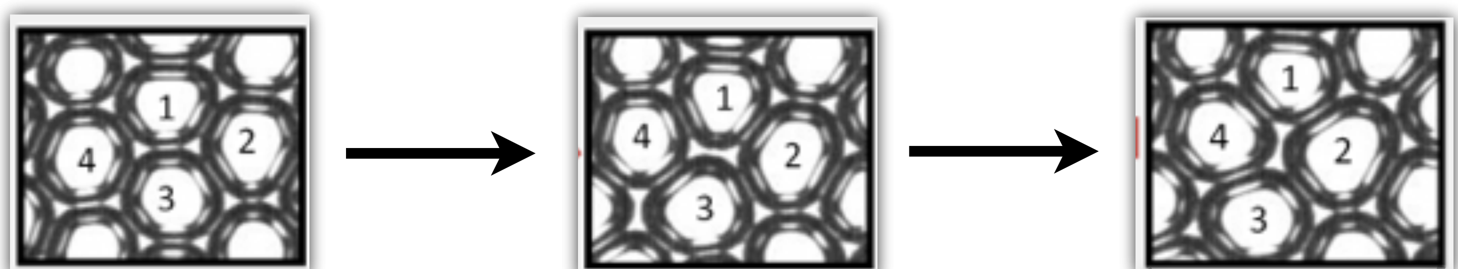
1. Strongly localised irreversible mechanical instabilities

Schematic representation of shear-induced mechanical instabilities in the potential-energy surface

increase of shear stress

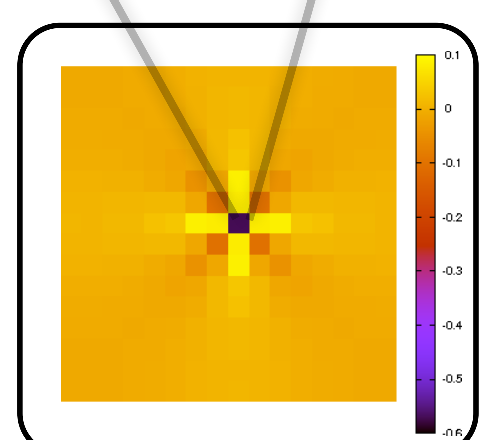
Malandro & Lacks PRL 1998

2. Accompanied by rearrangement of at most some tens of particles, small shear strains (1-10%)



3. Surrounding materials responds in a first order approximation in a purely elastic way

$$G(\vec{r}, t) = \frac{1}{\pi r^2} \cos(4\theta)$$



Stress change due to a single plastic event (2d incompressible medium)

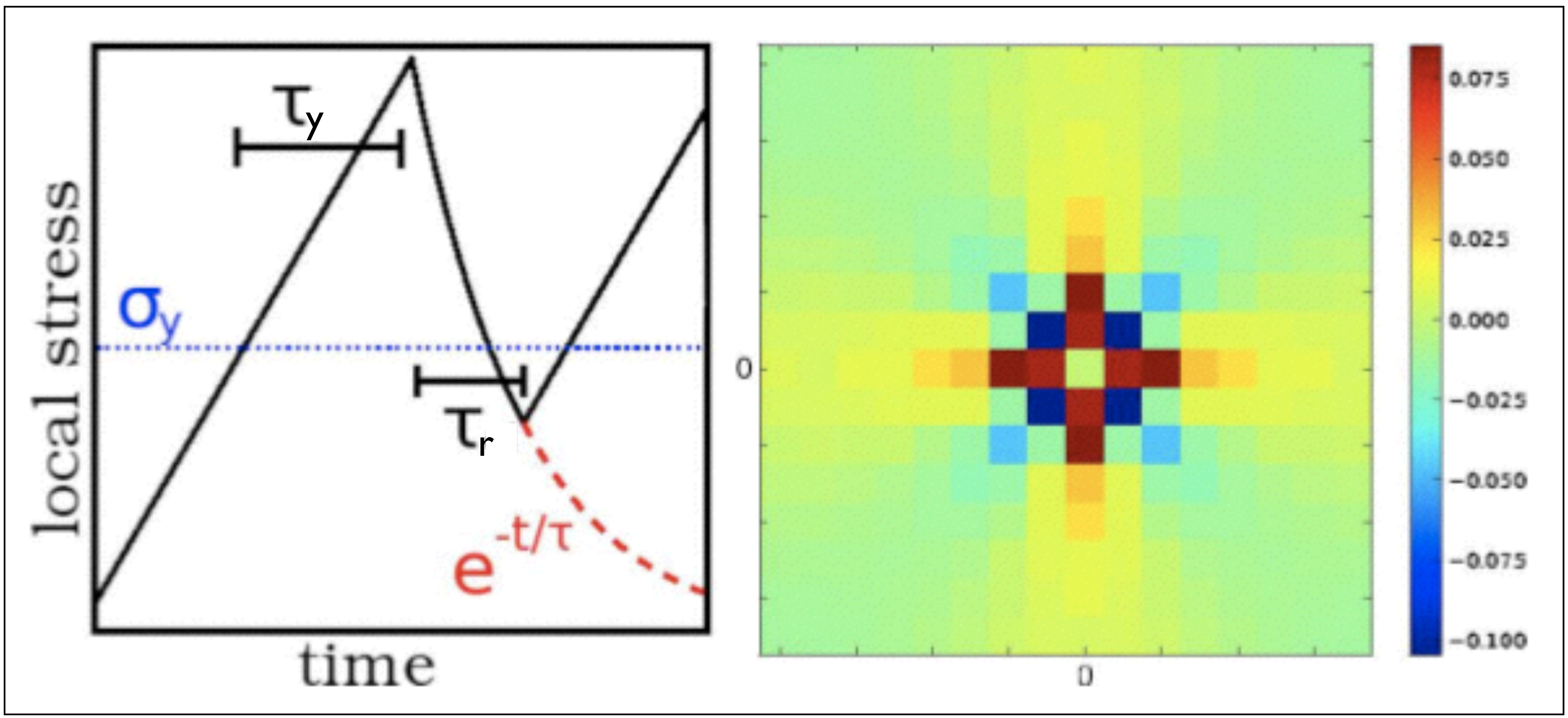
J.D. Eshelby (1957)

Transition to shear banding?

G. Picard et. al. (EPJE 2004, PRE 2005)

Stress dynamics (without fluctuations)

Stress propagator



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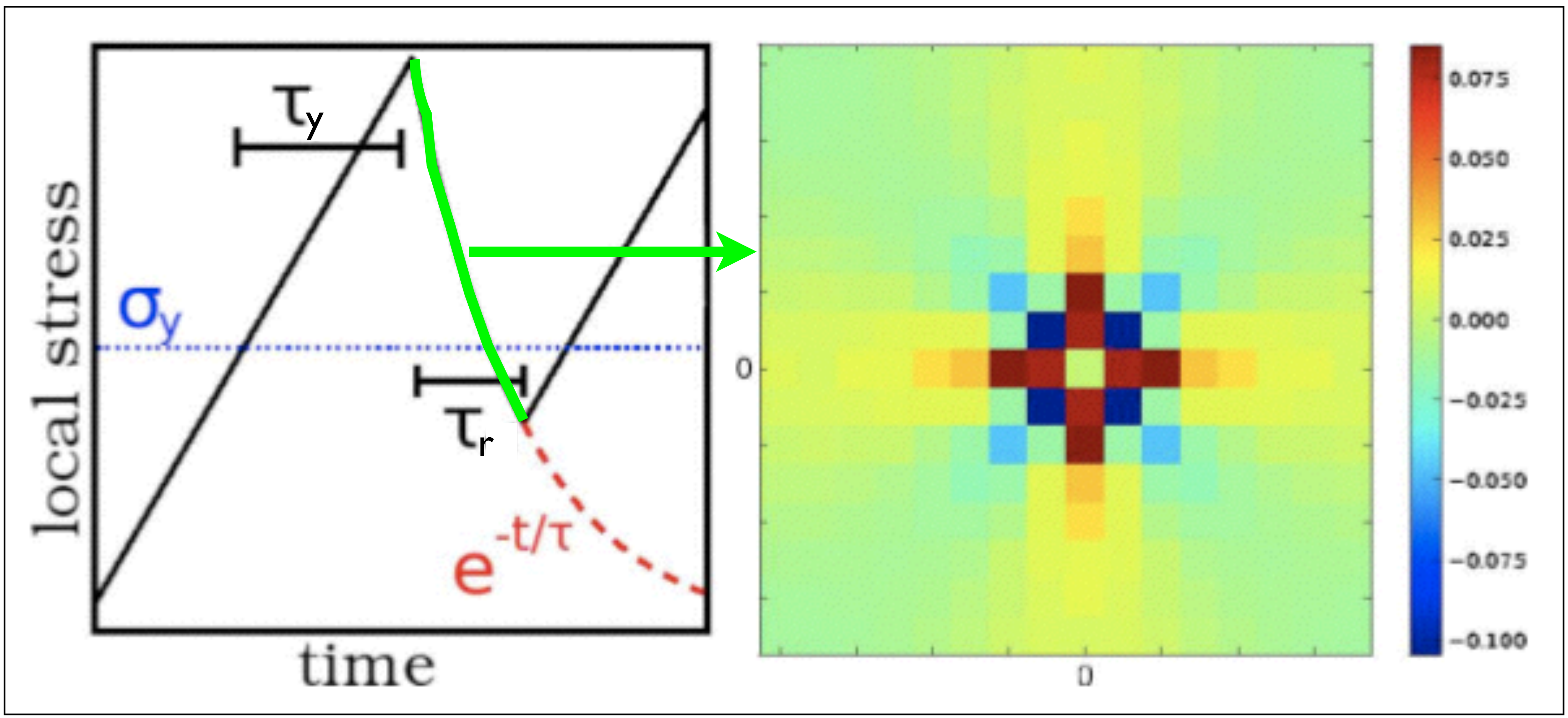

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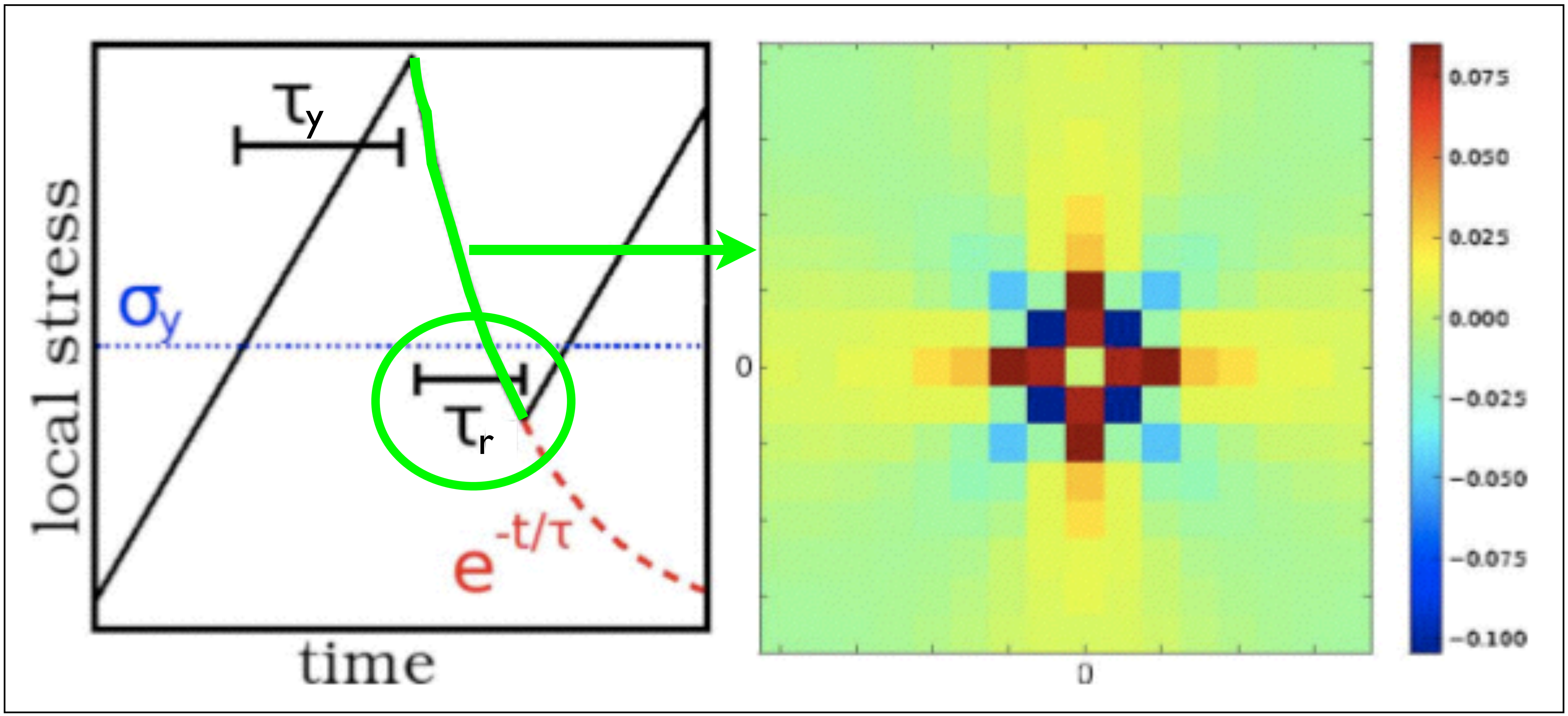
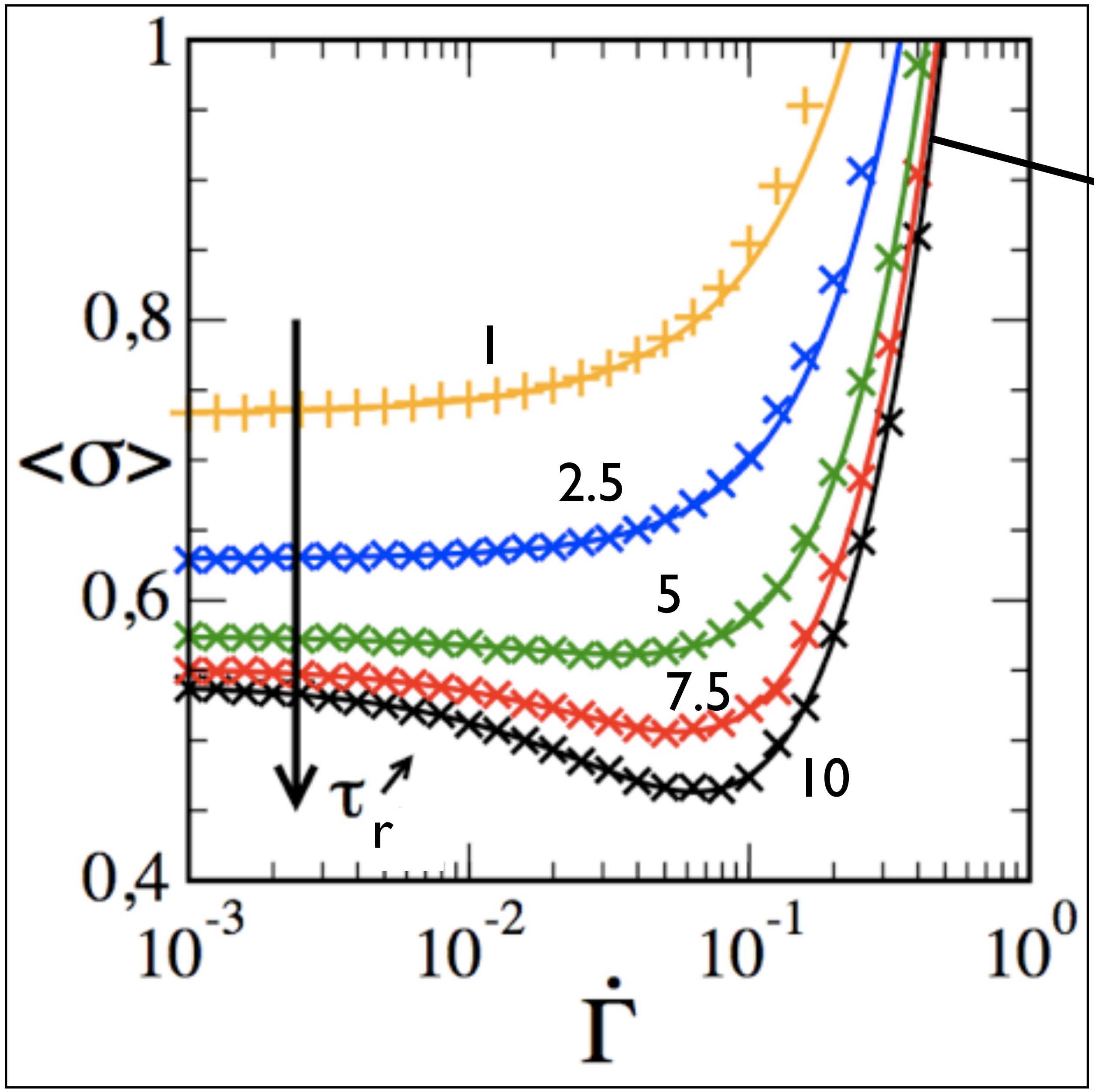


