New Heterotic GUT and Standard Model Vacua

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based on: R.B., S. Moster, T. Weigand (hep-th/0603015),
R.B., S. Moster, R. Reinbacher, T. Weigand (hep-th/0609nnn)

Motivation

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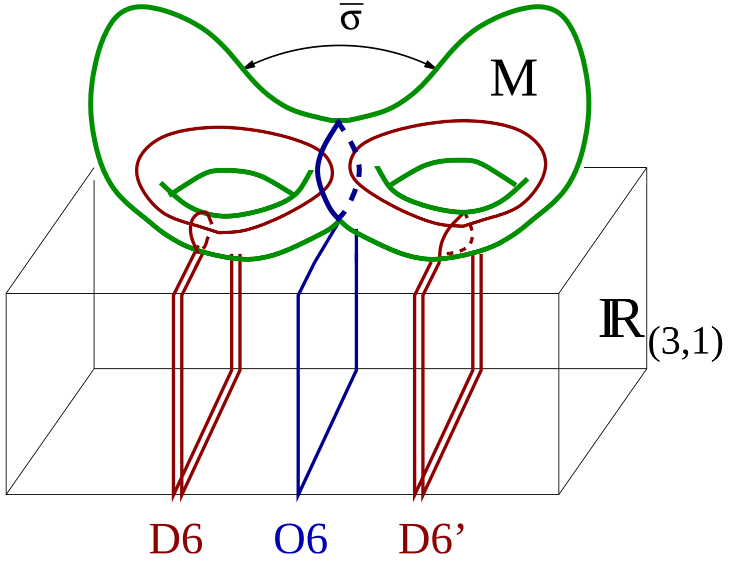

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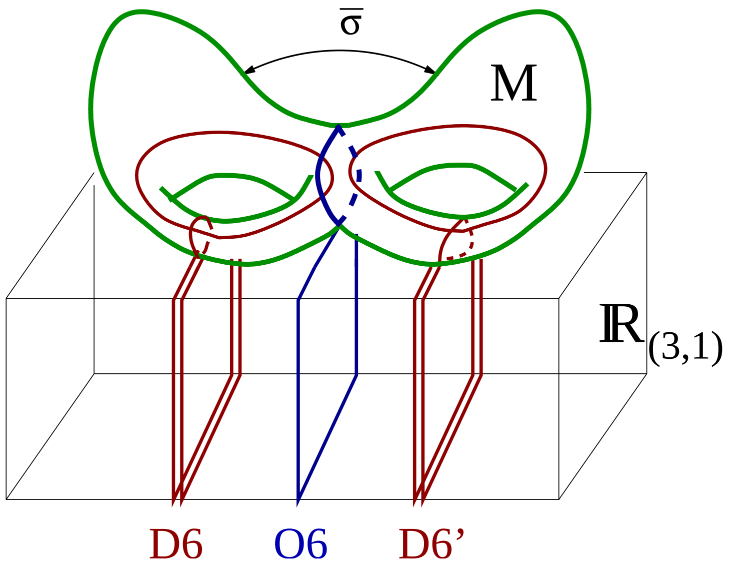

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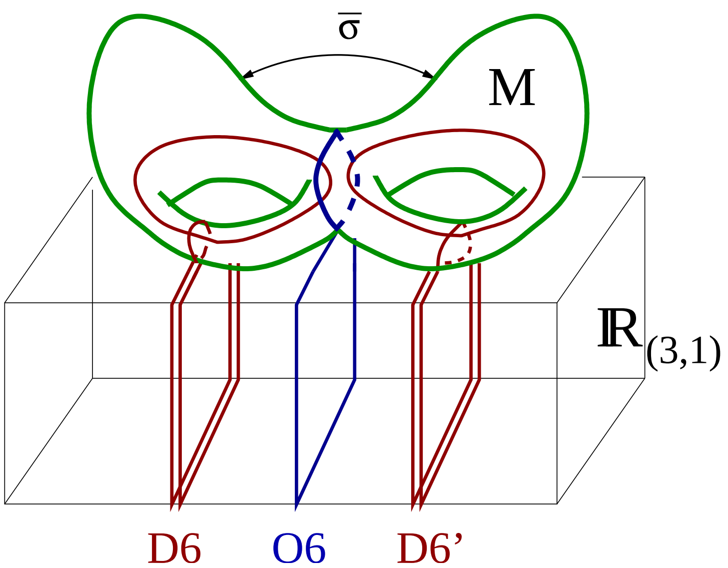

