Compactification Effects in String Inflation

Liam McAllister
Princeton University

Daniel Baumann, Anatoly Dymarsky, Igor Klebanov,
Juan Maldacena, L.M., and Arvind Murugan,
hep-th/0607050.

Daniel Baumann and L.M.,
hep-th/0610285.

Key Question:

What predictions can string theory make about the early universe?

Status:

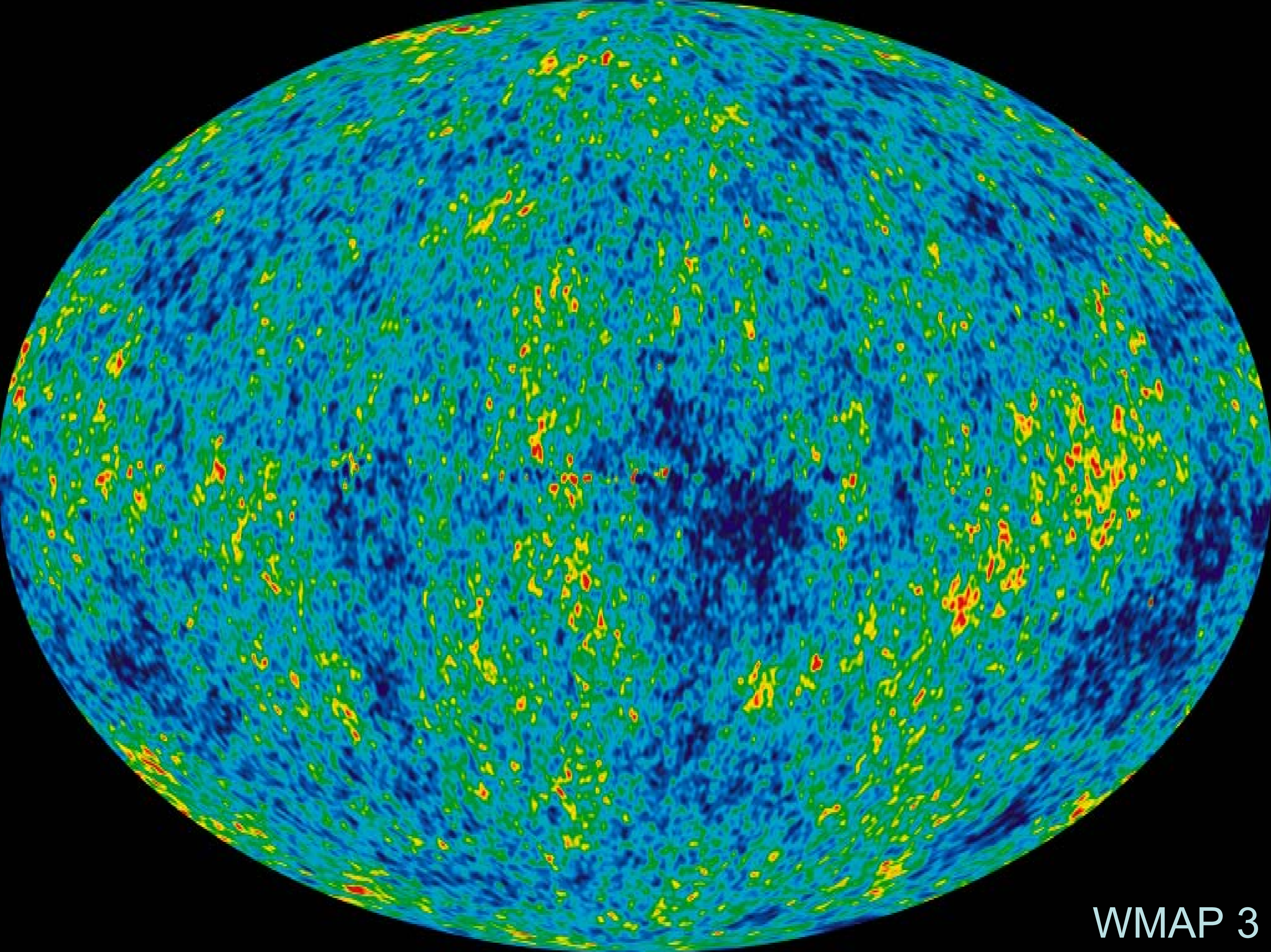
- Predictions are possible in concrete models.
- Given a model, deriving the effective theory is nontrivial.
- No general answer to date.

Inflation

A period of accelerated expansion

$$\boxed{ds^2 = -dt^2 + e^{Ht} d\vec{x}^2 \qquad H \approx const.}$$

- Solves horizon, flatness, and monopole problems.
- *i.e.* explains why universe is so large, so flat, and so empty.
- Predicts minute variations in CMB temperature:
  - approximately, but in general not exactly, scale-invariant
  - approximately Gaussian



WMAP 3

Wish List

- ☐ Rigorous context with clear rules
  - for better predictivity
- ☐ UV control; ideally full quantum gravity
- ☐ Specific, reliable, non-debatable predictions
  - loose predictions → observation may not discriminate!

Obvious wishes. What's new since early 80's?

- Theoretical tools (strings, D-branes, moduli stabilization, ...)
- Precise observations! Time limit on predictions.

String Inflation Assessment

- ☑ Rigorous context with clear rules
- ☑ Full quantum gravity theory
- ➡ Specific, reliable, non-debatable predictions
  - Achievable in concrete string inflation models.
  - Achieved in very few.
  - Lots of work to do!

