Relativistic Heavy Ion Collisions and String Theory

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1. Introduction

The Relativistic Heavy Ion Collider (RHIC), operating at Brookhaven National Laboratory, collides gold on gold.

- Total center of mass energy is about $39 \text{ TeV}$.
- There is good evidence that a thermalized quark-gluon plasma (QGP) forms with temperature above the confinement scale, $T_C \approx 170 \text{ MeV} \approx 2 \times 10^{12} \text{ K}$
The theoretical understanding of RHIC physics is imperfect.

- The QGP is strongly coupled, so perturbative QCD is of limited utility.
- Lattice calculations provide good information about static properties, e.g., $T_C$ for confinement, but not transport properties (like viscosity).
- String theory, in particular AdS/CFT, offers an alternative description of strongly coupled gauge theory.

Two main themes of the AdS/CFT - RHIC connection are

A The viscosity bound $\eta/s \geq \hbar/4\pi$.

B Jet-quenching and the drag force on hard partons, especially heavy quarks.

A has been under discussion for about 5 years. B is a relatively new development, and the focus of our contribution.
2. **What happens at RHIC?**

RHIC accelerates beams of heavy nuclei (gold, copper, etc.) in opposite directions around a large circular ring and collides them.

Gold nuclei are nearly spherical with radius of about 7 fm in rest frame; Lorentz contraction reduces front-to-back length to $\sim 0.07$ fm.

**Figure 1:** Before-and-After shots of ultra-relativistic dynamics simulation of a gold-gold collision [1].
• The main ring is \(3.8\) km in circumference.
• Beam CM energy per nucleon per nucleon is \(\sqrt{s_{NN}} = 200\) GeV.
• RHIC’s design luminosity is \(2 \times 10^{26}\) cm\(^{-2}\)s\(^{-1}\). Integrated luminosity to date is in the ballpark of \(4\) nb\(^{-1}\).
Experimentalists claim that a thermalized QGP gets formed. Hadron yields follow nearly Boltzmann distributions (Kaneta 2004 [2]):

\[
\text{Ratio} = \frac{N_{\text{hadrons}}}{N_{\text{expected}}} \quad \text{for } 200 \text{ GeV Au+Au, } <N_{\text{part}}> = 322
\]

\[
\begin{align*}
T_{\text{ch}} &= 157 \pm 3 \quad \text{[MeV]} \\
\mu_q &= 9.4 \pm 1.2 \quad \text{[MeV]} \\
\mu_s &= 3.1 \pm 2.3 \quad \text{[MeV]} \\
\gamma_s &= 1.03 \pm 0.04 \\
\chi^2/\text{dof} &= 19.9 / 10
\end{align*}
\]

Figure 2: Yellow lines are thermal model predictions, icons represent experimental data. $T_{\text{ch}}$ is chemical freeze-out temperature, $\mu_q$ is up/down chemical potential, $\mu_s$ is strange chemical potential, and $\gamma_s$ is strangeness saturation.
Furthermore, theoretical predictions from lattice simulations find that deconfinement happens at $T_c \approx 170$ MeV, and that $\frac{\varepsilon}{T^4}$ has a plateau at 80% of the free field value (Karsch 2001 [3]):

![Graph](image)

**Figure 3:** Lattice results for the equation of state of QCD.
Jet-quenching refers to the rapid loss of energy of a hard parton propagating through the hot dense matter created in a gold-gold collision. The prima facie evidence for jet quenching is the the suppression of high $p_T$ jets (more precisely, high $p_T$ hadrons) relative to expectations from “binary collision scaling.”

