The Growth of Negativity

Geometry, Topology, and Dimensionality in String theory

- ES hep-th/0510044, PRD 73 086004
- J. McGreevy ES, D. Starr hep-th/061xxxx
- D. Green, A. Lawrence, JMcG, D. Morrison, ES in progress

cf O. Aharony, ES hep-th/?

Consider moduli potential in a string compactification X of volume V

$$\underset{\substack{4d \\ \text{Einstein frame}}}{\mathcal{U}}(g_s, V, \ldots) = \frac{g_s^2 (D - D_c)}{V} + \frac{g_s^2}{V}\int_X \frac{\sqrt{G_X}(-R)}{V}$$

+ fluxes + orientifolds + branes + loops + non-perturbative

From the worldsheet point of view

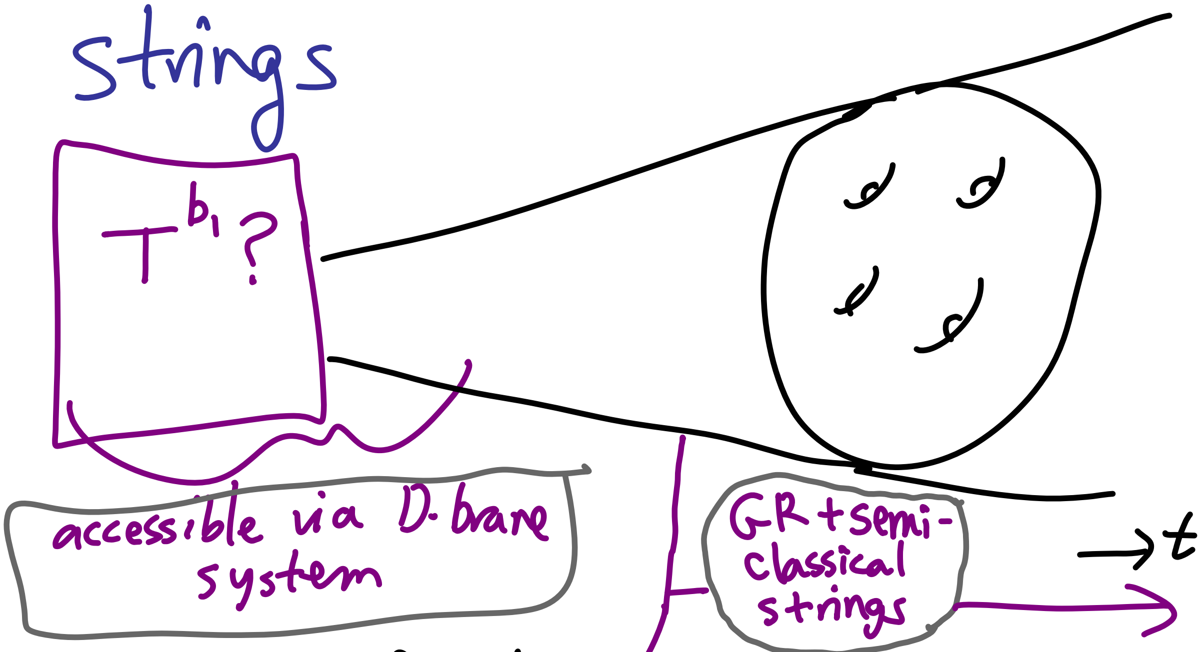
$$\beta_{\log g_{eff}^2} \sim D - D_c + \frac{\int -R}{V} + \cdots$$

↳ Raises the question:

Q Are negatively curved compactifications of 10d string theory effectively supercritical?

This talk:

Evidence for a new duality ("D duality") between "critical" strings on compact negatively curved space, and supercritical strings



Small radius: D-probes, AdS/CFT suggest natural transition to Jacobian torus

Large radius: exponential density of winding strings $\Rightarrow$ "effective central charge" $> c_{crit}$

String theory on small spaces does interesting things:

Familiar examples include

- T-duality: $S^1_R \cong S^1_{\frac{1}{R}}$
  - winding modes build up momentum modes on dual circle
  - But $\underline{c_{eff}}$ same for all R

$$\int \frac{d^2\tau}{\tau_2^2} Z_1(\tau) = \mathrm{Tr} \int \frac{d^2\tau}{4\tau_2} q^{L_0} \bar{q}^{\bar{L}_0}$$

$$Z_1 \xrightarrow[UV]{\tau_2 \to 0} \sim e^{\frac{c_{eff}\,\pi}{6\tau_2}}$$

c_{eff} counts net effective # of dimensions in which string can oscillate.

e.g. type 0 $c_{eff} = 12$, bosonic $c_{eff} = 24$, susy $c_{eff} = 0$

- M theory on small $T^2 \simeq$ IIB, with wrapped membrane building up 10th dimension

- Small CY crosses over to e.g. Landau-Ginsburg phase, yields topology changing transitions, etc.

- D-branes on small tori have winding modes which build up extra worldvolume dimensions wrapped D-branes on tori have Wilson lines which describe positions of T-dual pointlike branes. ...

Above was all for flat target spaces.

<u>Curved</u> target spaces are generic.

e.g. $d=2$

and at large radius, expand slowly with time controlled by GR e.g. constant curvature vacuum sol'n $-dt^2 + t^2 ds^2_{\mathbb{H}_n/\Gamma}$

Relevant for basic physical questions:

- compactifications (most are curved)
- 4d space may be globally compact

$\hookrightarrow$ initial singularity, measure?

- <u>Claim</u>: new type of duality - dimensions from topology

Back to our question.

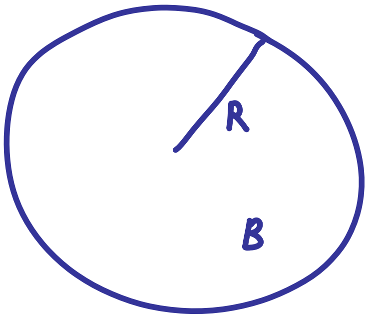
Are negatively curved compactifications of 10d string theory effectively supercritical?

Answer : Yes, for compact spaces M_n with negative sectional curvature.

because there is an exponential density of winding modes!

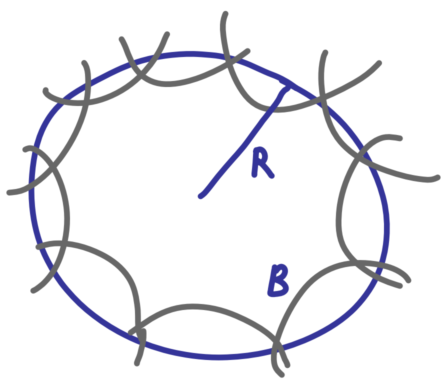
- Milnor '68 : $\pi_1(M_n)$ has exponential growth (in "word metric")
- Margulis '69 : the number of closed geodesics (up to homotopy) grows exponentially. $\rho(L) \propto e^{L/L_0}$

The covering space $\tilde{M}_n$ has



$$\mathrm{Vol}\big(B(R)\big) \propto e^{(n-1)\frac{R}{R_0}}$$

$M_n = \tilde{M}_n / \Gamma$ compact $\Rightarrow$



exponential # of closed geodesics of length L

see http://www.ma.utexas.edu/~hausel/m392cr/kang.pdf for a plagiarized version available online...

