Can we live without them?

- Entropy Bounds:
  - Bekenstein Bound: \( S < \alpha R E \)
  - Holographic Bound: \( S < A/4 l_p^2 \)

Based on work with R. Sorkin, also with S. Ross and D. Minic

A common story

Black Hole Thermodynamics

Loophole!

Entropy Bounds

\[ S \leq A/4 l_p^2 \]
\[ S \leq \alpha R E \]
Outline:

2. Loopholes: Preserving the 2\textsuperscript{nd} law without Bekenstein’s bound ($S < \alpha RE$).
3. The observer dependence of entropy.
4. Save holographic bound ($S < A/4\rho^2$) for questions.

1. Bekenstein’s Concern:

Adiabatic:
- No kinetic energy
- Energy redshifted to almost zero.

$S_{\text{infinity}} > 0$
$E_{\text{infinity}} = mc^2$

$S_{\text{box}} = S_{\text{infinity}}$
$E_{\text{box}} = E_{\text{infinity}}/z$
\sim 0$
After

\[ S = S_{BH} \]

Entropy has decreased???

Before

\[ S = S_{BH} + S_{box} \]

K \approx 0

\[ E_{box} = E_{\text{infinity}} / z \]

\[ S_{box} = S_{\text{infinity}} \]

\[ R = 2MG/c^2 \]

M, R, and \( S_{BH} \) are same before and after!

\[ S_{BH} = \frac{4\pi R^2}{4L_p^2} \]

Bekenstein's Concern: Entropy (1973)

The Bekenstein Bound:

\[ E_{box} > 0 \]

Black Hole grows!

All OK if

\[ S_{box} < 2\pi RE_{\text{infinity}}/ch \]

Before

\[ S = S_{\text{bigger BH}} \]

After

\[ S = S_{\text{bigger BH}} \]

\[ E_{\text{top}} > 0 \]

\[ E_{\text{bottom}} = 0 \]

What about the Hawking radiation?
Weak or Strong?

More accurately: a thermal fluid

Box Floats! (Archimedes)
- Box does not reach horizon.
- Even for $R_{box} = 0$, $E_{box}$ does not reach zero!
- Black Hole grows if box dropped in.
- BH Entropy increases!

Claim: Takes care of Bekenstein’s concern w/o new bounds!
Arguments about reflectivity problem!!
And what about the equivalence principle?

Lab in grav. field g

Lab in accel. rocket a = g

Supported by thermal physics is same in both!

2. Another response: (DM & RS)
- Bekenstein concerned about objects with large S.
- Common at thermal equilibrium!
- Our claim: Boxes are readily produced by thermal fluctuations. They are an important part of the thermal atmosphere and lead to new effects.

Why should you believe this?
Bekenstein’s Concern:
(80’s and 90’s)
- Let box fall freely from far away.
- Hawking radiation is a small effect.
  \( E_{\text{box}} \) is constant.
- \( S_{\text{BH}} \) grows. \( S_{\text{box}} \) gone!
- But \( \Delta S_{\text{BH}} \) depends only on \( E_{\text{box}} \) – not on \( S_{\text{box}} \)!

\[ S_{\text{infinity}} > 0 \]
\[ E_{\text{infinity}} = mc^2 \]

Small Box, so OK if
\[ S < \Delta S_{\text{BH}} = \Delta E_{\text{BH}} / T_{\text{BH}} \]
\[ = 4\pi R_{\text{BH}} E = 8\pi \zeta R E \]
\[ E_{\text{box}} = mc^2 \]
\[ \zeta = \frac{R_{\text{BH}}}{2R} \]
But Wald...

Boxes and Thermal Fluctuations
(DM & R Sorkin 2002)
- These boxes have lots of \( S \).
- What is their free energy?
  \( F \) is negative!

Violation: \( S > 8\pi \zeta R E \)
\[ T = 1/(4\pi R_{\text{BH}}) \]
\[ \zeta = \frac{R_{\text{BH}}}{2R} \]
\[ F = E - TS \]
\[ < E - \frac{8\pi \zeta R E}{4\pi R_{\text{BH}}} = E - E \zeta (2R/R_{\text{BH}}) = 0 \]
Negative Free Energy?

$$Z = \sum_{\text{macro}} e^{-S/kT} = \sum_{\text{macro}} e^{-E/kT}$$

Any macrostate of box w/ this E more likely to be present than not!

What happens to Bekenstein's Box?

- Bounce off,
- Annihilate,
- Or, if transparent, another box comes out.