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see also talks by Bianchi, Choi, Cvetic, Lüst, Marchesano, Schellekens, Taylor, Verlinde

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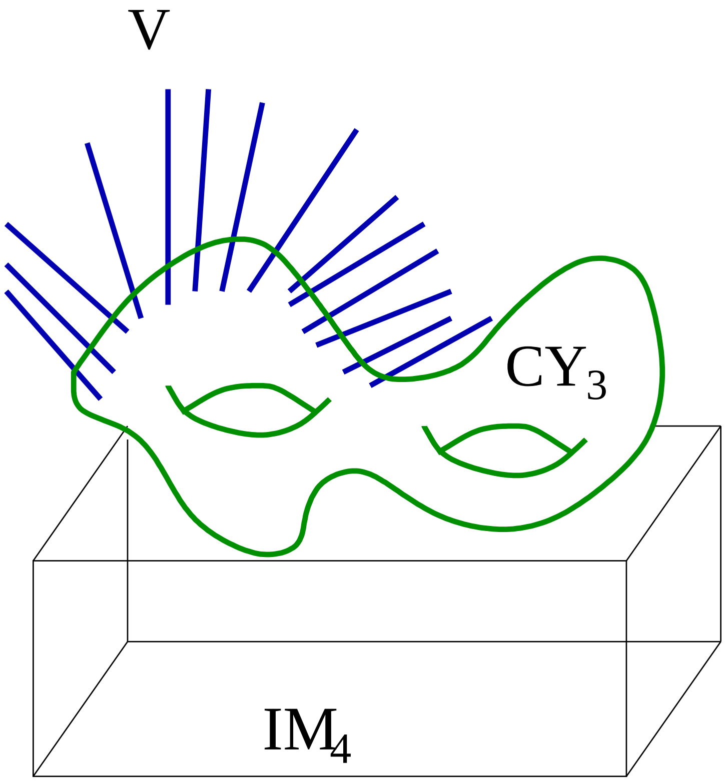

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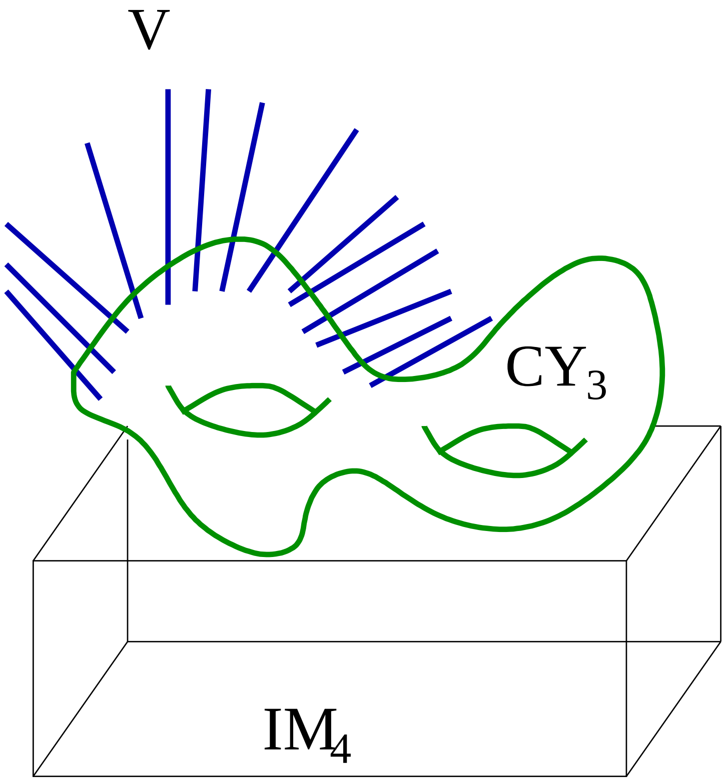

