Constructing 3 family models from the E(8)xE(8) heterotic string

Outline

- phenomenology
- orbifold GUTs
- heterotic string
    - 4D Pati-Salam
    - Discrete non-Abelian flavor sym's
    - Mini-landscape search for MSSM

Phenomenology

- charge quantization
- gauge coupling unification
- Yukawa unification
- neutrino masses

$$q = \begin{pmatrix} u & u & u \\ d & d & d \end{pmatrix} \qquad \bar{u} = \begin{pmatrix} \bar{u} & \bar{u} & \bar{u} \end{pmatrix} \qquad \bar{d} = \begin{pmatrix} \bar{d} & \bar{d} & \bar{d} \end{pmatrix}$$

$$l = \begin{pmatrix} \nu \\ e \end{pmatrix} \qquad \bar{e}$$

$$Q_{EM} = T_L^3 + Y/2$$

$$Q = \begin{pmatrix} u & u & u & \nu \\ d & d & d & e \end{pmatrix} \qquad \bar{Q} = \begin{pmatrix} \bar{u} & \bar{u} & \bar{u} & \bar{\nu} \\ \bar{d} & \bar{d} & \bar{d} & \bar{e} \end{pmatrix}$$

$$Y = (B - L) + 2T_R^3$$

Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$

$$SO(10) \to SU(5)\times U(1)$$

$$SO(10) \to SU(4)\times SU(2)_L \times SU(2)_R$$

$$16 \to 10_1 + \bar{5}_{-3} + 1_5$$

$$16 \to Q + \bar{Q}$$

$$16 / \overline{16} = (\pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}) \quad \text{even/odd no. - signs}$$

$$45 = (\underline{\pm 1, \pm 1, 0, 0, 0})$$

$$10 = (\underline{\pm 1, 0, 0, 0, 0})$$

Cartan-Weyl

Charge quantization

State	Y $= \frac{2}{3}\Sigma(C) - \Sigma(W)$	Color C spins	Weak W spins
$\overline{\nu}$	0	+ + +	+ +
$\overline{e}$	2	+ + +	− −
u_r	$\frac{1}{3}$	− + +	+ −
d_r		− + +	− +
u_b		+ − +	+ −
d_b		+ − +	− +
u_y		+ + −	+ −
d_y		+ + −	− +
$\overline{u}_r$	$-\frac{4}{3}$	+ − −	+ +
$\overline{u}_b$		− + −	+ +
$\overline{u}_y$		− − +	+ +
$\overline{d}_r$	$\frac{2}{3}$	+ − −	− −
$\overline{d}_b$		− + −	− −
$\overline{d}_y$		− − +	− −
ν	−1	− − −	+ −
e		− − −	− +

spinor repsn. of SO(10)

Phenomenology

- charge quantization
- gauge coupling unification
- Yukawa unification
- neutrino masses

Gauge coupling unification

$$M_G \approx 3\times10^{16} \quad \text{GeV}$$

Phenomenology

- charge quantization
- gauge coupling unification
- Yukawa unification
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Yukawa unification

$SU(5)$ $\lambda_u 10\ 10\ \mathrm{H_u} + \lambda_d 10\ \bar{5}\ \mathrm{H_d} + \lambda_\nu \bar{5}\ 1\ \mathrm{H_u}$

$$\lambda_{\mathrm{t}} \neq \lambda_{\mathrm{b}} \neq \lambda_{\nu}$$

$PS / SO(10)$ $\lambda\ \bar{\mathrm{Q}}\ \mathrm{H}\ \mathrm{Q}$

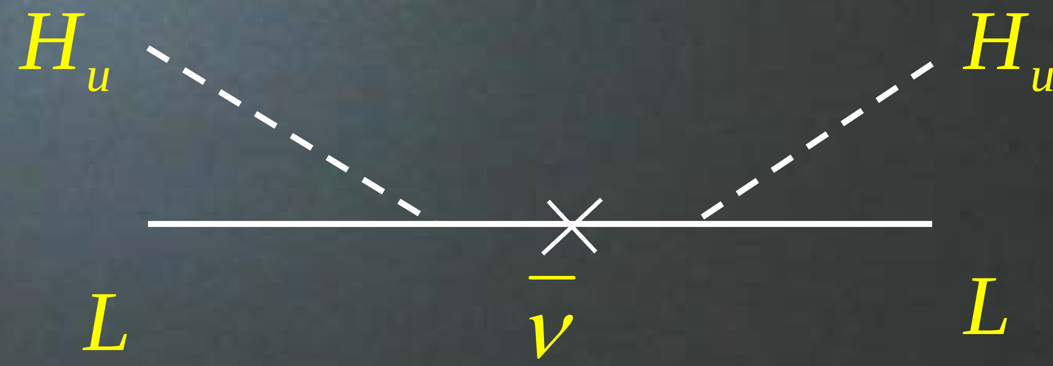
$$\lambda_t = \lambda_b = \lambda_\tau = \lambda_\nu \equiv \lambda$$

Phenomenology

- charge quantization
- gauge coupling unification
- Yukawa unification
- neutrino masses

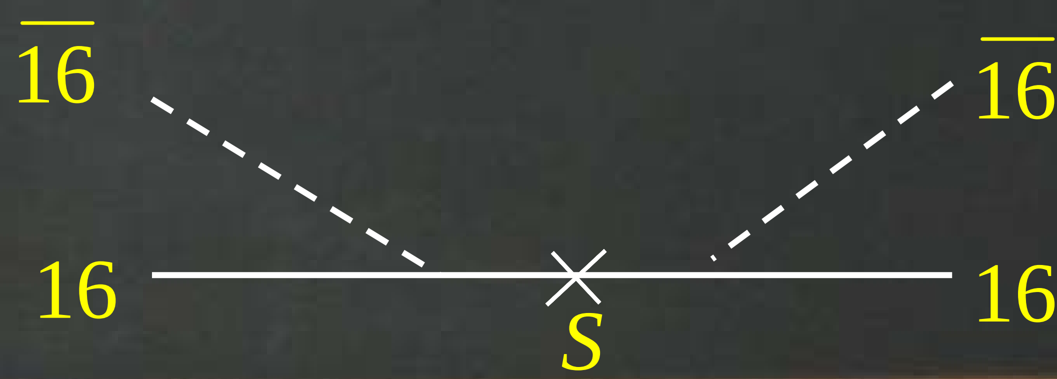
neutrino masses

$$\frac{\lambda_\nu^{\,2}}{M_{\bar{\nu}}}\left(H_u L\right)$$



$$\lambda_\nu \simeq \lambda_t \quad \Rightarrow \quad \mathrm{M}_{\bar{\nu}} \approx 10^{14}\ \mathrm{GeV} \ \simeq \ 10^{-2}\ \mathrm{M_G}$$

$$M_{\bar{\nu}} \approx \frac{M_G^{\,2}}{M_P}$$



Phenomenology

- charge quantization
- gauge coupling unification
- Yukawa unification
- neutrino masses

⇒ **SO(10) SUSY GUT w/ L.E. SUSY**

Quark & Lepton masses and mixing

Hierarchical $\lambda\, q_3\, \overline{u_3}\, H_u \qquad \lambda \sim O(1)$

$$q_i\, \overline{u_j}\, H_u \left(\frac{S}{M_P}\right)^{n(i,j)}$$ Froggatt-Nielsen

Flavor symmetry breaking

$U(1)$ or non-Abelian SU(2), SU(3)

$S_3 \approx D_3,\ D_4,\ A_4,\ \Delta(27),\ \Delta(54)$

SUSY flavor problem

~125 soft SUSY breaking parameters !

