Local Grand Unification

Michael Ratz

TUM

Santa Barbara, String Pheno, August 29, 2006

Based on:
W. Buchmüller, K. Hamaguchi, O. Lebedev & M.R. :
- Nucl. Phys. B 712, 139 &
- Phys. Rev. Lett. 96, 121602 &
- hep-ph/0512149 &
- hep-th/0606187

O. Lebedev, H. P. Nilles, S. Raby, S. Ramos-Sánchez, M. R., P. Vaudrevange & A. Wingerter :
- in preparation
Outline

1. Motivation

2. Local grand unification (using heterotic $\mathbb{Z}_3 \times \mathbb{Z}_2$ orbifold)

3. Brief discussion of the MSSM from the heterotic string

4. Summary and outlook
Beautiful and ugly aspects of GUTs

MSSM gauge coupling unification @ $M_{\text{GUT}} \sim 10^{16} \text{ GeV}$
Beautiful and ugly aspects of GUTs

🌞 MSSM gauge coupling unification @ $M_{\text{GUT}} \sim 10^{16}$ GeV

🌞 One generation of observed matter fits into $16$ of $\text{SO}(10)$

\[
\text{SO}(10) \rightarrow \text{SU}(3) \times \text{SU}(2) \times \text{U}(1)_Y = G_{\text{SM}} \\
16 \rightarrow (3, 2)_{1/6} \oplus (\overline{3}, 1)_{-2/3} \oplus (\overline{3}, 1)_{1/3} \\
\oplus (1, 1)_1 \oplus (1, 2)_{-1/2} \oplus (1, 1)_0
\]
MSSM gauge coupling unification @ $M_{\text{GUT}} \sim 10^{16}$ GeV

One generation of observed matter fits into $\mathbf{16}$ of $\text{SO}(10)$

However: Higgs only as doublet(s)

\[
\begin{align*}
\mathbf{10} &\rightarrow (\mathbf{1}, \mathbf{2})_{1/2} \oplus (\mathbf{1}, \mathbf{2})_{-1/2} \oplus (\mathbf{3}, \mathbf{1})_{-1/3} \oplus (\mathbf{\overline{3}}, \mathbf{1})_{1/3}
\end{align*}
\]

doublets: needed

triplets: excluded
Beautiful and ugly aspects of GUTs

😊 MSSM gauge coupling unification @ $M_{\text{GUT}} \sim 10^{16}$ GeV

😊 One generation of observed matter fits into $16$ of $\text{SO}(10)$

😢 However: Higgs only as doublet(s)

Convincing answer: ‘localized gauge groups’
Local grand unification (using small extra dimensions)

- Large(r) group $G$
- $G_{lt}$, $G_{rt}$, $G_{rb}$
- SO(10)

Standard model as an intersection of $G_{rb}$, $G_{rt}$, $G_{lt}$ & SO(10) in $G$

'Low-energy' effective theory

Higgs doublets:
- Live somewhere else

SM generation(s):
- Localized in region with SO(10) symmetry

Higher-dimensional GUTs vs. heterotic orbifolds

**Top-down**
→ Orbifold compactifications of the heterotic string

- Dixon, Harvey, Vafa, Witten (1985-86)
- Ibáñez, Nilles, Quevedo (1987)
- Ibáñez, Kim, Nilles, Quevedo (1987)
- Font, Ibáñez, Nilles, Quevedo (1988)
- Font, Ibáñez, Quevedo, Sierra (1990)
- Katsuki, Kawamura, Kobayashi, Ohtsubo, Ono, Tanioka (1990)
- Kawamura (1999-2001)
- Altarelli, Feruglio (2001)
- Hall, Nomura (2001)
- Hebecker, March-Russell (2001)
- Asaka, Buchmüller, Covi (2001)
- Hall, Nomura, Okui, Smith (2001)
- ... 

- has UV completion
- automatically consistent
- explain representations

**Bottom-up**
→ Orbifold GUTs

- simple geometrical interpretation
- shares many features with 4D GUTs

- Faraggi, Förste, Timirgaziu (2006)
- Kim, Kyae (2006)

**Combine both approaches**
implement field-theoretic GUTs in non-prime orbifold compactifications of the heterotic string
Higher-dimensional GUTs vs. heterotic orbifolds

combine both approaches

implement field-theoretic GUTs in non-prime orbifold compactifications of the heterotic string

What’s new?

- systematic analysis of non-prime orbifolds with Wilson lines
- geometric picture with various orbifold GUT limits
- anisotropic compactification may mitigate the discrepancy between GUT and string scales

- localized 16-plets as the origin of complete generations
Local Grand Unification

in

$$\mathbb{Z}_6 - \mathbb{II} = \mathbb{Z}_3 \times \mathbb{Z}_2$$

orbifolds
Compactification on $\mathbb{T}^6/\mathbb{Z}_6$ orbifold $(\mathbb{Z}_6 - II)$

$\mathbb{T}^6$ torus is defined by the root lattice

$$\Lambda_{G_2 \times SU(3) \times SO(4)} := \text{root lattice of Lie algebra of } G_2 \times SU(3) \times SO(4)$$

Compactification on $\mathbb{T}^6/\mathbb{Z}_6$ orbifold

$\mathbb{T}^6$ torus is defined by the root lattice

$$\Lambda_{G_2 \times SU(3) \times SO(4)} := \text{root lattice of Lie algebra of } G_2 \times SU(3) \times SO(4)$$

The $\mathbb{Z}_6$ action on $\Lambda_{G_2 \times SU(3) \times SO(4)}$ is

$$z_i \rightarrow e^{2\pi i v_6^i} z_i \quad \text{with} \quad v_6 = \frac{1}{6}(-1, -2, 3)$$
Compactification on $\mathbb{T}^6/\mathbb{Z}_6$ orbifold ($\mathbb{Z}_6 - \Pi$)

$\mathbb{T}^6$ torus is defined by the root lattice

$$\Lambda_{G_2 \times SU(3) \times SO(4)} := \text{root lattice of Lie algebra of } G_2 \times SU(3) \times SO(4)$$

$\circ = \mathbb{Z}_6$ fixed point

The $\mathbb{Z}_6$ action on $\Lambda_{G_2 \times SU(3) \times SO(4)}$ is

$$z_i \rightarrow e^{2\pi i v_6^i} z_i \quad \text{with} \quad v_6 = \frac{1}{6} (-1, -2, 3)$$

and has $\mathbb{Z}_k$ ($k = 2, 3, 6$) fixed points:

$$z_{\mathbb{Z}_k \text{f.p.}}^i - e^{2\pi i \frac{6}{k} v_6^i} z_{\mathbb{Z}_k \text{f.p.}}^i \in \Lambda_{G_2 \times SU(3) \times SO(4)}$$
Compactification on $\mathbb{T}^6/\mathbb{Z}_6$ orbifold 