Plan of the Talk

I. Background and Motivation

II. Tensors in D-brane Inflation
   i. Gravitational waves from inflation
   ii. D-brane inflation setup
   iii. Compactification effect: field range limit
   iv. An upper bound on tensors

III. Computation of the D3-brane Potential
   i. Moduli stabilization
   ii. Backreaction in supergravity
   iii. Result and Applications

IV. Conclusions

Part II.

Gravitational Waves from D-brane Inflation?

Daniel Baumann and L.M.,
hep-th/0610285.

Inflationary Gravitational Waves

Always present, but amplitude can be small.

$$\boxed{P_T = \frac{2}{\pi^2}\left(\frac{H}{M_p}\right)^2 = \frac{2}{3\pi^2}\left(\frac{V}{M_p^4}\right)}$$

$$r \equiv \frac{P_T}{P_S} = 16\varepsilon \qquad \varepsilon \equiv \frac{1}{2}M_p^2\left(\frac{V'}{V}\right)^2$$

Lyth Bound

$$3H\dot{\varphi} = -V'$$

$$\frac{d\varphi}{Hdt} = -M_p\sqrt{2\varepsilon}$$

$$\frac{\Delta\varphi}{M_p} = \frac{1}{\sqrt{8}}\int dN\,\sqrt{r(N)}$$

D.H. Lyth,
hep-ph/9606387

$$\frac{\Delta\varphi}{M_p} \equiv \sqrt{\frac{r_{CMB}}{8}N_{eff}}$$

depends on evolution of r

$$r_{CMB} \le \frac{8}{N_{eff}^{\ 2}}\left(\frac{\Delta\varphi}{M_p}\right)^2_{MAX}$$

microscopic input

Evolution of r

$$\boxed{\frac{d\ln r}{dN} = n_T - (n_s - 1) = -\left[(n_S - 1) + \tfrac{1}{8} r\right]}$$

experimental constraints on RHS give $\boxed{N_{eff} \geq 30}$

$$\boxed{\frac{r_{CMB}}{.01} \leq \frac{8}{9}\left(\frac{\Delta\varphi}{M_p}\right)^2_{MAX}}$$

cf. Easther, Kinney, & Powell;
Lyth and Boubekeur.

Useful constraint if we can compute $\Delta\phi_{max}$.

What's wrong with large ϕ?

In an effective field theory with cutoff M:

$$V = \frac{1}{2}m^2\phi^2 + \phi^4 \sum_{p=0}^{\infty} \lambda_p \left(\frac{\phi}{\mathbf{M}}\right)^p$$

Flatness over distance $\Delta\phi > M$ requires tuning **all** the λ's: "functional fine-tuning"!

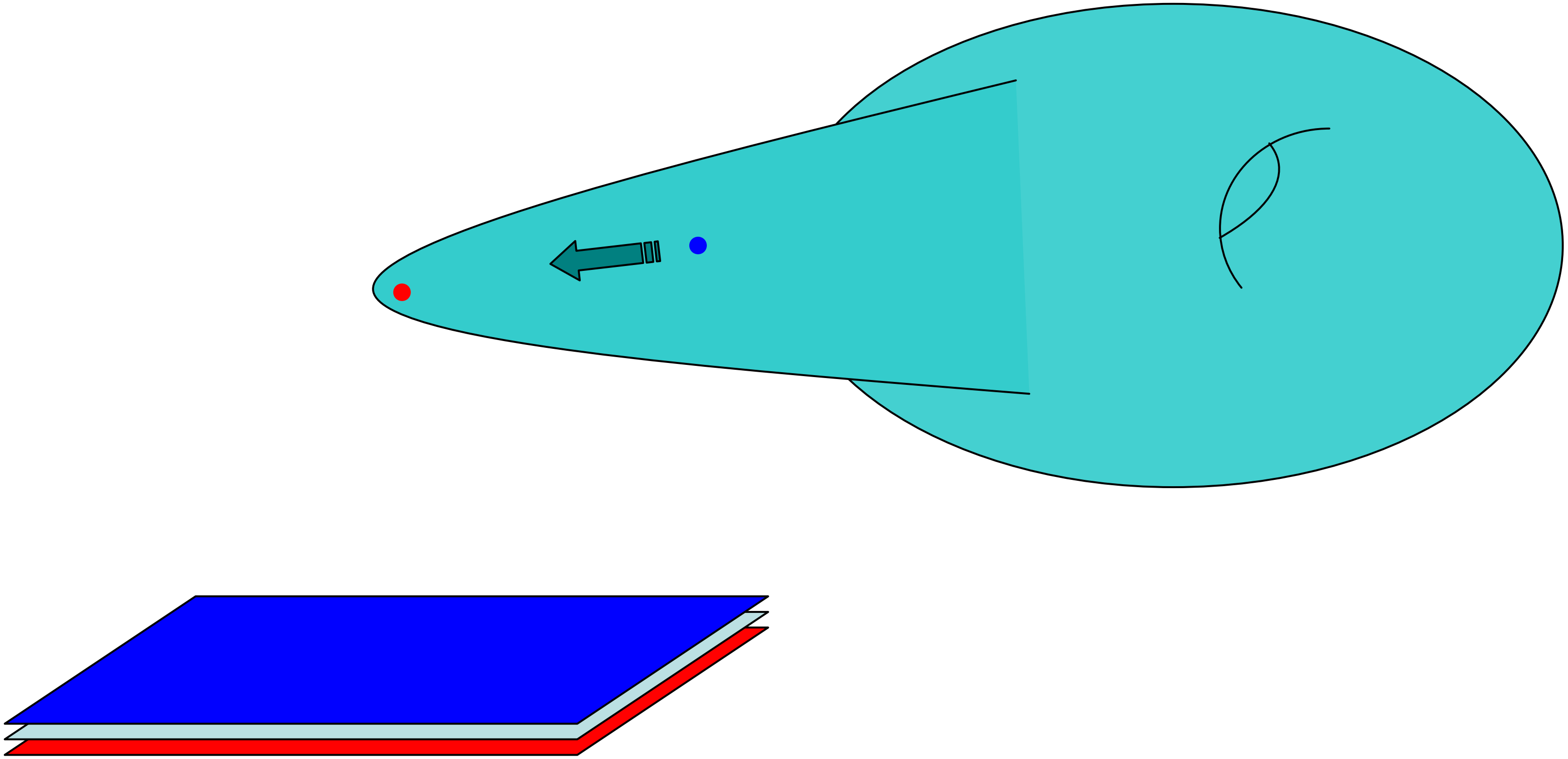
Clearly we can't take the cutoff $M \gg M_{pl}$

