Challenges for Brane Gas Cosmology

Outline

- Motivation
- Challenges for String Cosmology
- Brane Gas Cosmology
- Challenges for Brane Gas Cosmology
- Inflation from Brane Gases

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WMAP
(2003)

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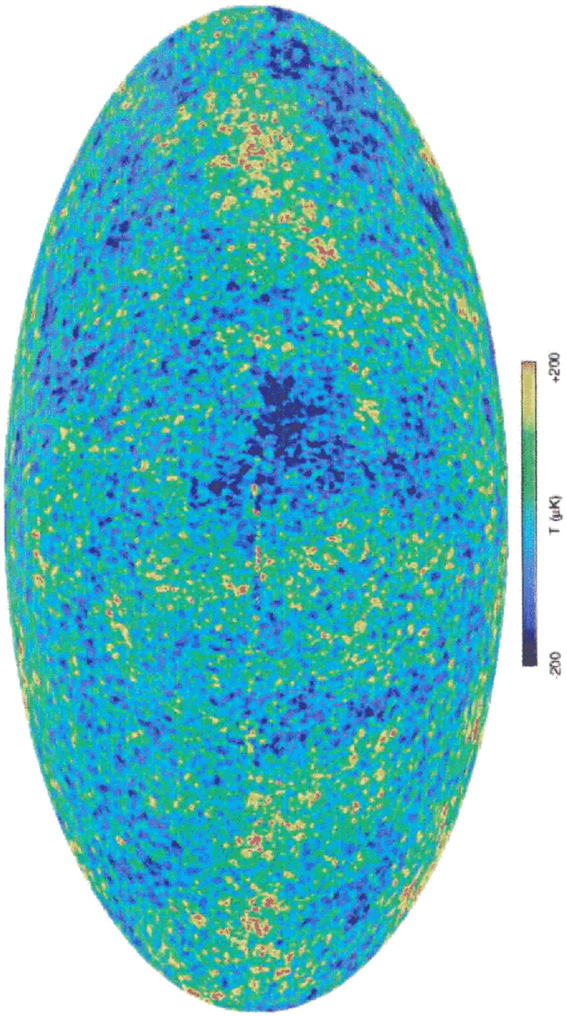

Fig. 11.— This "internal linear combination" map combines the five band maps in such as way as to maintain unity response to the CMB while minimizing foreground foreground contamination. For a more detailed description see Bennett et al. (2003c). For the regiop that covers the full sky outside of the inner Galactic plane, the weights are 0.109, −0.684, −0.096, 1.921, −0.250 for K, Ka, Q, V, and W bands, respectively. Note that there is a chance alignment of a particularly warm feature and a cool feature near the Galactic plane. As discussed in Bennett et al. (2003c), the noise properties of this map are complex, so it should not generally be used for cosmological analyses

WMAP
(2003)

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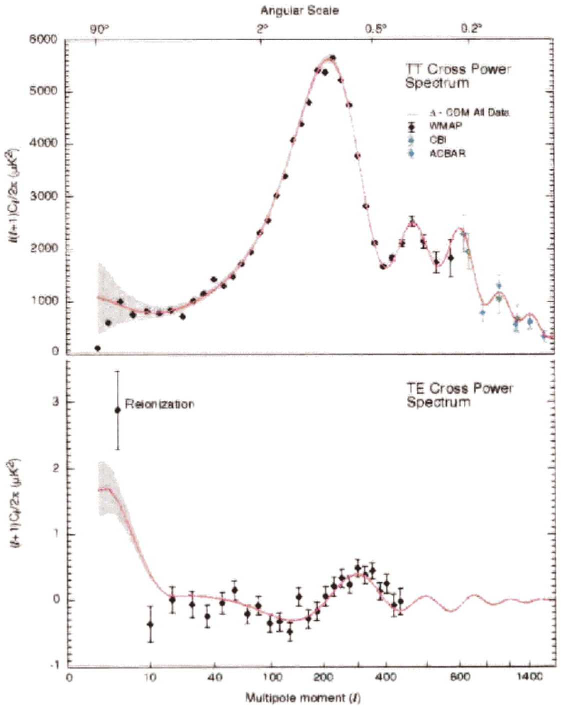


Fig. 12.— The *WMAP* angular power spectrum. (*top:*) The *WMAP* temperature (TT) results are consistent with the ACBAR and CBI measurements, as shown. The TT angular power spectrum is now highly constrained. Our best fit running index ΛCDM model is shown. The grey band represents the cosmic variance expected for that model. The quadrupole has a surprisingly low amplitude. Also, there are excursions from a smooth spectrum (e.g., at $\ell \approx 40$ and $\ell \approx 210$) that are only slightly larger than expected statistically. While intriguing, they may result from a combination of cosmic variance, subdominant astrophysical processes, and small effects from approximations made for this first year data analysis (Hinshaw et al. 2003b). We do not attach cosmological significance to them at present. More integration time and more detailed analyses are needed. (*bottom:*) The temperature-polarization (TE) cross-power spectrum, $(l+1)C_l/2\pi$. (Note that this is *not* multiplied by the additional factor of l.) The peak in the TE spectrum near $l \sim 300$ is out of phase with the TT power spectrum, as predicted for adiabatic initial conditions. The antipeak in the TE spectrum near $l \sim 150$ is evidence for superhorizon modes at decoupling, as predicted by inflationary models.