Mean-field predictions

KM, Boquet & Barrat (2011)
Nicolas et al. (2013)



analytical approximation

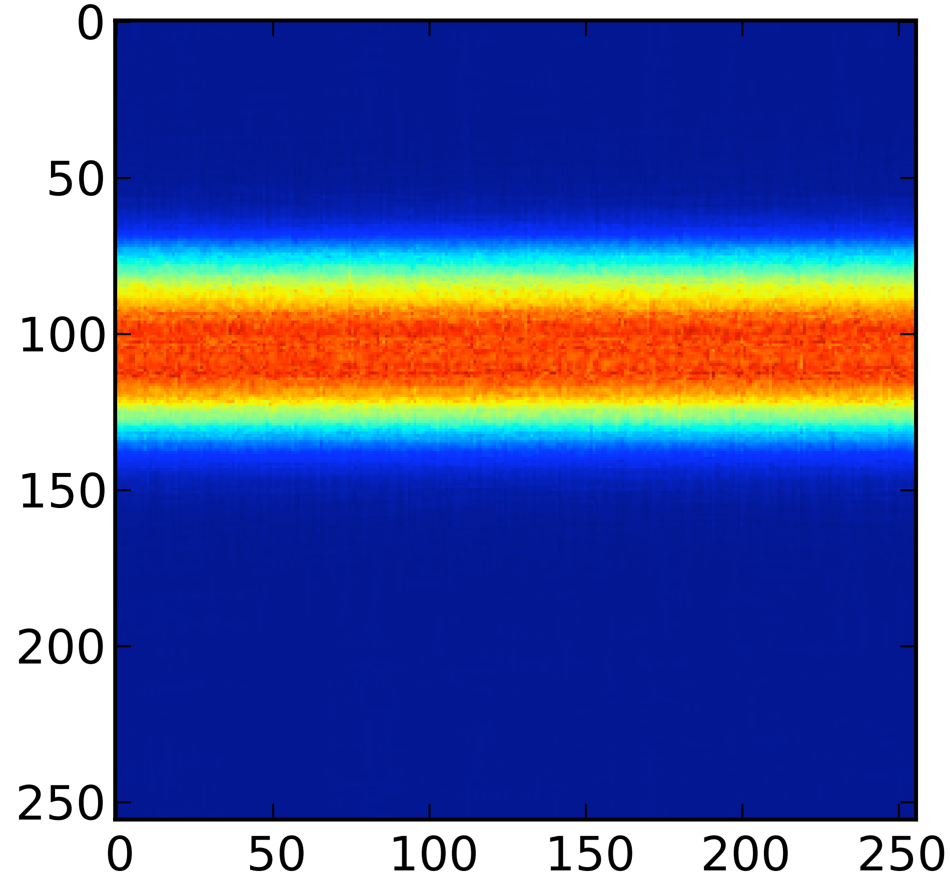
for $\dot{\Gamma} < g$

and $\tau_r > \tau_y$

Flow localised for large restructuring time

Full spatial model

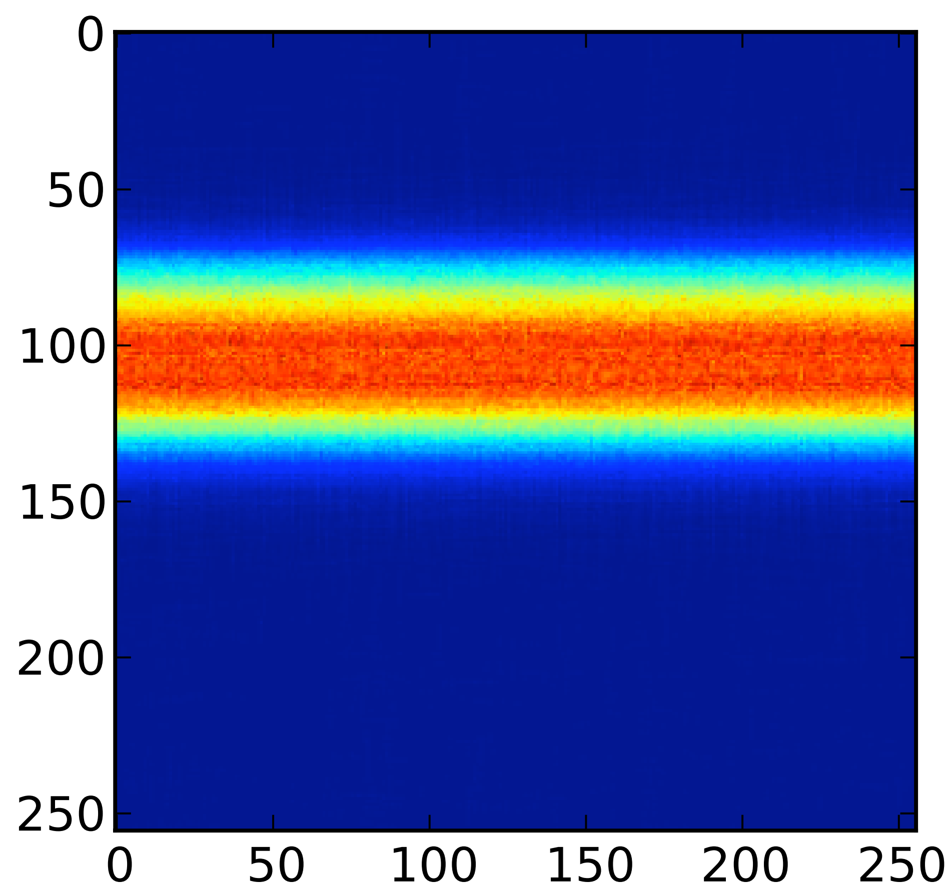
Cumulated plasticity



small shear rate,
large restructuring time

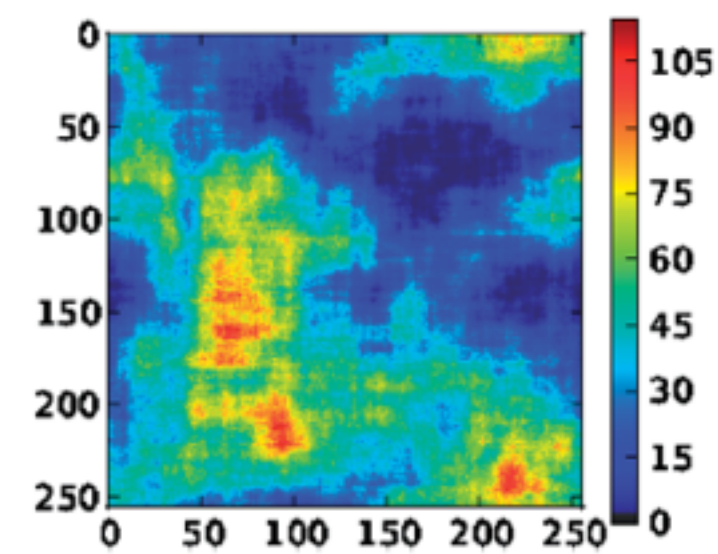
Full spatial model

Cumulated plasticity



short range anisotropic kernel

-1/4	1/2	-1/4
1/2	-1	1/2
-1/4	1/2	-1/4

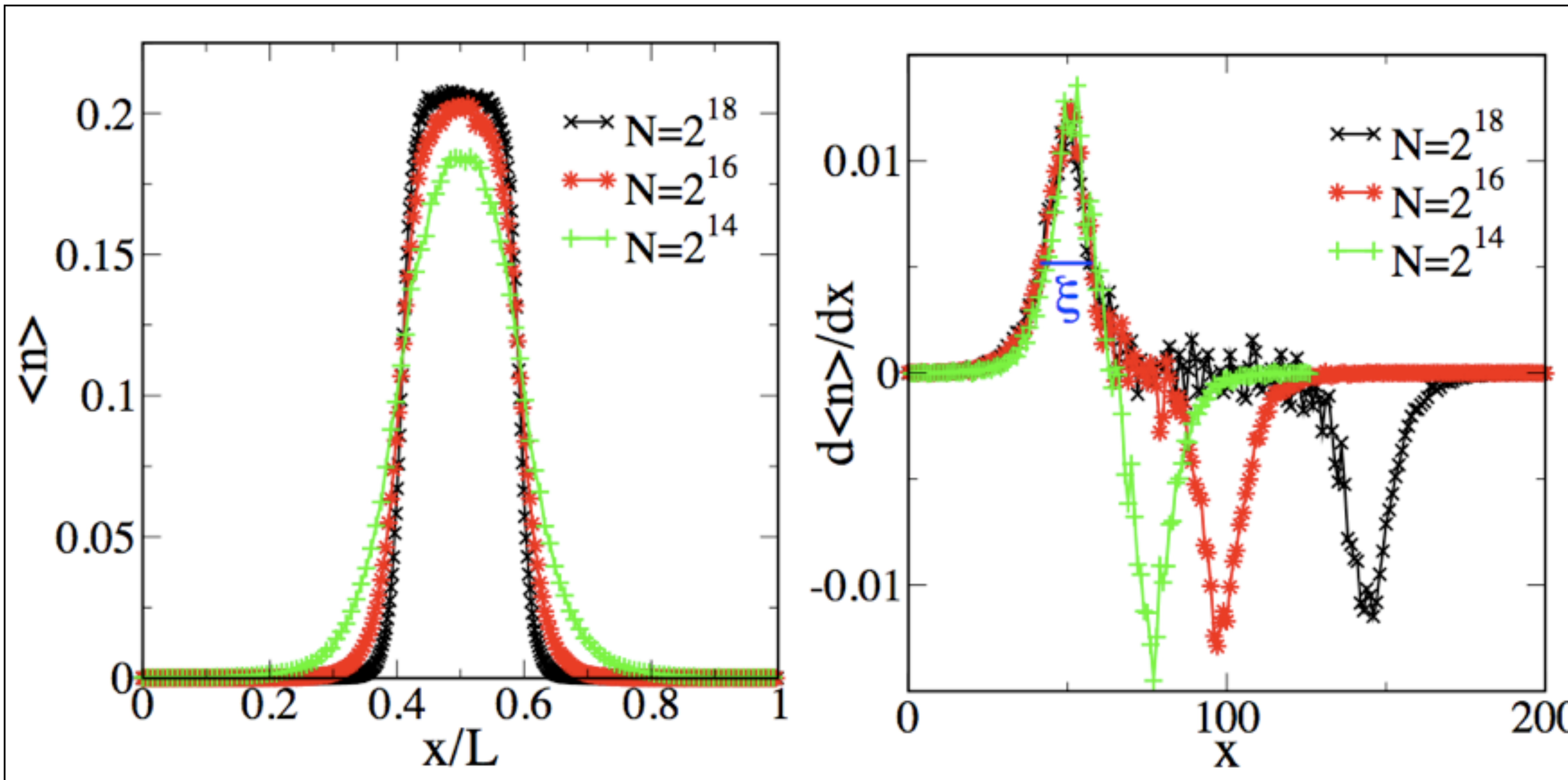


small shear rate,
large restructuring time

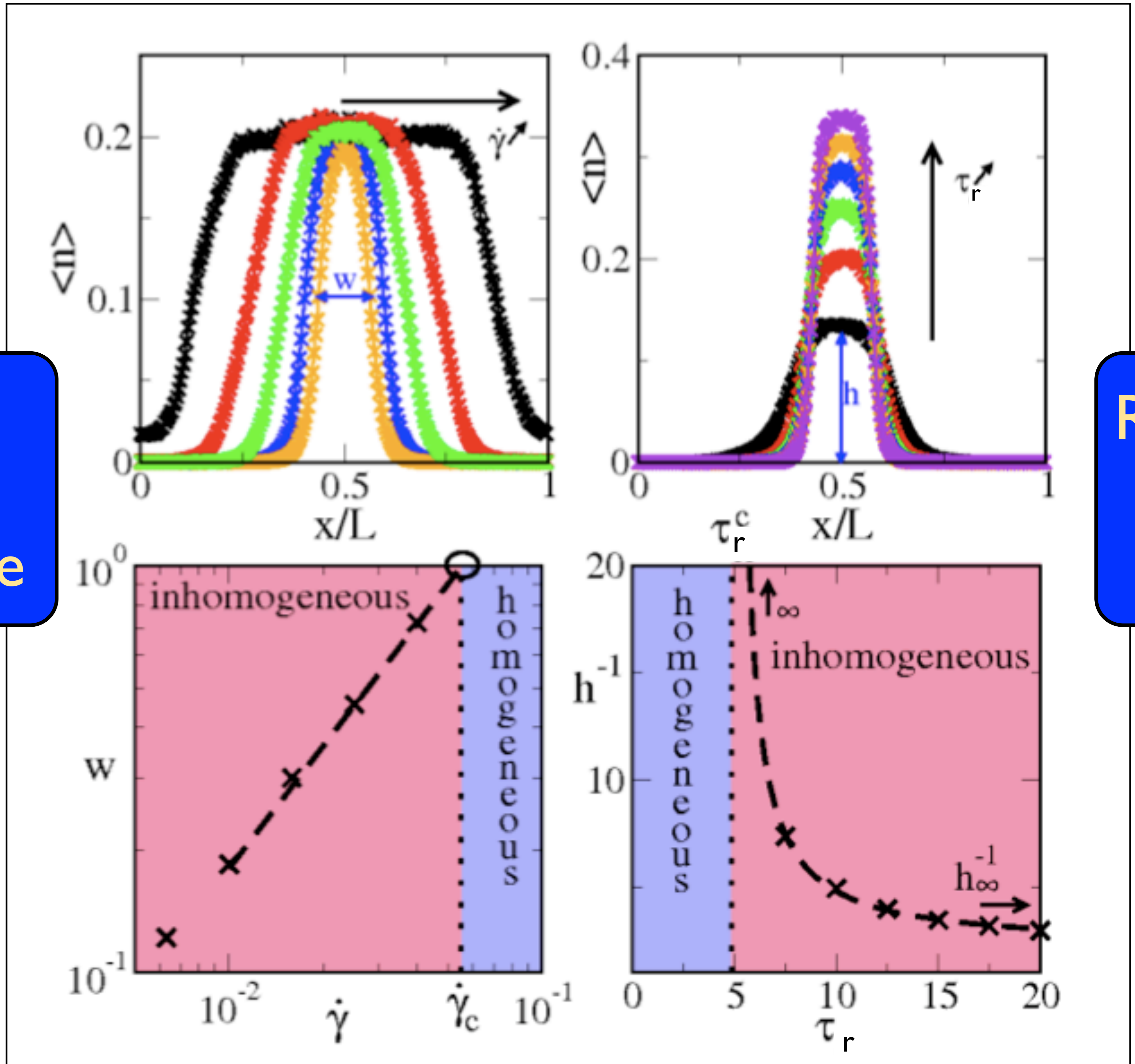
System size dependence

Width scales with system size

Interface width is constant



Steady state band characteristics



Shear rate dependence

Restructuring time dependence

Microscopic interpretation?

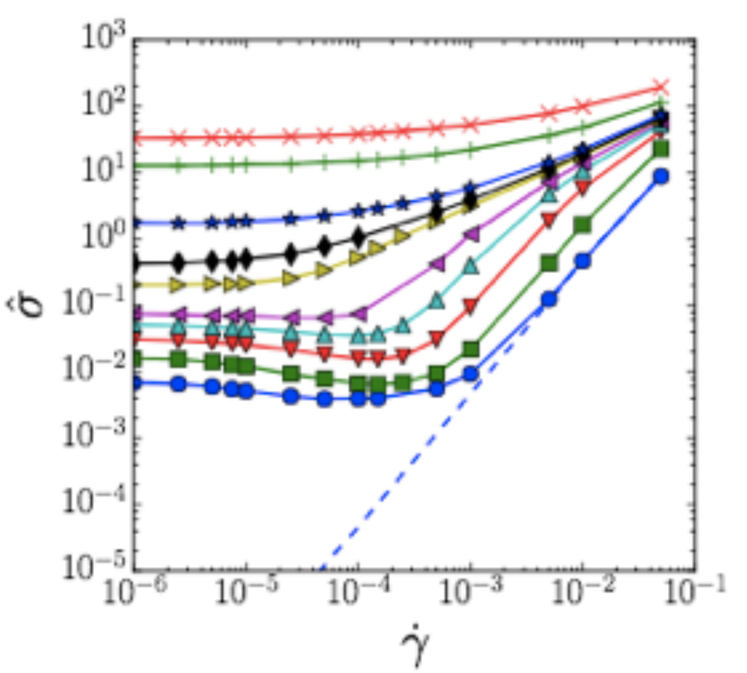
Potential and dynamics for particle based simulations?

Attraction alone is not sufficient in the jammed state

Chaudhuri, Berthier & Bocquet, PRE 2012