Why is exponential growth relevant?

It leads to a Hagedorn density of winding strings and a new contribution to c_{eff}:

$$\rho(m) = e^{m\sqrt{\alpha'}\,\pi\sqrt{\frac{2c_{eff}}{3}}}$$

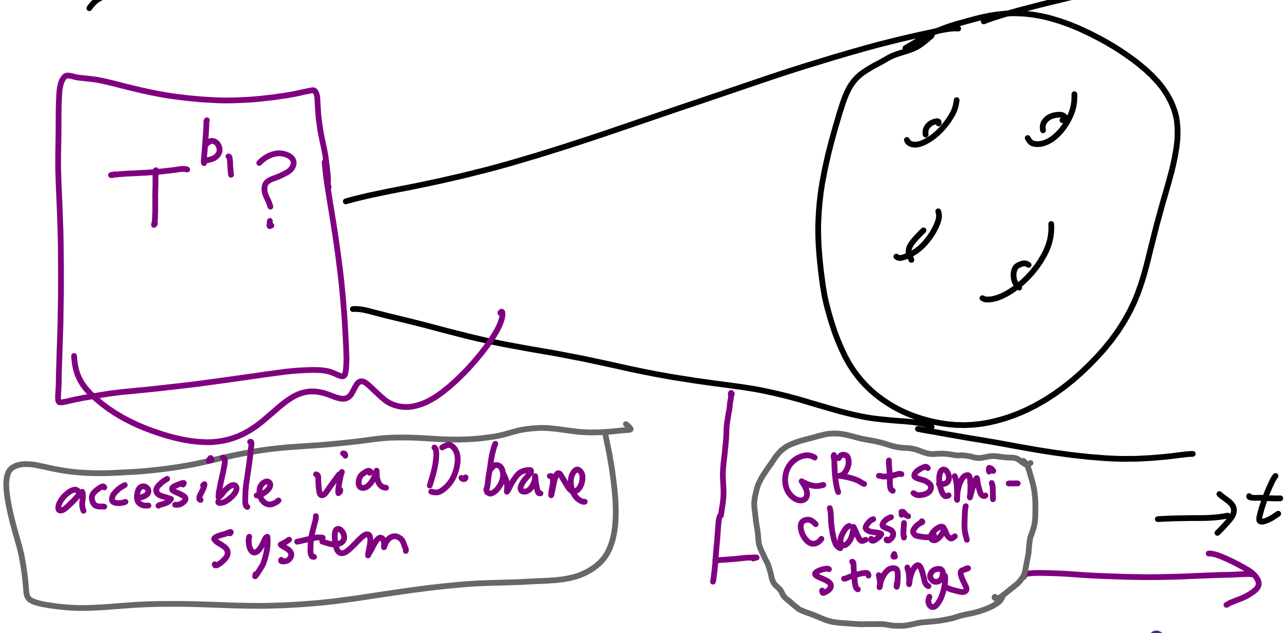
$$\underbrace{\int dm\, \rho(m)\, e^{-\pi\tau_2\alpha' m^2}}_{\text{Saddle point}} \underset{\tau_2\to 0}{\sim} e^{\frac{\pi c_{eff}}{6\tau_2}}$$

$$2m_*\pi\tau_2\alpha' = \sqrt{\alpha'}\,\pi\sqrt{\frac{2c_{eff}}{3}}$$

So exponential growth of (conjugacy classes of) π_1 will yield a contribution to c_{eff} from the sum over winding modes.

Outline of rest

$$C_{eff} = \frac{3\, g'(n-1)^2}{2t}$$



① Explicit computation of C_{eff} and check of Modular invariance for t-dependent $M_n = \mathbb{H}_n/\Gamma$

$$\underset{(UV)}{C_{eff}} \xleftrightarrow{\tau \to -\frac{1}{\tau}} \underset{(IR)}{\text{pseudo-tachyons}}$$

② • Nilmanifold case: power law growth, negative scalar curvature • Solmanifold case • old San Marcos Road case

③ very small radius and the Jacobian/Albionese variety (T^{b_1}) via AdS/CFT

A consistent worldsheet CFT must satisfy 1-loop modular invariance.

Recall standard static cases

$$\int \frac{d^2\tau}{\tau_2^2} \underbrace{Z_1(\tau)}_{= Z_1(-\frac{1}{\tau})} = \mathrm{Tr} \int \frac{d^2\tau}{4\tau_2} \underbrace{q^{\overbrace{L_0}^{L_0 = \tilde{L}_0 = (k^2+m^2)\frac{\alpha'}{4}}} \bar{q}^{\tilde{L}_0}}_{e^{-2\pi\tau_2 \mathcal{H}} \; e^{2\pi i \tau_1 P}}$$

$$Z_{IR} \sim e^{-\pi \tau_2 \alpha' m^2_{min}} \qquad \tau_2 \to \infty$$

$$Z_{UV} \sim e^{-\frac{\pi \alpha' m^2_{min}}{\tau_2}} \qquad \tau_2 \to 0$$

$$= e^{\frac{\pi c_{eff}}{6\tau_2}}$$

$\Rightarrow$ $c_{eff} \neq 0$ means IR divergence in path integral (Kutasov, Seiberg). However, in time dependent context, the instability does not always cause significant back reaction.

(Aharony, ES; cosmo theorists, early timelike linear dilaton works, ...)