Conceptual Problems of Scalar Field - Driven Inflationary Cosmology

a) Fluctuation Problem
b) Trans - Planckian Problem
c) Singularity Problem
d) Cosmological Constant Problem

a) Fluctuation Problem

inflation → density fluctuations on all scales

~ scale invariant spectrum

Mukhanov & Chibisov 81

quantum theory of cosmological perturbations

Mukhanov ; Sasaki

Mukhanov, Feldman & R.B.

classical fluctuations emerge via squeezing of initial vacuum state

↓

amplitude $\frac{\delta M}{M}(k, t_f(k)) \sim 10^2 \lambda^{1/2}$

$\uparrow$

$V(\varphi) = \lambda \varphi^4$

$\Rightarrow \lambda \leq 10^{-12}$ hierarchy problem

N.B. problem is generic

Adams, Freese & Guth

New twist to fluctuation problem :

Parametric amplification of super-Hubble cosmological perturbations during reheating

F. Finelli & R.B., Phys. Rev. Lett. (99)

D. Kaiser, B. Bassett & R. Maartens, Phys. Lett. B (99)

- allowed by causality (F.F. & R.B.) PRL 82 (1999)
- does not occur in single matter field models

  (N. Afshordi & R.B., gr-qc/0011075)

  (W. Lin, X. Meng & X. Zhang, hep-ph/9912510)
- occurs in multi-matter field models in which entropy perturbations are not suppressed during inflation

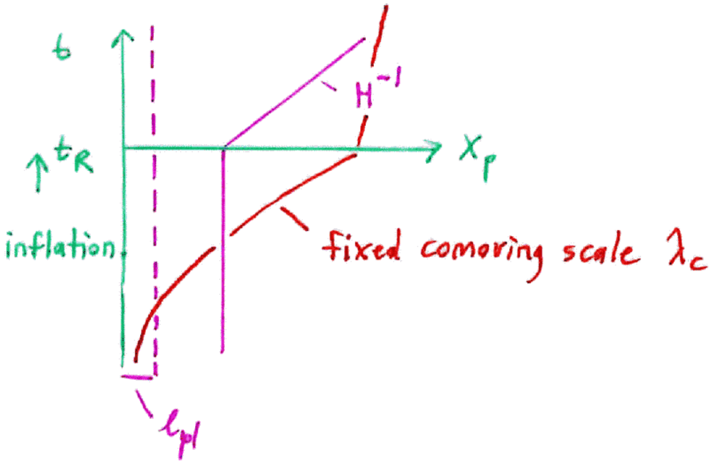
  (F. Finelli & R.B., Phys. Rev. D62, (00))
- results in $\frac{\delta M}{M}(k, t_f(k)) \gtrsim \mathcal{O}(1)$ even when taking back reaction into account

  (J. Zibin, R.B. & D. Scott, Phys. Rev. D63 (~~00~~ 01))

Window of Opportunity

b) Trans-Planckian ~~Problem~~

R.B 1999



Success of inflation: at early times $\lambda_c(t) < H^{-1}(t)$

$\downarrow$

causal generation mechanism for fluctuations

$\downarrow$

Problem: if $\Delta t_{inflation} > 70\,H^{-1}$

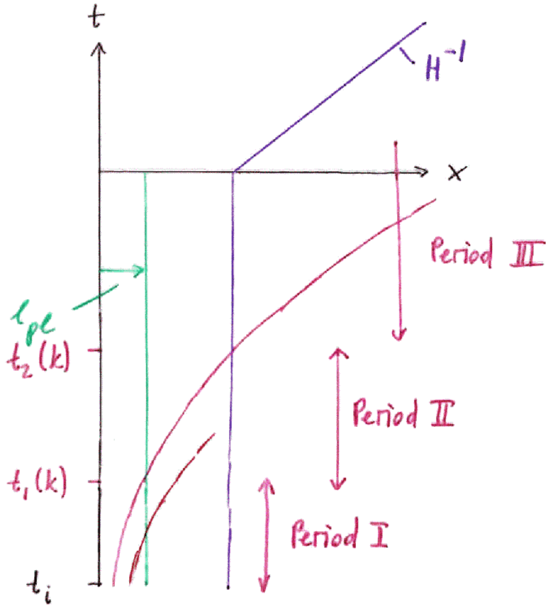
$\Downarrow$

$\lambda_c(t_i) < l_{pl}$

$\nwarrow$ beginning of inflation

$\Downarrow$

new physics MUST enter into calculation of fluctuations



If evolution in Period I non-adiabatic then $P(k)$ likely not scale invariant.