Only below the critical packing fraction shear bands can be created through an additional attractive force

Irani, Chaudhuri & Heussinger, PRL 2014



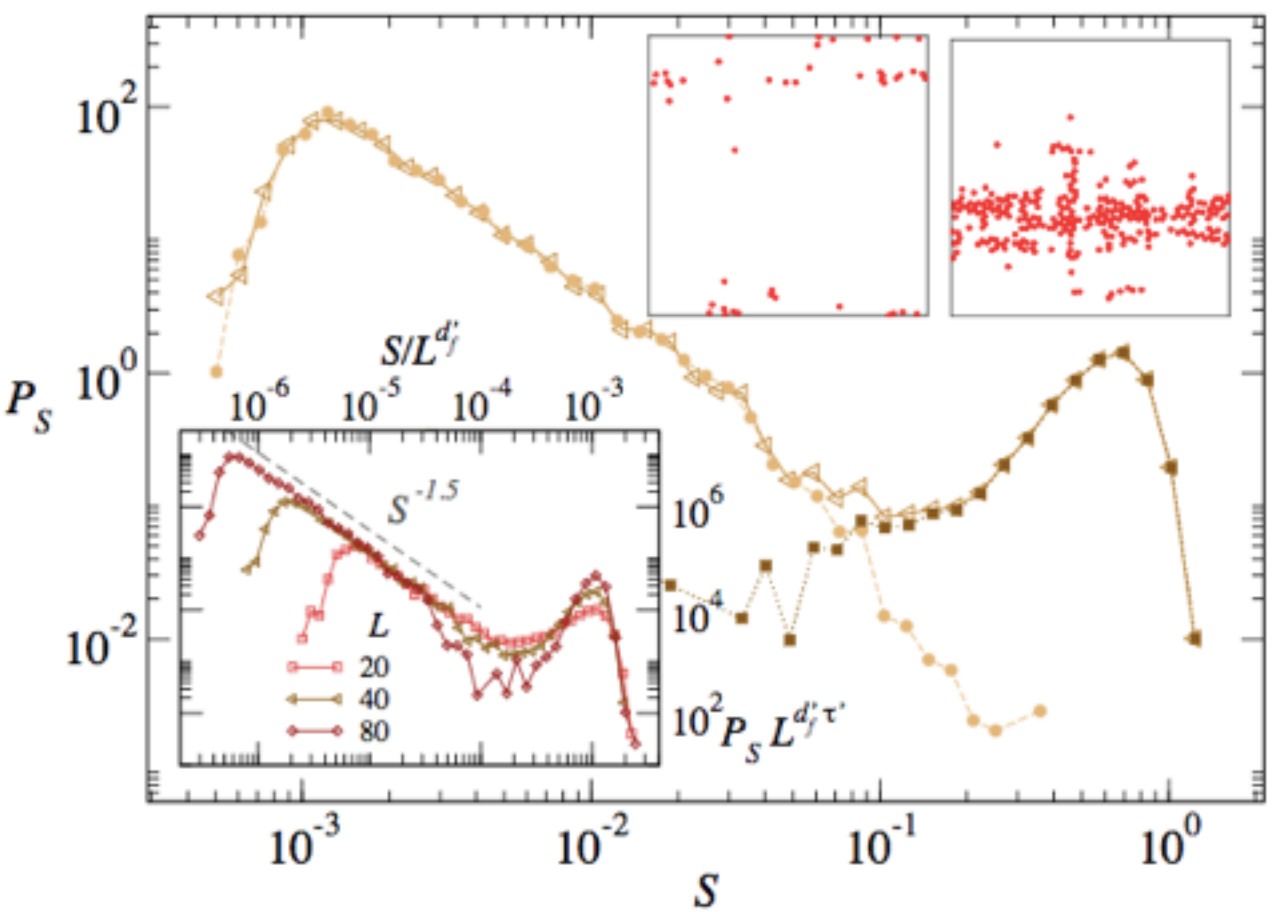
Is there a mechanism for dense particle systems?

Permanent shear bands due to inertia

Inertia matters

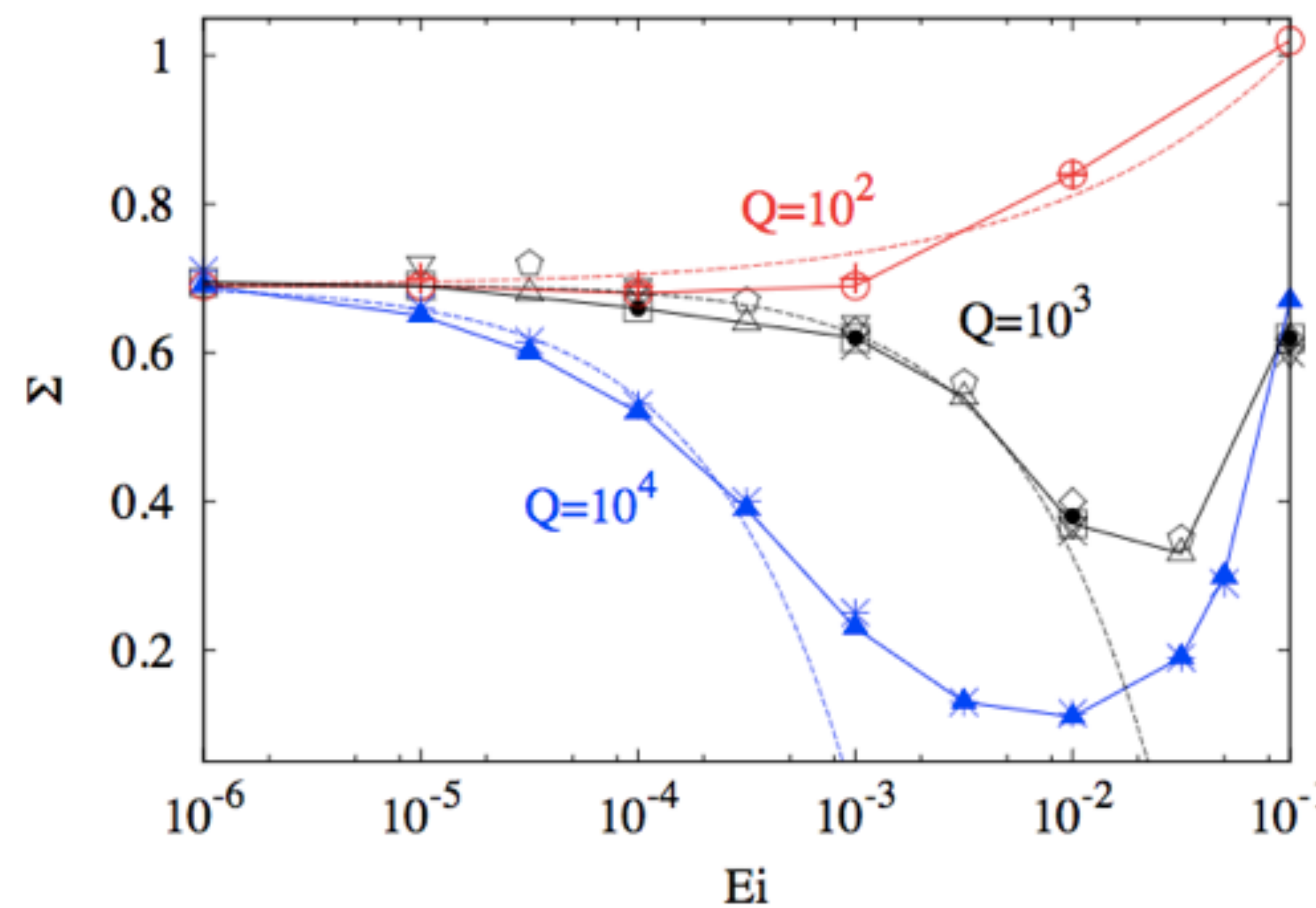
Avalanche statistics depend on inertia

Salerno & Robbins PRE 2013



Karimi, Ferrero & Barrat PRE 2017

Flow curve depends on inertia



Nicolas, Barrat & Rottler PRL 2017

Model system

Repulsive particles at 0.7 packing fraction, 10% polydispersity

Potential:

$$U(r_{ij}) = 4\epsilon\left[\left(\frac{\sigma_{ij}}{r_{ij}}\right)^{12} - \left(\frac{\sigma_{ij}}{r_{ij}}\right)^{6}\right] + \epsilon \quad \text{for } r_{ij} \leq 2^{1/6}\sigma_{ij}$$

$$\text{else } U(r_{ij}) = 0$$

Athermal dissipative dynamics

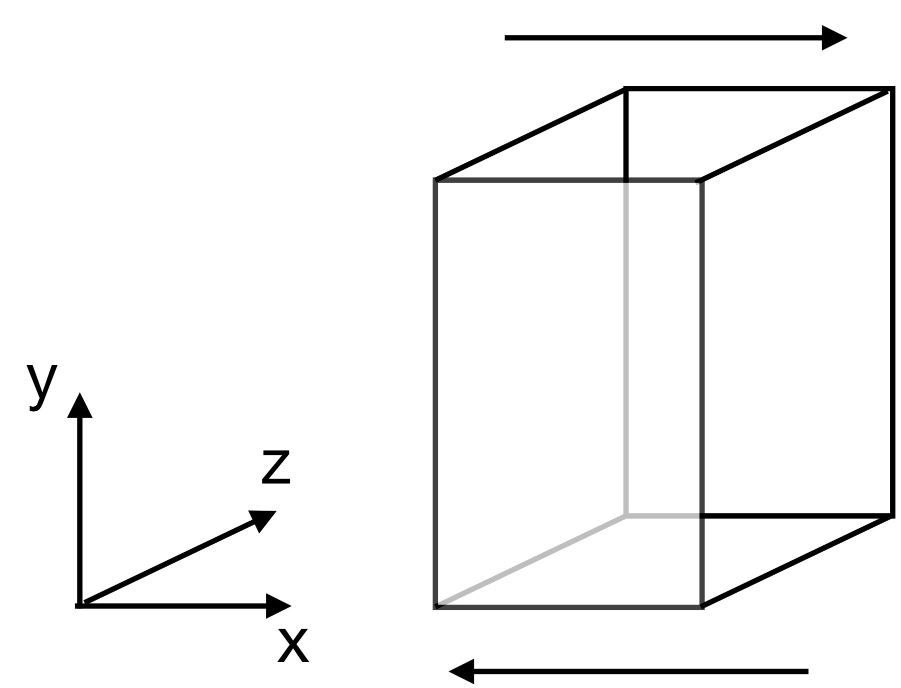
$$m\frac{d^2\vec{r}_i}{dt^2} = -\zeta_{DPD}\sum_{j(\neq i)} \omega(r_{ij})(\hat{r}_{ij}.v_{ij})\hat{r}_{ij} - \nabla_{\vec{r}_i} U$$