- Jet production from proton-proton collisions is well studied, as is photon production.
- Binary scaling means to multiply yields in proton-proton by the ratio of incident parton flux of a gold-gold collision to the analogous flux for proton-proton.
- This scaling basically works for high-energy photons ($2 \text{ GeV}/c < p_T < 14 \text{ GeV}/c$) (Adler 2005[4]).
- It doesn’t work for high $p_T$ hadrons: at mid-rapidity,

$$R_{AA} \equiv \frac{dN(\text{gold-gold})/dp_Td\eta}{\langle N_{\text{binary}} \rangle dN(\text{proton-proton})/dp_Td\eta} \approx 0.2 \quad (1)$$

where $\langle N_{\text{binary}} \rangle$ is the number of nucleon-nucleon collisions in a “factorized” gold-gold collision.
Figure 4: Nuclear modification factor $R_{AA}$ for photons and hadrons in 0 to 10% central gold-gold collisions.
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Can AdS/CFT explain this deficit?
The entropy and viscosity calculations have drawn the attention of RHIC phenomenologists, as well as DOE higher-ups:

“The possibility of a connection between string theory and RHIC collisions is unexpected and exhilarating.” — Ray Orbach, DOE Office of Science Director [5]

But before we proceed, some cautionary statements are worth noting — $\mathcal{N} = 4$ gauge theory misses several essential features of QCD:

- No confinement. Coupling doesn’t run: it’s a parameter you can dial.

  But is this so bad? We want to use AdS/CFT at finite temperature to model the QGP above $T_c$. Phenomenological studies of RHIC physics routinely set $v_s = 1/\sqrt{3}$ and $\epsilon \sim 1/t^{4/3}$ (both corresponding to conformal invariance) for the QGP, e.g. when the energy density $\epsilon$ is significantly above 1 GeV.

- No chiral condensate.

  But is this so bad? The chiral condensate turns off around $T_c$ according to lattice calculations.

- All fundamental matter fields are in adjoint representation: $A_\mu$, four Majorana fermions $\lambda_i$, six real scalars $X_I$.

  This looks kind of bad. Maybe gauge interactions dominate the dynamics anyway?
3. Jet Quenching in AdS/CFT

Figure 5: In blue: the trailing string of an external quark (Herzog et al, 2006 [6]; Gubser 2006[7]). The dashed line shows classical propagation of a graviton from the string to the boundary, where its behavior can be translated into the stress-energy tensor $\langle T_{mn} \rangle$ of the boundary gauge theory.

An analog of jet-quenching in AdS/CFT should involve a colored probe that we drag through the QGP, preferably at relativistic speeds. Readiest at hand are external quarks: strings with one end on the boundary.
Our background metric is

$$ds^2 = \frac{L^2}{z_H^2 y^2} \left( -h dt^2 + d\vec{x}^2 + z_H^2 \frac{dy^2}{h} \right)$$

$$h \equiv 1 - y^4$$

$$z_H = \frac{1}{\pi T}$$  (2)

We’re interested in non-zero quark masses, so consider splitting one D3-brane from the large stack and putting it at a height $y_* \geq 0$, and treat it in the test-brane approximation. (Alternatively, wrap a D7 on an $S^3 \subset S^5$, and have it fill three extended directions plus the interval $0 \leq y \leq y_*$.) Then a string with one endpoint at $y_*$, going straight down into the horizon has a mass

$$m_{\text{static}} = \frac{L^2}{2\pi \alpha'} \left( \frac{1}{z_*} - \frac{1}{z_H} \right) = \sqrt{g_{YM}^2 N} \frac{T}{2} \left( \frac{z_H}{z_*} - 1 \right)$$  (3)
In the QGP, $u$, $d$, and $s$ quarks are dominated by thermal mass, whereas electroweak contribution still dominates for $c$ and $b$. We use

$$m_u = m_d = m_s = 300\text{MeV} \quad m_c = 1400\text{MeV} \quad m_b = 4800\text{MeV} \quad (4)$$

Figure 6: (A) A finite mass quark moving at velocity $v$ through the QGP can be represented as a string hanging from a “flavor brane” (Herzog et al, 2006). This picture is best justified for heavy quarks like $c$ and $b$. In this figure and below, we use the radial coordinate $y = z/z_H$. (B) At $T = 0$, flavor branes can be realized by separating one D3-brane from several others. The massive $W$ boson is similar to a heavy quark. We also show an $R \bar{B}$ gluon.
3.1. A drag force computation

We need to know the shape of the trailing string and the momentum flow down it. We assume a “co-moving” ansatz, and parameterize the worldsheet as:

\[(t, x^1, x^2, x^3, y) = (\tau, vt + \xi(y), 0, 0, \sigma)\]  \hspace{1cm} (5)