Consider compact hyperbolic manifold $M_n = \mathbb{H}_n / \Gamma$ $\leftarrow ds^2 = dy^2 + \sinh^2 y \, d\Omega^2$

$$ds^2 = -dt^2 + t^2 ds^2_{\begin{cases}\mathbb{H}_n \\ \mathbb{H}_n/\Gamma \equiv M_n \text{ compact}\end{cases}} + ds^2_{\perp, q-n}$$

$\mathbb{R}^{n,1}$ (negative curvature FRW)

some Ricci-flat space with moduli

Satisfies Einstein's equations = leading order worldsheet $\beta = 0$ equations

↪ time-dependent worldsheet CFT

Modular invariance $\Rightarrow$

$$\begin{cases} \mathbb{H}_n \text{ theory}: \ c_{eff} = 0 \Leftrightarrow m^2_{min} = 0 \quad \text{gap in spatial modes} \\ \mathbb{H}_n/\Gamma \text{ theory}: \ c_{eff} \neq 0 \Leftrightarrow m^2_{min} < 0 \quad \text{zero mode normalizable} \end{cases}$$

A check of modular invariance for $M_n = \mathbb{H}_n/\Gamma$:

First consider covering space $(\mathbb{R}^{n,1})$

$$ds^2 = -dt^2 + t^2 ds^2_{\mathbb{H}_n} + ds^2_\perp$$

UV $\quad C_{eff} = 0$ (SUSY cancellation)

IR $\quad m^2_{min} = 0$: gapped spectrum on $\mathbb{H}_n$ spatial slices, compensated by t-dependence, as follows.

$$ds^2_{\mathbb{H}_n} = dy^2 + \sinh^2 y \, d\Omega$$

$$\eta = u(t)\, Y(\Omega)\, f(y)$$

$$\nabla^2 \eta = -k^2 \eta$$

$$\underset{y\to\infty}{\longrightarrow} \eta \sim e^{\lambda_\pm y} \qquad \lambda_\pm = \frac{-(n-1)}{2} \pm \sqrt{(n-1)^2 - 4k^2}$$

Normalizability: $\int^\infty \sqrt{G}\, f^*(y) f(y) = 1 \Rightarrow \boxed{k^2 > \frac{(n-1)^2}{4}}$ gap

$ds^2 = -dt^2 + t^2 ds^2_{\mathbb{H}_n}$ is flat $\mathbb{R}^{3,1}$, so gap must be compensated by t dependence.

The full t-dependent Laplacian is

$$\mathcal{H} = \nabla^2 = \frac{1}{t^2}\left(-(t^2\partial_t^2 + n t\partial_t) + \nabla^2_{\mathbb{H}_n}\right)$$

Work in basis of modes satisfying

$$\nabla^2 \eta = \frac{\omega^2 - k^2 + \frac{(n-1)^2}{4}}{t^2}\,\eta \qquad \eta = t^{\frac{1-n}{2}} e^{i\omega \ln t} f_{k,L}(y)\, Y_L(\Omega)$$

$\to$ IR limit of partition function is

$$\int d^dx \sqrt{-g}\,\Lambda(t) = \int d^dx\sqrt{-g}\,\mathrm{Tr}\log(\mathcal{H})$$

$$= \int dt\, dy\, d\Omega\, t^n \sinh^{n-1} y\, V_{S^{n-1}} \int \frac{d\tilde{\omega}}{2\pi} \sum_{L,k^2 > \frac{(n-1)^2}{4}} f^* f\, Y^* Y$$

$$\times \int \frac{d\tau_2}{\tau_2}\, e^{-\pi\alpha' \frac{\tau_2}{t^2}\underbrace{\left(\tilde{\omega}^2 + k^2 - \frac{(n-1)^2}{4}\right)}_{m^2_{min} = 0 \text{ as it must be for flat space}}}$$

So far recovered expected behavior for flat $\mathbb{R}^{n,1}$ (sliced by $\mathbb{H}_n$).

We can now see the effect of the projection Γ: $M_n = \mathbb{H}_n/\Gamma$

Modular invariance requires an $\underline{\text{IR}}$ divergence as $\tau_2 \to \infty$, since $c_{\text{eff}} \neq 0$

IR: no gap in spatial spectrum (k=0 mode is normalizable)

$$\Rightarrow \quad m^2_{min}(t) = -\frac{(n-1)^2}{4t^2} < 0$$

$$Z_1\Big|_{\tau_2 \to \infty} \sim \int dt\, t^n e^{\pi\alpha' \tau_2 \frac{(n-1)^2}{4t^2}} \quad ✓$$

This is consistent with modular invariance, but does $m^2_{min} < 0 \Rightarrow$ catastrophic instability?

cf Kutasov, Seiberg

<u>No</u> : This 0-mode with $m^2_{min} = -\frac{(n-1)^2}{4t^2}$ condenses, but does <u>not</u> cause significant back reaction :

$$S_0 = \int dt\, t^n\, \dot{\eta}^2$$

Solutions $\langle \eta \rangle = \eta_0 + \frac{\eta_1}{t^{n-1}}$

$\Rightarrow$ • energy density $\rho_\eta \sim \frac{1}{t^{2n}} \ll R \sim \frac{1}{t^2}$

so negligible effect on metric

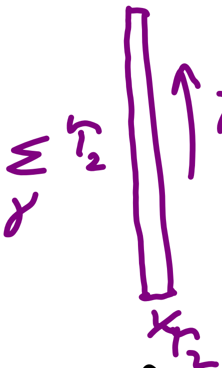
• Hubble friction damps η motion

• $\langle \eta \rangle$ doesn't change c_{eff}

Many smooth t-dependent backgrnds have this feature (e.g. inflationary density perturbations): "pseudotachyons"

(... O. Aharony & ES)

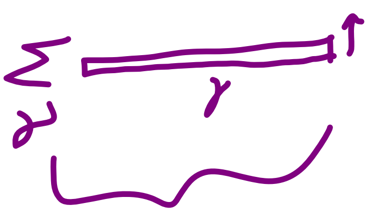
The IR & UV limits of Z_1 for the compact space $M_n = \mathbb{H}_n/\Gamma$ are thus



$$Z_1\Big|_{\tau_2\to\infty} \sim \int dt\, t^n e^{\pi\alpha'\tau_2\frac{(n-1)^2}{4t^2}}$$

pseudo-tachyon

Modular invariance $\Rightarrow$



$$Z_1\Big|_{\tau_2\to 0} \sim \int dt\, t^n e^{\frac{\pi\alpha'(n-1)^2}{4t^2\tau_2}}$$

winding modes (exponential growth)

$$\Rightarrow \boxed{c_{eff} = \frac{3\alpha'(n-1)^2}{2t^2}}$$

Can indeed write $s\to\infty$ limit of path integral as sum over windings (Selberg Gutzwiller ...)

$$\int d\ell \frac{1}{(\sinh\frac{\ell}{2})^{n-1}}\rho(\ell)\, e^{-\frac{\alpha'\ell^2}{4\pi}\tau_2}$$

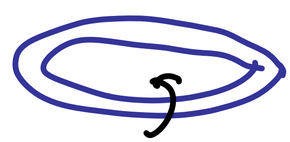
$\leftarrow e^{(n-1)\frac{\ell}{t}}$ (cf Margulis)

SUSY structure

If we apply this in the "critical" superstring, must address issue of (asymptotic) SUSY.

- Fixed winding, $L_0 \to \infty$: SUSY breaking sensed by paths around nontrivial cycles introduced by orbifolding, suppressed by e^{-tm}

- $\ell \to \infty$, L_0 fixed: ~~SUSY~~ sensed directly



Also, enhanced IR & UV divergences in the bosonic case require the winding string contribution.