R.B. & J. Martin (2000)

$\Downarrow$

Planck scale physics testable in observations

c) Singularity Problem

standard cosmology: Penrose-Hawking theorems

↓

initial singularity ("Big Bang")

↓

incompleteness of theory

Penrose-Hawking theorems

Ass: Einstein action

(*) weak energy condition

$\rho > 0$, $\rho + 3p \geq 0$

} ⇒ space-time is geodesically incomplete

inflationary cosmology: weak energy condition violated ($p = -\rho$)

↓

Penrose-Hawking theorems do not apply

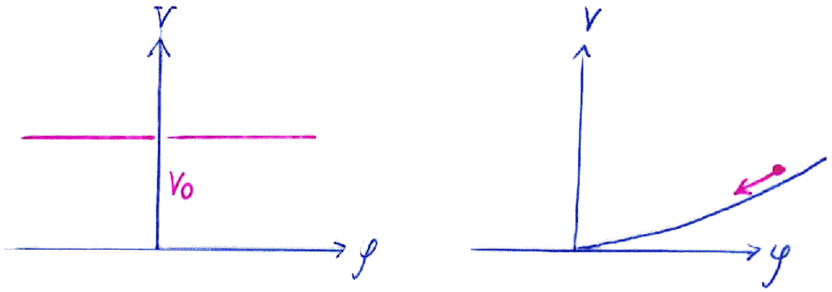
but: Theorem: In a chaotic inflation model initial singularity persists

[Borde & Vilenkin]

↓

incompleteness of theory

d) Cosmological Constant Problem



Quantum vacuum energy of matter fields do not gravitate

$$\frac{V_0}{\Lambda_{obs}} > 10^{122} \;!!$$

Cosmological Constant Problem

Why does the unknown mechanism which renders V_0 gravitationally inert also render $V(\varphi)$ gravitationally inert?

driving inflation

Why String Theory can Help (?)

a) Fluctuation Problem

string theory moduli $\overset{?}{\rightarrow}$ small parameters required for $\frac{\delta\rho}{\rho} \sim 10^{-5}$

b) Trans-Planckian Problem

string theory determines physics when $\lambda_{ph} \sim l_{pl}$

→ determines initial conditions for fluctuations in classical regime

c) Singularity Problem

string theory → nonsingular cosmology

↑

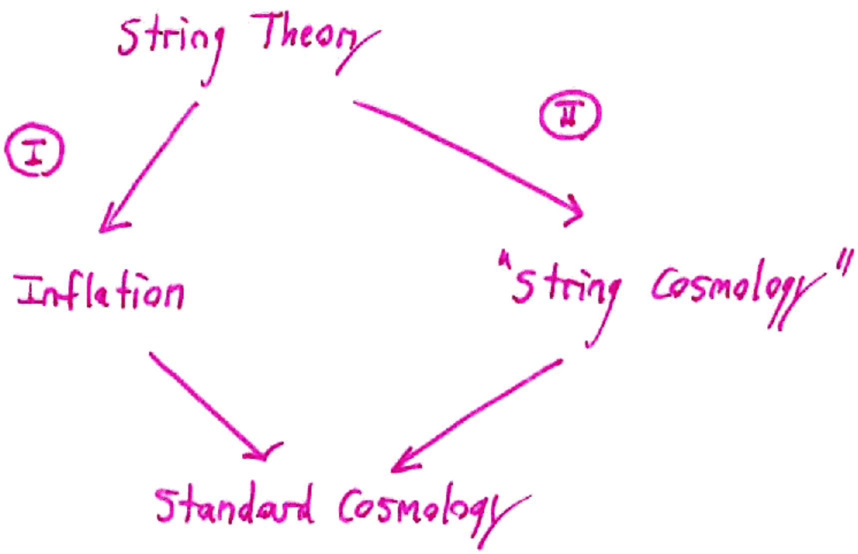
goal

R.B. & C. Vafa Nucl. Phys. B 1989

S. Alexander, R.B. & D. Easson 2000

d) Cosmological constant problem ??

Possibilities



I : maintains successes of infl. cosmology

II : must provide new solutions to the problems which inflation solves

Dimensionality Problem

Critical, perturbative Superstring Theory
is mathematically consistent only in
$d = 9+1$ space-time dimensions

fatal problem ?

