F- and D-terms from D7-branes

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in collaboration with

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Type IIB Orientifolds:

We consider type IIB orientifold compactifications with background spaces \( \mathcal{M}_{10} = R^{3,1} \otimes Y/O \),

\( Y \) : compact CY,

\( O \) : O3/O7 orientifold projection.

They offer several phenomenologically attractive features:

- Semi-realistic models for particle physics: MSSM from open strings on D7-branes.
- Moduli stabilization from 3-form G-fluxes and non-perturbative effects on D7-branes.
  - KKLT: possibility of dS-vacua with positive cosmological constant!
- (Possibly, cosmological models with D3/D7-brane inflation.)
(Intersecting) D-brane model building

(Bachas (1995); Blumenhagen, Görlich, Körs, Lüst (2000);
Angelantonj, Antoniadis, Dudas Sagnotti (2000); Ibanez,
Marchesano, Rabadas (2001); Cvetic, Shiu, Uranga (2001); ...)

Local D-brane module of the Standard Model by 4 stacks of intersecting D-branes:

D-brane compactifications: Wrapping the D-brane modules around cycles of the compact background space:

Standard Model module

G=SU(3)\times SU(2) \times U(1)

Soft SUSY breaking

Hidden sector module

Moduli stabilization

String Phenomenology 2006
Moduli Stabilization - KKLT

Step 1: F-terms: Fix all moduli (preserving SUSY) Dilaton $S$ and complex structure moduli $U$ are stabilized with 3-form fluxes, Kähler moduli $T$ are fixed by non-perturbative effects on D3/D7-branes $\rightarrow$ SUSY AdS vacuum.

$$W_{np} = \gamma(z)e^{-\alpha T}$$

Step 2: Lift the minimum of the potential to a metastable dS vacuum

(i) introducing $\overline{D3}$ branes

$$V_{D3} \sim \frac{\mu_3}{[\text{Re}(T)]^2}$$

(ii) D7-branes with F-flux $\rightarrow$ D-term potential:

$$V_D \sim \frac{q^2}{[\text{Re}(T)]^2}$$

(Kachru Kallosh, Linde, Trivedi (2002))

(Burgess, Kallosh, Quevedo (2003))
The talk will address two questions:

- **Can all moduli be fixed in AdS vacuum by F-terms from D3/D7 branes in blown-up orbifolds models?**
  
  (Lüst, Reffert, Scheidegger, Schulgin, Stieberger)
  

- **Can D-terms be obtained from D7-branes, i.e is the D-term uplift possible using D7-branes, or is there are clash between F- and D-terms?**
  
  (Haack, Krefl, Lüst, Van Proeyen, Zagermann)
  
F-terms:

Non-perturbative superpotential:

- Euclidean D3-brane wrapped on 4-cycle \( C^j_4 \subset Y \) with volume \( T^j \):
  \[ W \sim \gamma_j(z)e^{-2\pi T^j} \]

Condition for non-vanishing superpotential:

\[
\frac{N_F}{2} = \chi(C_4) = h^{0,0}(C_4) - h^{1,0}(C_4) + h^{2,0}(C_4) = 1
\]

(Arithmetic genus \( \chi \) also depends on O7-plane and 3-form flux!)

- Gaugino condensation in gauge theory on D7-branes wrapped on 4-cycle \( C^j_4 \subset Y \):
  \[ W \sim \gamma_j(z)e^{-T^j_j/b_j} \]

Condition for non-vanishing superpotential: no massless adjoint chiral multiplets (open string moduli of D7!)

(Witten; Tripathy, Trivedi; Kallosh, Kashani-Poor, Tomasiello; Saulina; E. Bergshoeff, R. Kallosh, A. K. Kashani-Poor, D. Sorokin, A. Tomasiello; J. Park)

(Gomis, Mateos, Marchesano)
Prefactor in gaugino condensate superpotential:

- Gauge kinetic function for D7-brane wrapped on divisor $C_4^j$:

$$f_j = T^j + \Delta(S, U) \implies \gamma_j \sim e^{\Delta(S, U)} \sim \eta(U)^2$$

  - tree level
  - loop corrections (Lüst, Stieberger)

- additional instantons in the D7-gauge theory:

$$\gamma_j \sim e^{-S} \int_{C_4} F \wedge F$$

- Charged matter fields: see next section.
F-terms $\Rightarrow$ AdS vacua

All together:

Total effective $\mathcal{N}=1$ superpotential:

$$ W = W_{\text{flux}}(S, U) + W_{\text{np}}(T) = \kappa_{10}^{-2} \int G_3 \wedge \Omega + \sum_{j=1}^{h_{(1,1)}^+(X_6)} \gamma_j(S, U) e^{a_j} T^j $$

Kähler potential:

$$ K = -\ln(S - \overline{S}) - \ln \int \Omega \wedge \overline{\Omega} + 2 \ln V $$

F-term scalar potential:

$$ V = e^{\kappa_4^2 K} \left( |D_S W|^2 + \sum_i |D_{Ti} W|^2 + \sum_j |D_{Uj} W|^2 - 3|W|^2 \right) $$

Supersymmetric vacua $\Rightarrow$ impose F-term SUSY-conditions:

$$ D_i W = \partial_i W + \kappa_4^2 W \partial_i K = 0, \quad i = S, U^i, T^i $$

All known SUSY vacua with all moduli fixed are AdS.
Fixing all moduli in resolved orbifolds

We consider IIB orientifolds with D3/D7-branes, compactified on resolved orbifolds $\mathcal{M}_6 = T^6/Z_N, T^6/(Z_N \times Z_M)$ (smooth Calabi-Yau!) using the methods of toric geometry:

- Describe the local patches near the singularities with toric methods. Resolve the singularities via blow-ups.
- Glue the patches to get a smooth Calabi-Yau
- Perform an orientifold projection on the smooth CY $\Rightarrow$ O-planes and D-branes
- Determine divisor topologies to decide whether a n.p.-superpotential is generated
- Determine the CY triple intersection no. $\Rightarrow$ Kähler potential:
- Fix the dilaton and the complex structure moduli with 3-form flux
- Look for critical points of the scalar potential
- Stable dS-uplift: $(\text{mass}_{\text{moduli}})^2 > 0$
Results for F-term moduli stabilization to AdS on blown-up orbifolds:

- Not in every model all the divisors have the correct topology to contribute the n.p. superpotential.

- “No-go Theorem” for models with no complex structure moduli: a stable uplift to a dS vacuum is not possible!

- In models with odd cohomology under the orientifold projection, \((h_{(1,1)}^{(-)} \neq 0)\), the F-terms cannot fix all moduli. However these can be stabilized by additional D-terms.

- Conclusion: Moduli stabilization is model dependent! However in particular examples like \(\mathbb{Z}_4, \mathbb{Z}_6, \mathbb{Z}_2 \times \mathbb{Z}_2, \mathbb{Z}_2 \times \mathbb{Z}_4\), all moduli can be fixed!
D-terms:

A D-term potential arises from the existence of a gauged shift symmetry:

(Anomalous) $U(1)_f$ gauge transformation:

$$ V_f \rightarrow V_f + i (\Lambda - \bar{\Lambda}). \quad (V_f : U(1)_f \text{ vector superfield}) $$

$V_f$ mixes in the Kähler potential with a chiral superfield $T_c$:

$$ K = - \log (T_c + \bar{T}_c + \delta_{GS} V_f). $$

Hence to keep $K$ invariant, $T_c$ has to shift under $U(1)_f$ as:

$$ T_c \rightarrow T_c + i \delta_{GS} \Lambda, \quad a_c \rightarrow a_c + \delta_{GS} \Lambda, \quad (a_c = Im(T_c)) $$

Hence $a_c$ appears with a gauge covariant derivative:

$$ \mathcal{L} \sim (\partial_\mu a^c + q A^{(f)}_\mu)^2, \quad q = \delta_{GS} $$
D-terms:

The \((a_c \leftrightarrow U(1)_f)\) mixing leads to the following FI D-term potential:

\[
V_D \sim \frac{1}{g_f^2} \xi_{FI}^2, \quad \xi_{FI} = g_f^2 \left( \frac{\partial K}{\partial V_f} \right)_{V_f=0}.
\]

Hence:

\[
V_D \sim \delta_{GS}^2 \frac{g_f^2}{(ReT_c)^2}.
\]

\(V_D\) arises from the standard expression:

\[
V_D \sim D^2, \quad D \sim \eta^{T_c} \partial_{T_c} K
\]

with constant Killing vector \(\eta^{T_c} = i\delta_{GS}\) \(\leftrightarrow\) gauged shift symmetry.
There can be two non $U(1)_f$ invariant terms in the action:

(i) If there is a gauge group $G_c = U(1)_c \times SU(N_c)$
    
    with gauge coupling $1/g_c^2 \sim ReT_c$

    then the term $\text{Im}(T_c) \text{tr}[F_c \wedge F_c]$ is not $U(1)_f$ invariant.