A “reduced” lagrangian follows from the Nambu-Goto action:

\[\mathcal{L} = -\frac{1}{y^2} \sqrt{1 + h\xi'^2} - \frac{v^2}{z_H} \]  \hspace{1cm} (6)

And the solution is

\[\xi' = -\frac{vz_Hy^2}{1 - y^4} \quad \xi = -\frac{vz_H}{4i} \left( \log \frac{1 - iy}{1 + iy} + i \log \frac{1 + y}{1 - y} \right) \]  \hspace{1cm} (7)

This is deceptively real, since the argument for the first \(\log\) is just a phase.
3.1 A drag force computation

Figure 7: The drag force is computed by measuring the momentum flux down the string. The position of $I$ is arbitrary because the energy-momentum current is conserved.

Momentum and energy drains down the string:

$$\Delta P_1 = - \int_I dt \sqrt{-g} P^y_{x^1} = \frac{dp_1}{dt} \Delta t. \quad (8)$$

d$p_1/dt$ is precisely the drag force:

$$F \equiv \frac{dp}{dt} = - \frac{\pi \sqrt{g_{YM}^2 N T^2}}{2} \frac{v}{\sqrt{1 - v^2}}. \quad (9)$$
The expression for the string shape $\xi$ holds for any mass (i.e., $y_*$) – just chop off the string above $y_*$. The drag force expression holds for heavy quarks, $m \gg T$, so that $y_*$ is near the boundary, and we can use standard relativistic expressions like $E = \sqrt{p^2 + m^2}$ and $p = mv/\sqrt{1 - v^2}$.

$$F = \frac{dp}{dt} = -\frac{\pi \sqrt{g_{YM}^2 NT^2}}{2} \frac{v}{\sqrt{1 - v^2}} \approx -\frac{\pi \sqrt{g_{YM}^2 NT^2}}{2m} p.$$  \hspace{1cm} (10)

Simply integrate this to find

$$p(t) = p_0 e^{-t/t_0}, \hspace{1cm} t_0 = \frac{2}{\pi \sqrt{g_{YM}^2 NT^2}} m.$$  \hspace{1cm} (11)

Plug in $T = 318$ MeV and $\lambda = 10$, we find that $t_0 = 0.6 \text{ fm/c}$ for charm, and $t_0 = 1.9 \text{ fm/c}$ for bottom, compared to $t_{QGP} \approx 6 \text{ fm/c}$ for the typical lifetime of the QGP. This temperature is also a significant overestimate, convenient so that $z_H = 1/\pi T = 1 \text{ GeV}^{-1}$. A more realistic temperature would reduce the quenching effect – QGP also cools substantially as it expands.
3.2. Graviton perturbations

A good measure of the energy loss is $\langle T_{mn} \rangle$ in the boundary gauge theory. Here we’ll attempt a concise description of the calculation.

$\langle T_{mn} \rangle$ is determined by the behavior near the boundary of linearized graviton perturbations of $AdS_5$-Schwarzschild:

$$ds^2_{(0)} = G^{(0)}_{\mu\nu} dx^\mu dx^\nu = \frac{L^2}{z_H^2 y^2} \left( -h dt^2 + d\vec{x}^2 + z_H^2 \frac{dy^2}{h} \right) \quad h \equiv 1 - y^4. \quad (12)$$

$$G_{\mu\nu} = G^{(0)}_{\mu\nu} + h_{\mu\nu}, \quad (13)$$

The Einstein equations are

$$R^{\mu\nu} - \frac{1}{2} G^{\mu\nu} R - \frac{6}{L^2} G^{\mu\nu} = \tau^{\mu\nu}, \quad (14)$$

where $\tau^{\mu\nu}$ is the stress-energy of the trailing string.
A priori, this leaves us with 15 equations for 15 perturbative modes. The stress tensor involves delta functions at the location of the string, so move to Fourier space. This gives a series of coupled ordinary differential equations in $y$ for the co-moving Fourier components $h^{\mu\nu}_K$. Then:

- Choose “axial gauge,” $h^{\mu y}_K = 0$. Now there are 10 independent quantities $h^{mn}_K$, where $0 \leq m, n \leq 3$.