⟹ **FCNC**

Solution

1. heavy 1st & 2nd generation scalars
2. degenerate squark & slepton at M_G
3. alignment of fermion/sfermions

2 & 3 non-Abelian flavor sym.

Phenomenology (summary)

SO(10) SUSY GUT w/ L.E. SUSY

Discrete non-Abelian flavor symmetry

4 D GUT problems

GUT symmetry breaking

Higgs doublet – triplet splitting

Orbifold GUTs

Orbifold SUSY GUTs

- N=2 -> N=1 SUSY
- gauge symmetry breaking
- chiral gauge theory
- Higgs doublet-triplet splitting

All this first observed in heterotic string in 10D

$E_8 \times E_8$ heterotic string

Constructing MSSM / Caveats

SUSY solution at M_s

100's moduli (geometric & more)

gauge & Yukawa couplings fcns. of moduli

tune to desired values

moduli stabilization & SUSY breaking ??

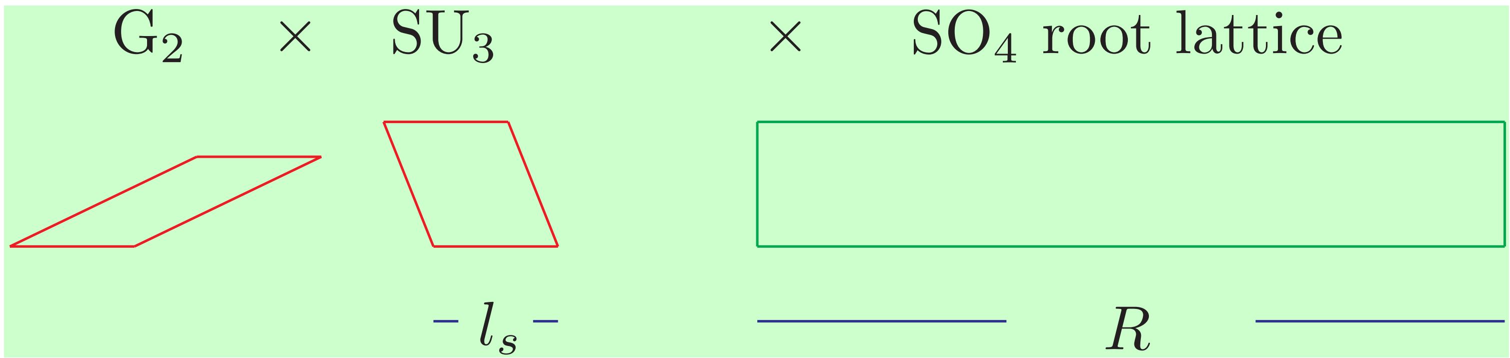
save for later

Kobayashi, SR, Zhang

Kobayashi, Nilles, Ploeger, SR & Ratz

LNRRRVW

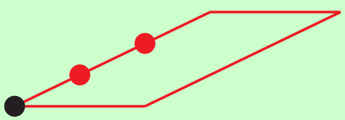
Compactify 6D on $(T^2)^3$

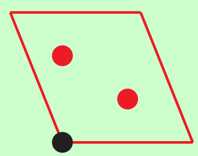


Then mod by $Z_6 = (Z_3 \times Z_2)$ and
Add Wilson lines

consistent with mod. inv. !

G_2 SU_3 $SO(4)$





$$V,\ \Sigma\ \in E(6)$$

$$(\mathbf{27} \oplus \overline{\mathbf{27}})$$

$$3(\mathbf{27} \oplus \overline{\mathbf{27}})$$

E_6 GUT in 5D

$$P \in E(8)$$

$$P = (n_1, n_2, \ldots, n_8), \ (n_1 + \tfrac{1}{2}, n_2 + \tfrac{1}{2}, \ldots, n_8 + \tfrac{1}{2})$$

$$n_i \in Z \quad (i = 1, \ldots, 8) \qquad \sum_{i=1}^{8} n_i = 2Z$$

Massless U sector $P^2 = 2$

GSO $\quad V_6 = \tfrac{1}{6}(2,2,2,0,0,0,0,0), \ \ V_3 = 2V_6$

$$P \cdot V_3 - r \cdot v_3 \in Z, \qquad r = (0, \underline{1, 0, 0})$$

$$v_3 = \tfrac{1}{3}(1, -1, 0) \qquad \text{twist}$$

Consider $P \cdot V_3 = Z$

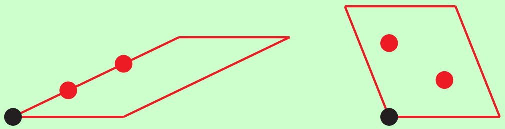
$P =$ $(0,0,0,\underline{\pm 1, \pm 1, 0, 0, 0})$ 45

$\pm(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \underline{\pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}})$ $16 \diagup 16$