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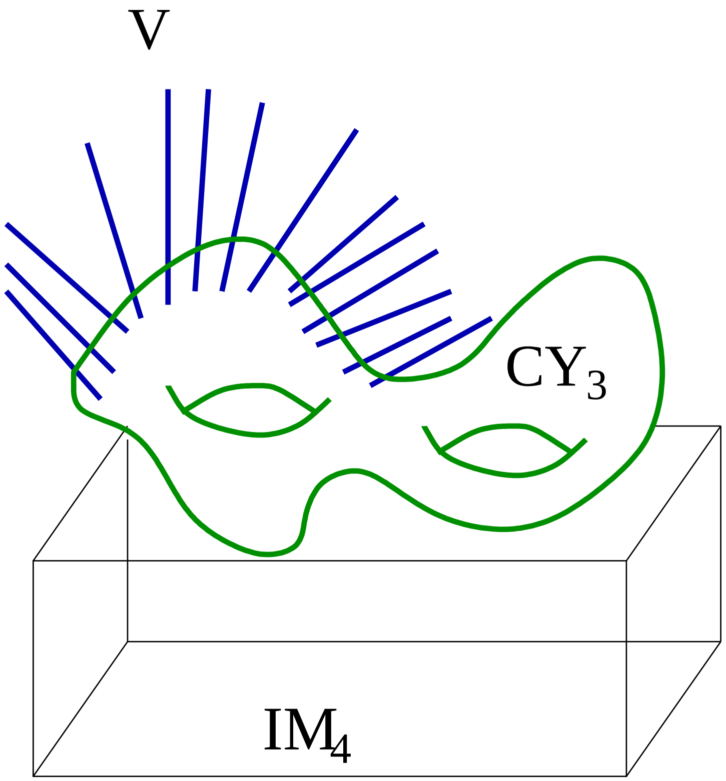

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- Consider the $E_8 \times E_8$ heterotic string equipped with the specific class of bundles

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$$W = V \oplus L$$

  with structure group $G = SU(4) \times U(1)$.
- Embedding this structure group into one of the E_8 factors leads to the breaking to $H = SU(5) \times U(1)_X$, where the adjoint of E_8 decomposes as follows into $G \times H$ representations.

Motivation

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$$\mathbf{248} \longrightarrow \left\{ \begin{array}{c} (\mathbf{15},\mathbf{1})_0 \\ (\mathbf{1},\mathbf{1})_0 + (\mathbf{1},\mathbf{10})_4 + (\mathbf{1},\overline{\mathbf{10}})_{-4} + (\mathbf{1},\mathbf{24})_0 \\ (\mathbf{4},\mathbf{1})_{-5} + (\mathbf{4},\overline{\mathbf{5}})_3 + (\mathbf{4},\mathbf{10})_{-1} \\ (\overline{\mathbf{4}},\mathbf{1})_5 + (\overline{\mathbf{4}},\mathbf{5})_{-3} + (\overline{\mathbf{4}},\overline{\mathbf{10}})_1 \\ (\mathbf{6},\mathbf{5})_2 + (\mathbf{6},\overline{\mathbf{5}})_{-2} \end{array} \right\} .$$

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reps.	Cohomology
$\mathbf{10}_{-1}$	$H^*(\mathcal{M}, V \otimes L^{-1})$
$\mathbf{10}_{4}$	$H^*(\mathcal{M}, L^{4})$
$\overline{\mathbf{5}}_{3}$	$H^*(\mathcal{M}, V \otimes L^{3})$
$\overline{\mathbf{5}}_{-2}$	$H^*(\mathcal{M}, \bigwedge^2 V \otimes L^{-2})$
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Table 1: Massless spectrum of $H = SU(5) \times U(1)_X$ models.

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Table 1: Massless spectrum of $H = SU(5) \times U(1)_X$ models.

Candidate for a flipped $SU(5)$ model $\rightarrow$ need to understand structure of $E_8 \times E_8$ compactification with $U(N)$ bundles.

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$SU(3) \times SU(2) \times U(1)_Y$	Cohom.
$(\mathbf{3}, \mathbf{2})_{\frac{1}{3}}$	$H^*(V)$
$(\mathbf{3}, \mathbf{2})_{-\frac{5}{3}}$	$H^*(L^{-1})$
$(\overline{\mathbf{3}}, \mathbf{1})_{\frac{2}{3}}$	$H^*(\bigwedge^2 V)$
$(\overline{\mathbf{3}}, \mathbf{1})_{-\frac{4}{3}}$	$H^*(V \otimes L^{-1})$
$(\mathbf{1}, \mathbf{2})_{-1}$	$H^*(\bigwedge^2 V \otimes L^{-1})$
$(\mathbf{1}, \mathbf{1})_2$	$H^*(V \otimes L)$
$(\mathbf{1}, \mathbf{1})_1$	$H^*(L^{-1})$

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We choose

$$W_i = V_{N_i} \oplus \bigoplus_{m_i=1}^{M_i} L_{m_i}$$

with $U(N_i)$ bundle V_{N_i} and the complex line bundles L_{m_i}.