- We are left with 10 second order equations of motions, $\mathcal{E}^{mn} = 0$, and 5 first order constraints, $\mathcal{E}^{\mu y} = 0$.

- The differential equations may be partially decoupled and simplified by making a series of field redefinitions. They are still complicated—see below.

- Since hep-th/0607022, we have generalized by allowing the trailing string to end on a flavor brane at $y = y_*$. This is accomplished simply by including a factor of $\theta(y - y_*)$ in $\tau_{\mu\nu}$.

- We take $\vec{K} = (K_1, K_\perp, 0) = K(\cos \theta, \sin \theta, 0)$. 
Here’s the full problem:

\[
\begin{align*}
    h^K_{\mu\nu} &= \frac{\kappa^2}{2\pi\alpha'} \frac{1}{\sqrt{1-v^2}} \frac{L}{z_H y^2} \left( 
        \begin{array}{cccc}
            H_{00} & H_{01} & H_{02} & H_{03} \\
            H_{10} & H_{11} & H_{12} & H_{13} \\
            H_{20} & H_{21} & H_{22} & H_{23} \\
            H_{30} & H_{31} & H_{32} & H_{33}
        \end{array}
    \right), \\
    K &= \sqrt{K_1^2 + K_2^2} \quad \theta = \tan^{-1} \frac{K_2}{K_1} \\
    A &= \frac{-H_{11} + 2\cot\theta H_{12} - \cot^2\theta H_{22} + \csc^2\theta H_{33}}{2v^2} \\
    B_1 &= \frac{H_{03}}{K^2 v} \quad B_2 = -\frac{H_{13} + \tan\theta H_{23}}{K^2 v^2} \\
    C &= -\sin\theta H_{13} + \cos\theta H_{23} \\
    D_1 &= \frac{H_{01} - \cot\theta H_{02}}{2v} \quad D_2 = -\frac{H_{11} + 2\cot\theta H_{12} + H_{22}}{2v^2}
\end{align*}
\]
3.2 Graviton perturbations

\[ D'_1 - hD'_2 = \frac{y^3}{ivK_1} e^{-iK_1 \xi/z} \vartheta(y - y_*) \]  

(26)

\[ E_1 = \frac{1}{2} \left( -\frac{3}{h} H_{00} + H_{11} + H_{22} + H_{33} \right) \quad E_2 = \frac{H_{01} + \tan \theta H_{02}}{2v} \quad E_3 = \frac{H_{11} + H_{22} + H_{33}}{2} \quad E_4 = \frac{-H_{11} - H_{22} + 3 \cos \theta (-H_{11} + H_{22}) + 2H_{33} - 6 \sin \theta H_{12}}{4} \]

(27)

\[
\begin{pmatrix}
\partial_y^2 + \left( \begin{array}{cccc}
-\frac{3}{y} + \frac{3h'}{2h} & 0 & 0 & 0 \\
0 & -\frac{3}{y} & 0 & 0 \\
0 & 0 & -\frac{3}{y} + \frac{h'}{h} & 0 \\
0 & 0 & 0 & -\frac{3}{y} + \frac{h'}{h} \\
\end{array} \right) \partial_y \\
+ K^2 \left( \begin{array}{cccc}
-2h & 12v^2 \cos^2 \theta & 6v^2 \cos^2 \theta + 2h & 0 \\
0 & 2h & 0 & h \\
0 & 0 & -2h & -h \\
2h & -12v^2 \cos^2 \theta & 0 & 3v^2 \cos^2 \theta + h \\
\end{array} \right) \end{pmatrix} \left( \begin{array}{c}
E_1 \\
E_2 \\
E_3 \\
E_4 \\
\end{array} \right)
\]