Can we compute $\Delta\phi$ in a string inflation model?

D-Brane Inflation

- Brane-Antibrane Dvali&Tye; Alexander; Dvali,Shafi,Solganik; Burgess,Majumdar,Nolte,Rajesh,Zhang; Sarangi&Tye.
- Branes at Angles. Garcia-Bellido, Rabadan, Zamora; Blumenhagen, Kors, Lust, Ott.
- D3-D7. Dasgupta,Herdeiro,Hirano, Kallosh; Hsu,Kallosh, Prokushkin; Hsu&Kallosh.
- warped brane-antibrane Kachru,Kallosh,Linde,Maldacena,L.M.,Trivedi; Firouzjahi&Tye; Burgess,Cline,Stoica,Quevedo; Iizuka&Trivedi; Berg,Haack, Körs; Cline&Stoica; Barnaby, Burgess, Cline; Kofman&Yi; Frey, Mazumdar, Myers; Chialva, Shiu, Underwood; Shandera&Tye; Baumann, Dymarsky, Klebanov, Maldacena, L.M.,Murugan; Burgess, Cline, Dasgupta, Firouzjahi.
- DBI. Silverstein&Tong; Alishahiha,Silverstein,Tong; Chen; Chen; Kecskemeti, Maiden, Shiu, Underwood.
- Giant Inflaton. DeWolfe,Kachru,Verlinde.
- Warped tachyonic. Cremades, Quevedo, Sinha.

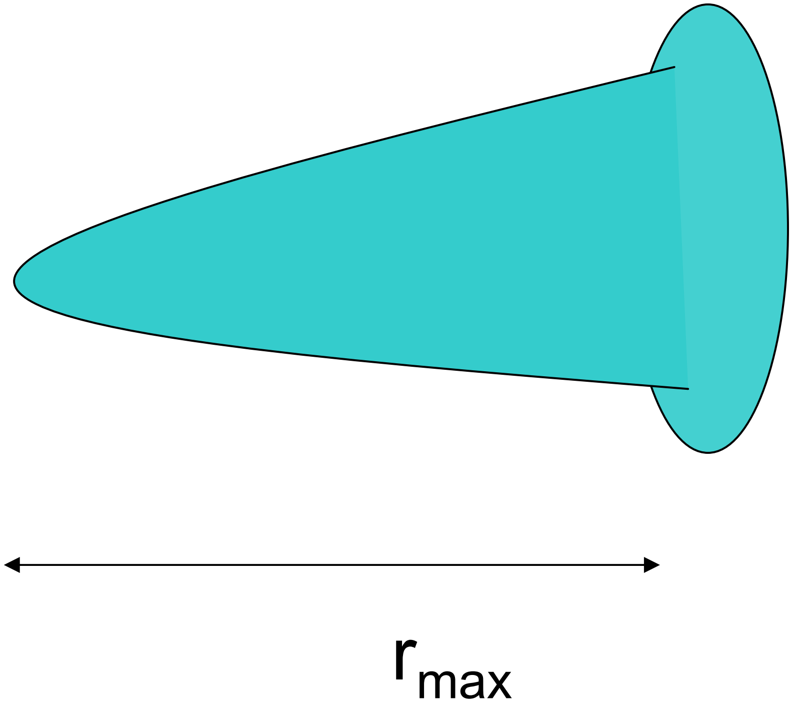
Warped Brane Inflation



Why Warped?

- Coulomb potential too steep in unwarped space ⇒ slow roll inflation hard to achieve.
- Exponential warping makes potential ‘exponentially flat’ (before moduli stabilization)
- Local model, explicit metrics, hence computable.
- RS-like hierarchy, so can adjust scales.
  - in particular, allows cosmic superstrings.
  - rich, novel reheating (involving KK modes and highly-excited strings)

Computing the Field Range



warped cone over X_5

$$dr^2 + r^2 ds^2_{X_5}$$

$$\boxed{ds^2 = h^{-\frac{1}{2}}(Y) g_{\mu\nu} dx^\mu dx^\nu + h^{\frac{1}{2}}(Y) g_{ij} dY^i dY^j}$$