Traditional approach :

extra dimensions compactified
ad hoc ?
stabilization ?

New approach :

brane world scenarios :
we live on a $d=3+1$ brane

ad hoc
why 3+1 brane ?
.....

⇓ my conclusion

key challenge for string cosmology

Some Challenges for String Cosmology

* Resolve Cosmological Singularities

* Predict Dimensionality

* Make contact with observations

either : provide convincing theory
of inflation

or : yield alternative cosmology
which maintains successes
of inflation

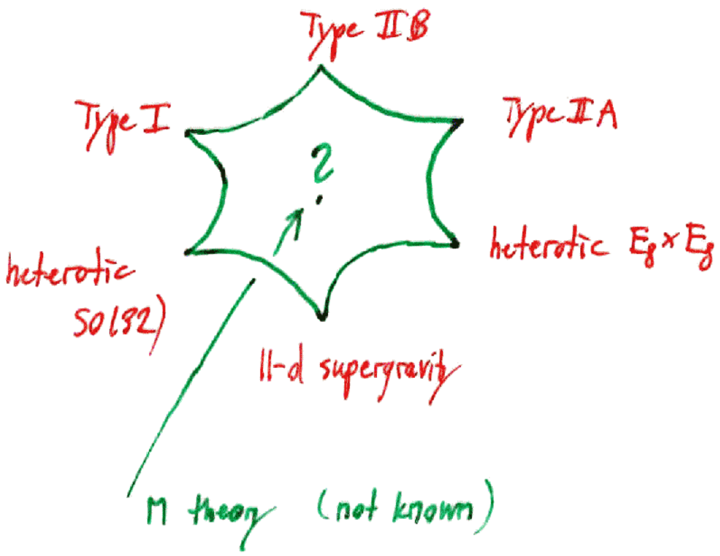
— — — — — —

* Address cosmological constant problem

- Fluctuations mathematically consistent
Evolution equations physically consistent

The Problem

String cosmology does not exist because nonperturbative string theory is not yet known.



Most work on string cosmology starts in one corner of string theory moduli space.

Essential Tools of String Cosmology

Ex: toroidal background space, radius R

New degrees of freedom

strings
- momentum modes $E_m = \frac{m}{R}$
- oscillatory modes* E indep. of R
- winding modes $E_n = n R$

* density of states grows exponentially with E
→ maximal temperature
Hagedorn temperature

New Symmetries

t-duality $R \to \frac{1}{R}$

$(n, m) \to (m, n)$

mass spectrum of closed strings is invariant

The "Rules" for String Cosmology

A Hot Beginning

Expect gas of strings & branes of all dimensionalities

see Brane gas cosmology

B Cold Beginning

Expect special initial state determined by symmetry

see PBB
" Ekpyrotic

Some Approaches

Brane gas cosmology	type A
Pre-big-bang scenario	type B
Ekpyrotic	type B
Brane world scenario	?

- Could come from type A
  M. Majumdar & A.-C. Davis
- Moving branes → variations of fund. constants

↑

Mirage cosmology

Brane Gas Cosmology

Theory, Background & Initial Conditions

Theory : Type II superstring theory

Background : $\underset{\text{time}}{\underset{\uparrow}{\mathbb{R}}} \times \underset{\text{space}}{\underset{\uparrow}{T^9}}$

"democratic"

Initial Conditions :
- hot gas
  all degrees of freedom excited
- $R_i = R = 1$

"conservative"

R.B. & C. Vafa (89)

T Duality

for fundamental strings

oscillatory modes : E ind. of R

momentum modes : $E = \frac{n}{R}$

winding modes : $E = mR$

n, m integer

$$M^2 = \frac{n^2}{R^2} + m^2 R^2 + 2(N + \tilde{N} - 2)$$

$$\boxed{\begin{array}{ll} \text{t-duality :} & R \rightarrow \frac{1}{R} \\ & (n, m) \rightarrow (m, n) \end{array}}$$

spectrum of states invariant under t-duality

vertex operators " "

$\Downarrow$

t-duality is symmetry of perturbative string theory

K. Kikkawa & M. Yamasaki (84)

...

for branes

T. Boehm & R.B. (02)

Note: t-duality is used to argue for the existence of branes

Polchinski (95)

T-duality in direction // to p-brane:

p-brane → p-1 brane

T-duality in direction ⊥ to p brane:

p-brane → p+1 brane

$$\boxed{\begin{array}{l}\text{T-dualizing in all spatial dimensions}\\ \quad p\text{-brane} \rightarrow 9-p \text{ brane}\end{array}}$$