- Nil manifold : power law growth

The geometry

$$ds^2_{Nil} = \left(dx + \tfrac{1}{2} y\,dy - \tfrac{1}{2} z\,dy\right)^2 + \left(dy^2 + dz^2\right)$$

has $R < 0$ const, but indefinite sectional curvatures.

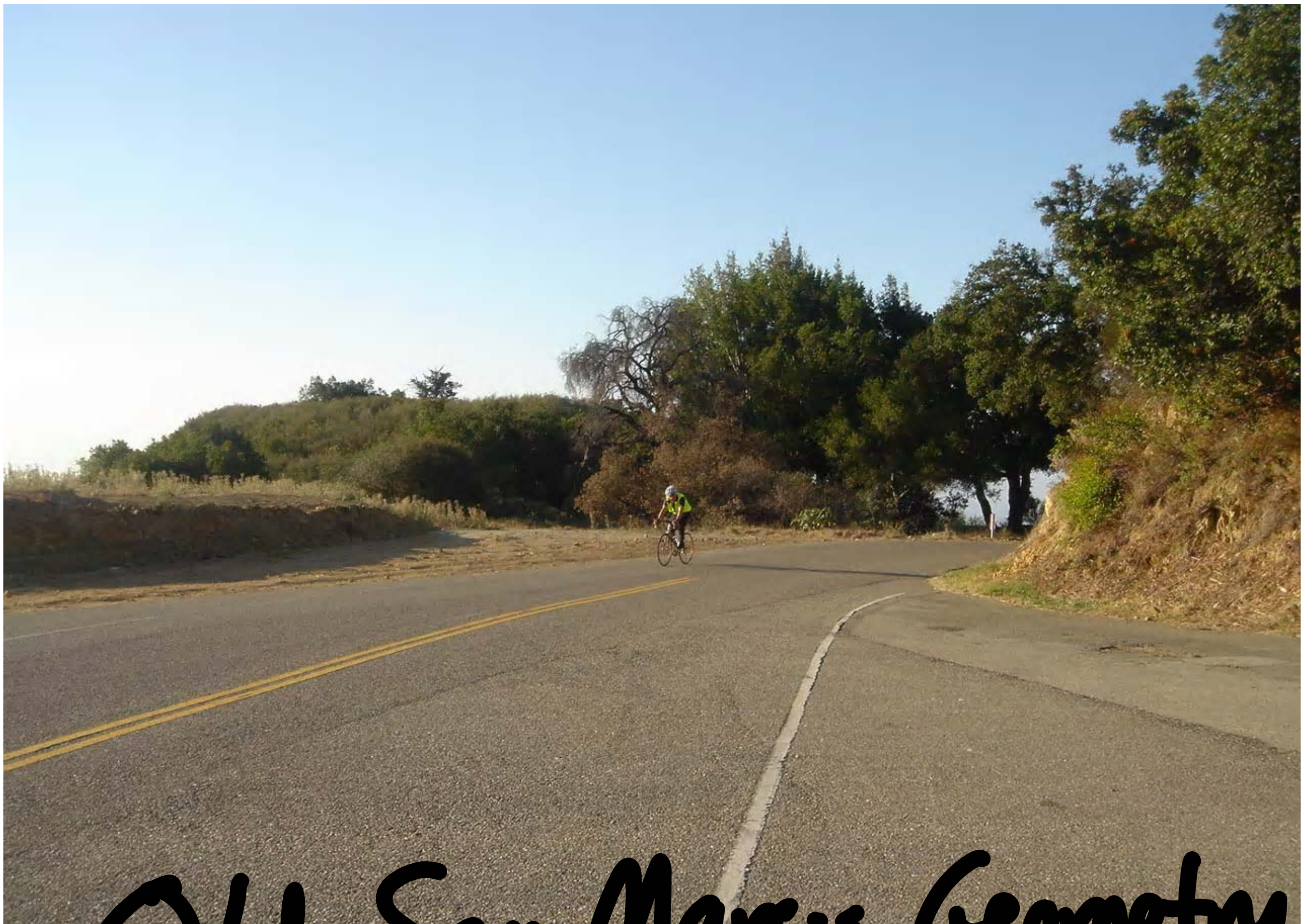
- π_1 is of power law growth, but higher power than any non-negatively curved space
- Late-time Ricci flow solution (= time evolution for $D \gg D_c$ theory) yields a mode spectrum without stronger IR divergence.

- Sol manifolds $T^2 \overset{\Gamma}{\to} \tilde{T}^2$ $\xrightarrow{\quad S^1 \quad}$ Γ = hyperbolic element of $SL(2,\mathbb{Z})$
  - π_1 has exponential growth
  - no gap, but still find enhanced IR divergence

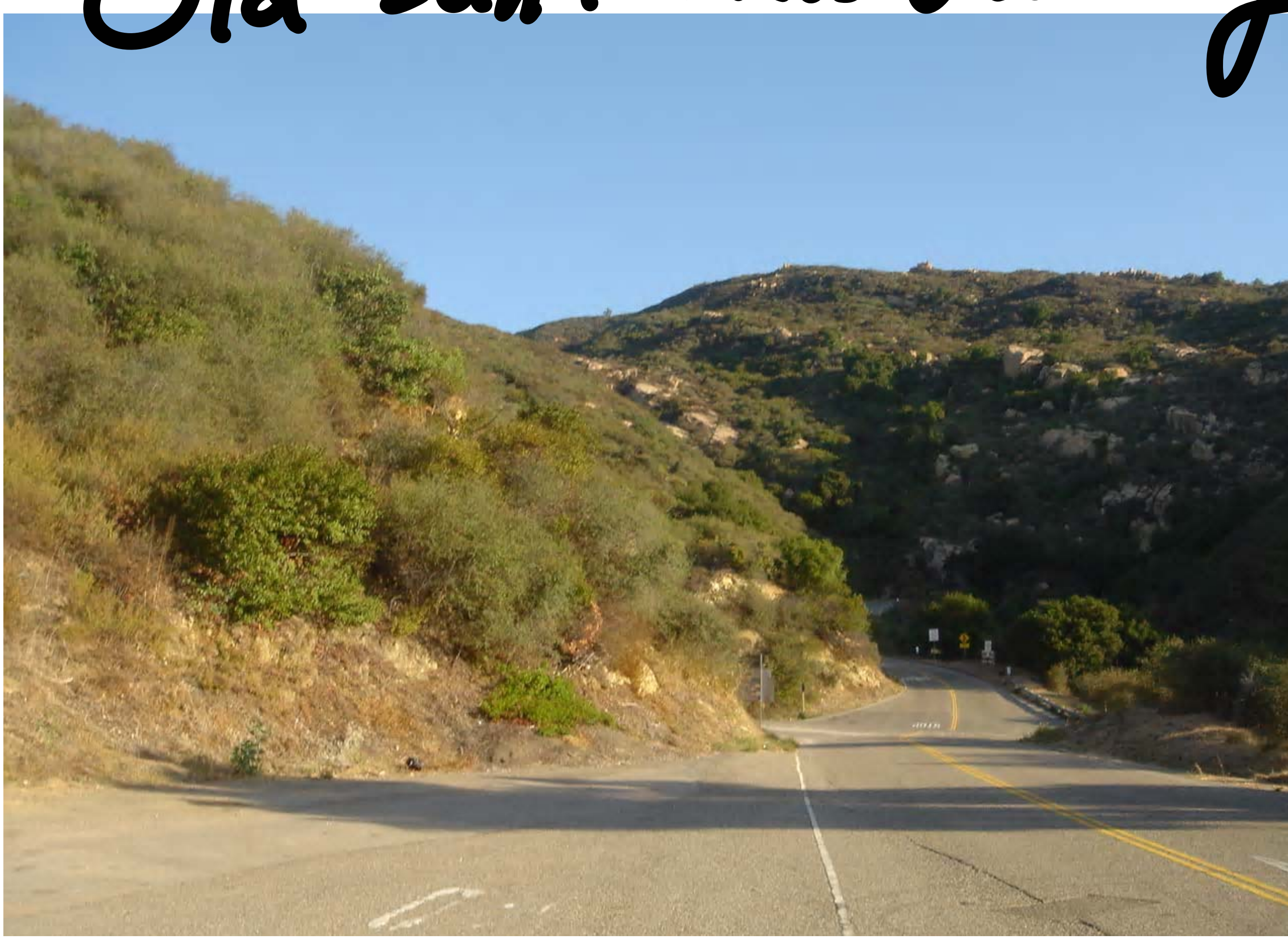
- More generally, string backgrounds with sectional curvature < 0 are effectively supercritical