$\mathbb{T}^6$ torus is defined by the root lattice

$$\Lambda_{G_2 \times SU(3) \times SO(4)} := \text{root lattice of Lie algebra of } G_2 \times SU(3) \times SO(4)$$

- $\mathbb{Z}_6$ fixed point

Twist action is embedded into the gauge degrees of freedom

$$X^I \rightarrow X^I + \pi V^I_6$$  \hspace{1cm} (where $6V_6 \in \Lambda_{E_8 \times E_8}$)

Torus translations are associated to Wilson lines, e.g.

$$z_3 \rightarrow z_3 + 1 \quad \leftrightarrow \quad X^I \rightarrow X^I + \pi W^I_2$$  \hspace{1cm} (where $2W_2 \in \Lambda_{E_8 \times E_8}$)
Light states of effective field theory \((k \equiv 0\) for untwisted sector\)

\[
\Psi_{r,s}(x; z_1, z_2, z_3)
\]

\[
r = p + k V_6
\]

\[
s = q + k v_6
\]

- **Heterotic string**
  - **Untwisted sector** = strings closed on the torus

\[T_k\] **Twisted sector** = strings which are only closed on the orbifold

**Field theory**
- Extra components of gauge fields
- ‘brane fields’

(hard to understand in field-theoretical framework)
Local gauge symmetry (breaking)

Analyze invariance conditions \textit{locally} \,(for illustration just in \,SO(4) plane)
Local gauge symmetry (breaking)

Analyze invariance conditions \textbf{locally} (for illustration just in SO(4) plane)

\[ p \cdot V_{bl} \in \mathbb{Z} \]

\[ e^{2\pi i p \cdot V_{bl}} A^p_{\mu} (x; \ldots, e^{2\pi i/2 z_3}) = A^p_{\mu} (x; \ldots, z_3) \]

\[ \text{projection @ } z_3 = 0: \]

local shift

gauge group generator
Local gauge symmetry (breaking)

Analyze invariance conditions **local**ly (for illustration just in SO(4) plane)

Emerging **local** gauge group:

\[ G_{\text{bl}} \subset E_8 \times E_8 \]
Local gauge symmetry (breaking)

Analyze invariance conditions locally (for illustration just in SO(4) plane)

\[ \text{projection:} \]

\[ p \cdot V_{\text{br}} \in \mathbb{Z} \]

\[ V_{\text{br}} = V_{\text{bl}} + W_2 \]
Local gauge symmetry (breaking)

Analyze invariance conditions locally (for illustration just in SO(4) plane)

Emerging local gauge group: $G_{br} \neq G_{bl}$
Local gauge symmetry (breaking)

Analyze invariance conditions **locally** (for illustration just in $SO(4)$ plane)

$$\begin{align*}
\text{Im} z_3 & \quad W' \\
\text{Re} z_3 & \quad W_2 \\
G_{\text{tl}} & \quad G_{\text{bl}} \\
G_{\text{tr}} & \quad G_{\text{br}}
\end{align*}$$
Local gauge symmetry (breaking)

Analyze invariance conditions \textit{locally} (for illustration just in SO(4) plane)

\[ W'_2 \]
\[ \text{Im} z_3 \]

\[ G_{\text{bl}} \cap G_{\text{br}} \cap G_{\text{tl}} \cap G_{\text{tr}} \sim G_{\text{SM}} \]

but \( G_{\text{bl}} \not\supseteq G_{\text{SM}} \) etc.

\[ G_{\text{bl}} \text{ etc. : 'local GUTs'} \]
Local gauge symmetry (breaking)

Analyze invariance conditions \textbf{locally} (for illustration just in SO(4) plane)

\begin{center}
\begin{tikzpicture}
\draw[thick,blue,->,>=latex] (0,0) -- (5,0) node[below] {\(W_2\)};
\draw[thick,blue,->,>=latex] (0,0) -- (0,5) node[left] {Im\(z_3\)};
\draw[thick,blue,->,>=latex] (0,0) -- (3,3) node[midway,below] {\(G_{\text{tl}}\)};
\draw[thick,blue,->,>=latex] (0,0) -- (3,0) node[midway,below] {\(G_{\text{bl}}\)};
\draw[thick,blue,->,>=latex] (0,0) -- (0,3) node[midway,right] {\(G_{\text{tr}}\)};
\draw[thick,blue,->,>=latex] (0,0) -- (0,0) node[below] {\(G_{\text{br}}\)};
\draw[thick,blue,->,>=latex] (0,0) -- (0,0) node[below] {\textbf{fundamental region}};
\end{tikzpicture}
\end{center}
Local gauge symmetry (breaking)

Analyze invariance conditions \textit{locally} (for illustration just in $SO(4)$ plane)

- ‘pillow’
- ‘folding edge’
- fundamental region

Diagram with labels:
- $G_{tl}$
- $G_{tr}$
- $G_{bl}$
- $G_{br}$

Mathematical expressions:
- $\text{Im } z_3$
- $W'_2$
- $W_2$

Sections of the text:
- Introduction
- Local grand unification in heterotic orbifolds
- MSSM from the heterotic string
- Summary & outlook

Notes:
- $T^6/\mathbb{Z}_6$ orbifold ($\mathbb{Z}_6 - II$) and ‘orbifold construction kit’
- The role of localized 16-plets of $SO(10)$
- 3 vs. 2+1 family models
- Orbifold vacua, decoupling and $U(1)$ breaking
The ‘orbifold construction kit’

- Local gauge group $G_{\text{local}}$
- Local shift $V_{\text{local}}$

Basic structure: one ‘corner’ with shift $V$
The ‘orbifold construction kit’

The simplest possibility: consider identical corners
The ‘orbifold construction kit’

The combination corresponds to an
The ‘orbifold construction kit’

$\mathbb{P}^6/\mathbb{Z}_6$ orbifold ($\mathbb{Z}_6 - II$) and ‘orbifold construction kit’

The role of localized 16-plets of $SO(10)$

3 vs. 2+1 family models

Orbifold vacua, decoupling and $U(1)$ breaking

orbifold without Wilson lines
The ‘orbifold construction kit’

one can combine different ‘corners’
The ‘orbifold construction kit’

\[ V_{tl} = V + W \]
\[ V_{tr} = V + W + W' \]
\[ V_{bl} = V + W \]
\[ V_{br} = V + W' \]

\[ E_8 \times E_8 \]

this leads to an orbifold with Wilson lines

where the Wilson lines correspond to the differences of local shifts and

\[ G_{low-energy} = G \cap G' \cap G'' \cap G''' \]
The ‘orbifold construction kit’

but there are restrictions from modular invariance
i.e., one may combine the ‘corners’ not arbitrarily
The role of localized 16-plets of $SO(10)$

- **basic observation**: the states of the 1st twisted sector appear as complete multiplets of the local gauge group.