$(\underline{\pm 1, \pm 1, 0}, 0, 0, 0, 0, 0)$

⇒ E(6) x SU(3) $V, \Sigma = 78$

Similarly $27, \quad \overline{27} \quad \in \quad U_1, U_2$



$V \in PS \quad (F_3^c + \bar{\chi}^c) \in \Sigma$

$$F_3 \in \mathbf{27} + \mathcal{H} \in \overline{\mathbf{27}}$$

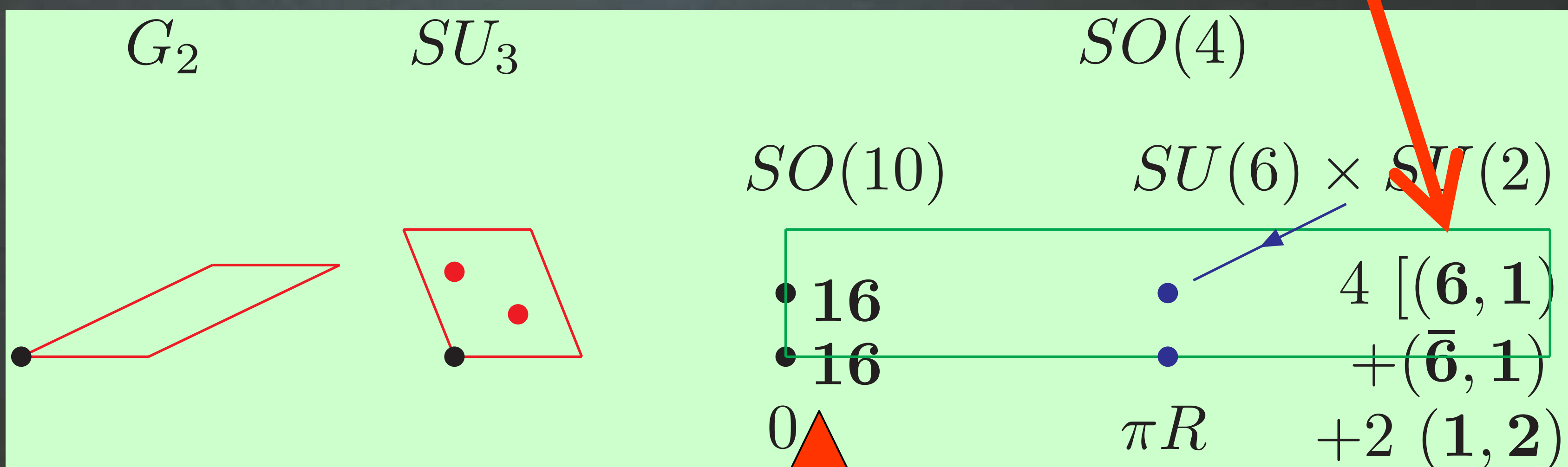
$2(\chi^c) + \bar{\chi}^c + 3C \in 3(\mathbf{27} \oplus \overline{\mathbf{27}})$

$E_6 \Rightarrow SO(10) \Rightarrow SU(4) \times SU(2)_L \times SU(2)_R$

$F_3 = (4,2,1), \quad F_3^c = (4^c,1,2), \quad H = (1,2,2)$

T_1 twisted sector

Vector-like exotics



2 Light fam's

gauge-Yukawa unification

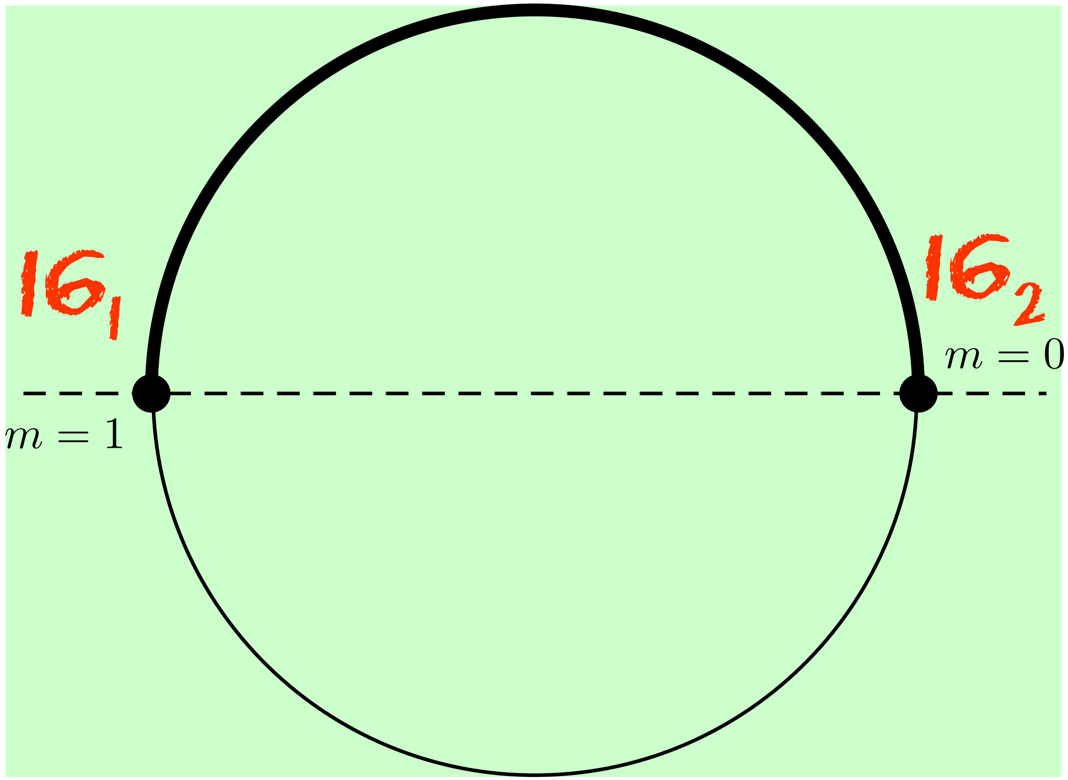
$$\frac{g_5}{\sqrt{\pi R}} \int_0^{\pi R} dy\, \overline{27}\, \Sigma\, 27 = g H F_3^c F_3$$

D_4 family symmetry

$$2\left(\chi^c + \overline{\chi^c}\right) \quad +3\ C \left(= T + \overline{T}\right)$$

Higgs for PS symmetry breaking

S_1/Z_2



D_4 family symmetry

$$D_4 = \{\pm 1, \pm\sigma_1, \pm\sigma_3, \mp i\sigma_2\}$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} : f_1 \leftrightarrow f_2 \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} : f_2 \leftrightarrow -f_2$$

geometry

space group sel. rule

$$\begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$$ doublet

$$f_3$$ singlet

Fermion mass hierarchy

PS breaking VEVs

$$O_i = \langle \chi^c_\alpha \; \bar{\chi}^c_i \rangle, \quad i = 1,2$$

- Fermion mass matrix [simple form]

$$(f_1 \;\; f_2 \;\; f_3) \; h \; \mathcal{M} \begin{pmatrix} f^c_1 \\ f^c_2 \\ f^c_3 \end{pmatrix}$$

$$\mathcal{M} = \begin{pmatrix} (O_2 \; \tilde{S}_e + S_e) & (O_2 \; \tilde{S}_o + S_o) & (O_1 \; O_2 \; \phi_e + \tilde{\phi}_e) \\ (O_2 \; \tilde{S}_o + S_o) & (O_2 \; \tilde{S}_e + S_e) & (O_1 \; O_2 \; \phi_o + \tilde{\phi}_o) \\ \phi'_e & \phi'_o & 1 \end{pmatrix}$$