Inertial quality factor

$$Q = \frac{E_i}{W_i} = \frac{\tau_{vib}\dot{\gamma}}{\tau_{diss}\dot{\gamma}}$$

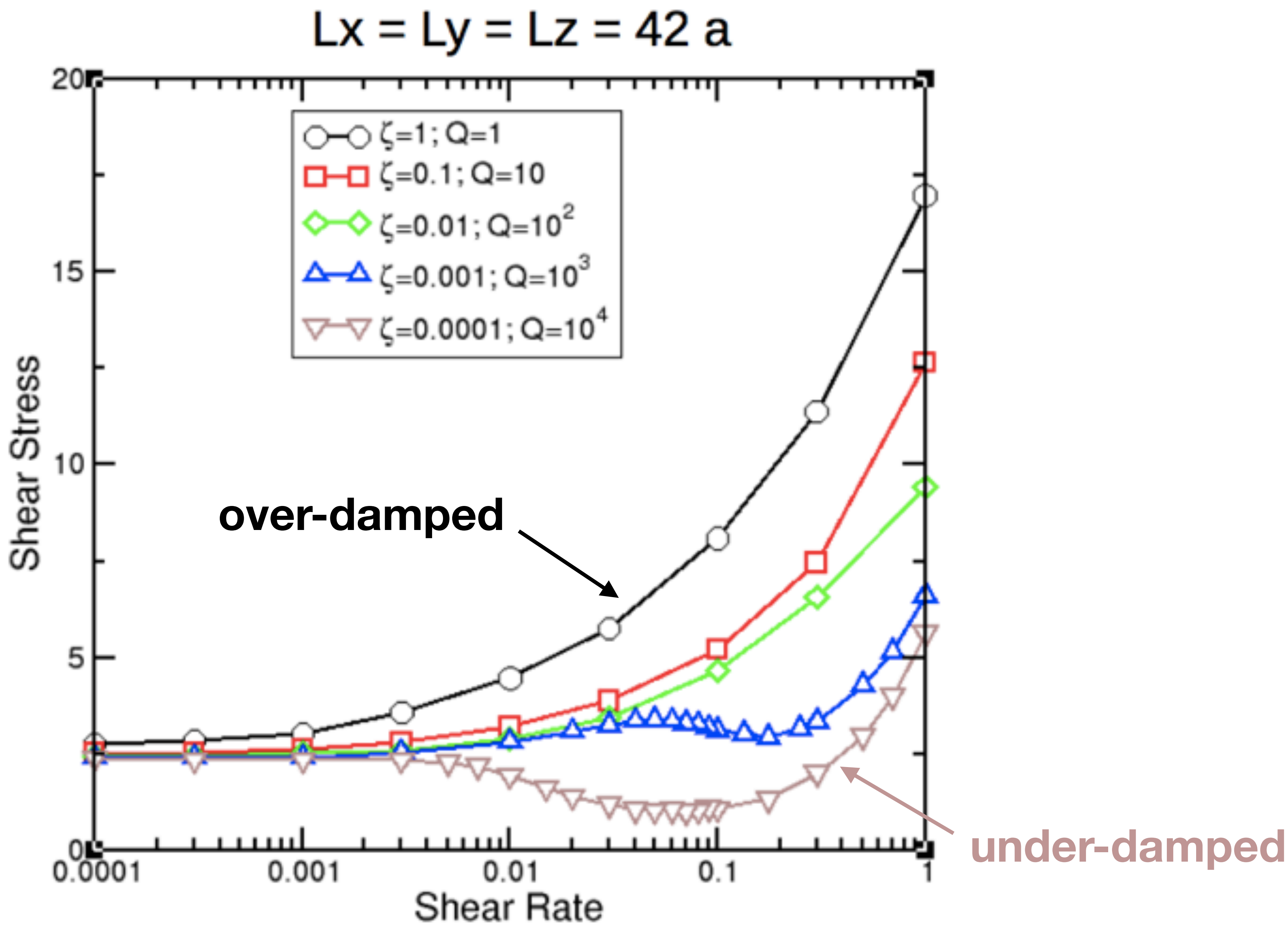
$Q \leq 1$ **"overdamped dynamics"**

Homogeneous shear
with periodic boundary
conditions at constant volume



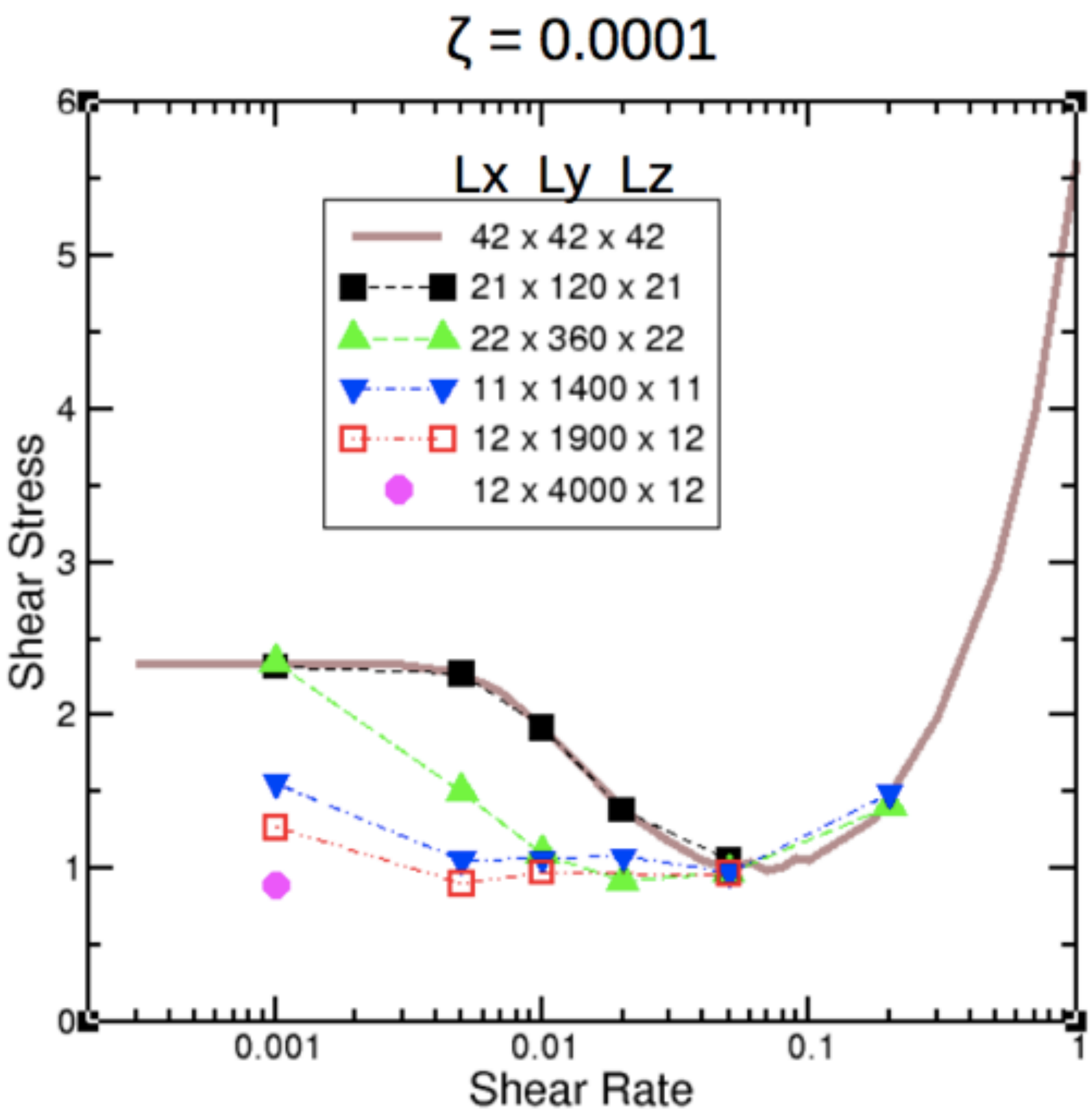
Flow curves

Different damping coefficients



Flow curves

System size dependence for the inertial regime



Formation of shear bands

Start-up flow with instabilities due to stress overshoot

Onset of a second permanent shear band instability

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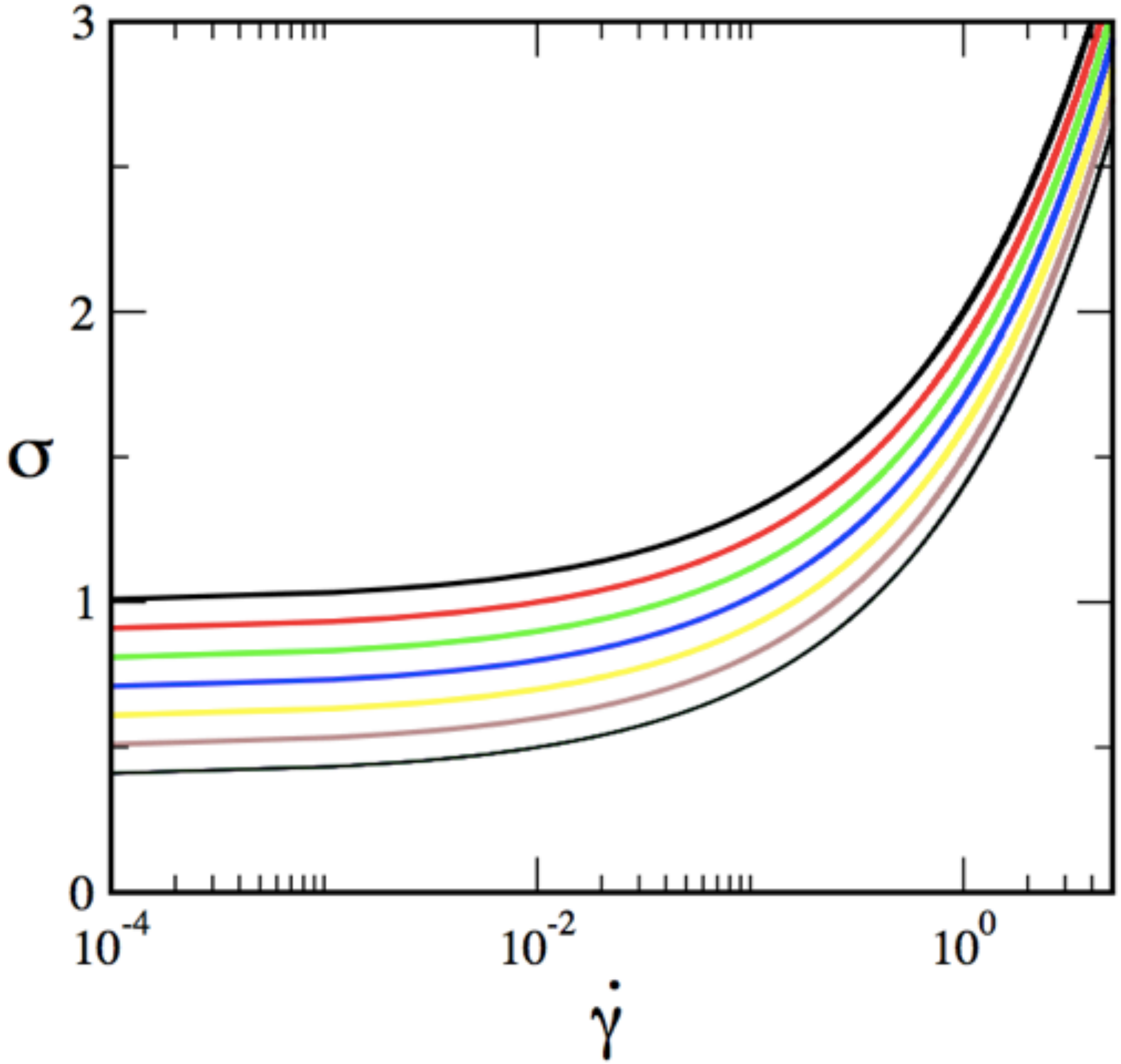
Effective temperature

Overdamped flow curves at different temperatures



Effective temperature

$$k_B \tilde{T} = E_{\mathrm{kin}}$$

depends on driving rate

$$\tilde{T} = \tilde{T}(\dot{\gamma})$$

Resulting in

$$\sigma(\dot{\gamma}; Q, T = 0)$$
$$\Leftrightarrow \sigma(\dot{\gamma}; Q = 1, \tilde{T}(\dot{\gamma}))$$