The Throat Volume

$$V_6^{throat;w} = \int dr\, r^5 d\Omega\ h(r) \equiv V_X \int dr\, r^5 h(r)$$

$$= V_X \int^{r_{\max}} dr\, r^5 \left(\frac{R}{r}\right)^4 = \tfrac{1}{2} V_X R^4 r_{\max}^2 = 2\pi^4 g_s \mathbb{N}\, r_{\max}^2 (\alpha')^2$$

$$R^4 = 4\pi g_s (\alpha')^2 \mathbb{N} \left(\frac{\pi^3}{V_X}\right)$$

S.Gubser, hep-th/9807164

Field Range Limit

$$M_p^2 = \frac{V_6^w}{\kappa_{10}^2}$$

$$M_p^2 > \frac{V_6^{throat;w}}{\kappa_{10}^2}$$

$$V_6^w \equiv V_6^{bulk;w} + V_6^{throat;w}$$

$$\varphi^2 = T_3 r^2$$

$$\left(\frac{\Delta\varphi}{M_p}\right)^2 < \frac{T_3 r_{\max}^2 \kappa_{10}^2}{V_6^{throat;w}} = \frac{T_3 r_{\max}^2 \kappa_{10}^2}{2\pi^4 g_s \mathbb{N} r_{\max}^2 (\alpha')^2} = \frac{4}{\mathbb{N}}$$

$$\left(\frac{\Delta\varphi}{M_p}\right)^2 < \frac{4}{\mathbb{N}}$$

Microscopic Bound on r

$$\left(\frac{\Delta\varphi}{M_p}\right)^2 < \frac{4}{\mathbb{N}}$$

compactification
constraint

$$r_{CMB} \le \frac{8}{N_{eff}{}^2}\left(\frac{\Delta\varphi}{M_p}\right)^2_{MAX}$$

$$\left(\frac{r_{CMB}}{.009}\right) < \left(\frac{60}{N_{eff}}\right)^2 \frac{1}{\mathbb{N}} \le \frac{4}{\mathbb{N}}$$

observational
constraint

Implications

- Tensors larger in cases with small volumes, small warping, little flux.
- Detectable tensors virtually impossible in slow roll, independent of V.
- In DBI model, r can decrease *rapidly*. This may give $N_{eff} \sim 15$, so some window may exist.
- Expect similar field range bounds for most closed string models.

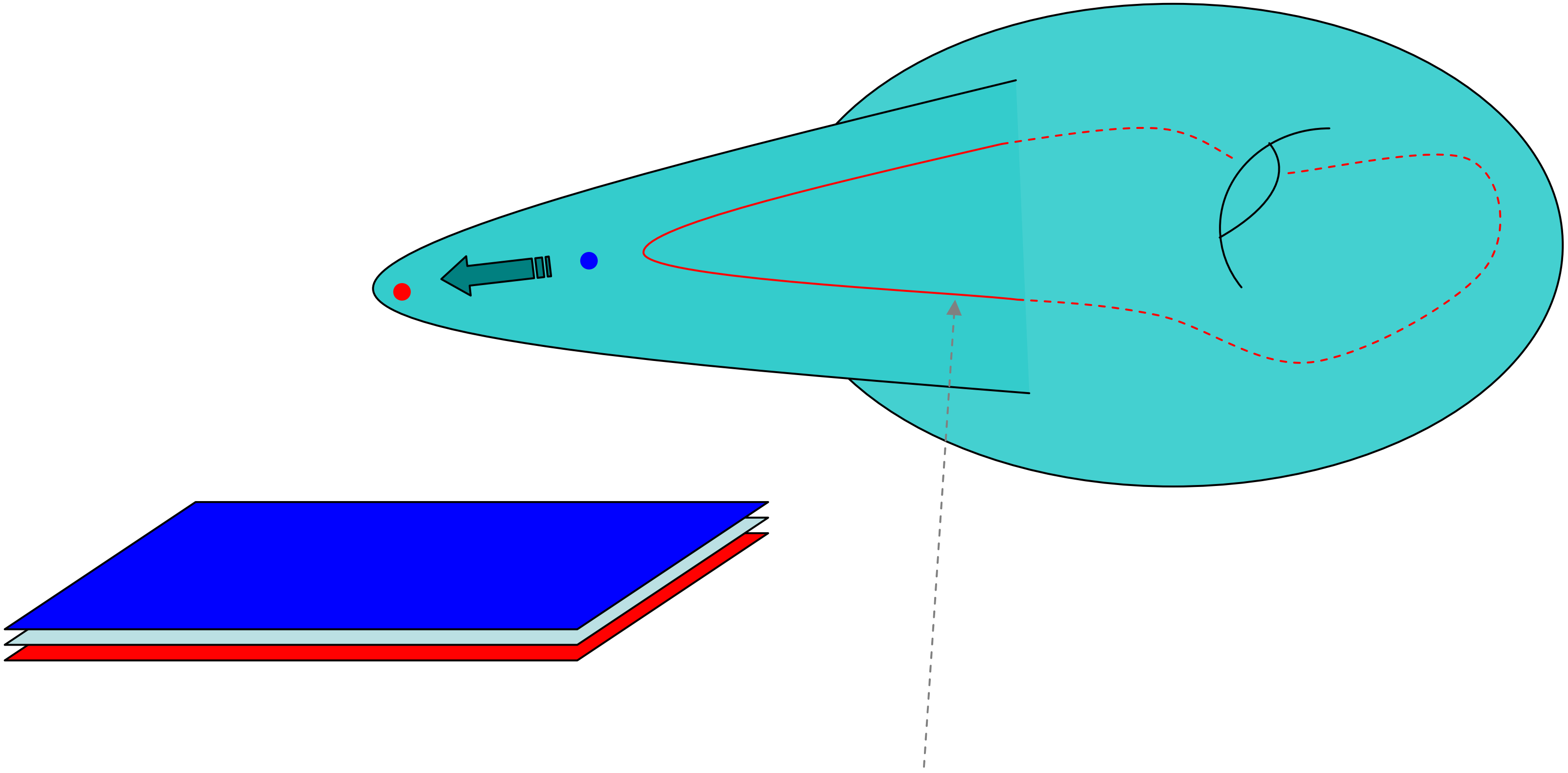
Part III.