Orbifolds preserving N=1 SUSY

(a) $\mathbb{Z}_N$

orbifold	twist
$\mathbb{Z}_3$	$(1,1,-2)/3$
$\mathbb{Z}_4$	$(1,1,-2)/4$
$\mathbb{Z}_6$-I	$(1,1,-2)/6$
$\mathbb{Z}_6$-II	$(1,2,-3)/6$
$\mathbb{Z}_7$	$(1,2,-3)/7$
$\mathbb{Z}_8$-I	$(1,2,-3)/8$
$\mathbb{Z}_8$-II	$(1,3,-4)/8$
$\mathbb{Z}_{12}$-I	$(1,4,-5)/12$
$\mathbb{Z}_{12}$-II	$(1,5,-6)/12$

(b) $\mathbb{Z}_N \times \mathbb{Z}_M$

orbifold	v^1	v^2
$\mathbb{Z}_2 \times \mathbb{Z}_2$	$(1,0,-1)/2$	$(0,1,-1)/2$
$\mathbb{Z}_2 \times \mathbb{Z}_3$	$(1,0,-1)/2$	$(0,1,-1)/3$
$\mathbb{Z}_2 \times \mathbb{Z}_4$	$(1,0,-1)/2$	$(0,1,-1)/4$
$\mathbb{Z}_2 \times \mathbb{Z}_6$	$(1,0,-1)/2$	$(0,1,-1)/6$
$\mathbb{Z}_2 \times \mathbb{Z}_6'$	$(1,0,-1)/2$	$(1,1,-2)/6$
$\mathbb{Z}_3 \times \mathbb{Z}_3$	$(1,0,-1)/3$	$(0,1,-1)/3$
$\mathbb{Z}_3 \times \mathbb{Z}_6$	$(1,0,-1)/3$	$(0,1,-1)/6$
$\mathbb{Z}_4 \times \mathbb{Z}_4$	$(1,0,-1)/4$	$(0,1,-1)/4$
$\mathbb{Z}_6 \times \mathbb{Z}_6$	$(1,0,-1)/6$	$(0,1,-1)/6$

Table 1: (a) $\mathbb{Z}_N$ and (b) $\mathbb{Z}_N \times \mathbb{Z}_M$ orbifold twists for 6D $\mathbb{Z}_N$ orbifolds leading to N=1 SUSY.

Orbifold + Twisted Sectors

$$x^i = x^i + n_a e_a^i, \quad (i = 1, .., d)$$

$$T^d = R^d \big/ \Lambda$$ torus

$n_a e_a^i \subset \Lambda$ lattice

$$\theta\Lambda = \Lambda \qquad \theta^N = 1$$

$$T^d \big/ \mathbf{Z}_N$$ orbifold

$$f = \left(\theta^k f\right) + \Lambda$$

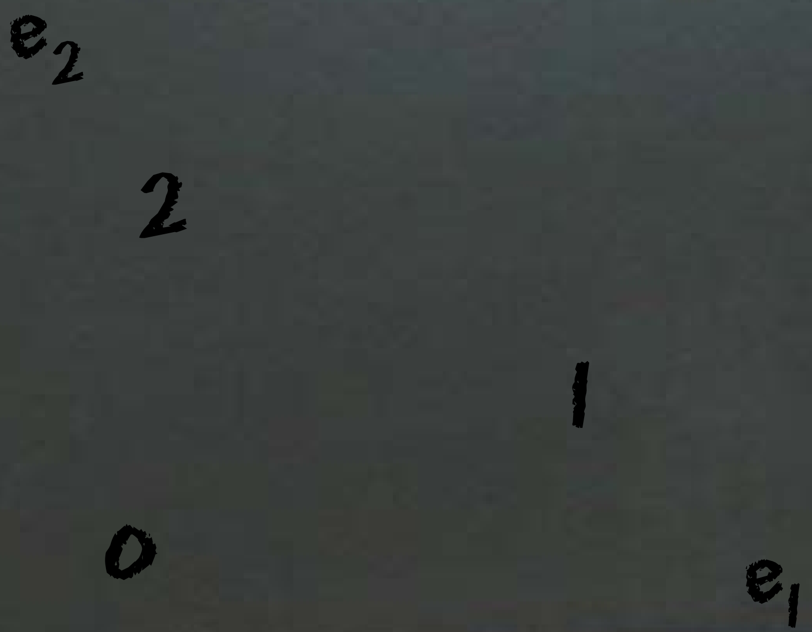
Fixed points, k^{th} twisted sector

$$\left(1 - \theta^k\right) f = \left(1 - \theta^k\right)\Lambda \equiv \Lambda_k$$

Conj. classes

T^2/Z_3 orbifold

$(1-\theta)\Lambda$ spanned by $3e_1,\ e_2$-e_1



fixed points

$$\left(\theta,\ m_1\ e_1\right)$$

$$m_1 = 0, 1, 2$$

Stringy Selection Rules

Eg. : Yukawa couplings of n states in first twisted sector

$$\prod_i^n \left(\theta,\ m_1^{(i)} e_1\right) = \left(\theta^n,\ \sum_i m_1^{(i)} e_1\right) = \left(1,\ (1-\theta)\Lambda\right)$$

$$\text{(I)} \quad n = 3\mathbf{Z}, \qquad \text{(II)} \quad \sum_i m_1^{(i)} = 0 \ (\text{mod } 3)$$

Space group selection rules

$$\text{(I)}\quad \begin{pmatrix} |(\theta,0)\rangle \\ |(\theta,e_1)\rangle \\ |(\theta,2e_1)\rangle \end{pmatrix} \Rightarrow \begin{pmatrix} \omega & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega \end{pmatrix} \begin{pmatrix} |(\theta,0)\rangle \\ |(\theta,e_1)\rangle \\ |(\theta,2e_1)\rangle \end{pmatrix} \qquad \omega = e^{2\pi i/3}$$

$$\text{(II)}\quad \begin{pmatrix} |(\theta,0)\rangle \\ |(\theta,e_1)\rangle \\ |(\theta,2e_1)\rangle \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \begin{pmatrix} |(\theta,0)\rangle \\ |(\theta,e_1)\rangle \\ |(\theta,2e_1)\rangle \end{pmatrix}$$