Nicolas, Barrat & Rottler PRL 2017

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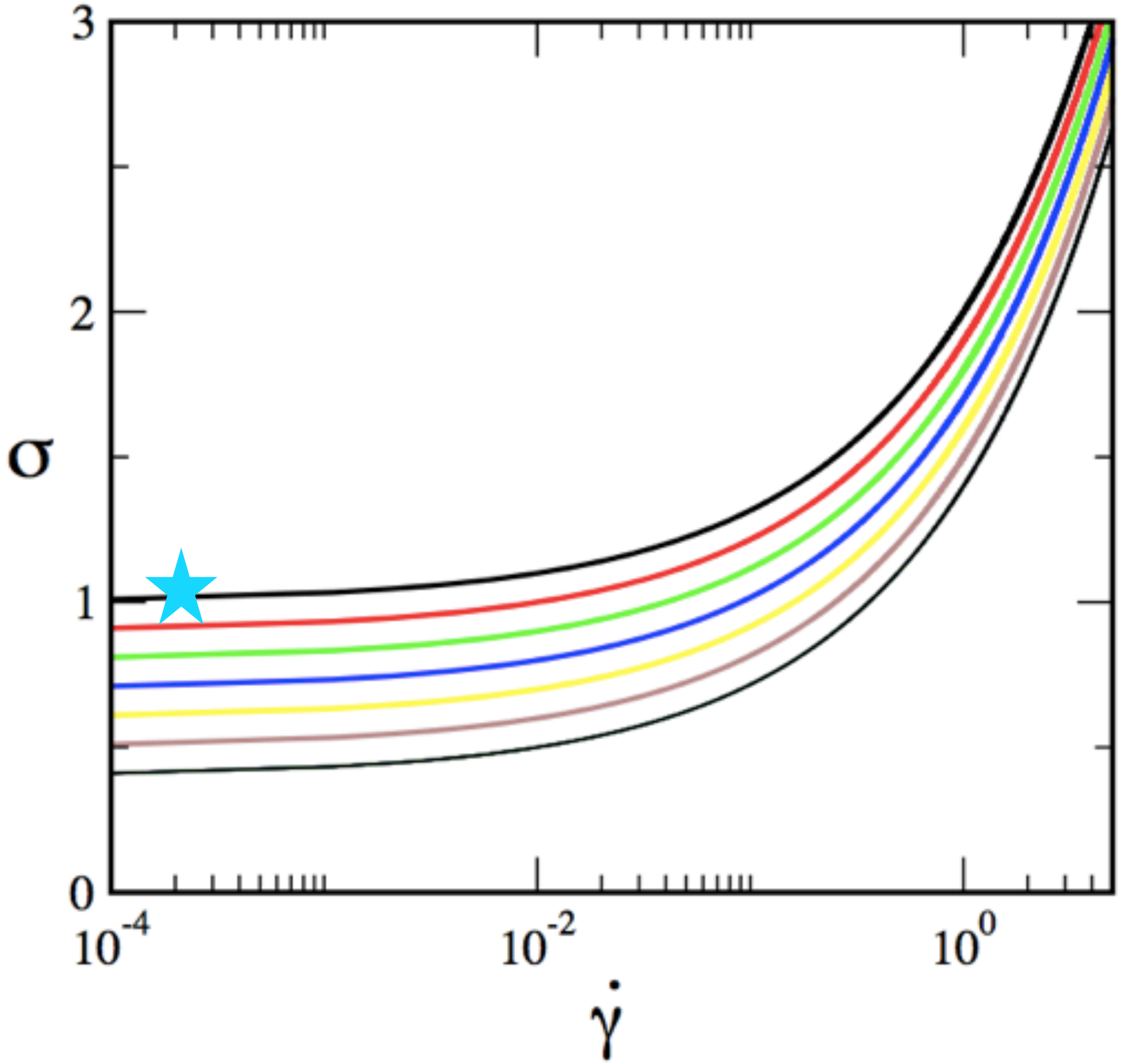
Overdamped flow curves at different temperatures



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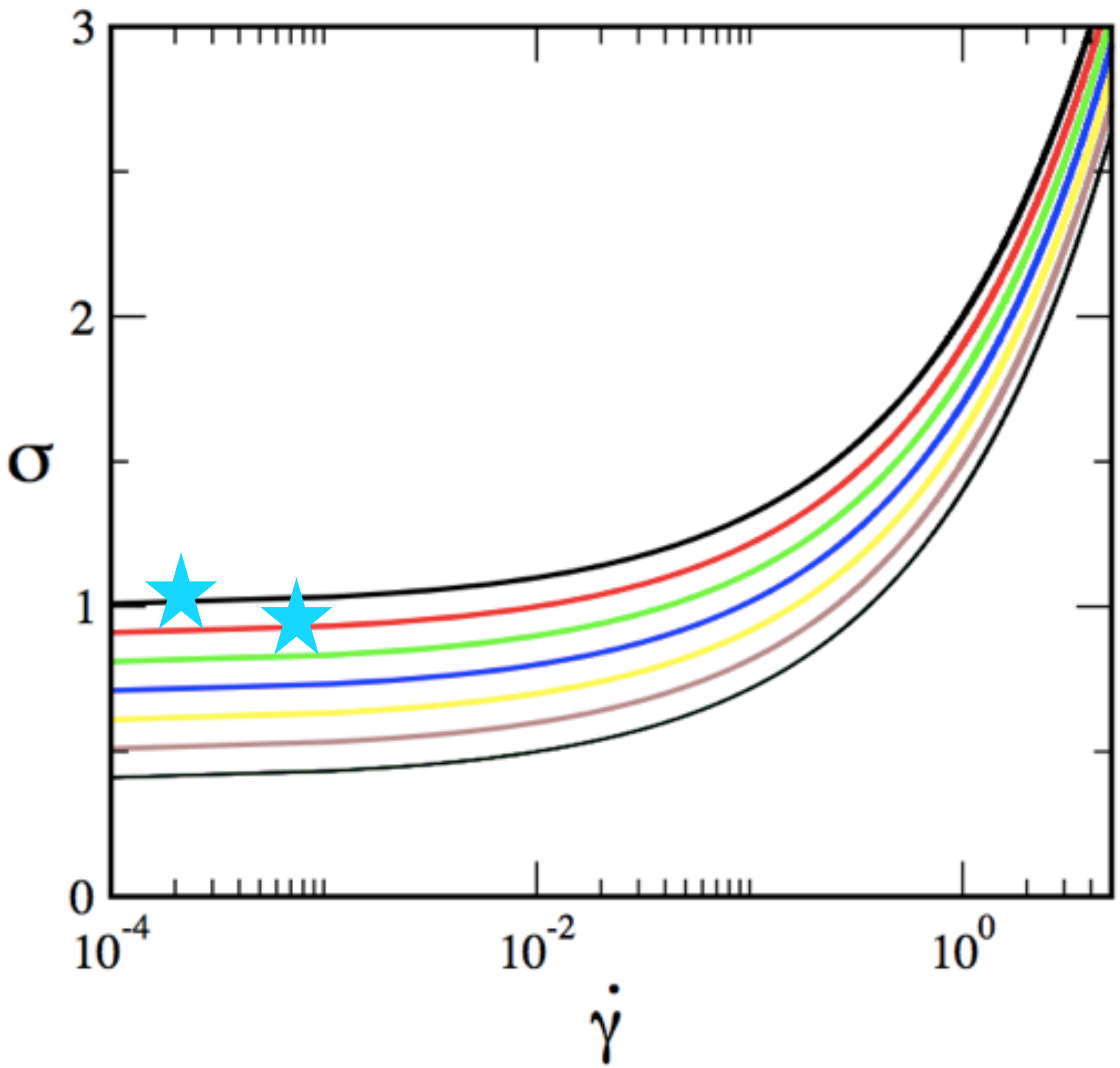

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Nicolas, Barrat & Rottler PRL 2017

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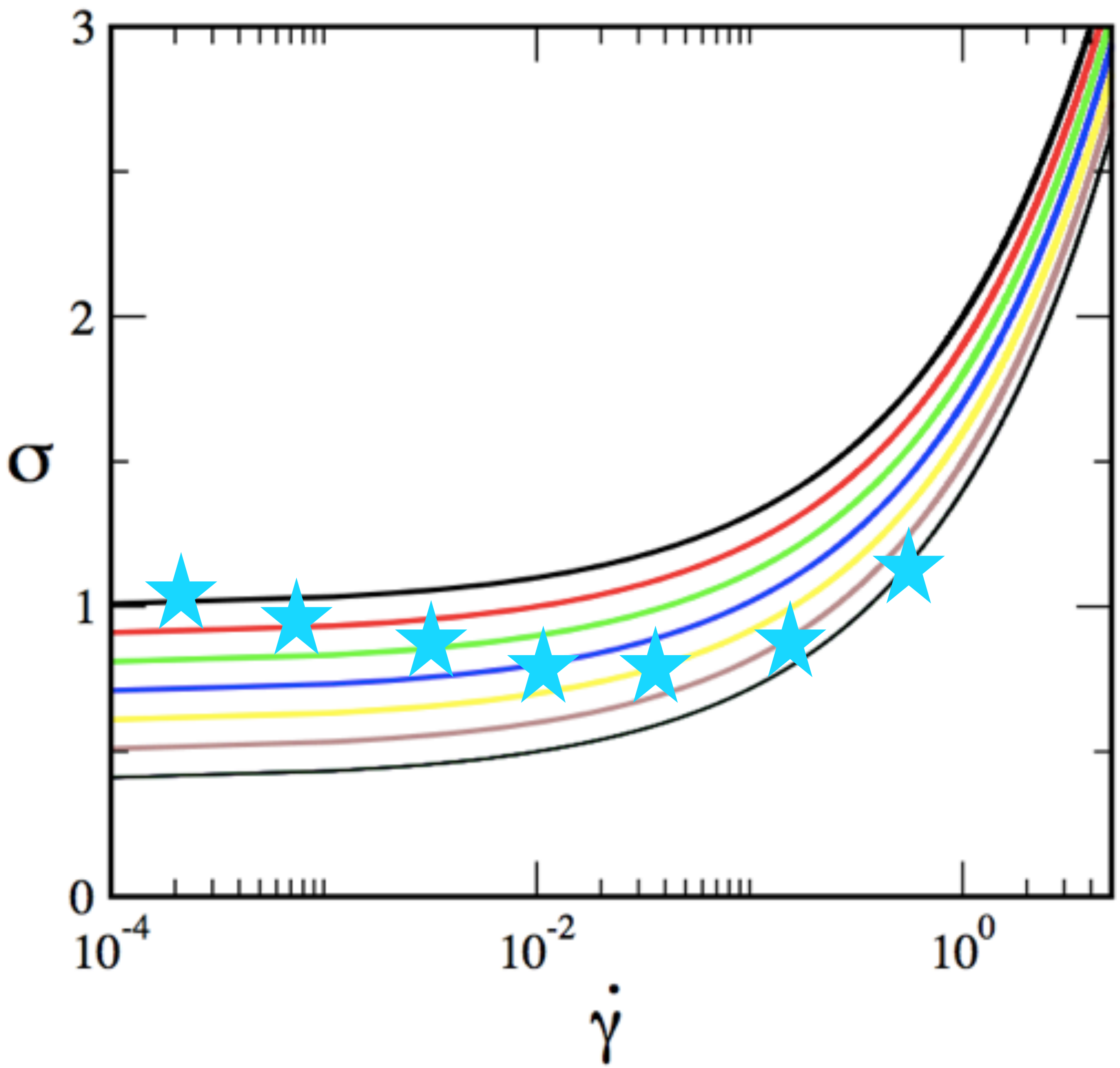


Effective temperature

$$k_B\tilde{T} = E_{\rm kin}$$

depends on driving rate

$$\tilde{T} = \tilde{T}(\dot{\gamma})$$

Resulting in

$$\sigma(\dot{\gamma}; Q, T=0)$$
$$\Leftrightarrow \sigma(\dot{\gamma}; Q=1, \tilde{T}(\dot{\gamma}))$$

Nicolas, Barrat & Rottler PRL 2017

Continuum model

Athermal overdamped dynamics: $\sigma(\dot{\gamma}) = \sigma_y + A\dot{\gamma}^{0.5}$

Athermal underdamped dynamics: $\sigma(\dot{\gamma}, \tilde{T}(\dot{\gamma})) = \sigma_y + A\dot{\gamma}^{0.5} - B\tilde{T}^{\alpha}$

Time evolution for the shear component of the velocity
and the effective temperature

$$\rho \frac{\partial v_x}{\partial t} = \frac{\partial \sigma}{\partial y} = \frac{\partial \sigma}{\partial \tilde{T}} \frac{\partial \tilde{T}}{\partial y} + \frac{\partial \sigma}{\partial \dot{\gamma}} \frac{\partial^2 v_x}{\partial y^2}$$

$$c_V \frac{\partial \tilde{T}}{\partial t} = \lambda_T \frac{\partial^2 \tilde{T}}{\partial y^2} + \frac{\partial v_x}{\partial y} \sigma\left(\frac{\partial v_x}{\partial y}, \tilde{T}\right) - \frac{c_V}{\tau}\tilde{T}$$

ρ density

c_V heat capacity

λ_T thermal conductivity

τ dissipative timescale (thermostat)

(reminder: x shear, y gradient, z vorticity direction)

Hinkle, Rycroft, Shields & Falk PRE 2017

Linear stability analysis

Linear velocity profile stable against small amplitude perturbations?

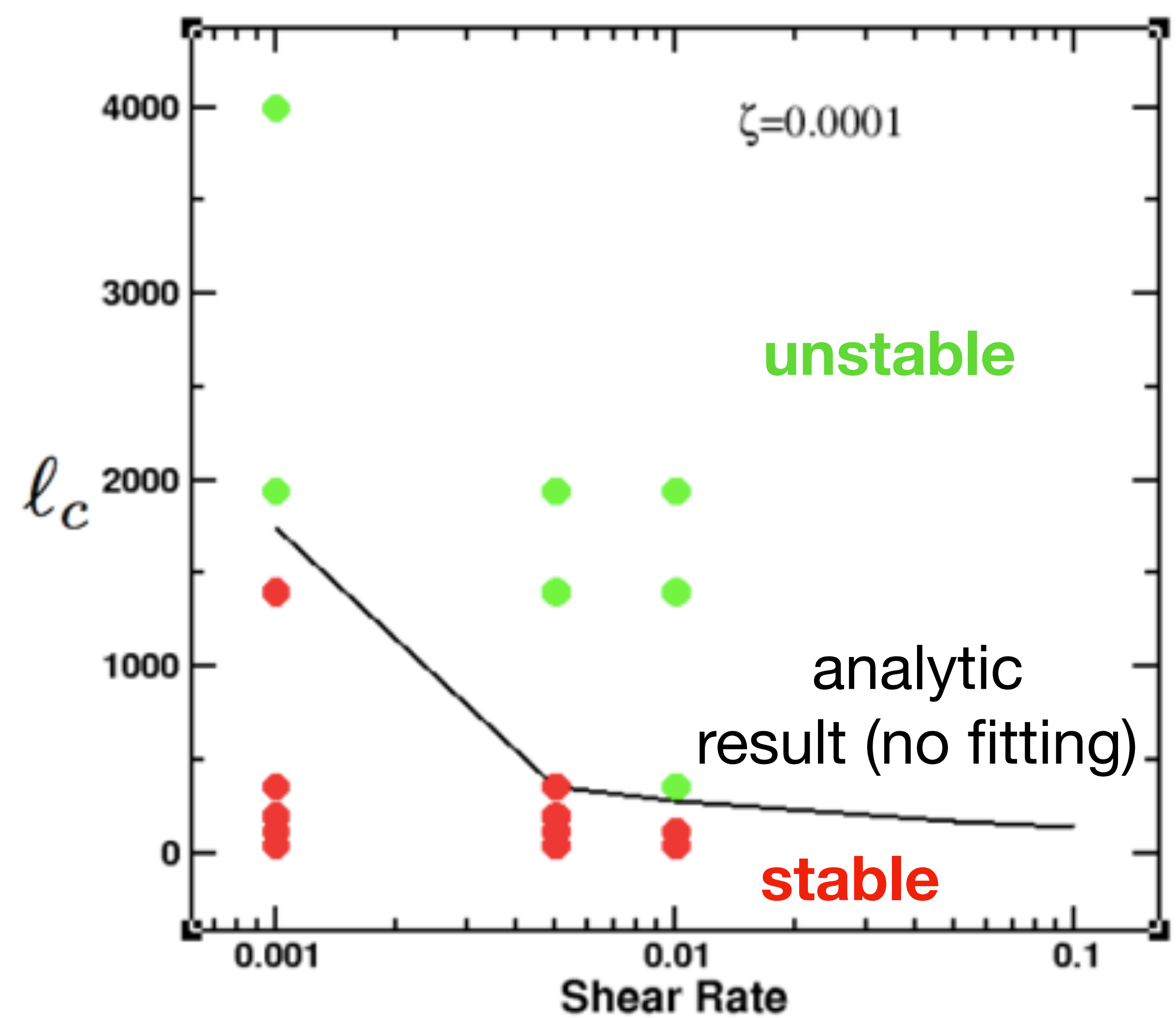
Only below a critical length:

$$\ell_c = 2\pi\sqrt{\lambda_T}\left(-\sigma\frac{\partial_T\sigma}{\partial_{\dot{\gamma}}\sigma} - \frac{c}{\tau}\right)^{-\frac{1}{2}}$$