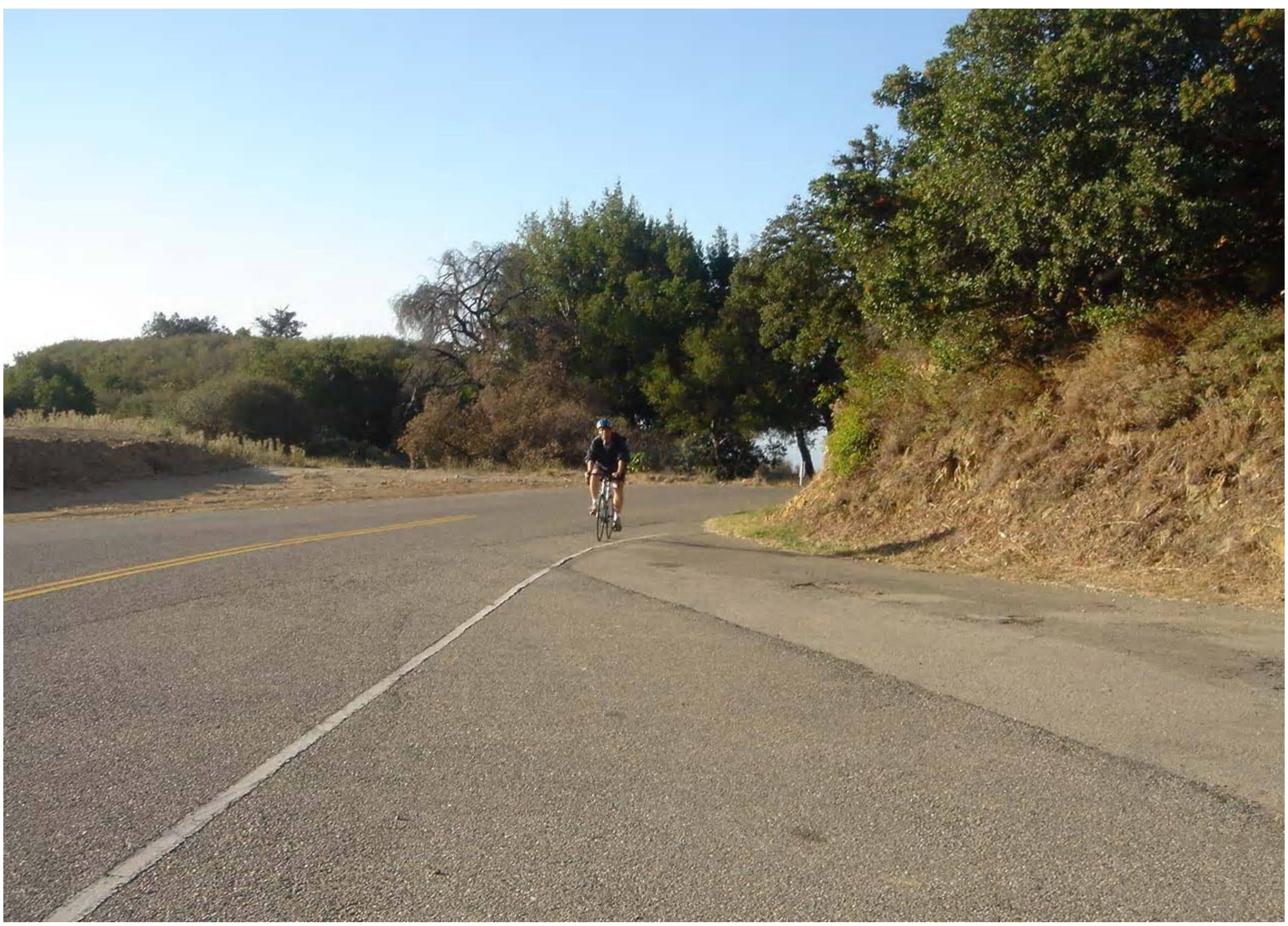
$$
\begin{tikzcd}
\text{Geometry} \arrow[r, leftrightarrow, "\text{Milnor,}\ldots"] \arrow[d, Rightarrow] & \text{Topology} \arrow[d, Rightarrow] \\
\begin{array}{l}\text{pseudotachyonic} \\ \text{perturbation} \\ \text{spectrum}\end{array} \arrow[r, leftrightarrow, "\tau \leftrightarrow -\frac{1}{\tau}"] & c_{eff} > 0 \text{ dimensionality}
\end{tikzcd}
$$

- All results consistent with idea that positive potential energy $\leftrightarrow$ more worldsheet degrees of freedom