- **main idea**: use localized 16-plets of $SO(10)$ to explain generations.

- $\mathbb{Z}_3 \times \mathbb{Z}_2$: there are two shifts which ‘produce’ local $SO(10)$ and 16-plet in 1st twisted sector.

\[
V_6 = \frac{1}{6} (3, 3, 2, 0, 0, 0, 0, 0) (2, 0, 0, 0, 0, 0, 0, 0)
\]

\[
V'_6 = \frac{1}{6} (2, 2, 2, 0, 0, 0, 0, 0) (1, 1, 0, 0, 0, 0, 0, 0)
\]

No-Go for three sequential families

- **simplest implementation**: three `sequential` 16-plets
- Only possible in $\mathbb{Z}_3 \times \mathbb{Z}_2$ etc. but not in $\mathbb{Z}_3$, $\mathbb{Z}_4$, $\mathbb{Z}_2 \times \mathbb{Z}_2$ etc.
No-Go for three sequential families

- **simplest implementation:** three ‘sequential’ 16-plets

- Only possible in $\mathbb{Z}_3 \times \mathbb{Z}_2$ etc. but not in $\mathbb{Z}_3$, $\mathbb{Z}_4$, $\mathbb{Z}_2 \times \mathbb{Z}_2$ etc.

- **however:** in all models there are **chiral exotics** (at least when hypercharge is correctly normalized)
No-Go for three sequential families

- **simplest implementation**: three ‘sequential’ 16-plets
- Only possible in $\mathbb{Z}_3 \times \mathbb{Z}_2$ etc. but not in $\mathbb{Z}_3, \mathbb{Z}_4, \mathbb{Z}_2 \times \mathbb{Z}_2$ etc.
- **however**: in all models there are chiral exotics (at least when hypercharge is correctly normalized)

**bottom-line**: not possible in $\mathbb{Z}_{N \leq 7}$ orbifolds
2+1 family models

Features:

- Two families come from two equivalent fixed points
- 3rd family has to come from ‘somewhere else’ (untwisted sector, $T_{k>1}$)
2+1 family models

Features:

☞ Two families come from two equivalent fixed points

☞ 3rd family has to come from 'somewhere else' (untwisted sector, $T_{k>1}$)

☞ $O(100)$ models with:
  • $E_8 \rightarrow G_{SM} \times U(1)^4$
  • 3 generations + vector-like exotics $X_i, \bar{X}_j$
  • $X_i, \bar{X}_j$ have couplings

$X_i \bar{X}_j \quad s_{i_1} \ldots s_{i_n}$

$\text{vev} \rightarrow \text{mass term}$
2+1 family models

Features:

☞ Two families come from two equivalent fixed points

☞ 3rd family has to come from ‘somewhere else’ (untwisted sector, \(T_k > 1\))

☺ \(O(100)\) models with:

- \(E_8 \rightarrow G_{\text{SM}} \times U(1)^4\)
- 3 generations + vector-like exotics \(X_i, \overline{X}_j\)
- \(X_i, \overline{X}_j\) have couplings

\(X_i \overline{X}_j \stackrel{s_{i_1} \ldots s_{i_n}}{\rightarrow} \text{vev} \rightarrow \text{mass term}\)

? are the exotics’ mass terms consistent with supersymmetry?
orbifold point is ‘saddle point’

\[ V_D = g^2 \left( \sum q_{\text{anom}}^{(i)} |\phi_i|^2 + g \xi \right)^2 + \ldots \]

\[ \xi = \frac{M_P^2}{192\pi^2} \sum q_{\text{anom}}^{(i)} \]

\[ \sim 10^{-2} \cdots 1 M_P^2 \]

\[ \xi \sim \sqrt{\xi} \]

Exotics’ masses are \( \sim \langle \phi_1 \ldots \phi_n \rangle \)
orbifold point is ‘saddle point’

\[ V_D = g^2 \left( \sum q^{(i)}_{\text{anom}} |\phi_i|^2 + g \xi \right)^2 + \ldots \]

\[ \xi = \frac{M_P^2}{192\pi^2} \sum q_{\text{anom}}^{(i)} \]

\[ \sim 10^{-2} \ldots \sim 10^{-1} M_P^2 \]

\[ \sim \sqrt{\xi} \sim \sqrt{\xi} \]

\[ 0 \phi_2 \]

\[ V \xi^2 \]

\[ \text{some } \langle \phi_i \rangle \sim 0.1 M_P \]

Exotics’ masses are \( \sim \langle \phi_1 \ldots \phi_n \rangle \)

Scale of vector-like exotics’ masses is \( \sim \text{few} \times M_{\text{GUT}} \sim 10^{16} \text{ GeV} \)
Orbifold vacua, decoupling and $\mathbb{U}(1)$ breaking

Orbifold point is ‘saddle point’

$$V_D = g^2 \left( \sum q^{(i)}_{\text{anom}} |\phi_i|^2 + g \xi \right)^2 + \ldots$$

$$V_F = \sum_i \left| \frac{\partial W}{\partial \phi_i} \right|^2 + \text{SUGRA} + \text{non-perturbative} + \ldots$$

It is possible to ‘rescale’ solutions of $\frac{\partial W}{\partial \phi_i} = 0$ to $V_D = 0$ by ‘complexified gauge transformations’

e.g. Wess & Bagger … Gray, He, Jejjala, Nelson (06)

These solutions are manifolds or points in field space
orbifold point is ‘saddle point’

\[ V_D = g^2 \left( \sum q_{\text{anom}}^{(i)} |\phi_i|^2 + g \xi \right)^2 + \ldots \]

\[ V_F = \sum_i \left| \frac{\partial W}{\partial \phi_i} \right|^2 + \text{SUGRA + non-perturbative +} \ldots \]