$\Delta(54) =$

$S_3 \ltimes (Z_3 \mathrm{x} Z_3')$

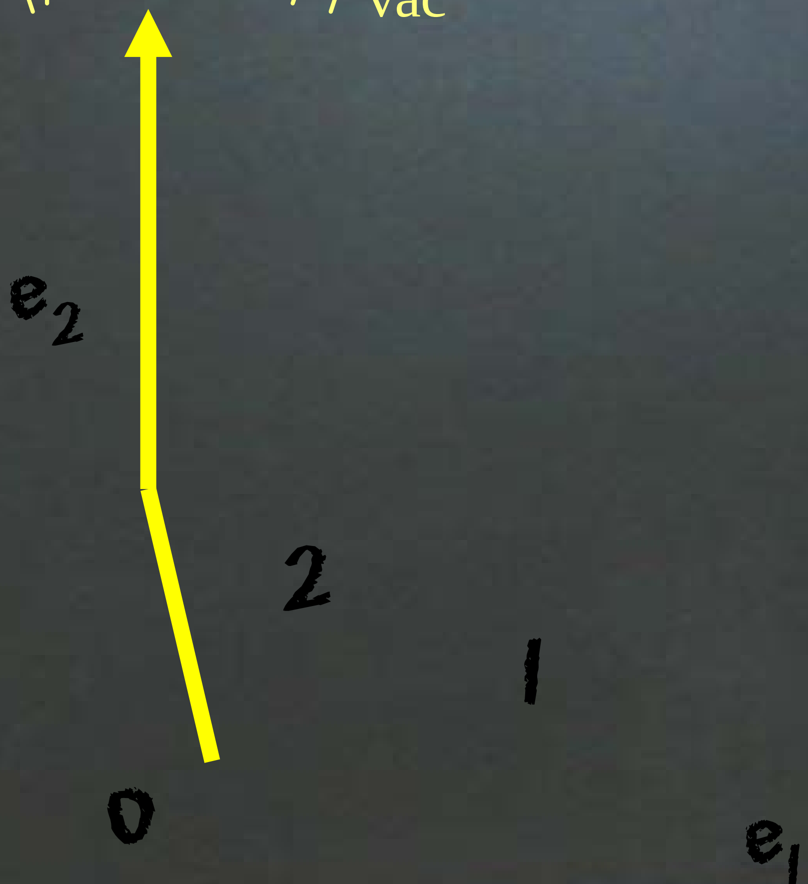
$$\text{(III)}\quad \begin{pmatrix} |(\theta,0)\rangle \\ |(\theta,e_1)\rangle \\ |(\theta,2e_1)\rangle \end{pmatrix} \Rightarrow \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} |(\theta,0)\rangle \\ |(\theta,e_1)\rangle \\ |(\theta,2e_1)\rangle \end{pmatrix}$$

S_3 perm's.

geometry

Flavor Symmetry Breaking

$$\left\langle \left|(\theta,0)\right\rangle \right\rangle_{\text{vac}} \neq 0 \qquad \Delta(54) \to D_3 = S_2 \bigcup \mathbf{Z}_3$$

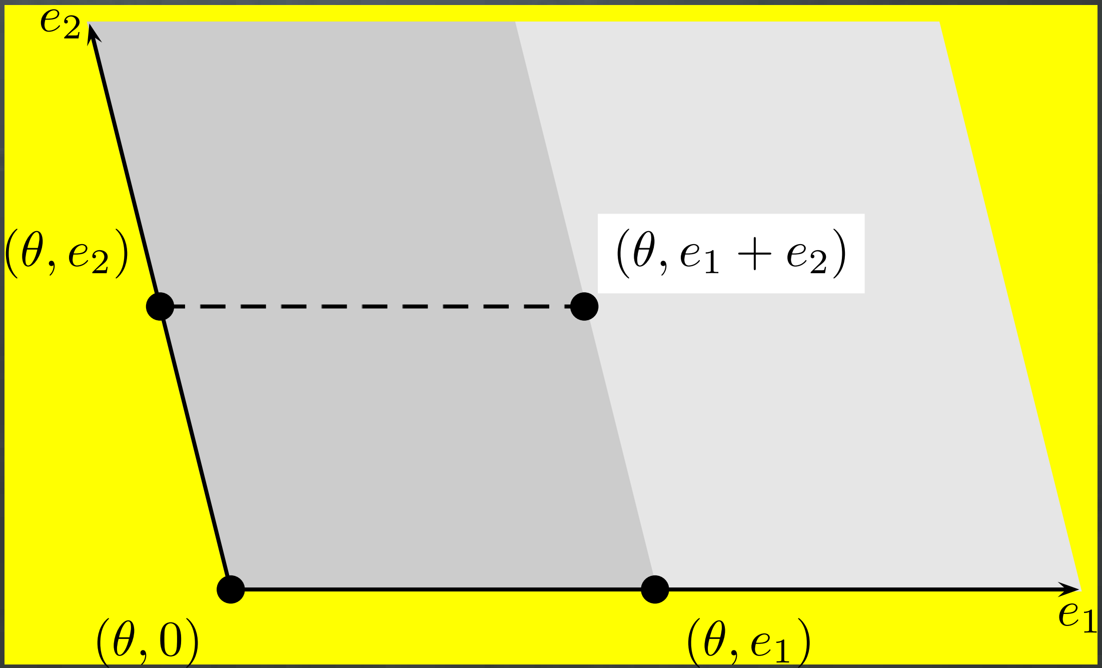


$$\begin{pmatrix} \left|(\theta, e_1)\right\rangle \\ \left|(\theta, 2e_1)\right\rangle \end{pmatrix} \quad \mathrm{D}_3$$