Computing the Potential in D-brane Inflation

Difficulty from Moduli Stabilization

- Moduli stabilization crucial for realistic model.
  - unfixed moduli can spoil BBN; overclose universe; allow runaway decompactification; spoil slow-roll inflation.
- But moduli stabilization (e.g. by KKLT mechanism) spoils flatness of the potential!
- Obliged to compute corrected potential in stabilized vacuum (today's task)

Stabilized Warped Brane Inflation



‘wrapped brane’: Euclidean D3-brane, or D7-brane stack, on a four-cycle

Question:

What is the potential for motion of a D3-brane in a nonperturbatively-stabilized flux compactification?

Has implications beyond D-brane inflation, for:

- particle-physics models with D3-branes
- open string moduli stabilization

Related Work

- O. Ganor, hep-th/9612007
  - insight into form of superpotential correction
- S. Kachru, R. Kallosh, A. Linde, J. Maldacena, L.M., and S. Trivedi, hep-th/0308055
  - described role of correction in inflaton potential
- M. Berg, M. Haack, and B. Körs, hep-th/0404087
  - explicitly computed correction in toroidal orientifolds, in open string channel
- S. Giddings and A. Maharana, hep-th/0507158
  - presented general closed string channel method
- D. Baumann, A. Dymarsky, I. Klebanov, J. Maldacena, L.M., A. Murugan, hep-th/0607050
  - explicitly computed correction in general classes of warped throats

Warmup: D3 in flux background

Consider type IIB on a CY_3 orientifold with G_3 flux.

EOM: $*G_3 = i\, G_3$ (ISD)

But scalars governing motion of a spacetime-filling D3-brane couple **only** to

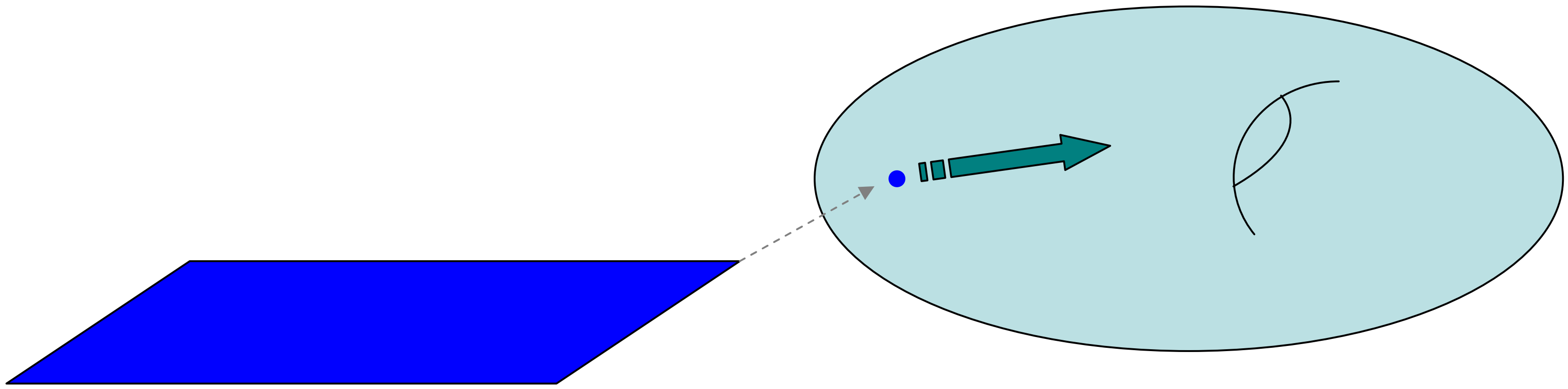
$*G_3 - i\, G_3$ (IASD flux)

Graña;
Graña, Grimm, Jockers, Louis

'No-force' condition.

- Can also see as a DBI-CS cancellation.
- A 'BPS-like' property (GKP).
- But, does not require unbroken SUSY.

D3-brane in ISD flux



D3-brane scalars are free fields.

D3-brane moduli space is the CY.

Is this property preserved in more complicated cases?

Moduli Stabilization

- In type IIB, generic fluxes lift complex structure moduli and dilaton.
- Kähler moduli are unlifted by flux.
- KKLT scenario: stabilize Kähler moduli by incorporating nonperturbative effects in a flux compactification.
- The same nonperturbative effects also lift the D3-brane moduli.
- Our task: compute resulting potential.

Part III.
Computing the D3-brane Potential

1. Nonperturbative effects
2. Backreaction in warped backgrounds
3. Computation
4. Result: ‘the superpotential prefactor is the embedding condition’

Nonperturbative Effects

$$\boxed{ds^2 = h^{-\frac{1}{2}}(Y)g_{\mu\nu}dx^\mu dx^\nu + h^{\frac{1}{2}}(Y)g_{ij}dY^i dY^j}$$

$$\boxed{V^w_{\Sigma_4} \equiv \int_{\Sigma_4} d^4Y\sqrt{g}\, h(Y)}$$

- Gaugino condensation on N D7-branes wrapping a four-cycle Σ_4

$$W_{\lambda\lambda} = \exp\left(-\frac{8\pi^2}{g_{YM}^2 N}\right)$$

- Euclidean D3-branes wrapping a four-cycle Σ_4

$$W_{np} = \exp\left(-T_3 V^w_{\Sigma_4}\right)$$

Either case: can write

$$W_{np} = \exp\left(-\frac{T_3 V^w_{\Sigma_4}}{N}\right)$$

KKLT Proposal

$$W_{KKLT} = \int G \wedge \Omega + \exp\left(- \frac{T_3 V^{w}_{\Sigma_4}}{N} \right)$$