R large		R small
Type II A brane gas $\mathcal{B}$	$\xleftrightarrow{T}$ duality	Type II B brane gas $\mathcal{B}^*$
0, 2, 4, 6, 8 branes		1, 3, 5, 7, 9 branes
string coupling g		string coupling g'

$$g' = \left(\frac{\alpha'^{1/2}}{R}\right)^9 g$$

effects tensions

$$\tilde{\tau}_p = e^{-\phi} T_p = \frac{1}{g} T_p = (2\pi)^{-p} g^{-1} a'^{-\frac{p+1}{2}}$$

Ex.:

$$M_4 = (2\pi)^4 \tau_4 R^4 = g^{-1} \alpha'^{-5/2} R^4$$

$$M_5' = (2\pi)^5 \tau_5' R'^5 = M_4$$

using

$$R' = \frac{l_s}{R}$$

$$g' = \left(\frac{l_s}{R}\right)^9 g \qquad \left[l_s = \alpha'^{1/2}\right]$$

$$M_{9-p}' = M_p$$

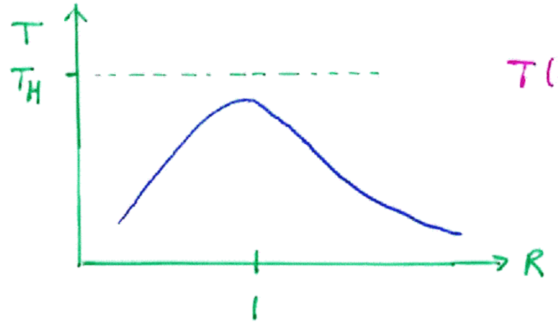
$$\Downarrow$$

$$\boxed{\begin{array}{l}\text{mass spectrum of stable brane states}\\ \quad\quad\text{invariant under T-duality}\end{array}}$$

T-duality → Nonsingular Cosmology

Consider adiabatic change in R

A. Thermodynamics



$T(R) = T(\frac{1}{R})$

⇒ temperature nonsingular as $R \downarrow 0$

B. "Physical" Length

$R > 1$ ℓ measured in terms of x

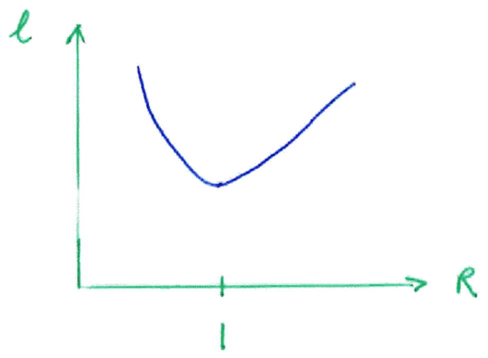
$$|x\rangle = \sum_p e^{ixp} |p\rangle$$

↖ momentum eigenstates

$R < 1$ ℓ measured in terms of $\tilde{x}$

$$|\tilde{x}\rangle = \sum_{p_w} e^{i\tilde{x}p_w} |p_w\rangle$$

↑ winding eigenstates



⇒ physical length nonsingular as $R \to 0$

R.B. & C. Vafa (89)

N.B. Obtain bouncing Universe!

Equations of Motion

Dilaton gravity!
(else no T-duality)

$$S = \int d^{10}x \sqrt{-g}\, e^{-2\phi}\left[R + 4\partial^{\mu}\phi\,\partial_{\mu}\phi\right]$$

$$ds^2 = dt^2 - a(t)^2 d\underline{x}^2$$

$$\lambda = \log a(t)$$

$$\varphi = 2\phi - d\lambda \qquad [d = 9]$$

$\Downarrow$ variational EOM

$$\boxed{\begin{aligned} -d\dot{\lambda}^2 + \dot{\varphi}^2 &= e^{\varphi}E \\ \ddot{\lambda} - \dot{\varphi}\dot{\lambda} &= \frac{1}{2}e^{\varphi}P \\ \ddot{\varphi} - d\dot{\lambda}^2 &= \frac{1}{2}e^{\varphi}E \end{aligned}}$$

winding modes → $P < 0$
→ confining potential for λ

Veneziano (91)
Tseytlin & Vafa (92)

duality: $\phi \rightarrow \phi - d\lambda$
$\lambda \rightarrow -\lambda$
$\Rightarrow \varphi$ invariant

Cosmic Loitering

R.B., D. Easson & D. Kimberly (01)