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with $U(N_i)$ bundle V_{N_i} and the complex line bundles L_{m_i}.

$$c_1(W_i) = c_1(V_{N_i}) + \sum_{m_i=1}^{M_i} c_1(L_{m_i}) = 0.$$

W can be embedded into an $SU(N_i + M_i) \subset E_8$.

Tadpole cancellation

Tadpole cancellation

- The Bianchi identity for the three-form H implies the tadpole cancellation condition

$$0 = \frac{1}{4(2\pi)^2}\left(\mathrm{tr}(\overline{F}_1^2) + \mathrm{tr}(\overline{F}_2^2) - \mathrm{tr}(\overline{R}^2)\right) - \sum_a N_a \overline{\gamma}_a,$$

to be satisfied in cohomology. Here $\overline{\gamma}_a$ are Poincare dual to two-cycles Γ_a wrapped by the N_a M5-branes.

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This can be written as

$$\sum_{i=1}^{2}\left(\mathrm{ch}_2(V_{N_i}) + \frac{1}{2}\sum_{m_i=1}^{M_i} c_1^2(L_{m_i})\right) - \sum_a N_a \overline{\gamma}_a = -c_2(T).$$

Massless spectrum

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- The massless spectrum is determined by various cohomology classes

$$H^*(X, W),$$

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- The net-number of chiral matter multiplets is given by the Euler characteristic of the respective bundle $\mathcal{W}$

$$\chi(X, \mathcal{W}) \;=\; \int_X \left[\mathrm{ch}_3(\mathcal{W}) + \frac{1}{12}\, c_2(T_X)\, c_1(\mathcal{W}) \right].$$

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- All non-abelian cubic gauge anomalies do cancel, whereas the mixed abelian-nonabelian, the mixed abelian-gravitational and the cubic abelian ones do not. They need to be cancelled by a generalised Green-Schwarz mechanism involving the terms

$$S_{GS} = \frac{1}{24\,(2\pi)^5\,\alpha'} \int B \wedge X_8,$$

and

$$S_{kin} = -\frac{1}{4\kappa_{10}^2} \int e^{-2\phi_{10}}\, H \wedge \star_{10}\, H.$$

(Lukas, Stelle, hep-th/9911156), (R.B., Honecker, Weigand, hep-th/0504232)

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- At string tree level, the connection of the vector bundle has to satisfy the hermitian Yang-Mills equations

$$F_{ab} = F_{\overline{a}\overline{b}} = 0, \quad g^{a\overline{b}} F_{a\overline{b}} = \star [J \wedge J \wedge F] = 0.$$

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F has to be a holomorphic vector bundle.

- A necessary condition is the so-called Donaldson-Uhlenbeck-Yau (DUY) condition,

$$\int_X J \wedge J \wedge c_1(V_{N_i}) = 0, \qquad \int_X J \wedge J \wedge c_1(L_{m_i}) = 0,$$

to be satisfied for all n_i, m. If so, a theorem by Uhlenbeck-Yau guarantees a unique solution provided each term is $\mu\text{-}stable$.

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Computing the FI-terms, reveals a one-loop correction to the DUY equation in the presence of M5-branes, which leads to the conjecture (Bl.,Moster, Reinbacher, Weigand, alg-geom/0609nnn).

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There exists a corresponding stringy one-loop correction to the HYM equation of the form

$$\star_6 \Bigg[J \wedge J \wedge F_i^{ab} - \frac{\ell_s^4}{4(2\pi)^2}\, e^{2\phi_{10}}\; F_i^{ab} \wedge \Bigg(\mathrm{tr}_{E_{8i}}(F_i \wedge F_i) - \frac{1}{2}\mathrm{tr}(R \wedge R) \Bigg) + \ell_s^4 e^{2\phi_{10}} \sum_a N_a \left(\frac{1}{2} \mp \lambda_a \right)^2 F_i^{ab} \wedge \overline{\gamma}_a \Bigg] + (\mathrm{non-pert.\ terms}) = 0..$$