(28)

\[ = \frac{y}{h} e^{-iK_1 \xi/z} \vartheta(y - y_*) \left( \begin{array}{c}
1 + \frac{v^2}{h} \\
1 \\
-1 + v^2 - \frac{v^2}{h} \\
\frac{v^2}{h} + 3 \cos \theta \\
\end{array} \right) \]

\[
\begin{pmatrix}
0 & 1 & 1 & 0 \\
-h & -3v^2 \cos^2 \theta - h & 0 & 0 \\
h & 0 & 2 & 0 \\
\end{pmatrix} \partial_y \\
+ \frac{1}{6h} \left( \begin{array}{cccc}
0 & -6h' & 0 & 0 \\
-3hh' & 18v^2 \cos^2 \theta h' & 3(3v^2 \cos^2 \theta + h)h' & 0 \\
2K^2 yh & -12K^2 v^2 y \cos^2 \theta & -2K^2 y(3v^2 \cos^2 \theta - h) & 2K^2 yh \\
\end{array} \right) \left( \begin{array}{c}
E_1 \\
E_2 \\
E_3 \\
E_4 \\
\end{array} \right)
\]

(29)

\[ = \frac{h'}{4Ky} e^{-iK_1 \xi/z} \vartheta(y - y_*) \left( \begin{array}{c}
-v y \sec \theta \\
3iv \cos \theta (v^2 + h) \\
K(v^2 - h) \\
\end{array} \right). \]
To solve the 10 second order equations of motion for specified $K$, we must fix 20 integration constants.

- Think of 15 as being fixed at the boundary of $AdS_5$-Schwarzschild (that is, $y = 0$) and the remaining 5 at the horizon to suppress solutions describing gravitons coming out of the black hole. Each set of equations has exactly one non-vanishing oscillatory mode at the horizon whose frequency must have the correct sign.

- Of the 15 boundary conditions at $y = 0$, five come from imposing the first-order constraints. This is arbitrary: the constraints can be imposed anywhere.

- The 10 remaining boundary conditions come from requiring $H_{\mu\nu} \rightarrow 0$ as $y \rightarrow 0$, i.e., the metric in the boundary theory remains Minkowski.
In practice, to proceed we:

- Note that the $B$ and $C$ sets are odd under the $Z_2$ reflection in the $(x_1, x_2)$ plane spanned two comoving momenta, so these functions must be zero.

- For the seven functions $A$, $D$, and $E$, find solutions to the equations of motion that are asymptotically exact at the boundary and horizon. Near the boundary, each mode roughly takes the form $H \sim P + Qy^4$

- Pick some specific $\vec{K} = (K_1, K_\perp)$.

- Set $A$ for each mode to zero at the boundary ($n$ integration constants in each set with $n$ equations).

- Impose the constraint equations at the boundary, which relate the $Q'$s in each set – there are $n - 1$ first-order constraints per set. We’re left with one remaining integration constant $Q$ per set – call them $Q_A$, $Q_D$, $Q_E$.

- Adjust $Q_X$ until the one undesirable outgoing mode at the horizon goes away.
3.2 Graviton perturbations

Figure 8: Contour plots of $K_\perp |Q^K_E|$ for various values of $v$. $Q^K_E$ is proportional to the $K$-th Fourier component of the energy density after a near-field subtraction. The phase space factor $K_\perp$ arises in Fourier transforming back to position space. The green line shows the Mach angle. The red curve shows where $K_\perp |Q^K_E|$ is maximized for fixed $K = \sqrt{K_1^2 + K_\perp^2}$. The blue curves show where $K_\perp |Q^K_E|$ takes on half its maximum value for fixed $K$. For $T = 318 \text{ MeV}$, momenta axes are in units of GeV.
3.2 Graviton perturbations

Same plot as previous page in polar coordinates: $K_1 = K \cos \theta$, $K_\perp = K \sin \theta$
3.2 Graviton perturbations

Energy density for a charm quark in polar coordinates, without a near-field subtraction – qualitatively identical to infinite mass case with the near-field subtraction

Figure 9: $K_{\perp} Q_E(K, \theta)$ for $y_* = 0.26$, i.e., $m_c = 1400\text{MeV}$, $T = 318\text{MeV}$, and $\lambda = 10$
3.3. The wake of a quark

A much-discussed aspect of RHIC’s current experimental program hinges on the following picture:

Figure 10: Left: A di-jet event with significant away-side jet quenching. (Jacak [8]). Right: The away-side parton may generate a sonic boom, with $\theta_M = \cos^{-1}(c_s/v)$ the Mach angle. From (Casalderrey-Solana et al, 2004 [9]).