In models with discrete Wilson lines: there are usually no fields charged only under \( U(1)_{\text{anom}} \)

@ the minimum of \( V \): \( n > 1 \) gauge factors are broken (rank reduction)
Orbifold vacua, decoupling and $U(1)$ breaking

orbifold point is ‘saddle point’

$$V_D = g^2 \left( \sum q_{anom}^{(i)} |\phi_i|^2 + g \xi \right)^2 + \ldots$$

$$V_F = \sum_i \left| \frac{\partial W}{\partial \phi_i} \right|^2 + \text{SUGRA} + \text{non-perturbative} + \ldots$$

bottom-line

there are many ‘vacua’ without exotics

Remainder of this talk:

one specific model
The MSSM from the heterotic string
Lattice, shift and Wilson lines

\[ V_6 = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{3}, 0, 0, 0, 0, 0\right) \left(\frac{1}{3}, 0, 0, 0, 0, 0, 0\right), \]
\[ W_2 = \left(\frac{1}{2}, 0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, 0, 0\right) \left(-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, -\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, -\frac{1}{4}\right), \]
\[ W_3 = \left(\frac{1}{3}, 0, 0, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 1\right) \left(1, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, 0, 0, 0, 0\right), \]

Gauge group after compactification:
\[ SU(3) \times SU(2) \times U(1)_Y \times [SU(4) \times SU(2) \times U(1)^8] \]
The model exhibits 3 generations + vectorlike matter

<table>
<thead>
<tr>
<th>name</th>
<th>irrep</th>
<th>count</th>
<th>name</th>
<th>irrep</th>
<th>count</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_i$</td>
<td>$(3, 2; 1, 1)_{1/6}$</td>
<td>3</td>
<td>$\bar{u}_i$</td>
<td>$(\bar{3}, 1; 1, 1)_{-2/3}$</td>
<td>3</td>
</tr>
<tr>
<td>$\bar{d}_i$</td>
<td>$(\bar{3}, 1; 1, 1)_{1/3}$</td>
<td>3+4</td>
<td>$d_i$</td>
<td>$(3, 1; 1, 1)_{-1/3}$</td>
<td>4</td>
</tr>
<tr>
<td>$\bar{\ell}_i$</td>
<td>$(1, 2; 1, 1)_{1/2}$</td>
<td>1+4</td>
<td>$\ell_i$</td>
<td>$(1, 2; 1, 1)_{-1/2}$</td>
<td>3+1+4</td>
</tr>
<tr>
<td>$m_i$</td>
<td>$(1, 2; 1, 1)_0$</td>
<td>8</td>
<td>$\bar{e}_i$</td>
<td>$(1, 1; 1, 1)_{1}$</td>
<td>3</td>
</tr>
<tr>
<td>$s^+_i$</td>
<td>$(1, 1; 1, 1)_{1/2}$</td>
<td>16</td>
<td>$s^+_i$</td>
<td>$(1, 1; 1, 1)_{1/2}$</td>
<td>16</td>
</tr>
<tr>
<td>$s^0_i$</td>
<td>$(1, 1; 1, 1)_0$</td>
<td>69</td>
<td>$h_i$</td>
<td>$(1, 1; 1, 2)_0$</td>
<td>14</td>
</tr>
<tr>
<td>$f_i$</td>
<td>$(1, 1; 4, 1)_0$</td>
<td>4</td>
<td>$\bar{f}_i$</td>
<td>$(1, 1; \bar{4}, 1)_0$</td>
<td>4</td>
</tr>
<tr>
<td>$w_i$</td>
<td>$(1, 1; 6, 1)_0$</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Remarks:**
- Extra states vectorlike $\rightarrow U(1)_Y$ non-anomalous
- None of the oscillators is charged under $G_{SM}$

... and if all oscillators get vevs, $G = G_{SM} \times SU(4) \times U(1)_{hidden}$
Vacua with $B - L$ symmetry at high energies

? How to distinguish between lepton and Higgs doublets?

☞ one possibility

1 break at high scale to

$$G_{\text{SM}} \times U(1)_{B-L} \times [SU(4)]$$

- three generations
- one pair of $d + \bar{d}$ and $\ell + \bar{\ell}$
- one pair of Higgs doublets
- three extra pairs $d + \bar{d}$ and $\ell + \bar{\ell}$ with $B - L$ charges $\mp 2/3$ and 0

2 break $U(1)_{B-L}$ at a hierarchically smaller scale

<table>
<thead>
<tr>
<th>field</th>
<th>$B - L$ charges</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_i$</td>
<td>$3 \times \left(\frac{+2}{3}\right)$</td>
</tr>
<tr>
<td>$\bar{u}_i$</td>
<td>$3 \times \left(-\frac{1}{3}\right)$</td>
</tr>
<tr>
<td>$\bar{d}_i$</td>
<td>$(3 + 1) \times \left(-\frac{1}{3}\right) + 3 \times \left(\frac{2}{3}\right)$</td>
</tr>
<tr>
<td>$d_i$</td>
<td>$1 \times \left(\frac{+2}{3}\right) + 3 \times \left(-\frac{2}{3}\right)$</td>
</tr>
<tr>
<td>$\ell_i$</td>
<td>$(3 + 1) \times (-1) + (1 + 3) \times 0$</td>
</tr>
<tr>
<td>$\bar{\ell}_i$</td>
<td>$1 \times (+1) + (1 + 3) \times 0$</td>
</tr>
<tr>
<td>$\bar{e}_i$</td>
<td>$3 \times (+1)$</td>
</tr>
</tbody>
</table>
Decoupling of exotics

mass terms for the exotic states

\[ W = x_i \bar{x}_j M_{ij}^{x}(\tilde{s}) \quad \text{with} \quad M_{ij}^{x}(\tilde{s}) = \sum \tilde{s}_{i_1} \cdots \tilde{s}_{i_n} \]

vector-like exotics

\[ B-L \text{ neutral singlets} \]
Decoupling of exotics

mass terms for the exotic states

\[ W = x_i \bar{x}_j M_x^{ij}(\tilde{s}) \quad \text{with} \quad M_x^{ij}(\tilde{s}) = \sum \tilde{s}_{i_1} \cdots \tilde{s}_{i_n} \]

\[ M_d^{ij}(\tilde{s}) = \begin{pmatrix}
0 & 0 & \tilde{s}^6 & 0 & 0 & \tilde{s}^6 & \tilde{s}^6 \\
0 & 0 & \tilde{s}^6 & 0 & 0 & \tilde{s}^7 & \tilde{s}^7 \\
0 & 0 & \tilde{s}^6 & 0 & 0 & \tilde{s}^7 & \tilde{s}^7 \\
\tilde{s}^8 & 0 & 0 & \tilde{s}^6 & \tilde{s}^6 & 0 & 0
\end{pmatrix} \]