In their language: $$W_{KKLT} = \int G \wedge \Omega + A e^{-a\rho}$$

Key point for today: $$A \rightarrow A(\varphi)$$

D3-brane position

Corrected Warped Volumes

$$ds^2 = h^{-\frac{1}{2}}(Y)g_{\mu\nu}dx^\mu dx^\nu + h^{\frac{1}{2}}(Y)g_{ij}dY^i dY^j$$

$$V^w_{\Sigma_4} \equiv \int_{\Sigma_4} d^4Y\sqrt{g}\,h(Y)$$ (probe approximation)

Including D3-brane backreaction:

$$h = h(Y,X) \leftarrow \text{D3-brane position}$$

$$h = h_0(Y) + \delta h(X,Y)$$

$$V^w_{\Sigma_4} = \underbrace{\int_{\Sigma_4} d^4Y\sqrt{g}\,h_0(Y)}_{V_0} + \underbrace{\int_{\Sigma_4} d^4Y\sqrt{g}\,\delta h(X,Y)}_{\delta V}$$

D3-brane Backreaction

$$h = h_0(Y) + \delta h(X,Y)$$

$$V_{\Sigma_4}^{w} = \underbrace{\int_{\Sigma_4} d^4Y \sqrt{g}\, h_0(Y)}_{V_0} + \underbrace{\int_{\Sigma_4} d^4Y \sqrt{g}\, \delta h(X,Y)}_{\delta V}$$

$$\nabla_Y^2 \delta h(X,Y) = -2\kappa_{(10)}^2 T_3 \left[\delta^6(X-Y) - \rho_{bg}(Y) \right]$$

- Solve for δh
- Integrate over Σ_4 to get δV(X)
- Read off δW(X)

$$W_{np} = \exp\left(-\frac{T_3 V_0}{N} \right) \exp\left(-\frac{T_3 \delta V(X)}{N} \right)$$

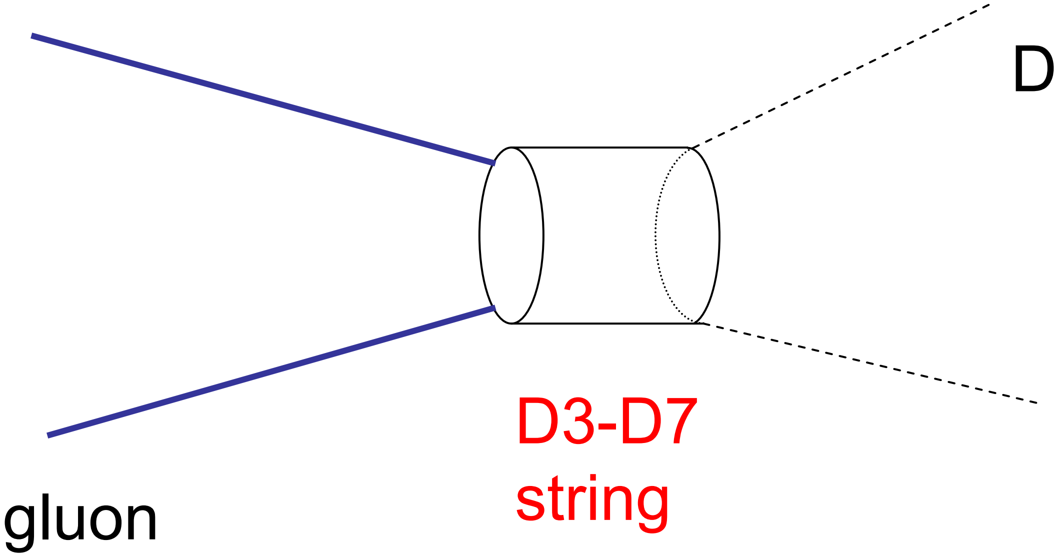
Comments

- D3-brane effect in exponent in W, so even minute effects important.
- This is the leading effect lifting the D3 moduli space.
- Effect vanishes if W_{np} does. Requires a topological condition on Σ_4.
- Our result is the D3-brane-dependence of the instanton fluctuation determinant (or, of D7-brane gauge theory threshold correction)
- Dependence on complex structure not known.

Open String Method

Berg, Haack, Körs `04 (BHK)

In case of gaugino condensation, they compute dependence on D3 position as threshold correction due to 3-7 strings.