To avoid overclosure: need all winding modes in 3 large dims. to annihilate

Kibble mechanism: $\gtrsim 1$ winding mode per Hubble volume persists

$\Downarrow$

need loitering $(H = 0)$

Consider R large

$$\rho_w(t) = \mu\,\tilde{\nu}(t)\,t^{-2} \qquad \text{winding modes}$$

$$\rho_e(t) = g(t)\,e^{-3(\lambda(t) - \lambda(t_0))}$$

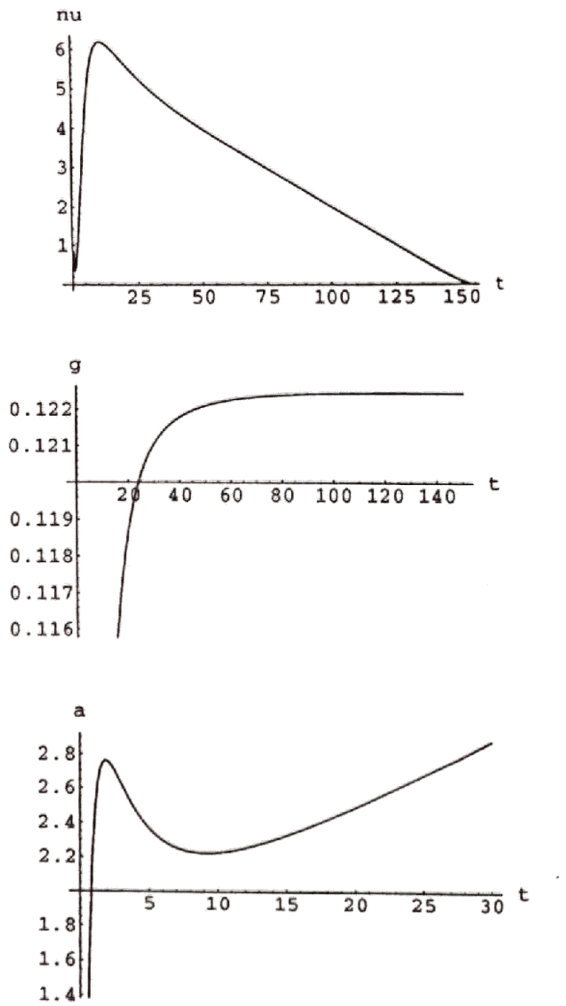
$$\boxed{\begin{aligned} \frac{d\tilde{\nu}(t)}{dt} &= 2\tilde{\nu}\left(t^{-1} - \ell\right) - cc'\tilde{\nu}^2 t^{-1} \\ \frac{dg(t)}{dt} &= cc'\mu\, t^{-3}\tilde{\nu}^2 e^{3(\lambda(t) - \lambda(t_0))} \end{aligned}}$$

A. Vilenkin (85)

$$\ell \equiv \frac{\dot{a}}{a}$$

energy transfer from winding modes to loops

see also: A. Campos (03)



Scenario

- * Brane winding modes prevent expansion
- * p branes : annihilate in ≤ d_c spatial dims.

$$2(p+1) = d_c + 1$$

Consider: $R(t_0) = 1$
$\dot{R}(t_0) > 0$

- * Heaviest branes fall out of equilibrium first
  └ largest p

no obstructions to $p = 4, \ldots$ brane annihilation

- * $p=2$ branes
  → $d_c = 5$ expand

S. Alexander, R.B. & D. Easson (00)

- * within these 5 dimensions :
  fundamental string winding modes
  → only 3 spatial dims. grow large

R.B. & C. Vafa (89)

→ Solution of Dimensionality Problem

see also : M. Sakellariadou (96)

S. Watson & R.B. (03)

Radion Stabilization

Q: How does radius of "small" dimensions evolve once the 3 "large" dimensions are expanding?

Ansatz: $ds^2 = dt^2 - e^{2\lambda} d\underline{x}^2 - e^{2\nu} d\underline{y}^2$

$\uparrow$ $\mathbb{R}^3$ large $\mathbb{R}^6$ small

Dilaton gravity action + perfect fluid matter

$$-3\ddot{\lambda} - 3\dot{\lambda}^2 - 6\ddot{\nu} - 6\dot{\nu}^2 + 2\ddot{\phi} = \frac{1}{2} e^{2\phi} \rho$$

$$\ddot{\lambda} + 3\dot{\lambda}^2 + 6\dot{\lambda}\dot{\nu} - 2\dot{\lambda}\dot{\phi} = \frac{1}{2} e^{2\phi} p_\lambda$$

$$\ddot{\nu} + 6\dot{\nu}^2 + 3\dot{\lambda}\dot{\nu} - 2\dot{\nu}\dot{\phi} = \frac{1}{2} e^{2\phi} p_\nu$$

$$4\ddot{\phi} - 4\dot{\phi}^2 - 12\dot{\lambda}\dot{\phi} - 24\dot{\nu}\dot{\phi}$$
$$+ 3\ddot{\lambda} + 6\dot{\lambda}^2 + 6\ddot{\nu} + 21\dot{\nu}^2 + 18\dot{\lambda}\dot{\nu} = 0$$