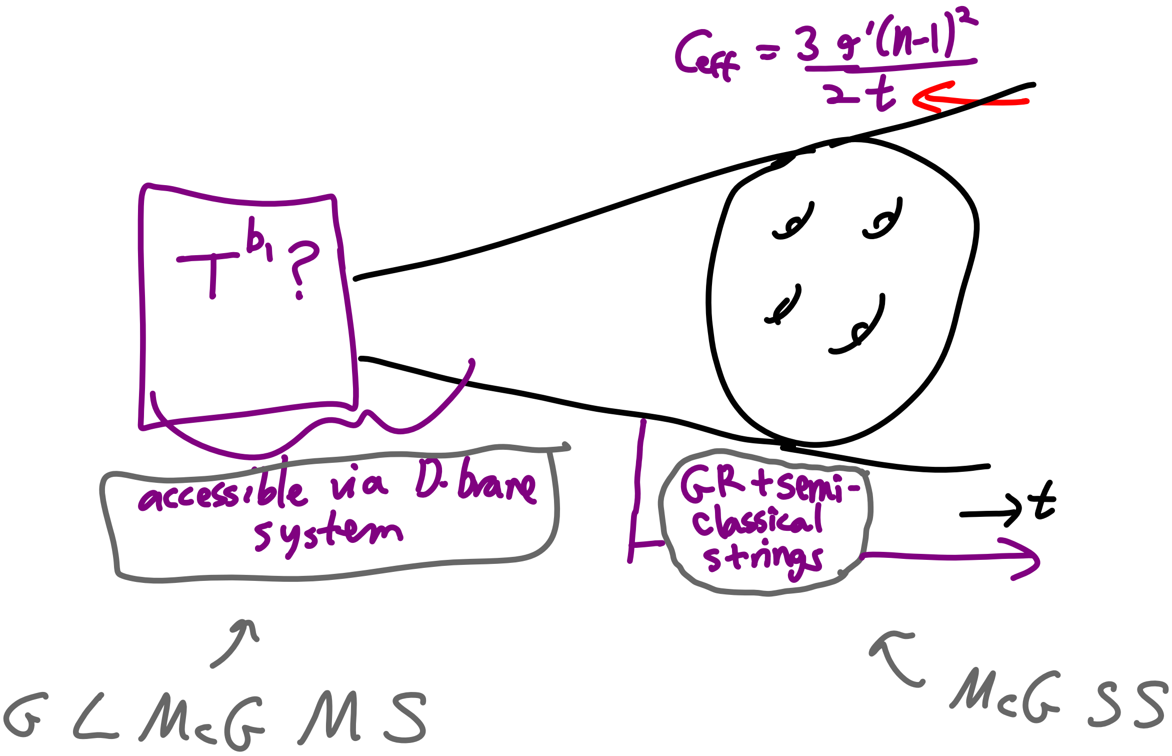
Old San Marcos Geometry





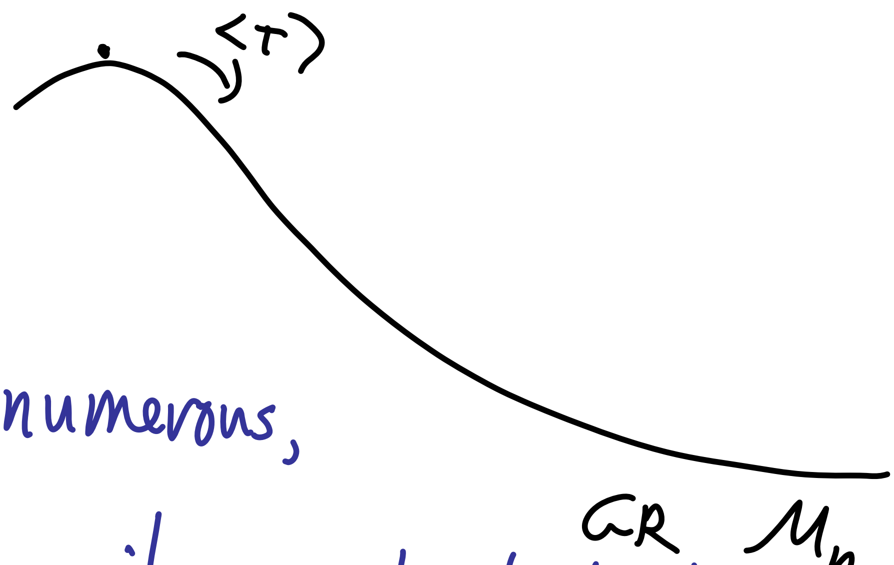
- as M_n shrinks, how large does c_{eff} get?
- Is there a way to control this regime?

No universal answer, but there is a natural starting point, Moreover one which appears to be amenable to AdS/CFT like definition



There are many ways to UV complete the nonlinear sigma model on M_n

e.g. gauged linear σ-model ((2,2) SUSY) describing embedding of M_n in higher-D space, with a tachyon (relevant perturbation) effecting the transition.



Though numerous,

Models easily constructed this way are <u>not</u> generic ...

... e.g. in case M_n = Riemann surface,
The Jacobian torus may provide a much more natural starting point. For starters, it

- is classical moduli space of D-probe

hol. 1-forms ω_i $i = 1 \dots h$

$$\left(\int_{p_0}^{p} \omega_1 , \dots , \int_{p_0}^{p} \omega_h \right)$$ traces out T^{2h}

- is singled out Mathematically :

$$T^n \longrightarrow \text{Jacobian}$$
$$\downarrow$$
$$\Sigma_h$$

- is singled out physically by symmetries : winding charge preserved by this map (1-cycles) → (1-cycles)

- Long wound strings trace out a lattice random walk in $2h$ dimensions.

Toward "D duality":

☆ We find an AdS/CFT setup where a small negatively curved space (= supercritical background of string theory) seems definable via a dual large-N QFT whose IR regime corresponds to the small $\mathcal{M}_n$ and has moduli space $(T^{b_1})^N/S_N$

Take e.g. $AdS_5 / \mathcal{N}=4$ (S)U(N) SYM on Poincare Patch, slice the $\mathbb{R}^{3,1}$ slices with $\mathbb{H}_3$, and mod by Γ to obtain compact space