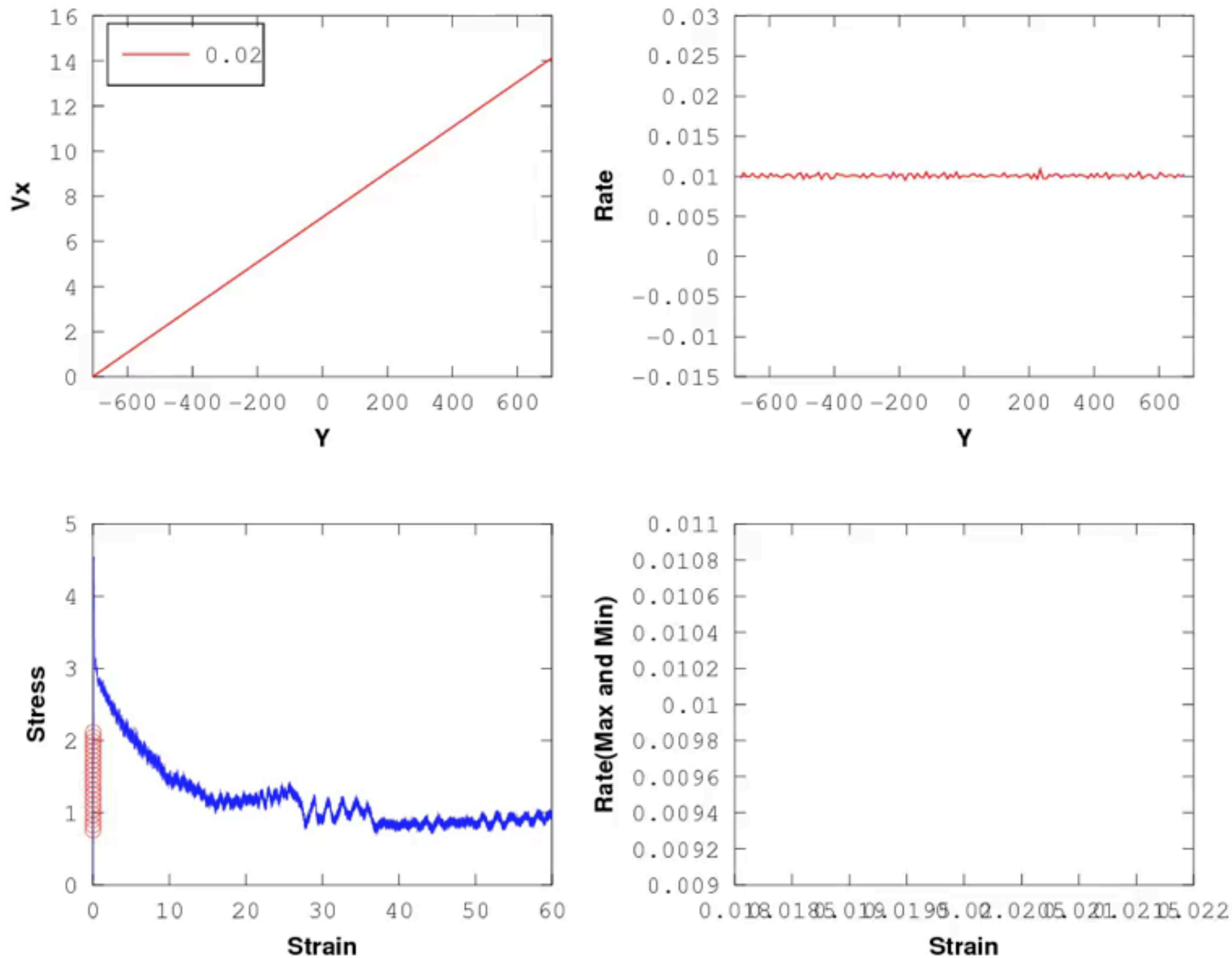
destabilising

stabilising

Simulation results



Coarsening dynamics



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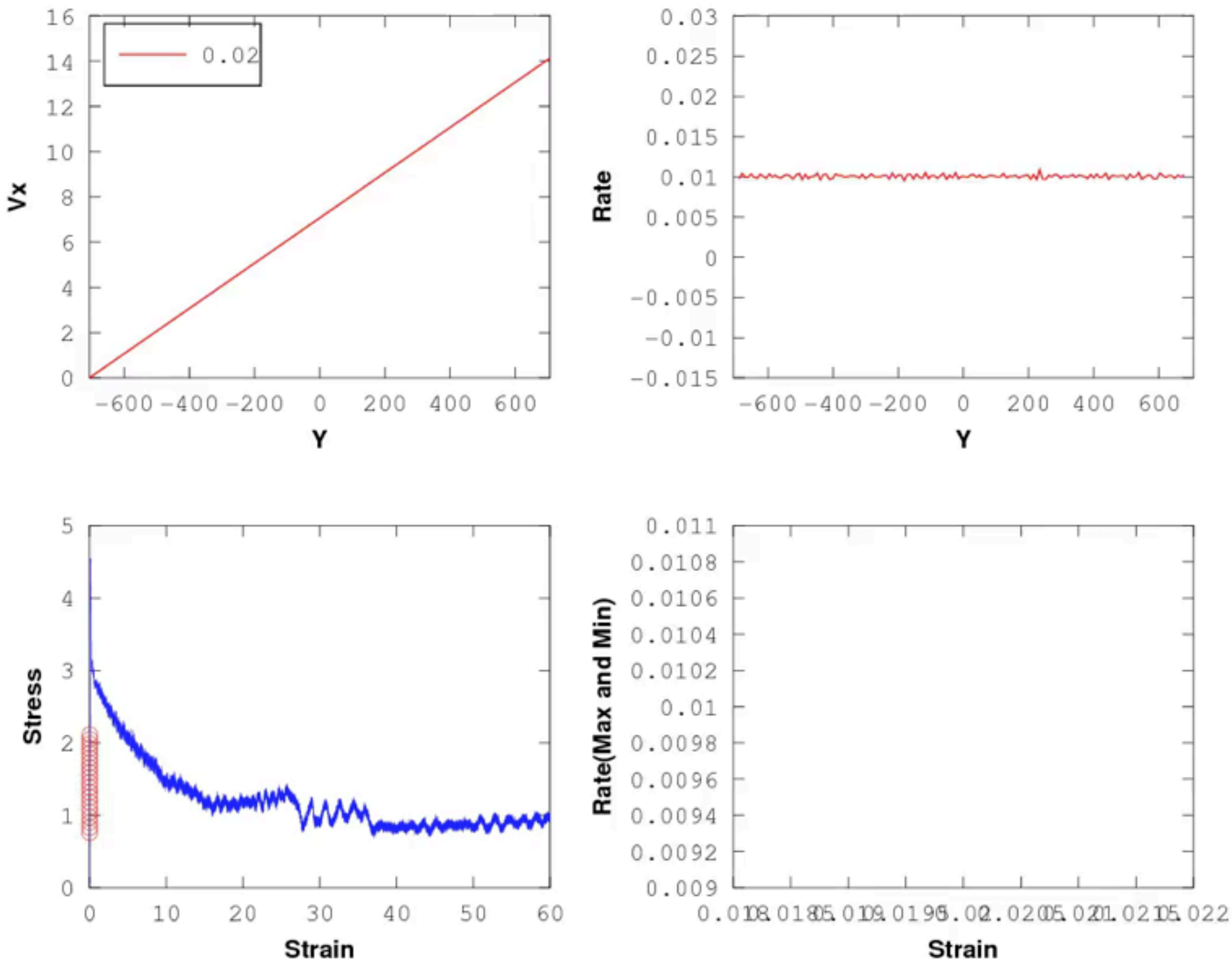

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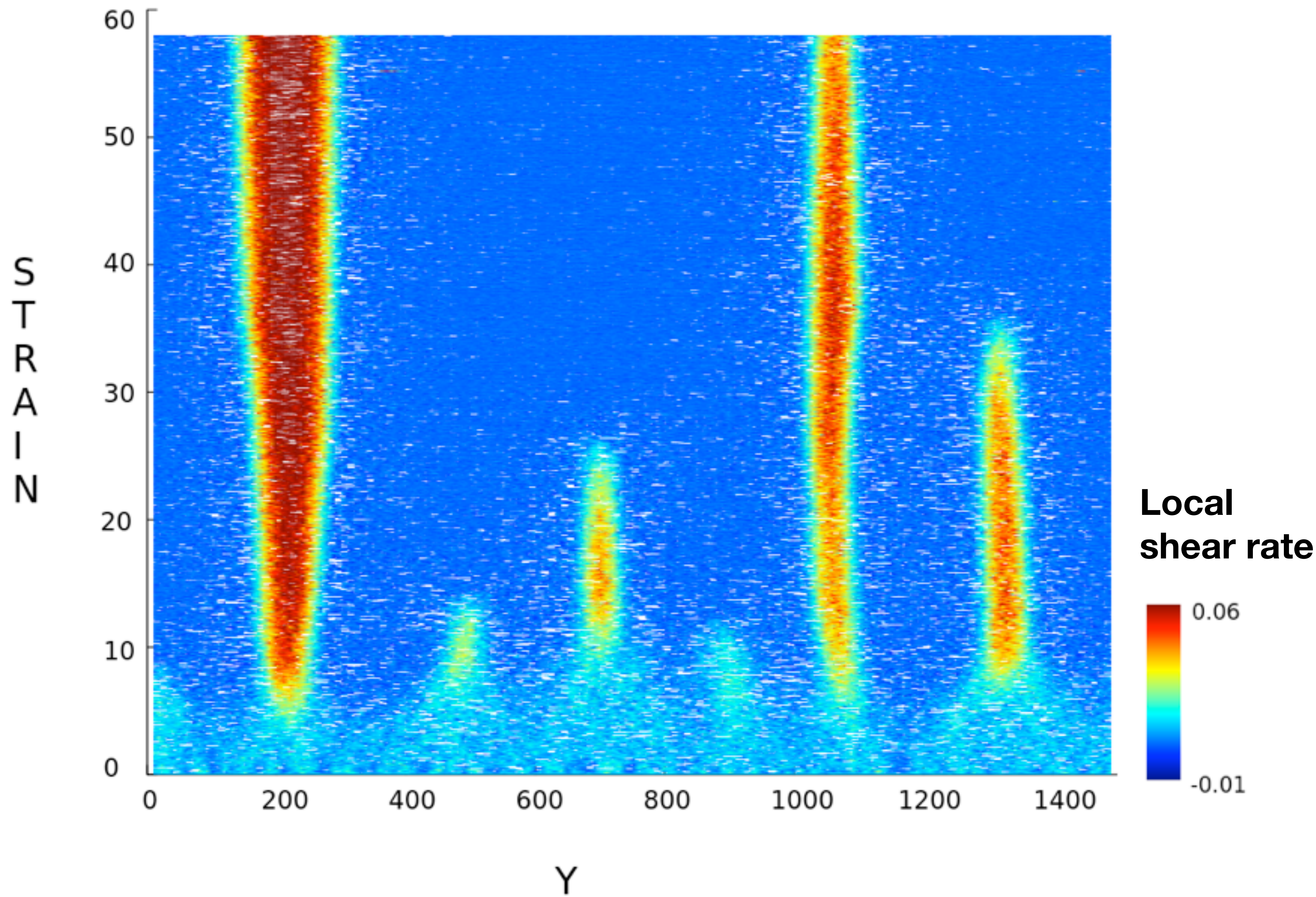
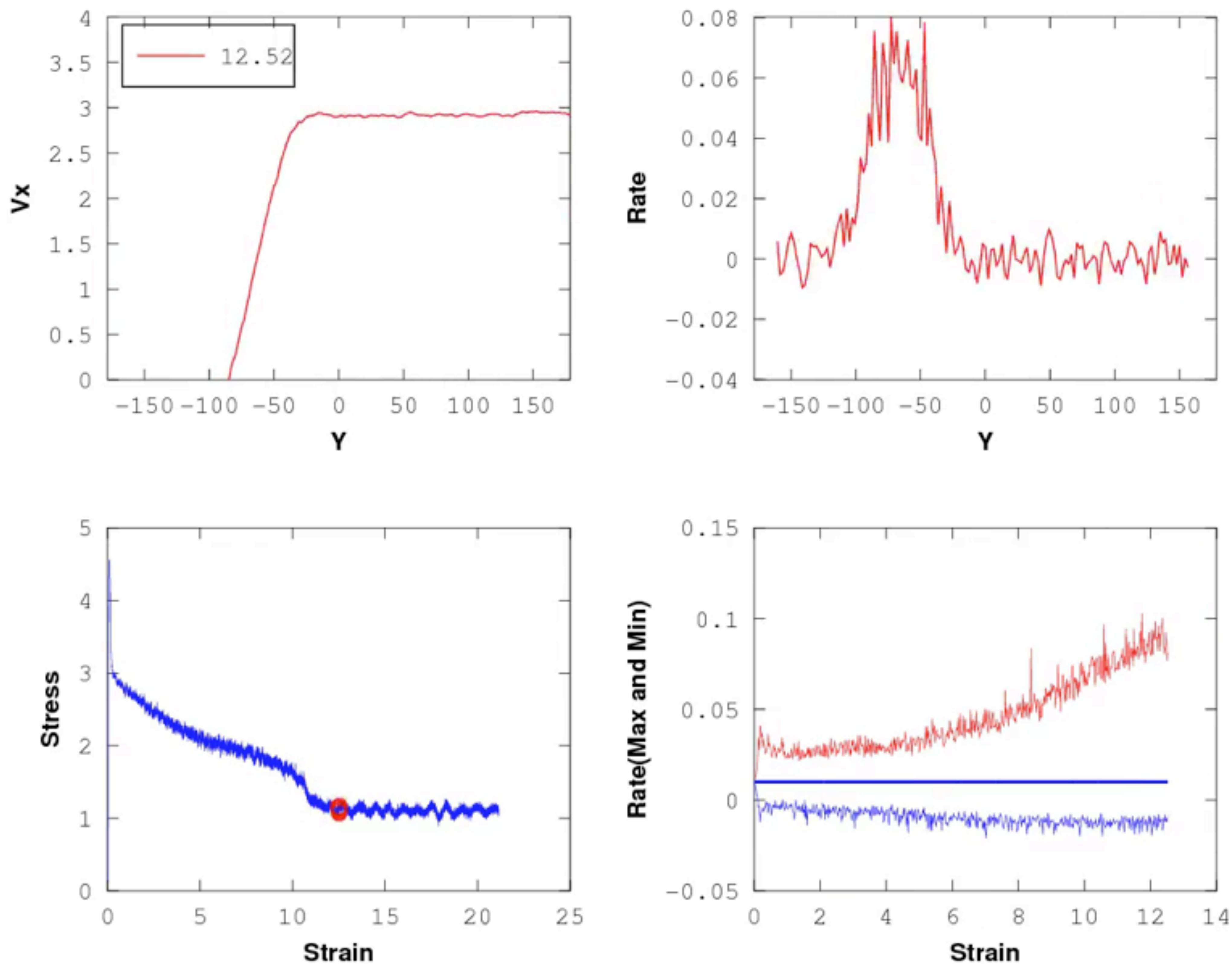