(ii) If the gauge group $SU(N_c)$ induces a gaugino condensate

    then the n.p. superpotential $W \sim e^{-T_c/b_c}$ is not $U(1)_f$ invariant.
D-terms:
Both non-invariances can be cancelled by the same mechanism if $\mathcal{N}_f$ matterfields $\Phi_I$ are present:

$U(1)_f$ charge: $Q_f (= 1)$; $U(1)_c$ charge: $Q_c$

$\mathcal{N}_c \oplus \bar{\mathcal{N}}_c$ representation of $SU(N_c)$

(i) Mixed triangle graph: $U(1)_f$ is anomalous

$$A_{(i)} \sim tr\left((Q_c^2 Q_f)\right) = tr(Q_f)$$

GS-mechanism: Together with the $T_c/U(1)_f$ FI mixing term, the $U(1)_f$ non-invariance (i) is cancelled.
D-terms:

(ii) The matter fields contribute to the effective superpotential:

\[ W_{n.p.} \sim \gamma_{1-loop}(U)(\det M) \frac{1}{N_c-N_f} e^{-\frac{8\pi^2 T_c}{N_c-N_f}}, \]

(Assume \( N_f < N_c \)) \quad \det M \equiv \det(\Phi \bar{\Phi})

Now the n.p. superpotential from gauge condensation is also \( U(1)_f \) invariant, if:

\[ \delta_{GS} = \frac{1}{8\pi^2} N_f \quad (\Phi \rightarrow e^{i\Lambda} \Phi). \]

D-term potential:

\[ V_D \sim \frac{1}{\text{Re}(T^f)} \left( \frac{\delta_{GS}}{\text{Re}(T^c)} + \sum |\Phi_i|^2 \right)^2 \]
Now we want to derive the D-term potential microscopically in type II orientifolds with D7-branes with F-fluxes:

**Set-up:** Consider two kinds of (stacks of) D7-branes:

(i) \( (n_f) D7_f \) :
- carry world volume 2-form flux \( F \)
- are wrapped around a divisor \( \Sigma \)
- gauge group: \( G_f = U(1)_f (\times SU(n_f)) \)

Intersect each other

Bi-fundamental matter fields:
\[
\Phi_I : Q_{\alpha \Sigma} \times (n_f, N_c \oplus \bar{N}_c) Q_f = 1
\]

Intersection no.

(ii) \( N^\alpha_c D7_c \) :
- are wrapped around divisors \( C_\alpha, C_\beta, \ldots \)
- gauge group: \( G_c = G_\alpha \times G_\beta \times \ldots \)
\[
G_\alpha = U(1)_c \times SU(N^\alpha_c)
\]
DBI and WZ actions:

D-terms correspond to unbalanced tensions (NS-tadpoles) of the D7-branes.

Consider the DBI actions for $D7_f$ with F-flux:

$$S_{DBI} = -\mu_7 \int_{\mathcal{W}} d^8\xi e^{-\phi} \sqrt{-\det(\iota^*g + \mathcal{F})}$$

$$\mathcal{W} = \mathcal{M}_4 \times \Sigma, \quad \mu_7 = \frac{1}{\sqrt{2}} (2\pi)^{-7} \alpha'^{-4}, \quad \mathcal{F} = (\iota^*B +) 2\pi \alpha' F.$$ 

WZ-action:

$$S_{WZ} = -\mu_7 \int_{\mathcal{W}} \sum_p \iota^*C_p \wedge e^{\mathcal{F}}$$

BPS calibration conditions on $\Sigma$:

$$\frac{1}{2} (\iota^*J + i\mathcal{F}) \wedge (\iota^*J + i\mathcal{F}) = e^{i\theta} \sqrt{\frac{\det(g_\Sigma + \mathcal{F})}{\det(g_\Sigma)}} \text{Vol}_\Sigma, \quad \mathcal{F}^{2,0} = \mathcal{F}^{0,2} = 0.$$
Scalar potential and gauge coupling:

Using the space-time ansatz \( \mathcal{M}_{10} = \mathcal{M}_4 \times Y \)
the DBI action can be rewritten as:

\[
S_{DBI} = -\mu_7 \int_{\mathcal{M}_4} d^4x e^{-\phi} \sqrt{-\det(g_{(4)})} \sqrt{\det \left( 1 + 2\pi\alpha' g^{-1}_{(4)}F_{(4)} \right)} \Gamma_{\Sigma}
\]

with \( \Gamma_{\Sigma} = \int_{\Sigma} d^4z \sqrt{\det(g_{\Sigma} + F)} \)

A low energy expansion provides:

- the 4D scalar potential
  \[
  V_{D7_f} = \mu_7 e^{3\phi} \mathcal{V}^{-2} \Gamma_{\Sigma}
  \]
- the 4D gauge coupling
  \[
  g_{D7_f}^{-2} = \mu_7 (2\pi\alpha')^2 e^{-\phi} \Gamma_{\Sigma}
  \]
Some elements of CY geometry and topology:

- **Kähler form** $J$ for ambient Calabi-Yau $Y$:

  \[
  J = v^\alpha \omega_\alpha \quad (\alpha = 1, \cdots, H^{(1,1)}(\Sigma))
  \]

  $v^\alpha$: geometrical 4-cycle volumes

- **CY volume and triple intersection numbers**:

  \[
  V = \frac{1}{6} (2\pi \sqrt{\alpha'})^{-6} \int_Y J \wedge J \wedge J = \frac{1}{6} K_{\alpha\beta\gamma} v^\alpha v^\beta v^\gamma
  \]

  $K_{\alpha\beta\gamma}$: triple intersection #’s

- **Cohomology of divisor** $\Sigma \subset Y$:

  2 type of $(1,1)$-forms: $i^* \omega_\alpha$: pullbacks of $(1,1)$-forms from $Y$

  $\tilde{\omega}_a$: harmonic only locally on $\Sigma$, lie in the cokernel of $i^*$

  Hence:

  \[
  H^{(1,1)}(\Sigma) = i^* H^{(1,1)}(Y) \oplus \tilde{H}^{(1,1)}(\Sigma)
  \]

  $\implies F = f^\alpha i^* \omega_\alpha + \tilde{f}^a \tilde{\omega}_a$
Relate D-term potential to CY geometry:

Use calibration condition:

\[ \Gamma_\Sigma = \tilde{\Gamma}_\Sigma e^{-i\theta}, \quad \tilde{\Gamma}_\Sigma \equiv \frac{1}{2} \int_\Sigma (i^* J \wedge i^* J - F \wedge F) + i \int_\Sigma (i^* J \wedge F) \]

\[ \text{Re}\tilde{\Gamma}_\Sigma = \frac{1}{2} \int_\Sigma (i^* J \wedge i^* J - F \wedge F) = \left( \frac{1}{2} v^K_{\alpha^\beta} K_{\alpha^\beta\Sigma} - f_\Sigma \right) (2\pi \sqrt{\alpha'})^4, \]

\[ \text{Im}\tilde{\Gamma}_\Sigma = \int_\Sigma i^* J \wedge F = v^K_{\alpha} Q_{\alpha\Sigma} (2\pi \sqrt{\alpha'})^4 \]

with

\[ Q_{\alpha\Sigma} = (2\pi \sqrt{\alpha'})^{-4} \int_\Sigma i^* \omega_{\alpha} \wedge F = f^K_{\beta} K_{\alpha^\beta\Sigma}, \]

\[ \Rightarrow N_f = (n_f) Q_{\alpha\Sigma} \]

\[ f_\Sigma = \frac{1}{2} \left( f^K_{\alpha} f^K_{\beta} K_{\alpha^\beta\Sigma} + \tilde{f}^a f^K_{ab} K^{(\Sigma)}_{ab} \right), \]

\[ K^{(\Sigma)}_{ab} = (2\pi \sqrt{\alpha'})^{-4} \int_\Sigma \tilde{\omega}_a \wedge \tilde{\omega}_b. \]
Relate D-term potential to CY geometry:

**SUSY condition:**
\[ \theta = 0 \iff \text{Im} \tilde{\Gamma}_\Sigma \int_\Sigma \iota^* J \wedge F = 0. \]

**Small SUSY breaking:**
\[ \theta \sim \text{Im} \tilde{\Gamma}_\Sigma \neq 0, \quad |\text{Im} \tilde{\Gamma}_\Sigma| \ll |\text{Re} \tilde{\Gamma}_\Sigma| \]

\[
V_{D7_f} = \mu_7 e^{3\phi} \mathcal{V}^{-2} \text{Re} \tilde{\Gamma}_\Sigma \sqrt{1 + \left( \frac{\text{Im} \tilde{\Gamma}_\Sigma}{\text{Re} \tilde{\Gamma}_\Sigma} \right)^2} \\
\approx \mu_7 e^{3\phi} \mathcal{V}^{-2} \text{Re} \tilde{\Gamma}_\Sigma + \frac{1}{2} \mu_7 e^{3\phi} \mathcal{V}^{-2} \frac{1}{\text{Re} \tilde{\Gamma}_\Sigma} (\text{Im} \tilde{\Gamma}_\Sigma)^2 .
\]

The first term vanished due to the RR-tadpole condition.

\[ \implies V_{D7_f} \equiv V_D = \frac{1}{2} \mu_7 e^{3\phi} \mathcal{V}^{-2} \frac{1}{\text{Re} \tilde{\Gamma}_\Sigma} (\text{Im} \tilde{\Gamma}_\Sigma)^2 . \]

Likewise for the gauge coupling constant:

\[
g_{\Sigma}^{-2} = \mu_7 (2\pi \alpha')^2 e^{-\phi} \text{Re} \tilde{\Gamma}_\Sigma = \mu_7 (2\pi)^6 \alpha'^4 \left( e^{-\phi} \frac{1}{2} v^\alpha v^\beta K_{\alpha\beta\Sigma} - e^{-\phi} f_\Sigma \right)
\]

String Phenomenology 2006
D-term potential in supergravity variables:

The $v^\alpha$ are the 4-cycle volumes in the geometrical string basis!

Define the following Kähler moduli in the SUGRA basis:

$$T^\alpha = \hat{V}^\alpha + ia^\alpha, \quad S = e^{-\phi} - iC_0.$$  

with $$\hat{V}^\alpha = e^{-\phi} \frac{1}{2} v^\beta v^\gamma K_{\beta\gamma\alpha}.$$  

Gauge coupling:  

$$g_\Sigma^{-2} = \mu_7 (2\pi)^6 \alpha'^4 \left( \text{Re} T^\Sigma - f_\Sigma \text{Re} S \right).$$

D-term potential:

$$V_D = \frac{1}{\text{Re}(Tf)} \left( \mu_7 (2\pi)^5 \alpha'^3 (\partial T^\alpha K) n_f Q_{\alpha\Sigma} \right)^2 \Rightarrow \delta_{GS} \sim n_f Q_{\alpha\Sigma}$$

This is of the familiar form from supergravity with a gauged U(1) symmetry.
Summary and remarks:

- Wrapped D7-branes branes can produce coexisting F- and D-terms: **Anomalous** $U(1)_f$ gauge symmetry.

$$D7_c : \quad W_{n.p.} \sim \gamma_{1-loop}(U)(\det M)^{-\frac{1}{N_c-N_f}} e^{-\frac{8\pi^2 T_c}{N_c-N_f}}$$

$$D7_f : \quad V_D \sim \frac{1}{\text{Re}(T^f)} \left( \frac{\delta_{GS}}{\text{Re}(T^c)} + \sum |\Phi_i|^2 \right)^2$$

- $Q_{\alpha \Sigma} \neq 0$: Only axions on 4-cycles that intersect $\Sigma$ can be charged under the gauged $U(1)$.

  If $\Sigma$ has self-intersections, then $T^\Sigma$ will be charged.

- One can construct a $Z2 \times Z2$ orientifold with F-flux as toy model.

- The $\Phi_I$ must be taken into account when minimizing the potential! ($\langle \Phi_I \rangle \neq 0$?) (see also Achucarro et al.)
- Generalizations:

\[ G_c = SU(N_c) \quad \text{with} \quad N_f = N_c \]

\[ \Rightarrow \text{Orientifolds with (anti)-symmetric repr. of} \quad SU(N_c) \]

\[ \Rightarrow \text{Orientifolds with} \quad G_c = SO(N_c)/Sp(N_c) \]

Thank You!