\[ M_d \] has full rank

⇒ all extra \( d_i - \bar{d}_j \) decoupled

☞ note: zeros partially dictated by \( B - L \)
Decoupling of exotics

- mass terms for the exotic states

\[ W = x_i \bar{x}_j M^{ij}_x(\tilde{s}) \quad \text{with} \quad M^{ij}_x(\tilde{s}) = \sum \tilde{s}_{i_1} \cdots \tilde{s}_{i_n} \]

\[ M^{ij}_\ell(\tilde{s}) = \begin{pmatrix}
\tilde{s}^3 & 0 & 0 & 0 & 0 & \tilde{s}^8 & 0 & 0 \\
0 & \tilde{s} & 0 & 0 & 0 & \tilde{s}^6 & 0 & 0 \\
0 & \tilde{s} & 0 & 0 & 0 & \tilde{s}^6 & 0 & 0 \\
0 & \tilde{s}^8 & \tilde{s}^8 & 0 & 0 & 0 & \tilde{s}^6 & \tilde{s}^6 \\
\tilde{s} & 0 & 0 & \tilde{s}^6 & \tilde{s}^6 & 0 & 0 & 0
\end{pmatrix} \]

- 1 eigenvalue is zero \( \sim \) Higgs

- note: we do not need to tune VEVs against each other in order to achieve doublet-triplet splitting

- mass-less \( \bar{\ell} \) eigenstate dominated by \( \bar{\ell}_1 \) (untwisted state)
### Untwisted sector (internal components of the gauge bosons)

<table>
<thead>
<tr>
<th>Field-theoretic description</th>
<th>State</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_1 \sim A_5 + iA_6$</td>
<td>$\bar{u}_1 + \ldots$</td>
</tr>
<tr>
<td>$U_2 \sim A_7 + iA_8$</td>
<td>$q_1 + \ldots$</td>
</tr>
<tr>
<td>$U_3 \sim A_9 + iA_{10}$</td>
<td>$\bar{\ell}_1 \sim H_u + \ldots$</td>
</tr>
</tbody>
</table>

**Renormalizable coupling**

$$y_t \, u_1 \, q_1 \, H_u$$

$$y_t \sim g \ @ \ M_{\text{comp}}$$

- all other Yukawa couplings are suppressed

---

**Graph:**

- $\alpha_1$
- $\alpha_2$
- $\alpha_3$

**Legend:**

- $\alpha_1$
- $\alpha_2$
- $\alpha_3$
Some taste of flavor

$$W_{\text{Yukawa}} = Y_{u}^{ij} (\tilde{s}) \phi_{u} q_{i} \bar{u}_{j} + Y_{d}^{ia} (\tilde{s}) \phi_{d} q_{i} \bar{d}_{a} + Y_{e}^{ib} (\tilde{s}) \phi_{d} \bar{e}_{i} \ell_{b},$$

- **matter**: right $B-L$ charges
- **Higgs**: massless $SU(2)$ doublets with zero $B-L$ charge
$W_{\text{Yukawa}} = Y_{u}^{ij}(\tilde{s}) \phi_u q_i \bar{u}_j + Y_{d}^{ia}(\tilde{s}) \phi_d q_i \bar{d}_a + Y_{e}^{ib}(\tilde{s}) \phi_d \bar{e}_i \ell_b,$

$Y_{u}^{ij}(\tilde{s}) = \begin{pmatrix} g & \tilde{s}^6 & \tilde{s}^4 \\ \tilde{s}^3 & 0 & \tilde{s}^7 \\ \tilde{s}^7 & \tilde{s}^7 & 0 \end{pmatrix}$
Some taste of flavor

\[ W_{\text{Yukawa}} = Y_{ij}^u (\tilde{s}) \phi_u q_i \bar{u}_j + Y_{d}^{ia} (\bar{s}) \phi_d q_i \bar{d}_a + Y_{e}^{ib} (\bar{s}) \phi_d \bar{e}_i \ell_b , \]

\[ Y_{d}^{ia} (\bar{s}) = \begin{pmatrix}
0 & \bar{s}^2 & \bar{s}^2 & 0 \\
\bar{s}^5 & \bar{s}^5 & \bar{s}^5 & 0 \\
0 & \bar{s} & \bar{s} & 0 \\
0 & \bar{s} & \bar{s} & 0
\end{pmatrix} \]

- \( Y_d \) becomes a 3 \times 3 matrix after integrating out the heavy \( d-\bar{d} \) pair
- \( Y_d \) has full rank
- Flavor structure à la Froggatt-Nielsen
Some taste of flavor

\[ W_{\text{Yukawa}} = Y^{ij}_u(\tilde{s}) \phi_u q_i \bar{u}_j + Y^{ia}_d(\tilde{s}) \phi_d q_i \bar{d}_a + Y^{ib}_e(\tilde{s}) \phi_d \bar{e}_i \ell_b, \]

\[ Y^{ib}_e(\tilde{s}) = \begin{pmatrix} 0 & \tilde{s}^6 & 0 & 0 \\ \tilde{s}^5 & 0 & 0 & 0 \\ 0 & \tilde{s}^5 & 0 & 0 \end{pmatrix} \]

☞ \( Y_e \) becomes 3 \times 3 matrix after integrating out the heavy \( d-\bar{d} \) pair

☞ \( Y_e \) has not full rank \( \Rightarrow \) electron massless

☞ \( \tau \) Yukawa coupling seems unrealistic
Summary and outlook
Guided by the idea of local grand unification we have obtained $\mathcal{O}(100)$ models with the following features:

1. $3 \times 16 + \text{Higgs} + \text{nothing}$

No exotics
\(O(100)\) models with:

1. \(3 \times 16 + \text{Higgs} + \text{nothing}\)

2. \(SU(3) \times SU(2) \times U(1)_Y \times G_{\text{hid}}\)
\( \mathcal{O}(100) \) models with:

1. \( 3 \times 16 + \text{Higgs} + \text{nothing} \)

2. \( \text{SU}(3) \times \text{SU}(2) \times \text{U}(1)_Y \times G_{\text{hid}} \)

3. unification
**Summary**

\( O(100) \) models with:

1. \( 3 \times 16 + \text{Higgs} + \text{nothing} \)
2. \( SU(3) \times SU(2) \times U(1)_Y \times G_{\text{hid}} \)
3. unification
4. \( y_t \approx g \) @ \( M_{\text{GUT}} \) & realistic flavor structures à la Froggatt-Nielsen

![Graph showing \( \alpha_i \) as a function of \( \log_{10}(\mu/\text{GeV}) \)]
O(100) models with:

1. $3 \times 16 + \text{Higgs + nothing}$
2. $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)_Y \times G_{\text{hid}}$
3. unification
4. $y_t \simeq g @ M_{\text{GUT}}$ & realistic flavor structures à la Froggatt-Nielsen
5. hidden sector gaugino condensation

→ Spontaneously broken SUSY with TeV scale soft masses
Introduction
Local grand unification in heterotic orbifolds
MSSM from the heterotic string
Summary & outlook

Summary
‘Orbifold landscape’

Outlook

‘Orbifold landscape’

orbifold models

$\mathbb{Z}_3 \times \mathbb{Z}_2$

16 other geometries
orbifold models

$\mathbb{Z}_3 \times \mathbb{Z}_2$

16 other geometries

$V_6, W_2, W_3$ as discussed

$O(10^{7 \pm 3})$ other gauge embeddings

orbifold landscape
Introduction
Local grand unification in heterotic orbifolds
MSSM from the heterotic string

Summary & outlook

Summary
‘Orbifold landscape’

Outlook

‘Orbifold landscape’

orbifold models

$\mathbb{Z}_3 \times \mathbb{Z}_2$

16 other geometries

$O(10^{7+3})$ other gauge embeddings

$V_6, W_2, W_3$ as discussed

SUSY vacuum with $B-L$ high energies

many other ‘vacua’
we’re studying the $O(100)$ MSSM\(_{(-\text{like})}\) models.

- Discrete symmetries \(\sim\) talk by S. Raby
- SUSY breakdown \(\sim\) talk by H.P. Nilles
- Neutrino masses. . . yes, we get see-saw

Further questions:
- Relation to the bundle constructions?
  - Braun, He, Ovrut, Pantev (2005); Bouchard, Donagi (2005);
  - Bouchard, Cveti\v{c}, Donagi (2006); Blumenhagen, Moster, Weigand (2006)
  \(\sim\) talks by R. Blumenhagen, M. Cveti\v{c}, B. Ovrut
- Relation to free fermionic models?
  \(\sim\) talk by A. Faraggi
- . . .
‘Appendix’
Gauge group topography & orbifold GUT limits
Hidden sector gaugino condensation
Flavor issues and proton decay
Pillow construction

Gauge group topography

$G_2$ $SU(3)$ $SO(4)$

$n_3 = 0$ $n_2 = 0$ $n_2 = 1$

$SO(10) \times SO(4)$ $SO(8) \times SO(6)$
Gauged group topography & orbifold GUT limits

Hidden sector gaugino condensation

Flavor issues and proton decay

Pillow construction

Gauge group topography

$G_2 \supset SU(3) \times SO(4)$

$n_3 = 1$

$W_3 \supset SO(12) \times SO(8) \times SO(6)$

$W_2$
Gauge group topography & orbifold GUT limits
Hidden sector gaugino condensation
Flavor issues and proton decay
Pillow construction

Gauge group topography

$G_2 \quad SU(3) \quad SO(4)$

$n_3 = 2$

$W_3 \quad W_2$

$SU(7) \quad SO(8)'' \times SO(6)''$
Motivation: anisotropic compactification may allow to understand why $M_{\text{GUT}} < M_{\text{string}}$

$SU(5) \subset SO(10)$     $SU(4) \times SU(2)_L$

$10 + 5 + 1 = 16$

$SU(6)$

$SU(5) \subset SO(10)$     $SU(4) \times SU(2)_L$

$10 + 5 + 1 = 16$

Hebecker, Trapletti (2005)

Witten (1996)
Orbifold GUT limit: $SU(3)$ plane `large`

\[
SU(4)''' \times SU(3)' \quad \quad \quad G_{PS} \subset SO(10) \quad \quad \quad SU(4)' \times SU(4)''
\]
Orbifold GUT limit: $G_2$ plane `large’

\[ SU(3)_C \times SU(3)'' \]

\[ SU(6) \times SU(2)' \times SU(2)'' \]

\[ G_{SM} \]

\[ SU(5)' \times SU(2)''' \]
Hidden sector stronger coupled

\[ g_{\text{obs/hid}}^{-2} = \text{Re} S \pm \varepsilon \text{Re} T + \cdots =: \text{Re} S \pm \Delta \]

\[ \log_{10} \left( \frac{\Lambda_4}{\text{GeV}} \right) \]

Ibáñez, Nilles 1986
Dixon, Kaplunovsky, Louis 1991
Mayr, Stieberger 1993
Hidden sector gaugino condensation

Hidden sector **stronger** coupled

\[ g_{\text{obs/hid}}^{-2} = \text{Re} S \pm \varepsilon \text{Re} T + \cdots =: \text{Re} S \pm \Delta \]

- **dilaton**
- **Kähler modulus**

Example:

- **Kähler stabilization:**
  - large coefficients in the relation

\[ m_{3/2} \approx \frac{\Lambda^3}{M_P^2} \]

**bottom-line:** \( m_{3/2} \) is fine

---

Ibáñez, Nilles 1986
Dixon, Kaplunovsky, Louis 1991
Mayr, Stieberger 1993

...
**$\mathbb{Z}_3$ and $\mathbb{Z}_2$ subtwists**

$\mathbb{Z}_3$ subtwist: $v_3 = 2v_6 = \frac{1}{3}(1, 2, -3)$

$\bigtriangleup = \mathbb{Z}_3$ fixed point

$\mathbb{Z}_2$ subtwist: $v_2 = 3v_6 = \frac{1}{2}(1, 2, -3)$

$\blacksquare = \mathbb{Z}_2$ fixed point
Decoupling of the extra states for ‘generic’ SM singlet vevs

The $\bar{d}_a d_b$ mass matrix (@ order 8)

- rank of the mass matrix is maximal
- four combinations $\bar{d}_a d_b$ disappear from the low-energy spectrum