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There exists a unique solution, once the bundle satisfies the corresponding integrability condition and the bundle is Λ-stable with respect to the slope

$$\Lambda(\mathcal{F}) = \frac{1}{\mathrm{rk}(\mathcal{F})}\left[\int_X J \wedge J \wedge c_1(\mathcal{F}) - \ell_s^4\, g_s^2 \int_X c_1(\mathcal{F}) \wedge \right.$$

$$\left. \left(\mathrm{ch}_2(V_{N_i}) + \frac{1}{2} \sum_{n_i=1}^{M_i} c_1^2(L_{n_i}) + \frac{1}{2}\, c_2(T) \right) + \ (\mathrm{npt}). \right.$$

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If, as for $SU(N)$ Bundles

$$\lambda(V) = \mu(V),$$

we can immediately conclude that a μ-stable bundle is also λ-stable for sufficiently small string coupling g_s.

Flipped $SU(5)$ vacua

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Consider heterotic string on a Calabi-Yau manifold X with bundle

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  (Tatar, Watari, hep-th/0602238), (Andreas, Curio, hep-th/0602247)
- Embed a second line bundle into the other E_8, such that a linear combination of the two observable $U(1)$'s remains massless

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- Concretely, we embed the line bundle L also in the second E_8, where it leads to the breaking $E_8 \to E_7 \times U(1)_2$ and the decomposition

$$\mathbf{248} \stackrel{E_7 \times U(1)}{\longrightarrow} \left\{ \ (\mathbf{133})_0 + (\mathbf{1})_0 + (\mathbf{56})_1 + (\mathbf{1})_2 + c.c. \ \right\}.$$

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- More general breakings are possible.

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- The linear combination

$$U(1)_X = -\frac{1}{2}\left(U(1)_1 - \frac{5}{2}\,U(1)_2\right)$$

remains massless if the following conditions are satisfied

$$\int_X c_1(L) \wedge c_2(V) = 0, \ \int_{\Gamma_a} c_1(L) = 0 \quad \text{for all M5 branes.}$$

Flipped $SU(5)$ vacua: spectrum

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reps.	bundle	SM part.
$(\mathbf{10},\mathbf{1})_{\frac{1}{2}}$ $(\mathbf{10},\mathbf{1})_{-2}$	$\chi(V) = g$ $\chi(L^{-1}) = 0$	$(q_L, d_R^c, \nu_R^c) + [H_{10}]$ $-$
$(\overline{\mathbf{5}},\mathbf{1})_{-\frac{3}{2}}$ $(\overline{\mathbf{5}},\mathbf{1})_{1}$	$\chi(V \otimes L^{-1}) = g$ $\chi(\bigwedge^2 V) = 0$	(u_R^c, l_L) $[(h_3, h_2) + (\overline{h}_3, \overline{h}_2)]$
$(\mathbf{1},\mathbf{1})_{\frac{5}{2}}$	$\chi(V \otimes L) + \chi(L^{-2}) = g$	e_R^c
$(\mathbf{1},\mathbf{56})_{\frac{5}{4}}$	$\chi(L^{-1}) = 0$	$-$

Table 2: Massless spectrum of $H = SU(5) \times U(1)_X \times E_7$ models with $g = \frac{1}{2}\int_X c_3(V)$.

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$$\lambda(V) = \mu(V) = \int_X J \wedge J \wedge c_1(V) = 0,$$

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- Gauge invariant Yukawa couplings

$$\mathbf{10}^i_{\frac{1}{2}}\,\mathbf{10}^j_{\frac{1}{2}}\,\mathbf{5}_{-1}, \quad \mathbf{10}^i_{\frac{1}{2}}\,\overline{\mathbf{5}}^j_{-\frac{3}{2}}\,\overline{\mathbf{5}}_{1}, \quad \overline{\mathbf{5}}^i_{-\frac{3}{2}}\,\mathbf{1}^j_{\frac{5}{2}}\,\mathbf{5}_{-1},$$

lead to Dirac mass-terms for the d, (u,ν) and e quarks and leptons after electroweak symmetry breaking.

Flipped $SU(5)$ vacua: couplings

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- Since the electroweak Higgs carries different quantum numbers than the lepton doublet, the dangerous dimension-four proton decay operators

$$\mathbf{l\,l\,e} \quad \in \quad \overline{\mathbf{5}}^i_{-\frac{3}{2}}\,\mathbf{1}^j_{\frac{5}{2}}\,\overline{\mathbf{5}}^k_{-\frac{3}{2}},\ \mathbf{q\,d\,l},\quad \mathbf{u\,d\,d} \quad \in \quad \mathbf{10}^i_{\frac{1}{2}}\,\mathbf{10}^j_{\frac{1}{2}}\,\overline{\mathbf{5}}^k_{-\frac{3}{2}}$$

are not gauge invariant.