doublet

T^2/Z_2 orbifold

$(D_4 \times D_4)/Z_2$ 4 - plet

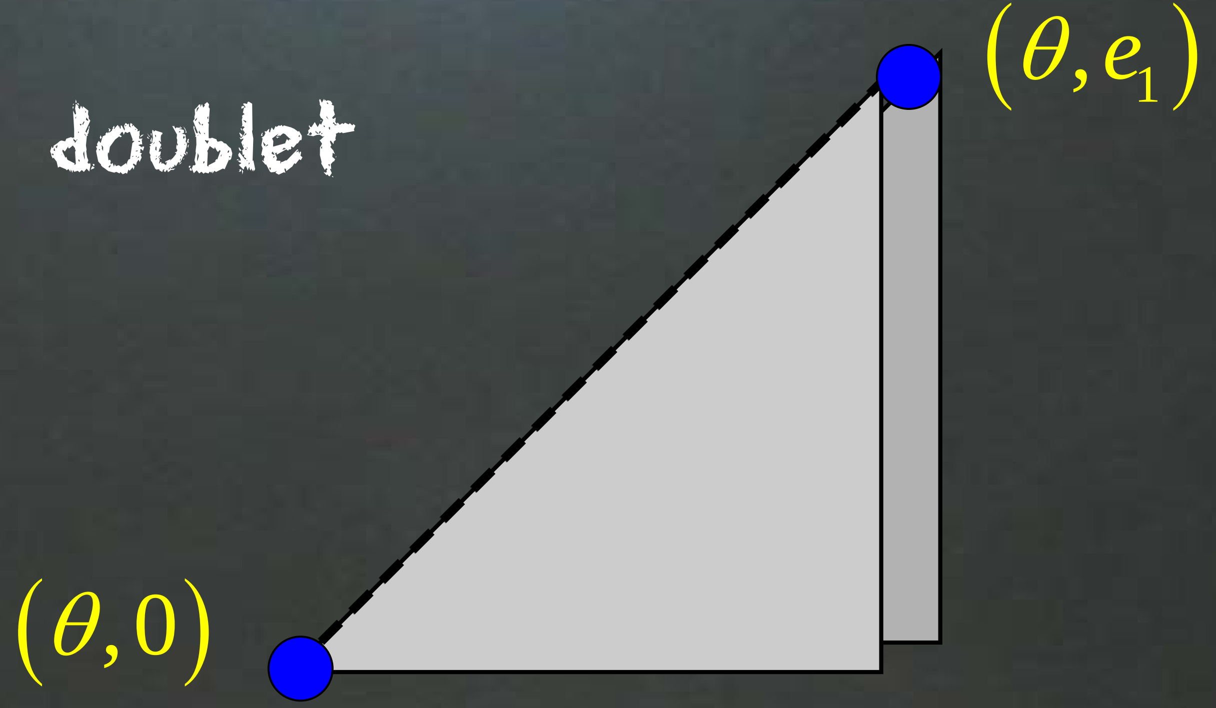


T^2/Z_4 orbifold (θ- twisted sector)

$(D_4 \times Z_4)/Z_2$

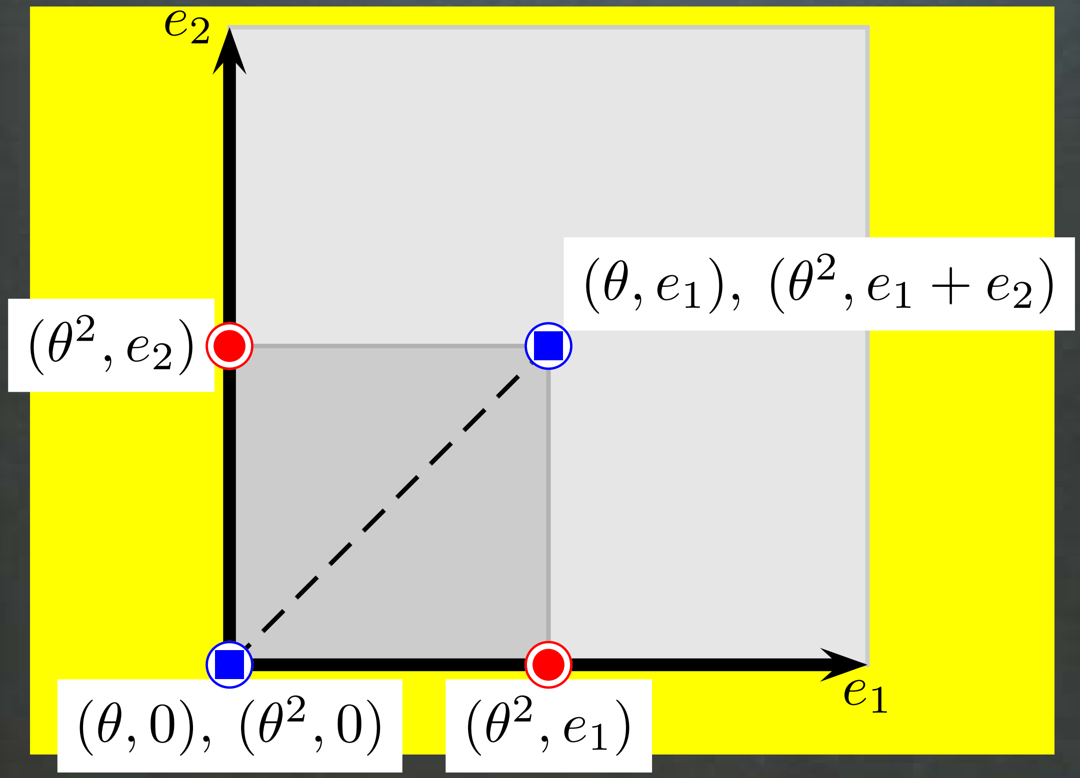
$$\left\{ \left| (\theta, 0) \right\rangle, \left| (\theta, e_1) \right\rangle \right\}$$

doublet



T^2/Z_4 orbifold (θ^2 – twisted sector)

$$\left\{ \left|(\theta^2,0)\right\rangle,\ \left|(\theta^2,e_1+e_2)\right\rangle,\ \left|(\theta^2,e_1)\right\rangle,\ \left|(\theta^2,e_2)\right\rangle \right\}$$



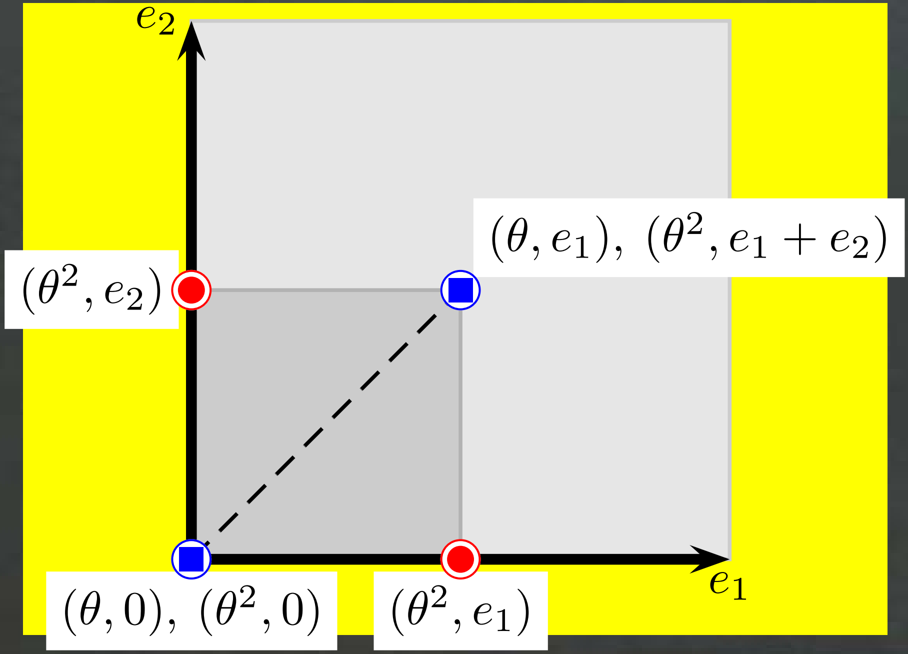
$(D_4 \times D_4)/Z_2$

4-plet

T^2/Z_4 orbifold ($\theta+\theta^2$ – twisted sectors)