<table>
<thead>
<tr>
<th></th>
<th>$\bar{d}_1$</th>
<th>$\bar{d}_2$</th>
<th>$\bar{d}_3$</th>
<th>$\bar{d}_4$</th>
<th>$\bar{d}_5$</th>
<th>$\bar{d}_6$</th>
<th>$\bar{d}_7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_1$</td>
<td>$s^5$</td>
<td>$s^5$</td>
<td>$s^5$</td>
<td>$s^5$</td>
<td>$s^5$</td>
<td>$s^3$</td>
<td>$s^3$</td>
</tr>
<tr>
<td>$d_2$</td>
<td>$s^1$</td>
<td>$s^1$</td>
<td>$s^3$</td>
<td>$s^3$</td>
<td>$s^3$</td>
<td>$s^3$</td>
<td>$s^3$</td>
</tr>
<tr>
<td>$d_3$</td>
<td>$s^1$</td>
<td>$s^1$</td>
<td>$s^3$</td>
<td>$s^3$</td>
<td>$s^3$</td>
<td>$s^3$</td>
<td>$s^3$</td>
</tr>
<tr>
<td>$d_4$</td>
<td>$s^6$</td>
<td>$s^6$</td>
<td>$s^6$</td>
<td>$s^3$</td>
<td>$s^3$</td>
<td>$s^6$</td>
<td>$s^6$</td>
</tr>
</tbody>
</table>

(An entry $s^n$ means that there is an allowed coupling $\bar{d}_a d_b s_{i_1}^0 \cdots s_{i_n}^0$)

- note: high powers of $s$ do not necessarily mean strong suppression

---

e.g. Cvetič, Everett, Wang (1998)
Decoupling of the extra states for ‘generic’ SM singlet vevs

The $\ell_a \ell_b$ mass matrix (at order 8)

- rank of the mass matrix is maximal
- How to get a rank 4 mass matrix? ... see later

<table>
<thead>
<tr>
<th></th>
<th>$\ell_1$</th>
<th>$\ell_2$</th>
<th>$\ell_3$</th>
<th>$\ell_4$</th>
<th>$\ell_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ell_1$</td>
<td>$s^3$</td>
<td>$s$</td>
<td>$s$</td>
<td>$s$</td>
<td>$s$</td>
</tr>
<tr>
<td>$\ell_2$</td>
<td>$s^4$</td>
<td>$s^2$</td>
<td>$s^2$</td>
<td>$s^2$</td>
<td>$s^6$</td>
</tr>
<tr>
<td>$\ell_3$</td>
<td>$s^4$</td>
<td>$s^2$</td>
<td>$s^2$</td>
<td>$s^2$</td>
<td>$s^6$</td>
</tr>
<tr>
<td>$\ell_4$</td>
<td>$s$</td>
<td>$s^5$</td>
<td>$s^5$</td>
<td>$s^5$</td>
<td>$s^3$</td>
</tr>
<tr>
<td>$\ell_5$</td>
<td>$s$</td>
<td>$s^5$</td>
<td>$s^5$</td>
<td>$s^5$</td>
<td>$s^3$</td>
</tr>
<tr>
<td>$\ell_6$</td>
<td>$s$</td>
<td>$s^3$</td>
<td>$s^3$</td>
<td>$s^6$</td>
<td>$s^6$</td>
</tr>
<tr>
<td>$\ell_7$</td>
<td>$s$</td>
<td>$s^3$</td>
<td>$s^3$</td>
<td>$s^3$</td>
<td>$s^3$</td>
</tr>
<tr>
<td>$\ell_8$</td>
<td>$s$</td>
<td>$s^3$</td>
<td>$s^3$</td>
<td>$s^3$</td>
<td>$s^3$</td>
</tr>
</tbody>
</table>
Decoupling of the extra states for ‘generic’ SM singlet vevs

The $m_a m_b$ mass matrix (@ order 8)

- rank of the mass matrix is maximal
- recall: $m_i$ are $SU(2)_L$ doublets with hypercharge 0

<table>
<thead>
<tr>
<th></th>
<th>$m_1$</th>
<th>$m_2$</th>
<th>$m_3$</th>
<th>$m_4$</th>
<th>$m_5$</th>
<th>$m_6$</th>
<th>$m_7$</th>
<th>$m_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_1$</td>
<td>*</td>
<td>*</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>*</td>
<td>-</td>
<td>*</td>
</tr>
<tr>
<td>$m_2$</td>
<td>*</td>
<td>*</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>*</td>
<td>-</td>
<td>*</td>
</tr>
<tr>
<td>$m_3$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>*</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$m_4$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$m_5$</td>
<td>-</td>
<td>-</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>-</td>
<td>*</td>
<td>-</td>
</tr>
<tr>
<td>$m_6$</td>
<td>*</td>
<td>*</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$m_7$</td>
<td>-</td>
<td>-</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>-</td>
<td>*</td>
<td>-</td>
</tr>
<tr>
<td>$m_8$</td>
<td>*</td>
<td>*</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

* means ‘there is a coupling’
Decoupling of the extra states for ‘generic’ SM singlet vevs

The $s_a^+ s_b^-$ mass matrix (@ order 8)

- rank of the mass matrix is maximal
- recall: $s_i^\pm$ are SU(3) × SU(2)$_L$ singlets with hypercharge ±1/2

|      | $s_1^-$ | $s_2^-$ | $s_3^-$ | $s_4^-$ | $s_5^-$ | $s_6^-$ | $s_7^-$ | $s_8^-$ | $s_9^-$ | $s_{10}^-$ | $s_{11}^-$ | $s_{12}^-$ | $s_{13}^-$ | $s_{14}^-$ | $s_{15}^-$ | $s_{16}^-$ |
|------|---------|---------|---------|---------|---------|---------|---------|---------|---------|------------|------------|------------|------------|------------|------------|
| $s_1^+$ | *       | *       |         | *       |         |         |         |         |         | *          | *          | *          | *          | *          | *          |
| $s_2^+$ | *       | *       | *       |         | *       |         |         |         |         | *          | *          | *          | *          | *          | *          |
| $s_3^+$ | *       | *       | *       | *       |         | *       |         |         |         | *          | *          | *          | *          | *          | *          |
| $s_4^+$ | *       | *       | *       | *       | *       |         | *       |         |         | *          | *          | *          | *          | *          | *          |
| $s_5^+$ | *       | *       | *       | *       | *       | *       |         | *       |         | *          | *          | *          | *          | *          | *          |
| $s_6^+$ | *       | *       | *       | *       | *       | *       | *       |         | *       | *          | *          | *          | *          | *          | *          |
| $s_7^+$ | *       | *       | *       | *       | *       | *       | *       | *       |         | *          | *          | *          | *          | *          | *          |
| $s_8^+$ | *       | *       | *       | *       | *       | *       | *       | *       | *       | *          | *          | *          | *          | *          | *          |
| $s_9^+$ | *       | *       | *       | *       | *       | *       | *       | *       | *       | *          | *          | *          | *          | *          | *          |
| $s_{10}^+$ | *   | *       | *       | *       | *       | *       | *       | *       | *       | *          | *          | *          | *          | *          | *          |
| $s_{11}^+$ | *  | *       | *       | *       | *       | *       | *       | *       | *       | *          | *          | *          | *          | *          | *          |
| $s_{12}^+$ | * | *       | *       | *       | *       | *       | *       | *       | *       | *          | *          | *          | *          | *          | *          |
| $s_{13}^+$ | * | *       | *       | *       | *       | *       | *       | *       | *       | *          | *          | *          | *          | *          | *          |
| $s_{14}^+$ | * | *       | *       | *       | *       | *       | *       | *       | *       | *          | *          | *          | *          | *          | *          |
| $s_{15}^+$ | * | *       | *       | *       | *       | *       | *       | *       | *       | *          | *          | *          | *          | *          | *          |
| $s_{16}^+$ | * | *       | *       | *       | *       | *       | *       | *       | *       | *          | *          | *          | *          | *          | *          |
Flavor issues (using $d$-type quarks as an example)