- Two hard partons collide near the surface of the QGP.
- One escapes and fragments into the “near side” jet.
- The other plows through the QGP and dissipates a lot of energy.
3.3 The wake of a quark

Figure 11: Numerical data for stress tensor at low $K$ for various velocities. Speed of sound is $c_s = 1/\sqrt{3} \approx 0.577$. Green lines indicate the Mach angle, as usual. Red lines are peak of distributions, and blue lines are locations of the half-maximum.
The sonic boom can also be immediately identified in the graviton calculation via a small $K$ expansion for the stress tensor, which can be found by matching boundary and horizon asymptotic solutions that are $K$-exact to small $K$ solutions for all $y$.

\[
\langle T^K_{00} \rangle \propto \frac{3iv(1 + v^2)\cos \theta}{2K (1 - 3v^2 \cos^2 \theta)} - \frac{3v^2 \cos^2 \theta [2 + v^2 (1 - 3 \cos^2 \theta)]}{2 (1 - 3v^2 \cos^2 \theta)^2} + O(K)
\]

\[
= \frac{3iv(1 + v^2)\cos \theta}{2K} \left( \frac{1}{(1 - 3v^2 \cos^2 \theta) (1 - \frac{ivK \cos \theta}{1+v^2})} - ivK \cos \theta \right) + O(K).
\]

(30)

First expression is clearly singular at each order in $K$ at the angle given by $\cos \theta = \frac{1}{v\sqrt{3}}$, which is the Mach angle. Second expression is equal, up to $O(K)$ terms, but it is regular and sharply peaked at the Mach angle, and appears to be a better fit to the numerics.
The sonic boom picture and related theoretical proposals suggest that high-angle emission carries away a lot of the energy. And data seems to confirm this:

![Histograms](image)

**Figure 12:** Histograms of the azimuthal angle between the trigger hadron (with $2.5 \text{ GeV/c} < p_T < 4 \text{ GeV/c}$) and the partner hadron (with $1 \text{ GeV/c} < p_T < 2.5 \text{ GeV/c}$). Away-side jet splitting, illustrated by the broad peak around $\Delta \phi = 2$, is evidence for high-angle emission in the QGP. (Adler 2005 [10]).
The numerical data from AdS/CFT agrees (at least) qualitatively with the RHIC data:

![Graphs showing data for different values of K: a) K_p |Q_E(K_1,K_p)| for K = 0.08, b) K_p |Q_E(K_1,K_p)| for K = 0.8, c) K_p |Q_E(K_1,K_p)| for K = 1.4, d) K_p |Q_E(K_1,K_p)| for K = 2.]

Figure 13: $K_\perp|Q_E^K|$ at fixed $K$ as a function of angle, for $v = 0.95$. $\Delta \phi = \pi - \theta$ where $\theta = \tan^{-1} K_\perp/K_1$. The dashed lines are from an analytic estimate, and the solid lines are from numerics. The green line is the Mach angle; the red dot is the peak; and the blue dots are at half the peak height. Plots (c) and (d) are in the ballpark of the experimental study summarized in the previous figure. $K$ is the total momentum, in units of GeV if $T = 318$ MeV.
4. Conclusions

- A simple type IIB string configuration helps elucidate the physics of jet quenching at RHIC.

- Broadly directional peaks agree qualitatively with observed splitting of the away-side jet.

- The string theory setup involves significant idealizations of the experimental setup, notably replacing QCD by $\mathcal{N} = 4$ super-Yang-Mills.

- Nevertheless, we hope that further improvements may lead to more precise comparisons of string theory predictions with data.
References

[1] The full MPEG can be found at http://www.bnl.gov/RHIC/images/movies/Au-Au_200GeV.mpeg, where it is attributed to the UrQMD group at Frankfurt, see http://www.physik.uni-frankfurt.de/~urqmd/.