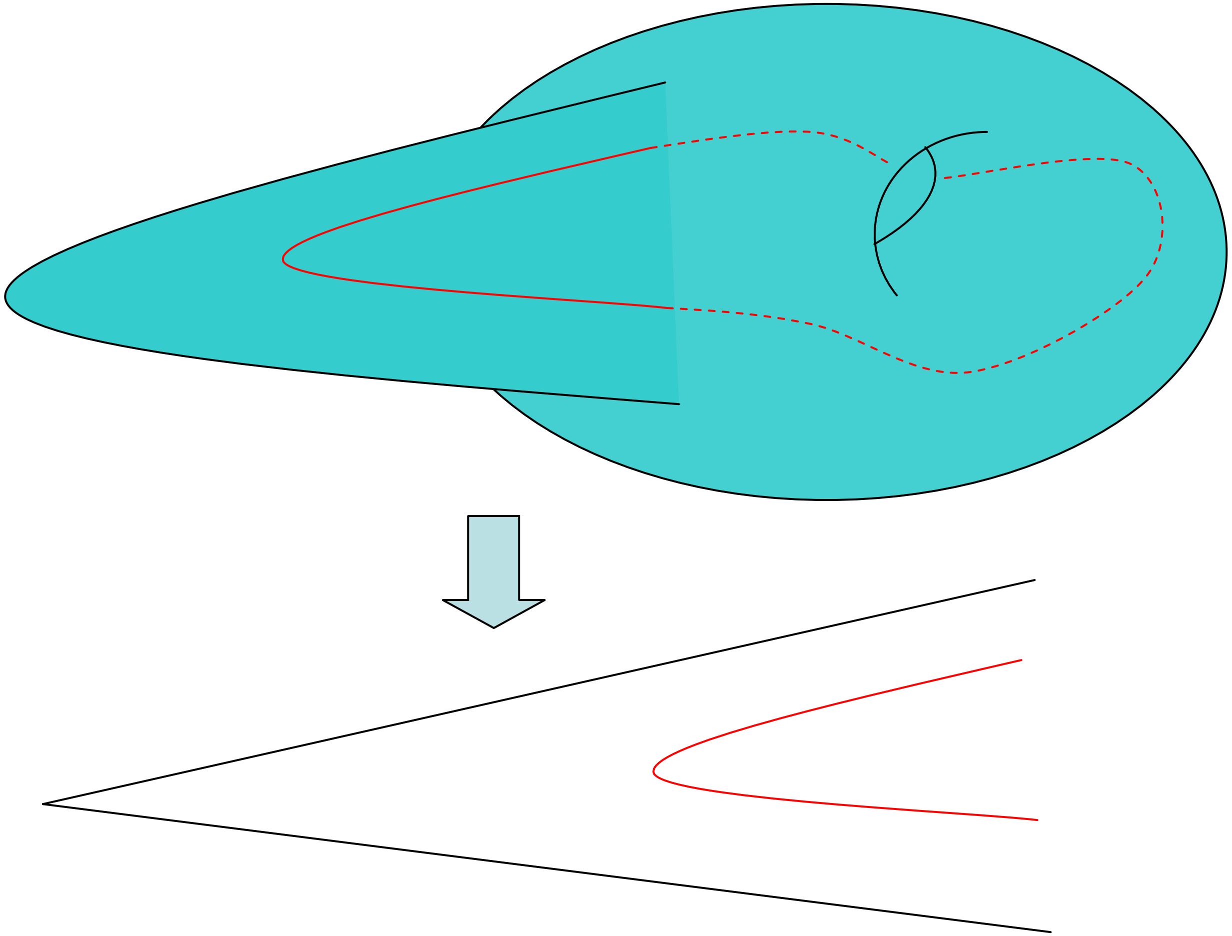


Comparison

Open String BHK	Closed String Giddings-Maharana; Us
One-loop open string	Tree-level SUGRA
Threshold correction	Backreaction on warping
Gaugino only	Gaugino or ED3
Hard (and impressive)	Comparatively easy
Toroidal cases only	More general geometries
Unwarped only	Warping ok

Perfect agreement where comparison is possible.

Asymptotically Conical Space



Wrapped Branes in Throats

$$\boxed{w_1 w_2 - w_3 w_4 = 0}$$

$$\boxed{w_i \in \mathbb{C}}$$

$$\boxed{w_1 = r^{\frac{3}{2}} \sin(\tfrac{\theta_1}{2}) \sin(\tfrac{\theta_2}{2}) \exp[\tfrac{i}{2}(\psi - \phi_1 - \phi_2)]}$$

SUSY embedding of D7:

$$\boxed{\prod_{i=1}^{4} w_i^{p_i} - \mu^P = 0}$$

Σ_4

Areán, Crooks, Ramallo,
hep-th/0408210: conifold

$$\boxed{p_i \in \mathbb{Z} \qquad P \equiv \Sigma p_i \qquad \mu \in \mathbb{C}}$$

Karch & Katz, hep-th/0205236
P. Ouyang, hep-th/0311084
S. Kuperstein, hep-th/0411097
Canoura, Edelstein, Pando Zayas, Ramallo, Vaman, hep-th/0512087

Example: Singular Conifold

$$ds^2 = dr^2 + r^2 ds^2_{T^{1,1}}$$

$$X = \{r, \psi_\alpha\}$$

$$\nabla^2_X G(X, X') = -\delta^6(X - X')$$

Solution:

$$G(X, X') = \sum_L N_L Y^*_L(\psi'_\alpha) Y_L(\psi_\alpha) \left(\tfrac{r'}{r}\right)^{-2+\sqrt{4+\Lambda_L}} r^{-4}$$

$$L = \{l_1, l_2, m_1, m_2, R\} \leftrightarrow SU(2) \times SU(2) \times U(1)_R$$

where

$$\nabla^2_\psi Y_L(\psi) = -\Lambda_L Y_L(\psi)$$

$$\Lambda_L = 6\left(l_1(l_1+1) + l_2(l_2+1) - \tfrac{R^2}{8}\right)$$

Now: integrate G(X,X') over SUSY Σ_4

$$G(X,X') = \sum_L N_L Y_L^*(\psi'_\alpha) Y_L(\psi_\alpha) \left(\tfrac{r'}{r}\right)^{-2+\sqrt{4+\Lambda_L}} r^{-4}$$

$$\int_\Sigma d^4X \; G(X,X') = \sum_L I_L$$

surprise:

$$I_L \neq 0 \Leftrightarrow l_1 = l_2 = \tfrac{R}{2}$$

Dual description

$$\boxed{\delta h = \frac{27\pi g_s \alpha'^2}{r^4}\left(\sum_i \frac{c_i f_i}{r^{\Delta_i}}\right)}$$

f_i: angular eigenfunction

Δ_i: conformal weight

c_i: coefficient of operator $\mathcal{O}_i$

(D3-brane position)

chiral subset:

$$\mathcal{O}_k = Tr[A_{\alpha_1} B_{\beta_1} \ldots A_{\alpha_k} B_{\beta_k}]$$

$$\boxed{l_1 = l_2 = \tfrac{k}{2}}$$

Only chiral operators contribute to δV!