Higher-order couplings giving rise to $d$-type Yukawa couplings

\[
W \supset \cdot \ell_4 \bar{d}_4 s^0_5 q_2 + \cdot \ell_5 \bar{d}_4 s^0_5 q_2 + \cdot \ell_4 \bar{d}_5 s^0_5 q_2 + \cdot \ell_5 \bar{d}_5 s^0_5 q_2 \\
+ \cdot \ell_4 \bar{d}_4 s^0_{12} q_3 + \cdot \ell_5 \bar{d}_4 s^0_{12} q_3 + \cdot \ell_4 \bar{d}_5 s^0_{12} q_3 + \cdot \ell_5 \bar{d}_5 s^0_{12} q_3
\]

omit coefficients

quark doublets from localized $16$

Promising: $h_d \sim \ell_1 \sim$ search for $\ell_1 \bar{d}_j q_k (s^0)^n$

appears at order 7, e.g.

\[
W \supset \ell_1 \bar{d}_1 q_2 s^0_{55} s^0_{56} s^0_7 s^0_5 + \ldots
\]

(15 terms at order 7)
Proton decay in orbifold GUTs

- Dimension 5 proton decay operators

\[
\begin{align*}
&d_L \quad H_3 \quad H_3 \\
&u_L \quad \tilde{q} \quad s_L \\
&\tilde{\ell} \quad \nu_L
\end{align*}
\]

\[
\begin{align*}
&d^C \quad H_5 \quad H_5 \\
&u^C \quad \tilde{u}^C \quad s_L \\
&\tilde{\ell}^C \quad \nu_L
\end{align*}
\]

- 4D GUT \( m_3 \ H_3 \ H_\frac{-3}{C0} \)

- 5D GUT: \( H_5 \oplus H_{\frac{-5}{C0}} \) →  dimensional reduction \( (H_5, H_{\frac{5}{C0}}^C) \oplus (H_{\frac{-5}{C0}}, H_{\frac{-5}{C0}}^C) \) in 4D

- 5D orbifold GUT: KK masses \( \frac{1}{R} \ H_3 \ H_{\frac{3}{C0}} \) and \( \frac{1}{R} \ H_{\frac{-3}{C0}} \ H_{\frac{-3}{C0}}^C \) in 4D

- Dimension 5 proton decay operator absent in orbifold GUT

- However: Constraints from dimension 6 operators!

see e.g. Hebecker, March-Russell ’02
Pillow construction (…using a $\mathbb{Z}_3$ orbifold as example)

Starting point is the torus

identify
Pillow construction (...using a $\mathbb{Z}_3$ orbifold as example)

The fundamental region of the orbifold is 1/3 of the fundamental region of the torus.
Pillow construction (...using a $\mathbb{Z}_3$ orbifold as example)

Rotating by $2\pi/3$ yields:
Pillow construction (...using a $\mathbb{Z}_3$ orbifold as example)

The rotated fundamental region covers a ‘new’ area
Pillow construction (...using a $\mathbb{Z}_3$ orbifold as example)

Further rotation yields:
Now the fundamental region covers the remaining area
The corners of the fundamental region are fixed under the $\mathbb{Z}_3$ rotation (on the torus)
Pillow construction (using a $\mathbb{Z}_3$ orbifold as example)

The edges are pairwise identified.
Pillow construction (...using a $\mathbb{Z}_3$ orbifold as example)

The geometry is that of a ‘pillow’
Gauge group topography & orbifold GUT limits
Hidden sector gaugino condensation
Flavor issues and proton decay
Pillow construction

$\mathbb{Z}_6$ pillow

Diagram with points labeled a, b, c, γ, δ, and lines connecting them.
$V_6 \text{ vs. } V'_6$

\[
V_6 = \frac{1}{6} (3, 3, 2, 0, 0, 0, 0, 0) (2, 0, 0, 0, 0, 0, 0, 0)
\]

\[
V'_6 = \frac{1}{6} (2, 2, 2, 0, 0, 0, 0, 0) (1, 1, 0, 0, 0, 0, 0, 0)
\]

<table>
<thead>
<tr>
<th>$G$</th>
<th>$V_6$</th>
<th>$V'_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SO(10) \times SO(4) \times U(1)$</td>
<td>$SO(10) \times SU(3) \times U(1)$</td>
<td></td>
</tr>
<tr>
<td>$U_1$:</td>
<td>$(16, 1, 2) \oplus \ldots$</td>
<td>$(16, \overline{3})$</td>
</tr>
<tr>
<td>$U_2$:</td>
<td>$(16, 2, 1) \oplus (10, 1, 1)$</td>
<td>$(10, 3) \oplus \ldots$</td>
</tr>
<tr>
<td>$U_3$:</td>
<td>$(10, 2, 2)$</td>
<td>$(16, 1) \oplus (\overline{16}, 1)$</td>
</tr>
</tbody>
</table>

\[\text{cf. Katsuki, Kawamura, Kobayashi, Ohtsubo, Ono, Tanioka (1989)}\]

\[\text{\textbf{Observation}}\]

For $V_6$ one can get Higgs pairs from $U_3$ (but not for $V'_6$)

\[\text{\textbf{Note}}\]

$U_3$ states always vector-like