Reaching the stationary state



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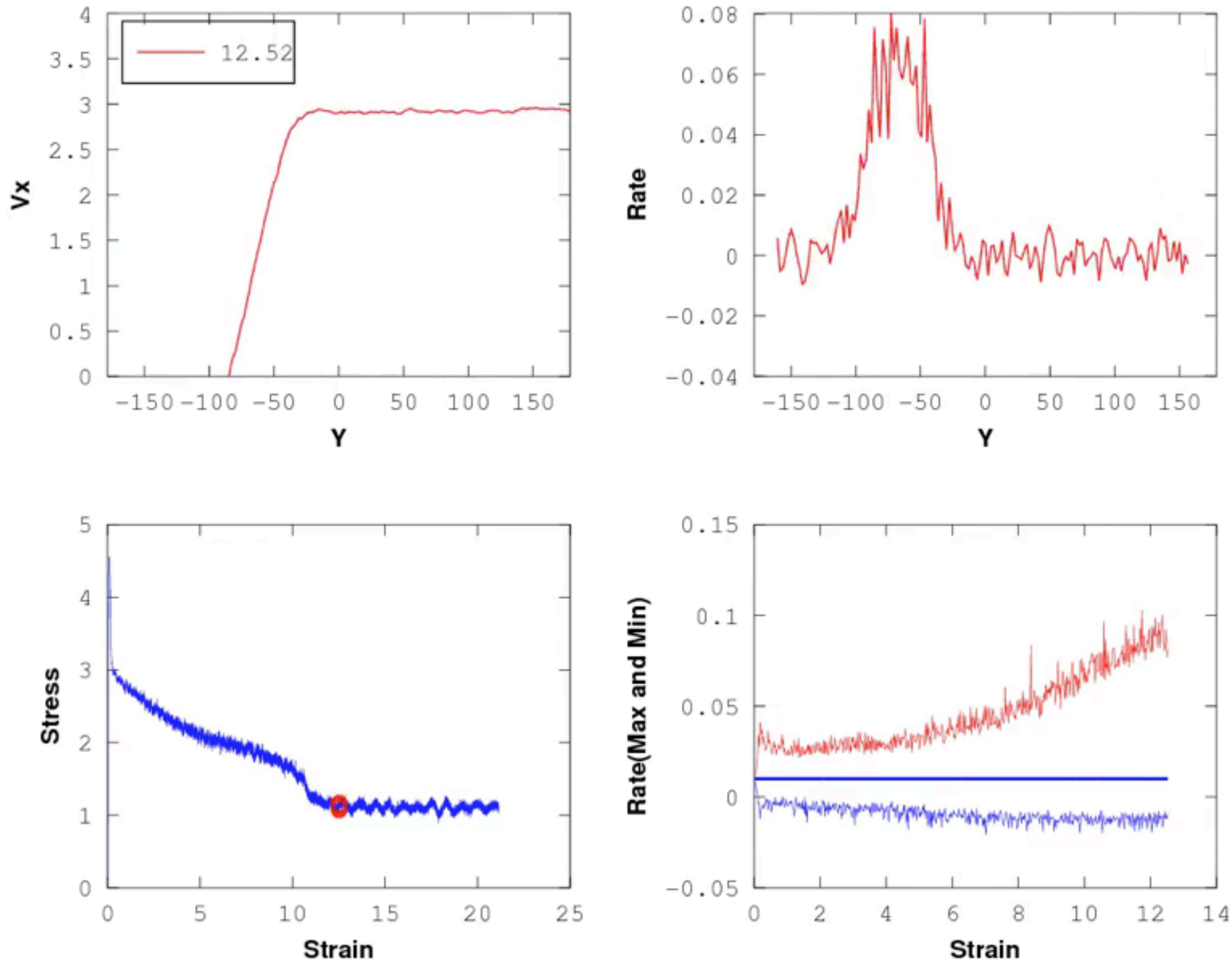


Numerical integration

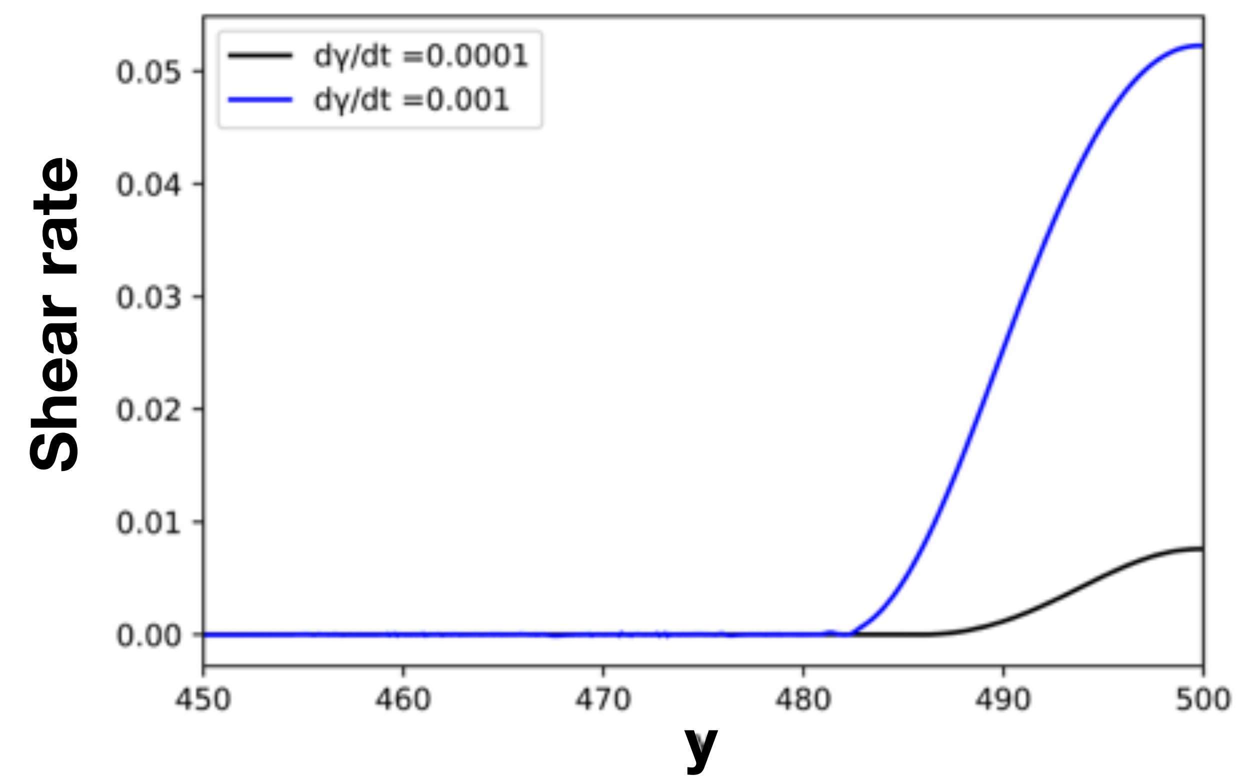
$$\rho \frac{\partial v_x}{\partial t} = \frac{\partial \sigma}{\partial y} = \frac{\partial \sigma}{\partial \tilde{T}} \frac{\partial \tilde{T}}{\partial y} + \frac{\partial \sigma}{\partial \dot{\gamma}} \frac{\partial^2 v_x}{\partial y^2}$$

$$c_V \frac{\partial \tilde{T}}{\partial t} = \lambda_T \frac{\partial^2 \tilde{T}}{\partial y^2} + \frac{\partial v_x}{\partial y} \sigma \left(\frac{\partial v_x}{\partial y}, \tilde{T} \right) - \frac{c_V}{\tau} \tilde{T}$$

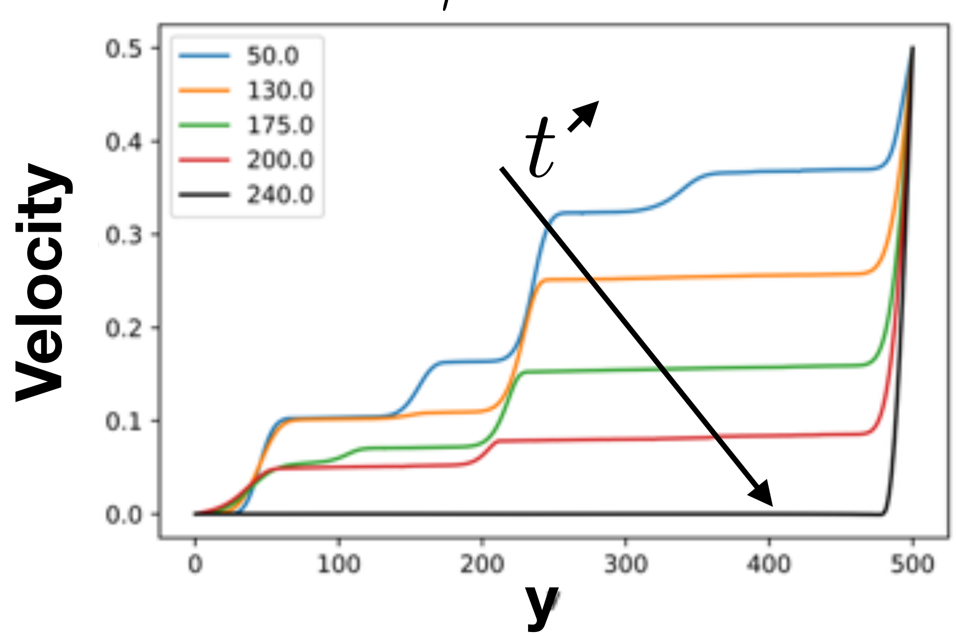
Boundary conditions:

$$v_x(0,t) = 0 \qquad v_x(L_y,t) = \dot{\gamma} L_y$$

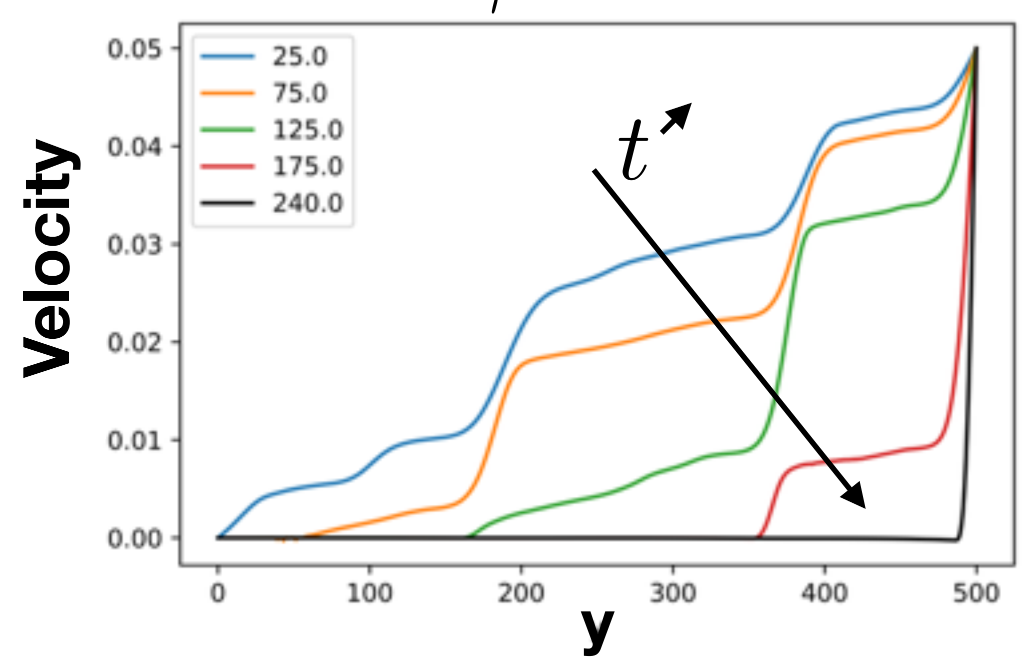
No-flux boundary condition
for the temperature