Matter (string gas) sources obeying T duality

$\downarrow$

include equal number of winding and momentum modes

$$E = 3\mu N^{(3)} e^{\lambda} + 3\mu M^{(3)} e^{-\lambda} + 6\mu N^{(6)} e^{\nu} + 6\mu M^{(6)} e^{-\nu}$$
$$p_\lambda = -\mu N^{(3)} e^{\lambda} + \mu M^{(3)} e^{-\lambda}$$
$$p_\nu = -\mu N^{(6)} e^{\nu} + \mu M^{(6)} e^{-\nu}$$

1st application: $N^{(3)} = M^{(3)} = N^{(6)} = M^{(6)}$.

$\lambda(t_i) = \nu(t_i) \neq 0$

$\Downarrow$

damped oscillations of radius about self-dual point

2nd application: $N^{(3)} = 0$; $N^{(6)} = M^{(6)}$

[after decompactification]

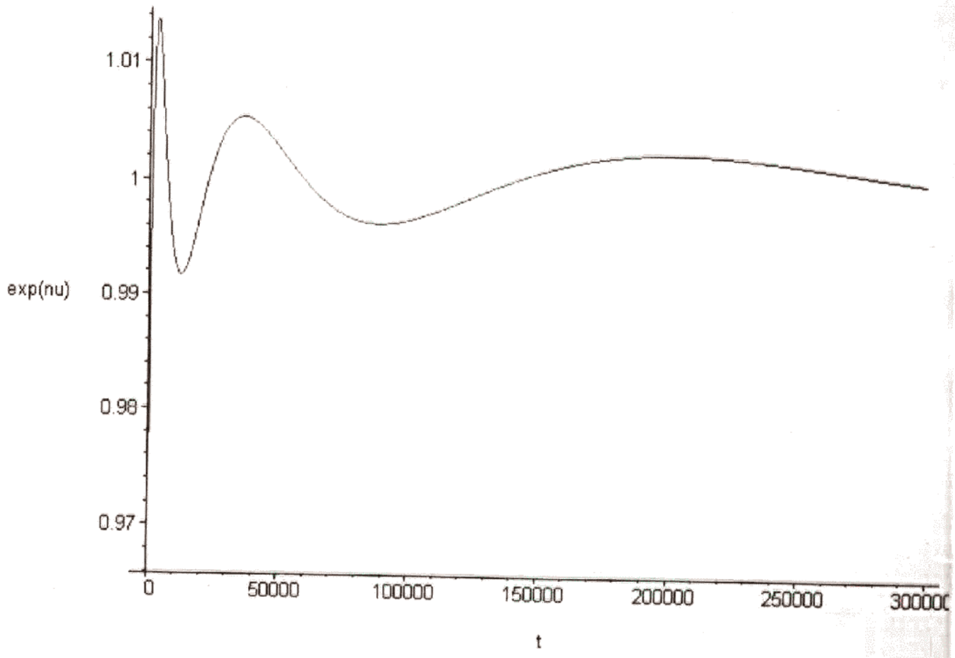
$\Downarrow$

λ increases

ν oscillates about $\nu = 0$

expansion of λ $\Rightarrow$ damping of ν oscillations radion stabilization

S. Watson & R.B. (03)



Challenges for Brane Gas Cosmology

Isotropy Problem

Are the 3 large dimensions isotropic ?

A: yes !

S. Watson & R.B. (02)

Isotropy is scaling fixed pt.

$R_i < R_j$

→ more efficient annihilation of i winding modes

→ $\frac{R_i}{R_j} \nearrow 1$

Dependence on Background

Existence of cycles not crucial

R. Easther et al. (01)

extension to orbifolds

dynamical obstruction to efficient annihilation

D. Easson (01)

Radion Stabilization

Interplay of winding & Momentum Modes
about internal tori
↓
$R_i \rightarrow 1$

S. Watson & R.B. (03)

Homogeneity of Internal Dimensions

??

S. Watson, R. Easther, B. Greene
in prep.

Extensions to Other Corners of M-Theory moduli space

M theory (11-d supergravity)

a) 2 branes
R. Easther et al. (02)
↓
hierarchy of dimensions

b) 2 brane / 5 brane intersections
↓
3 d get very large fastest
S. Alexander (02)

Connection with Observations

Need: i) large Universe
ii) mechanism to generate fluctuations

i) Flatness problem
↓
require inflation

ii) "only" viable mechanism now
is inflation

Q: Can we get inflation from
brane gas cosmology?