$(D_4 \times Z_4)/Z_2$ combined → smaller sym.

$$\left\{ \left|(\theta^2,0)\right\rangle \pm \left|(\theta^2,e_1+e_2)\right\rangle,\ \left|(\theta^2,e_1)\right\rangle \pm \left|(\theta^2,e_2)\right\rangle \right\}$$



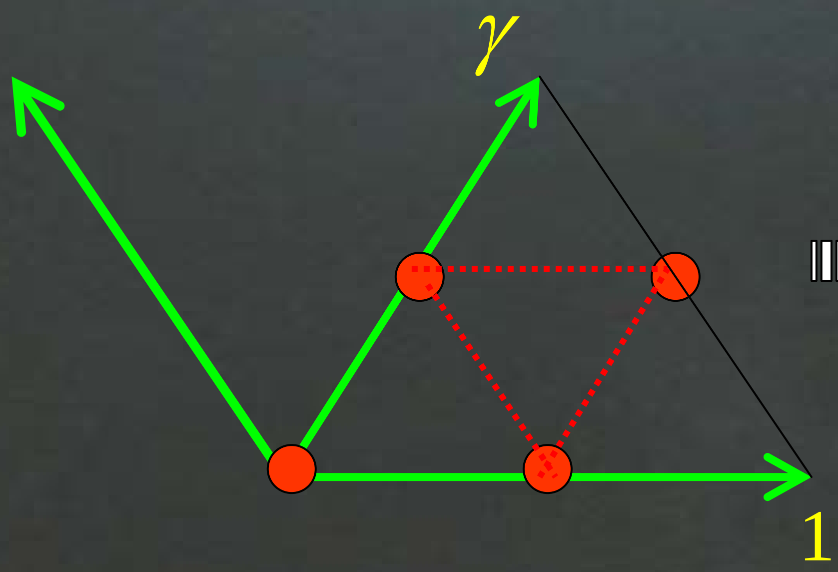
$$\left\{ \left|(\theta,0)\right\rangle,\ \left|(\theta,e_1)\right\rangle \right\}$$

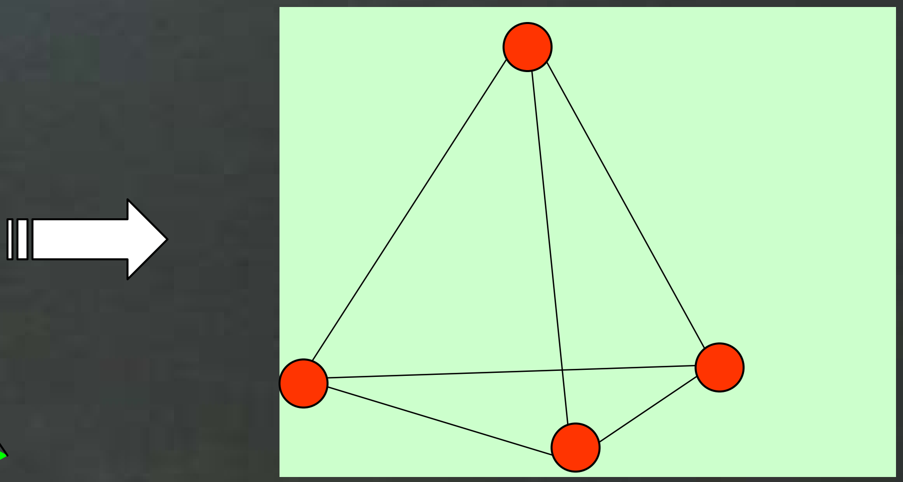
4- singlets !
+ doublet

T^2/Z_2 orbifold (SU(3) lattice)

$$z \to z+\gamma, \quad \gamma = e^{i\pi/3}$$

$$z \to z+1, \qquad z \to -z$$





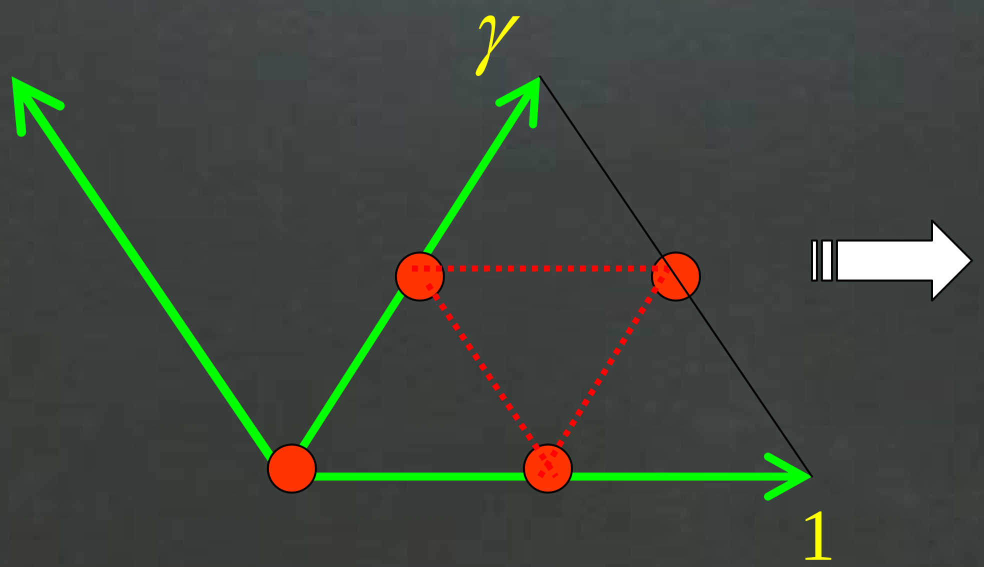
T^2/Z_2 orbifold (SU(3) lattice)