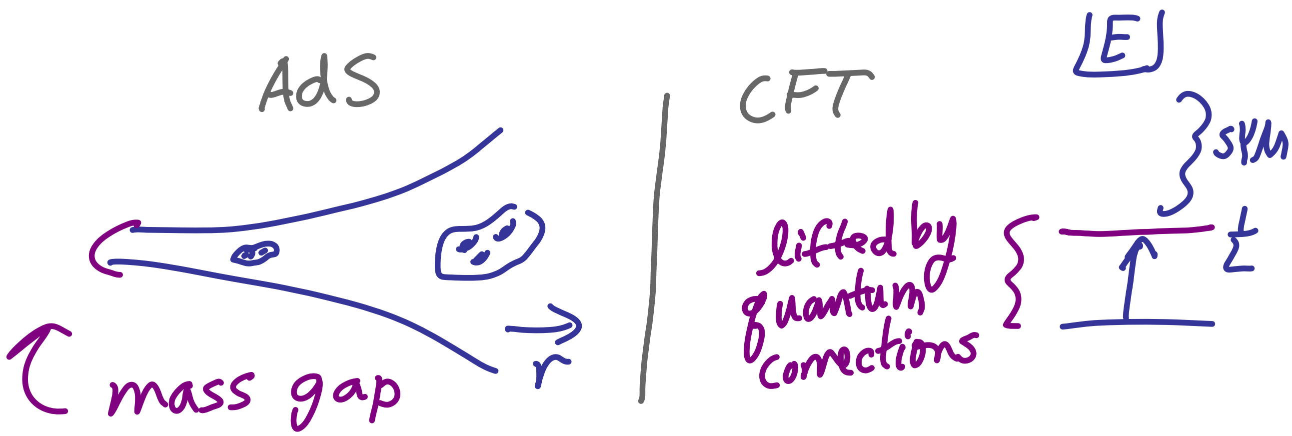
$$ds_0^2 = \left(\frac{r}{\ell}\right)^2 \left(-dt^2 + t^2 ds^2_{\underset{M_3 =}{\mathbb{H}_3/\Gamma}}\right) + \frac{\ell^2 dr^2}{r^2} + d\Omega_5$$

$$\ell^4 = (4\pi g_s N)\, \ell_s^4 = \lambda\, \ell_s^4$$

$\Rightarrow$ curvature radius of M_3 is $L(t) \sim t$. That is, string theory on ds_0^2 is equivalent to $\mathcal{N}=4$ SYM on $-dt^2 + t^2 ds^2_{M_3}$. Note M_3 proper size $L_{pr} \sim \frac{rL}{\ell}$ shrinks on the gravity side both toward $r \to 0$ and toward $t \to 0$.

$\Rightarrow$ ~~A~~ decoupling is a non-issue, so can focus on the IR physics, and on t evolution

So far, although classically the IR yields $(T^{b_1})^N/S_N$ as moduli space, quantum corrections will lift Wilson lines to scale $1/L$

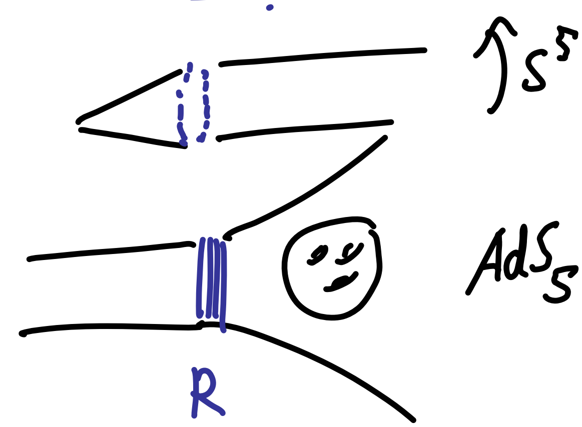


To access very small radius, go out on approximate Coulomb branch: cf Horowitz & E.S. '06 suppresses quantum corrections lifting moduli space spherical shell of N D3Bs at $r = R$.

$$ds^2 = h^{-1}(r)\left[-dt^2 + t^2 ds^2_{M_3}\right] + h(r)\left[dr^2 + r^2 d\Omega_5\right]$$

$$h(r) = \begin{cases} \dfrac{\ell^2}{r^2} & r > R \\[2ex] \dfrac{\ell^2}{R^2} & r < R. \end{cases}$$

Schematically:



inside shell, L_{pr} fixed at $L_{pr} = \dfrac{RL}{\ell}$

$\longrightarrow r$

There is a potential on the Coulomb branch, addressed below, so the shell (slowly) shrinks

On the QFT side, we can use this to suppress contributions to the potential on the moduli space V:

$U(N) \to U(1)^N$

Consider a given $U(1)$ factor. From the geometry of the shell, we can determine the number n of off-diagonal modes ("W bosons") have mass $m_W < \frac{1}{L}$. Only these contribute appreciably to V:

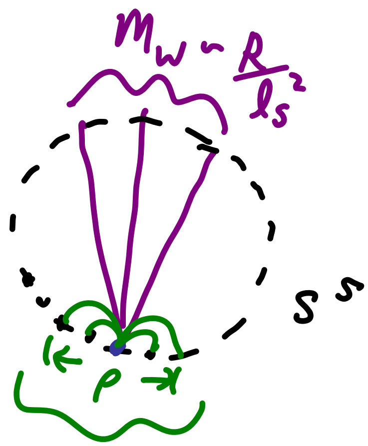
$$(*) \qquad V \sim \frac{g_s n}{L^2} a^2 + \frac{g_s n}{L^2} \mathcal{a}^2$$

a ← canonically normalized Wilson line scalar.

$\mathcal{a}$ ← Coulomb branch scalar

We will arrange $m_a^2 = \frac{g_s n}{L^2} << \frac{1}{L^2}$ so that we can preserve an IR region in the QFT, hence accessing the <u>small</u> M_3 regime.

N DBs are distributed uniformly on S^5 of radius $\frac{R}{l_s^2}$ in field space.



$$m_W < \frac{1}{L} \Rightarrow \rho = \frac{l_s^2}{L}$$

n = # of DBs close enough to a given one to yield $m_W < \frac{1}{L}$

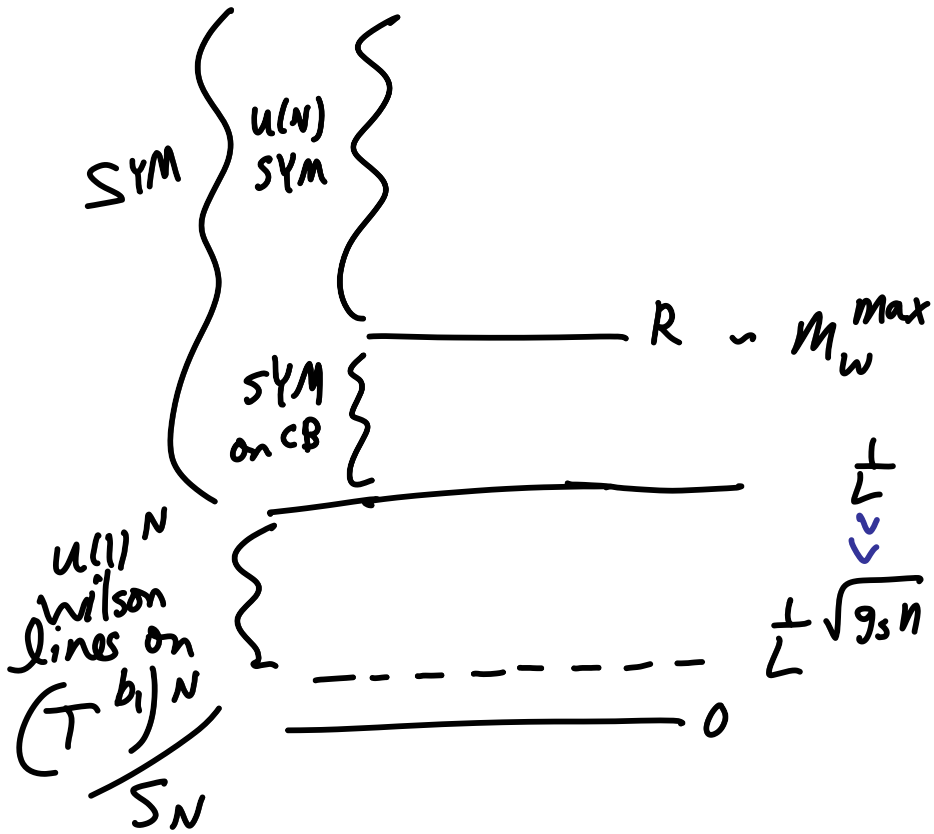
$$= N\left(\frac{\rho}{R}\right)^5 = N\frac{l_s^{10}}{(RL)^5}$$