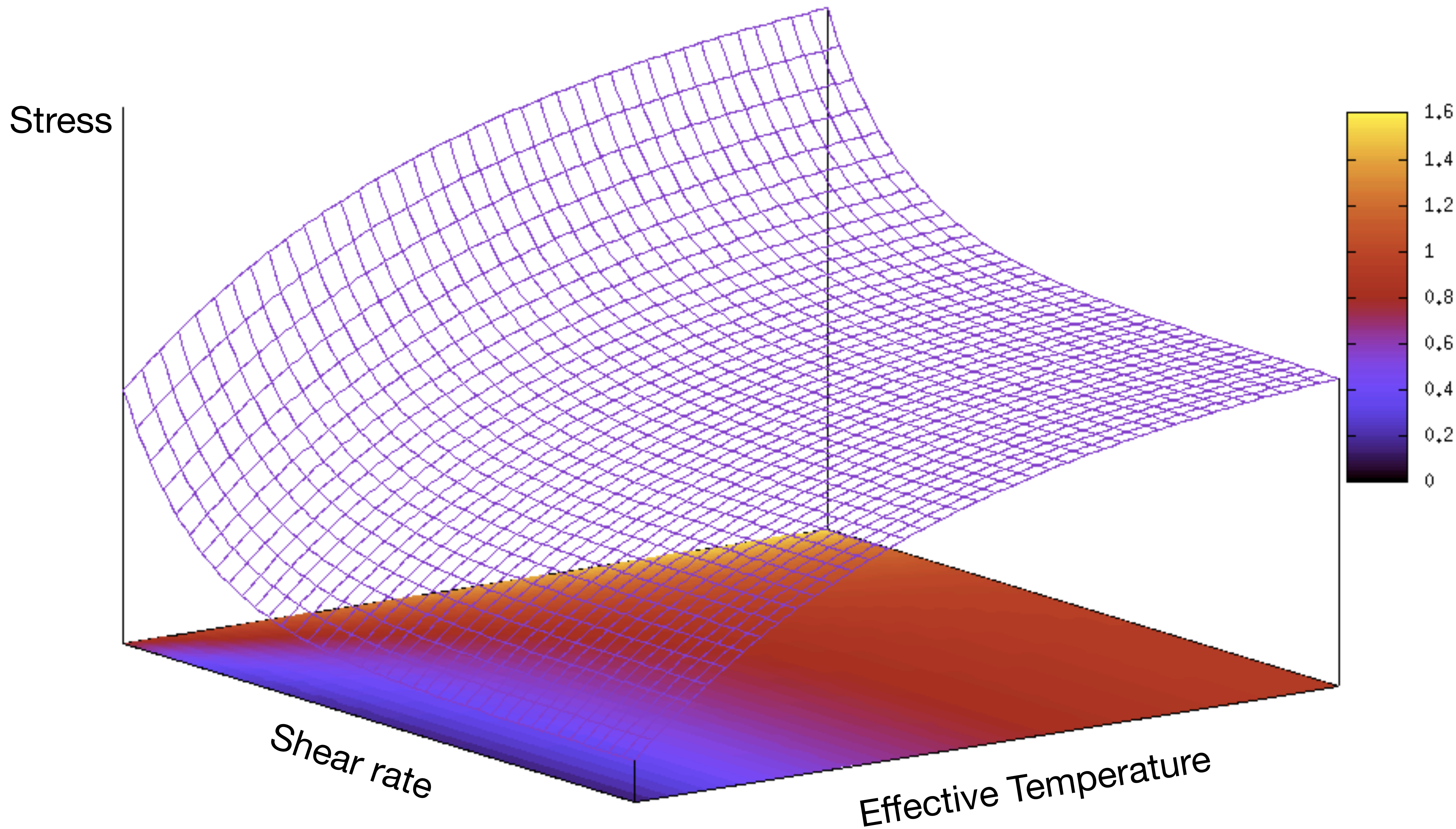
$\dot{\gamma} = 10^{-4}$



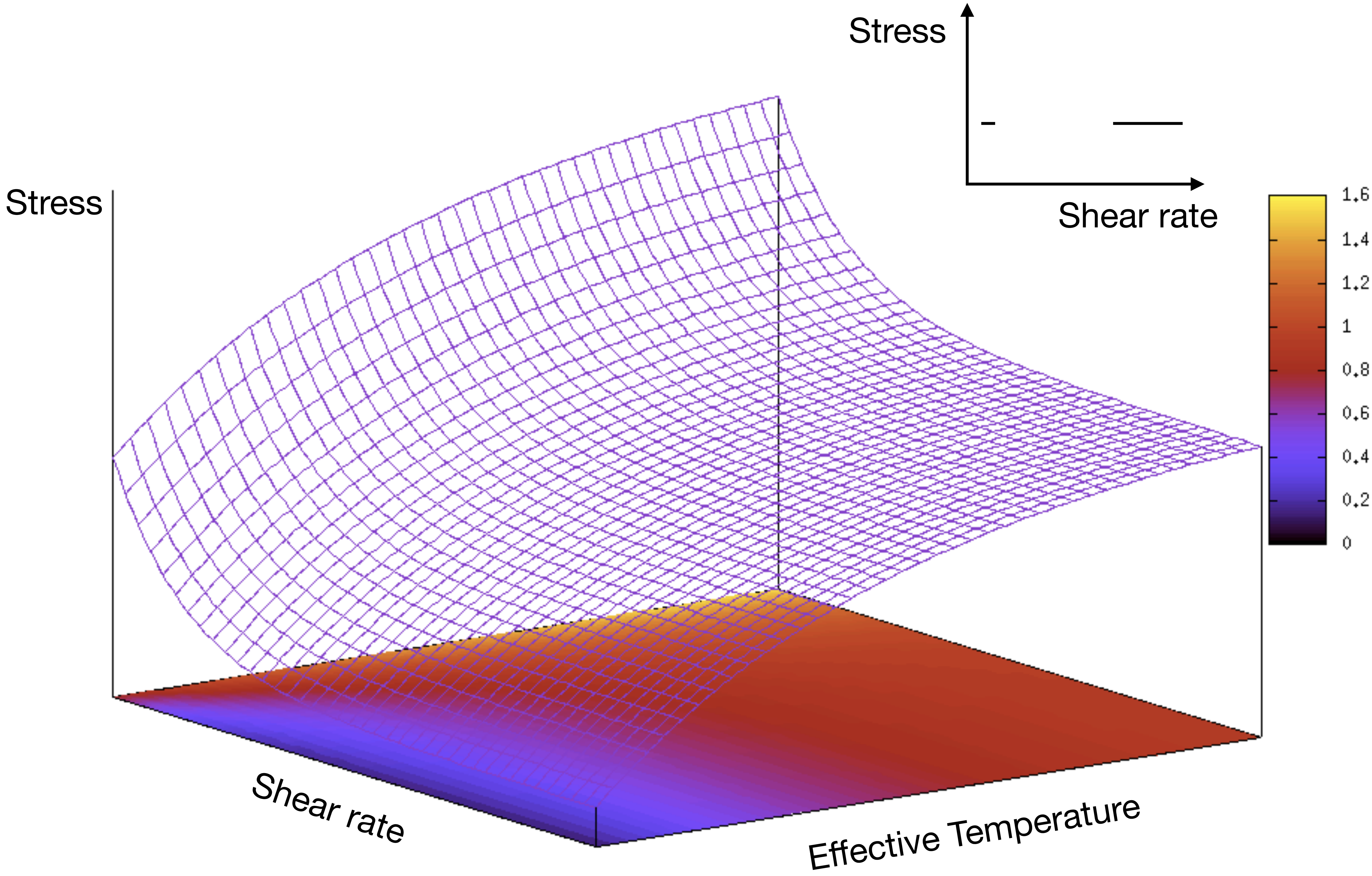
$\dot{\gamma} = 10^{-3}$



Stress selection rule?



Stress selection rule?

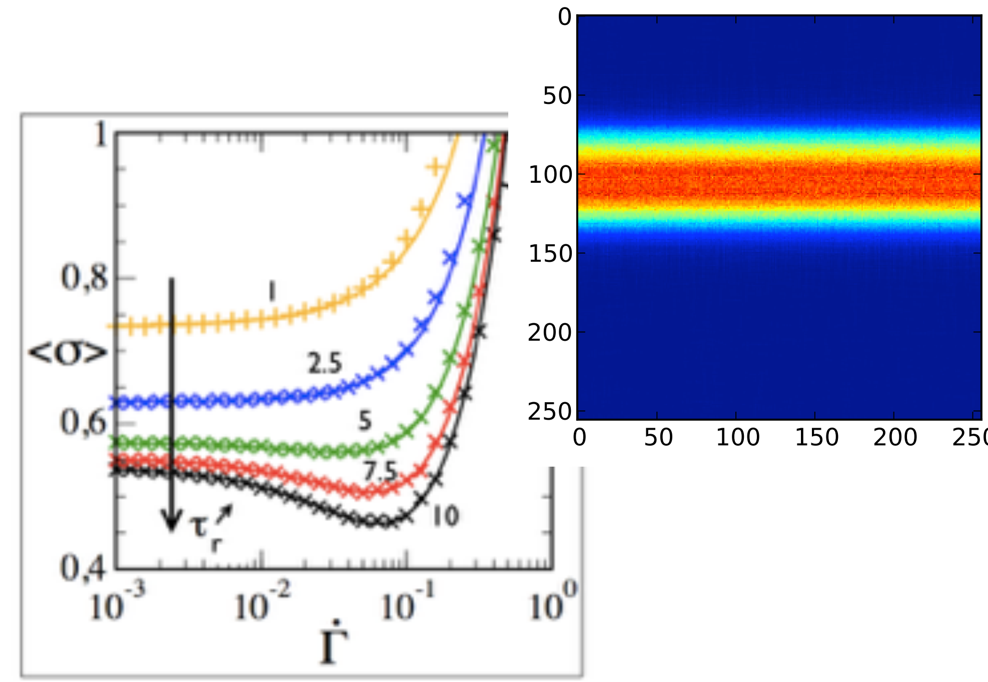


Conclusions

The question of permanent shear band formation in deeply jammed systems is still an open question

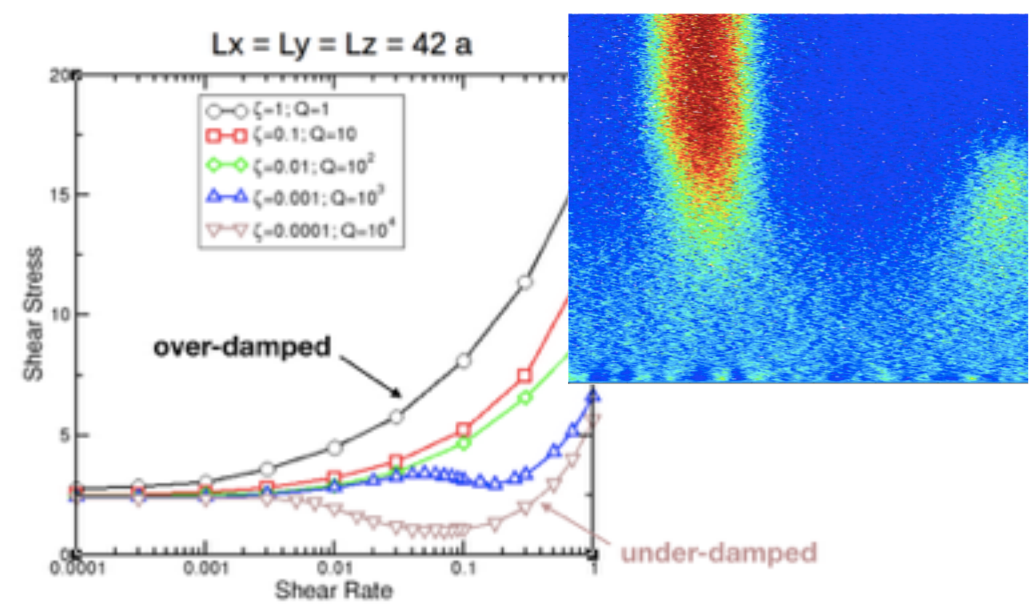
1) A long local restructuring time (local weakening) creates shear bands (lever rule, normal scalings)

But no simulation example of this phenomenon for jammed soft particle systems



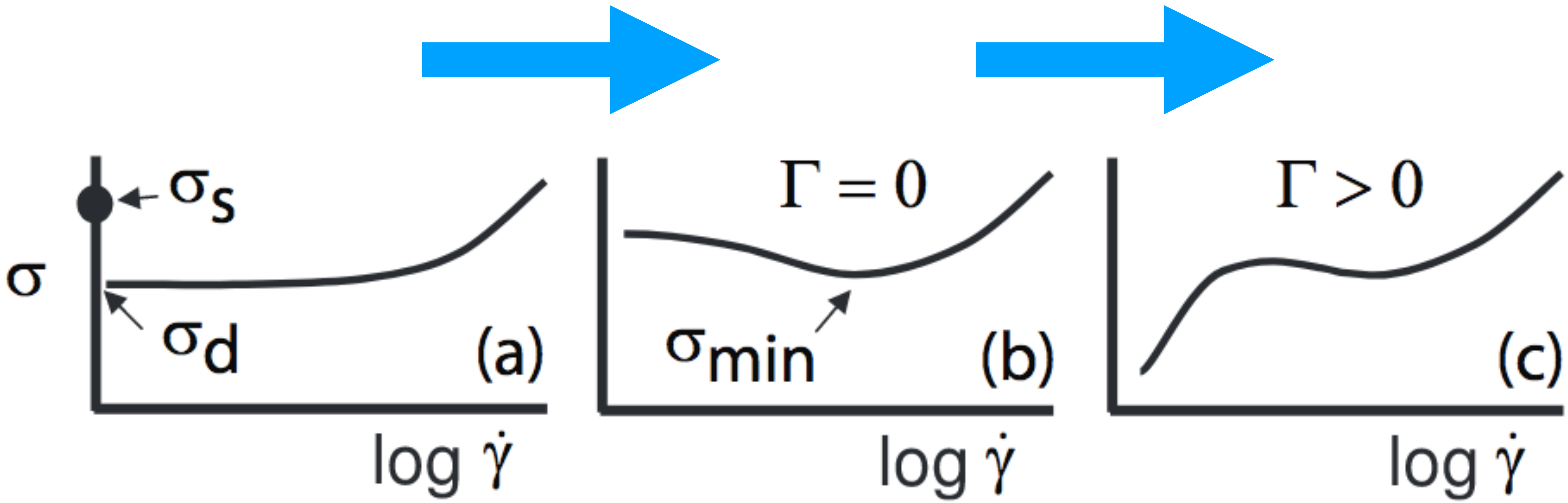
2) Inertia can create (or enhance) the instability in soft jammed particle systems

Here we are able to provide a complete quantitative comparison between continuum equations and microscopic simulations



Outlook

Weakening, inertia... **External vibrations**



Dijksman et al. PRL 2011

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