Brane Gas Inflation

R.B., D. Easson & A. Mazumdar (03)

(Inflation from Stabilized Embedded Defects)

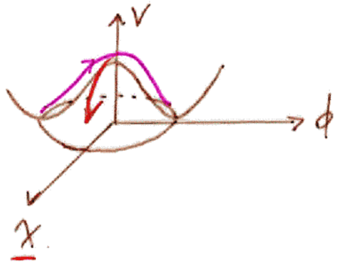
Embedded Defects

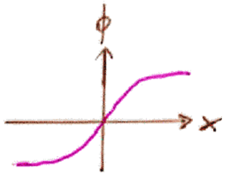
- solutions of field equations
- confined $V(\underline{\phi})$ (defect-like)
- unstable in vacuum
- topological defect of a theory with $\chi = 0$

$$\underline{\phi} = (\phi, \chi)$$

T. Vachaspati

Ex: Embedded Wall





" φ wall "

unstable to χ excitation

Plasma Stabilization of Embedded Defects

M. Nagasawa & R.B. (99, 02)
B. Carter, R.B. & A.-C. Davis (02)

Assume: $\left.\begin{array}{ll}\chi & \text{charged} \\ \phi & \text{neutral}\end{array}\right\}$ w.r.t. external gauge field

in thermal equilibrium

Ex: A) low energy sigma model for QCD

$$\phi = (\pi_0, \eta')$$

$$\chi = (\pi_+, \pi_-)$$

$$\phi(r, \theta) = \eta e^{i\theta} f(r)$$

polar coords. in plane ⊥ string

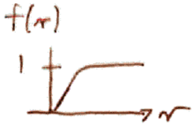
pion string

B) EW theory

$$\underline{\phi} = \begin{pmatrix} \phi \\ \chi \end{pmatrix} \quad \left.\begin{array}{l}\text{neutral} \\ \text{charged}\end{array}\right\} \text{Higgs}$$

$$\phi(r, \theta) = \eta f(r) e^{i\theta}$$

EW Z string



standard defect profile function

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi + \frac{1}{2}D_\mu\chi\,D^\mu\chi - V(\phi,\chi)$$

$$V(\phi,\chi) = \lambda\,(\phi^2+\chi^2-\eta^2)^2$$

consider $T_{rec} < T < \eta \equiv T_c$

A_μ in thermal equilibrium

$$D_\mu = \partial_\mu + ieA_\mu$$

Hartree approximation $\langle A_\mu A^\mu\rangle \sim T^2$

$\langle A_\mu\rangle = 0$

$\Downarrow$

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi + \frac{1}{2}\partial_\mu\chi\,\partial^\mu\chi - V_{eff}(\phi,\chi)$$

$$V_{eff}(\phi,\chi) = V(\phi,\chi) + \kappa T^2\chi^2$$

$\Downarrow$

stabilization

$\Downarrow$

stabilized embedded defect

Note: $T_0 < T < T_c$ symmetric defect $\chi = 0$

$T_{rec} < T \leq T_0$ core phase transition

$\chi \neq 0$ in core

Brane Gas Inflation

Assume: unstable branes stabilized by plasma effects

p branes in d spatial dimensions

$$\mathcal{P}_p = \left[\frac{p+1}{d}v^2 - \frac{p}{d}\right]\rho_p \qquad \text{T. Boehm \& R.B. (02)}$$

pressure, velocity, energy density

winding modes out of thermal equil. $\hookrightarrow v \to 0$ $\Big\}$: $\mathcal{P}_p = -\frac{p}{d}\rho_p$

$\Downarrow$

accelerated expansion if $p = d-1$

for $d = 3$ see Zel'dovich, Kobzarev & Okun (74)

D. Seckel (85)

stable branes → "domain wall" problem

unstable branes → finite duration inflation

but: • naively → wrong spectrum $n_s = 0$

curvaton to the rescue!

• too short (?) period

Scenario: Type IIB Brane Gas Cosmology

democratic, conservative initial conditions

Dilaton free

Stage 1: ↓ BV mechanism, 1-branes (stable)

hierarchy of dimensions

3 large

6 string scale

Assume:
- dilaton fixed ← ?
- unstable 2 branes stabilized
- $\tau_2^{\text{unstable}} < \tau_1^{\text{stable}}$

o.k. because of core phase transition

Stage 2: unstable 2 brane gas drives inflation

↓

flatness problem

Stage 3: ?

↓

$n_s \simeq 1$ spectrum

Conclusions

Brane gas cosmology: Cosmology of very early Universe based on
- we are in the bulk
- brane gas in dilaton gravity background
- hot, small beginning

Successes:
- nonsingular
- explains dimensionality problem

Problems:
- too heuristic
- instabilities
- not (yet) contact with late time cosmology