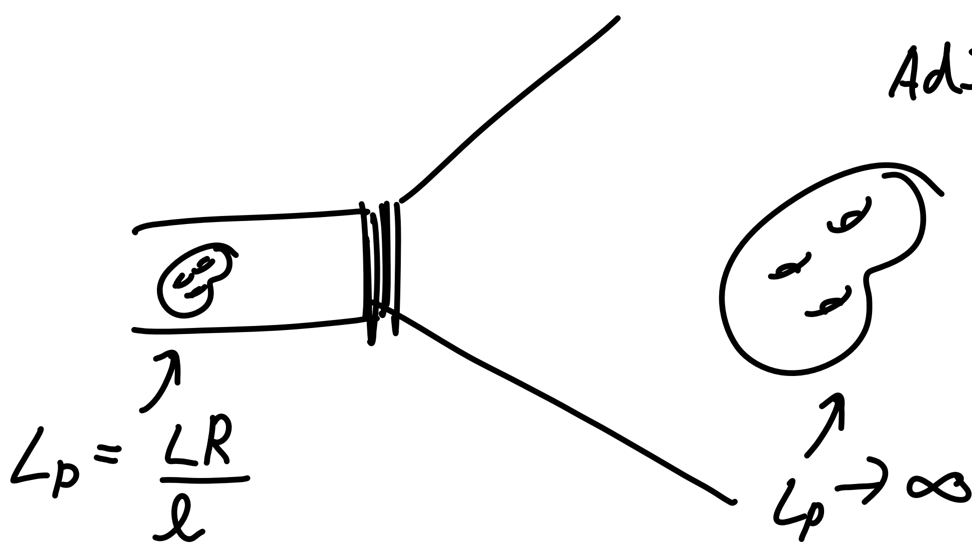
From $(*)$, if $n < N$ we can obtain

m_a^2, $m_a^2 \sim \frac{g_s n}{L^2} \ll L$ by taking

$$g_s n \ll 1 \ll (g_s N = 1)$$

So in the QFT we have energy scales





The shell contracts as a function of time t, governed by the equation

$$\ddot{\varphi} + \frac{3}{t}\dot{\varphi} = -\frac{\partial V}{\partial \varphi} \sim -\frac{g_s n \varphi}{t^2}$$

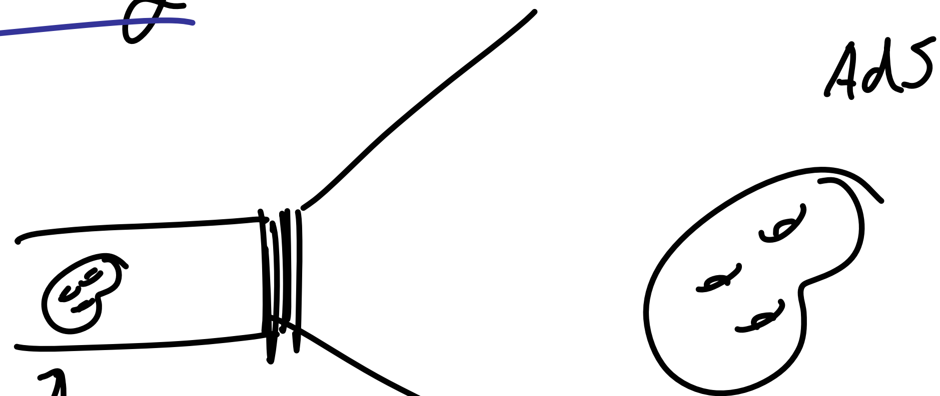
Hubble friction from negative curvature

Solution $\varphi = \varphi_+ t^{-1+\sqrt{1-g_s n}} + \varphi_- t^{-1-\sqrt{1-g_s n}}$

$$\simeq \varphi_+ t^{-\frac{1}{2}g_s n} + \varphi_- t^{-2}$$

$\boxed{\text{So at late times, the field is nearly constant because of Hubble friction + smallness of } g_s n\text{!}}$

· This AdS/CFT setup seems to access the regime interesting for D-duality:



$L_p = \dfrac{LR}{\ell}$ $\qquad$ $L_p \to \infty$

$\star$ T^{b_1} arises naturally in IR

D duality

$\langle T \rangle$ ← closed string tachyon grows with r and t

↪ gravity-side IR non-divergent

$\boxed{SC/T^{b_1}}$

In this theory, have quantum effects and $\langle T \rangle$ effects, to match to above...

Summary / Remarks

- Negative curvature & compactness $\Rightarrow$ Rich topology $\Rightarrow$ higher effective dimensionality (exponential density of winding strings; T^{b_1} in D-brane systems ...)

$\Bigg\}$ Novel mechanism for growing dimensions; suggests new "D duality"

- In string theory, "n-manifolds" are really $n + c_{eff}$-"dimensional"
  ↪ String theory probes geometry, topology, and dimensionality differently from point particle geometry
- applies to geometric group theory questions like intermediate growth

- String theory fits well with curved compactifications.

Phenomenology $\Rightarrow$ $\mathcal{N}=0$ or spontaneously broken $\mathcal{N}=1$ susy in 4d

Statistics, measure questions require both.

Amusing pattern in dS models:

ES, MSS, SS $m_{susy} \gtrsim m_{KK}$	KKLT... DK review $m_{susy} < m_{KK}$
• perturbative forces sufficient $\Rightarrow$ Fischler-Susskind worldsheet CFT?	• non-perturbative effects integral
• dS $\Rightarrow$ pseudo-tachyon	• dS $\Rightarrow$ pseudo-tachyon
• $c_{eff} > 0$ ($D>10$ models; Riemann surface models)	• c_{eff} not defined (also CY w/ H flux has subexponential growth)

- No observational signatures
  (yet ...)
  - what cosmological phases arise from curved compactifications?
    - relation to topology, geometric group theory?