Result for Conifold Case

$$I_k^{chiral} = \frac{1}{2k}\left(\prod_i \bar{w}_i^{\,p_i} \Big/ \bar{\mu}^P\right)^k$$

$$W_{np} = \exp\left(-\frac{T_3 V_0}{N}\right)\exp\left(-\frac{T_3\,\delta V_4}{N}\right)$$

$$T_3 \delta V_4 = \sum_k I_k = -\mathrm{Re}\left[\log\left(\mu^P - \prod_{i=1}^{4} \bar{w}_i^{p_i}\right) - \log\left(\mu^P\right)\right]$$

$$\exp(-T_3 \delta V_4 / N) = \left(\mu^P - \prod_{i=1}^{4} w^i\right)^{1/N} \mu^{P/N}$$

Result

for a general warped throat

If wrapped branes are embedded along

$$\boxed{f(w_i)=0}$$

then the superpotential correction is

$$\boxed{A=A_0\exp\left(-T_3\delta V_4/N\right)=\left[f(w_i)\right]^{1/N}}$$

Part IV.

Applications

Lifting of D3-brane Moduli

$$W_{KKLT} = \int G \wedge \Omega + A_0 f(w_i)^{1/N} e^{-a\rho}$$

$$K = -3\log\left(\rho + \bar{\rho} - k(w_i, \bar{w}_i)\right)$$

In general, D3-branes preserve SUSY only at special points in the CY.

Mass around a SUSY min: $m^2 \approx \frac{V_F^{\min}}{M_p^2}$

cf. Kachru et al.

Achieving Inflation

- Typically requires a scalar field ϕ with a rather flat potential $V(\phi)$.

$$\eta \equiv M_{pl}^{2} \frac{V''}{V} \ll 1 \quad \text{and} \quad \varepsilon \equiv \frac{1}{2} M_{pl}^{2} \left(\frac{V'}{V} \right)^{2} \ll 1$$

- Key goal of inflationary model-building: find such a field and such a potential in a controllable, well-motivated, natural setting.

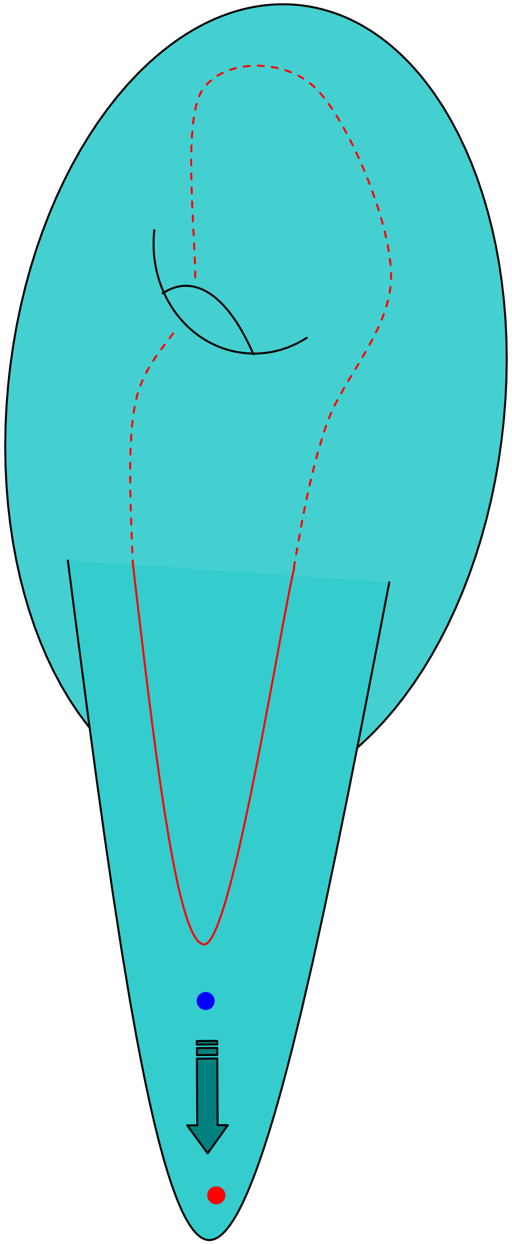
$$W_{KKLT} = \int G \wedge \Omega + A_0 \, f(w_i)^{1/N} e^{-a\rho}$$

$$K = -3\log\left(\rho + \bar{\rho} - k(w_i, \bar{w}_i)\right)$$

Neglecting f: $\eta = \frac{2}{3}$

Including f: $\eta \ll 1$

is possible, but not generic.



One can:

(1) reject model as fine-tuned, or

(2) search parameter space for small η and reassess

Status of Warped Brane Inflation

- Well-known: η is generically O(1).
- Our work gives substantially complete potential (including angular directions).
- One can check explicitly whether η is small for given microscopic parameters.
- Explicit fine-tuning gives important qualitative differences from uncorrected potential.
- Can change spectral index, tensor amplitude, cosmic string tension (in preparation).
- cf. recent work: Burgess, Cline, Dasgupta, Firouzjahi, hep-th/0610320

Conclusions

- We computed the interaction between D3-branes and wrapped branes, in warped throat backgrounds, using supergravity.
- Striking cancellations of non-chiral terms led to a simple result: ‘superpotential correction is the embedding equation’.
- Very explicit confirmation of Ganor’s result.
- Agrees with open-string method of BHK, but allows more complicated spaces; fluxes; warping.
- This gives the complete potential for D3-brane motion in a throat of a KKLT compactification.
- Hence, overcomes a technical obstacle for analysis of warped brane inflation.
- Implications for open string moduli stabilization.