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- Breaking a stringy $SU(5)$ or $SO(10)$ GUT model via discrete Wilson lines, the Standard Model tree level gauge couplings satisfy

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at the string scale.

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$$\alpha_3 = \alpha_2 = \frac{5}{3}\alpha_Y = \alpha_{GUT}$$

at the string scale.

- Since the $U(1)_X$ has a contribution from the second E_8, this relation gets modified to

$$\alpha_3 = \alpha_2 = \frac{8}{3}\alpha_Y = \alpha_{GUT}$$

Bundles on elliptically fibered CYs

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Elliptically fibered Calabi-Yau manifold X

$$\pi : X \to B$$

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- Friedman, Morgan and Witten have defined stable $SU(N)$ bundles on such spaces via the so-called spectral cover construction. (Friedman, Morgan, Witten, hep-th/9701162)

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Mathematically, such a prescription is realized by the Fourier-Mukai transform

$$V = \pi_{1*}(\pi_2^* \mathcal{N} \otimes \mathcal{P}_B)$$

with

$$\Big(X \times_B C, \, \mathcal{P}_B \otimes \pi_2^* \mathcal{N} \Big)$$

$$\swarrow^{\pi_1} \qquad \searrow^{\pi_2}$$

$$\Big(X, \, V \Big) \qquad\qquad \Big(C, \, \mathcal{N} \Big)$$

Fourier-Mukai transform

The idea is to use a simple description of $SU(n)$ bundles over the elliptic fibers and then globally glue them together to define bundles over X.
Mathematically, such a prescription is realized by the Fourier-Mukai transform

$$V = \pi_{1*}(\pi_2^* \mathcal{N} \otimes \mathcal{P}_B)$$

with

$$\Big(X \times_B C,\ \mathcal{P}_B \otimes \pi_2^* \mathcal{N} \Big)$$

$$\swarrow^{\pi_1} \qquad\qquad \searrow^{\pi_2}$$

$$\Big(X,\ V \Big) \qquad\qquad \Big(C,\ \mathcal{N} \Big)$$

Cohomology classes

In

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For example:

$$
\begin{aligned}
H^0(X, V_a \otimes V_b) &= 0, \\
H^1(X, V_a \otimes V_b) &= H^0(C_a \cap C_b, \mathcal{N}_a \otimes \mathcal{N}_b \otimes K_B), \\
H^2(X, V_a \otimes V_b) &= H^1(C_a \cap C_b, \mathcal{N}_a \otimes \mathcal{N}_b \otimes K_B), \\
H^3(X, V_a \otimes V_b) &= 0.
\end{aligned}
$$

For the special case $V_a = \mathcal{O}_X$ and $C_a = \sigma$, one finds agreement with (Donagi, He, Ovrut, Reinbacher, hep-th/0405014)

Three generation example

Three generation example

Using stable bundle extensions

$$0 \to V_1 \to V \to V_2 \to 0$$

we have so far found concrete flipped $SU(5)$ models with just three generations of MSSM quarks and leptons plus one vector-like GUT Higgs, i.e.

$$H^i(X,V) = (0,1,4,0).$$

Three generation example

Three generation example

Jumping over many technical details, the total spectrum of the "best" example we found so far reads

$SU(5)\times U(1)_X\times E_6$	Cohomology	χ
$(\mathbf{10},\mathbf{1})_{\frac{1}{2}}$	$(0,1,4,0)$	3
$(\mathbf{10},\mathbf{1})_{-2}$	$(0,0,0,0)$	0
$(\overline{\mathbf{5}},\mathbf{1})_{-\frac{3}{2}}$	$(0,0,3,0)$	3
$(\overline{\mathbf{5}},\mathbf{1})_{1}$	$(0,[51,55],[51,55],0)$	0
$(\mathbf{1},\mathbf{1})_{\frac{5}{2}}$	$(0,0,3,0)+(0,[0,2],[0,2],0)$	3
$(\mathbf{1},\mathbf{27})_{\frac{5}{6}}$	$(0,0,0,0)$	0
$(\mathbf{1},\mathbf{27})_{-\frac{5}{3}}$	$(0,0,0,0)$	0

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- Heterotic Landscape?