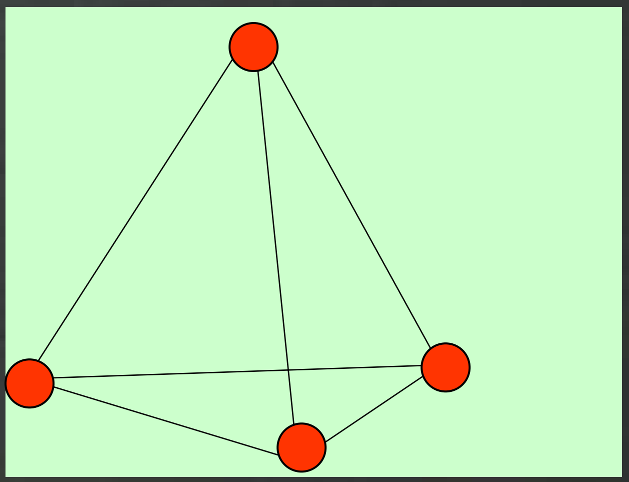
$$T: z \to \gamma^2 z$$

A_4 Altarelli, Feruglio, Lin

$$S: z \to z + \tfrac{1}{2}$$

$$S^2 = T^3 = (ST)^3 = 1$$

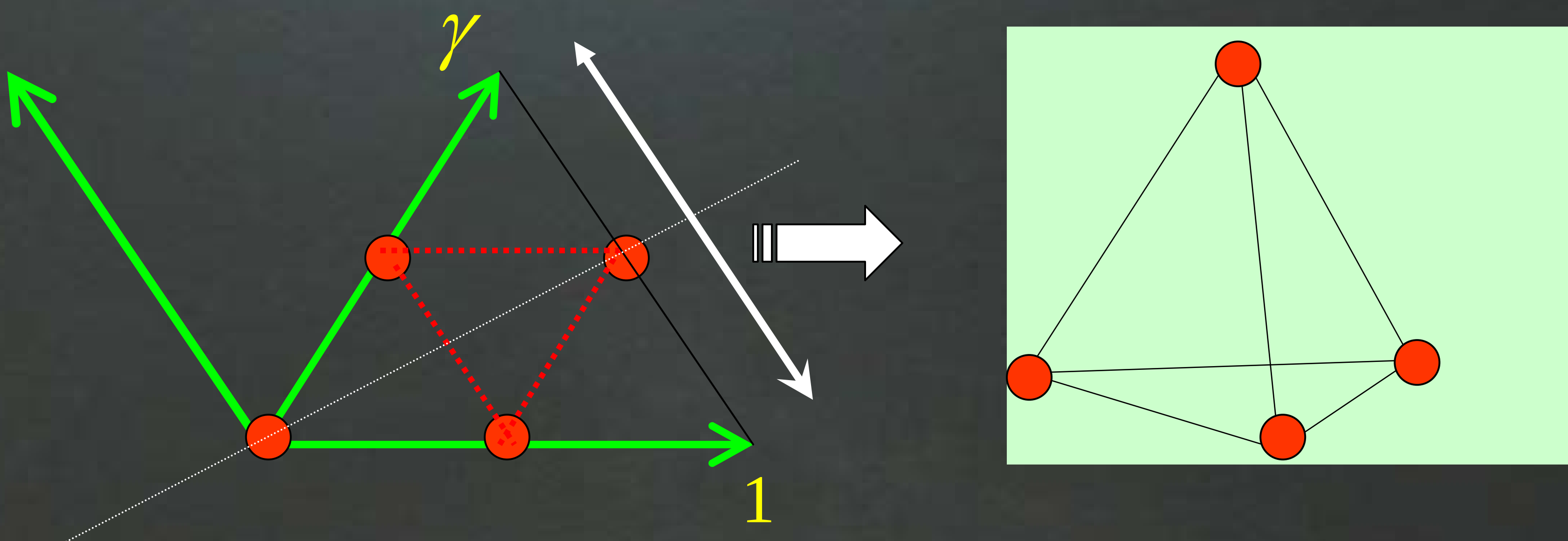




T^2/Z_2 orbifold (SU(3) lattice)

S_4

$S_4 \dagger (Z_2 \times Z_2 \times Z_2)$



Search for MSSM Spectra in heterotic orbifolds

Buchmuller, Hamaguchi, Lebedev & Ratz
Z_6-II orbifold

Looked for and found SM gauge group in 4D with 3 families and vector-like exotics

LNRRRVW - Mini-Landscape search

Mini-Landscape search Z_6-II orbifold

1. V_6 breaks to SO(10) (or E(6))
2. 2 families (16 (or 27)) in T_1 twisted sect.
3. Generate 2 Wilson lines W_3 , W_2
4. Identify 'inequivalent' models
5. Select models with $G_{SM} \subset SU(5) \subset SO(10)$
6. Select models with 3 net (3,2)
7. Select models with non-anom. $U_1(Y) \subset SU(5)$
8. Select models with 3 SM families + Higgses + vector-like exotics

Mini-Landscape search
LNRRRVW preliminary

criterion	$V^{\mathrm{SO}(10),1}$	$V^{\mathrm{SO}(10),2}$	$V^{\mathrm{E}_6,1}$	$V^{\mathrm{E}_6,2}$
(4) inequivalent models with 2 Wilson lines	22,142	7,843	675	1,694
(5) SM gauge group $\subset$ SU(5) $\subset$ SO(10) ($\subset \mathrm{E}_6$)	3563	1163	27	63
(6) 3 net $(\mathbf{3},\mathbf{2})$	1170	492	3	32
(7) non–anomalous $\mathrm{U}(1)_Y \subset \mathrm{SU}(5)$	528	234	3	22
(8) spectrum = 3 generations + vector-like	128	90	3	2

Table 1: Statistics of $\mathbb{Z}_6$-II orbifolds based on the shifts $V^{\mathrm{SO}(10),1}, V^{\mathrm{SO}(10),2}, V^{\mathrm{E}_6,1}, V^{\mathrm{E}_6,2}$ with two Wilson lines.

Mini-Landscape search
LNRRRVW preliminary

criterion	$V^{\mathrm{SO}(10),1}$	$V^{\mathrm{SO}(10),2}$	$V^{\mathrm{E}_6,1}$	$V^{\mathrm{E}_6,2}$
(9) heavy top	72	37	3	2
(10) exotics decouple at order 8	56	32	3	2

Table 1: A subset of the MSSM candidates.

Probability ~ 1/400
For MSSM-like models

Very 'fertile' patch in heterotic landscape

Future work

1. Check for $D = F = 0$ directions
2. Check all exotics get mass
3. Check fermion masses
4. Check R_{parity} conservation
5. Check neutrino masses (4 & 5 related)
6. SUSY breaking and lifting flat directions !

Conclusions

1. SO(10) SUSY GUT w/L.E. SUSY
2. E(8) x E(8) heterotic string

   'fertile' patch of Landscape
3. Find more 'fertile' patches

   Eg. $Z_2 \times Z_4$ search (RVW in progress)

Conclusions

What's new ? Why 3 families ?

Previously :

Z_3 multiples of 3 fixed points

problems :

NO GUTs (wrong Y + chiral exotics)

$Z_2 \times Z_2$ 3 twisted sectors +

SO(10) GUT (OK)

$Z_6 = Z_2 \times Z_3$ 3 = 2 + 1

localized SO(10) GUT + fam. sym.