Bek's box is already present in equilibrium. So are all similar boxes.
Complete Generality

Drop E, S into any black hole.
(Assume small change)

\[ \Delta S_{\text{total}} = \Delta S_{\text{BH}} - S \]
\[ = E/T - S \]
\[ = F/T \]

Sign of LHS = Sign of RHS

Boxes in thermal atmosphere
& New effects

“Holographic Bound” \( S < A/4\ell_p^2 \).

3. Equilibrium + 1?
Free fields (w/ S. Ross & D. Minic)

Illustrate issue in flat spacetime.

- Rindler observer sees \( S_{\text{Box}} \) disappear.
- Typically \( \delta S_{\text{Horizon}} = E_{\text{acc}}/T \).
- What if \( S_{\text{Box}} > E_{\text{acc}}/T \) ?
Let’s calculate E & S:

<table>
<thead>
<tr>
<th>Inertial observer</th>
<th>Accelerated observer</th>
</tr>
</thead>
<tbody>
<tr>
<td>No object</td>
<td>$</td>
</tr>
<tr>
<td>One object</td>
<td>$</td>
</tr>
<tr>
<td>(microstate)</td>
<td>$\rho = N^{-1} \Sigma</td>
</tr>
<tr>
<td>One object</td>
<td></td>
</tr>
<tr>
<td>(macrostate)</td>
<td>$E$</td>
</tr>
<tr>
<td>Change in E</td>
<td>$S_{box} = \ln N$</td>
</tr>
<tr>
<td>Change in S</td>
<td></td>
</tr>
</tbody>
</table>

Reminder

$$S_{acc} = - \delta \text{Tr}[\rho \ln \rho]$$

$$= - \text{Tr} [ \delta \rho (1 + \ln \rho)]$$

$$= - \text{Tr} [ \delta \rho (1 - H/T)]$$

$$= E_{acc}/T$$

Explicit calculations for one-particle state w/ N free fields in limit of large N:

$$S_{box} = \ln N \neq S_{acc} = E_{acc}/T$$

$E >> T : E_{acc} = E$

Take $\delta \rho$ small since $N > e^{E/T}$
Why does our intuition fail?

- Can't rely on intuition from distinguishable particles for $S - E/T > 0$ because...
  - QFT says to sum over # of particles at finite $T$.
  - $e^S$ states for one particle, $(e^S)^n$ states for $n$ particles.
  - $\Sigma_n e^{E_n / T} (e^S)^n = \Sigma_n e^{(S-E/T)n}$ -- diverges; i.e., infinite # of particles preferred.

Conclude: Indistinguishability critical in this regime (though details of statistics irrelevant).

Consider again fall into BH (free case):

- 2nd already violated?
- BEFORE object enters black hole!
- Like sending (unpolarized) low E photon into hot cavity.
- Other radiation (objects) must leak out!
1. Arguments for a fundamental "Bekenstein Entropy Bound", contain a loophole.

2. This loophole comes from the high probability that highly entropic objects will be created by thermal fluctuations.


4. Similar loophole in arguments for the "holographic bound", (Expands on Wald.)

My Viewpoint on Bekenstein bound ($S < \alpha R E$)

1. Not motivated by GSL.

2. Does not explain BH entropy or other properties. So, no real motivation at all.

3. Does not depend on G. No known definition that holds for all QFT's. In particular, violated w/ large # of species.

4. Hard to believe as a fundamental principle (and why bother?).

5. Nevertheless, not ruled out that some version holds in our Universe (at least at some scale). Experimental tests welcome.
My Viewpoint on holographic bound 
\( (S < A/4l_p^2) \)

1. Not motivated by GSL.
2. But would help to explain BH entropy if true.
3. Supported by AdS/CFT, Susskind/Witten in particular.
4. Depends on G. Much more robust than Bek. bound. No clear violations of Bousso form known, even w/ large # of species.
6. Experimental tests would be wonderful!!

4. The Holographic Bound \( S < A/4G \) 
   (Susskind, ’t Hooft)

1. Suppose Q,J=0 object with \( S > A/4G \).
2. Not a Black Hole, \( E < E_{BH} \) at same A.
3. Add shell of mass; make into BH of same A.

So, original S is smaller.
Wald: Some subtlety?

1. Suppose large S due to large number N of scalar fields.
2. Hawking radiation is N times as great!
3. Semiclassical result: Black Hole evaporates in T \sim R. Note that this is time for shell to fall.
4. BH is a fluctuation, not equilibrium.

Our Calculation

1. Suppose *any* object with S > A/4 = \pi R^2.
2. Not Black Hole, so E < M_{BH} = 2R.
3. Suggestion: BH may quickly produce a copy of object through Hawking-like process.
4. Can study this when E << M_{BH} and back-reaction is small.

\[ F = E - TS \]
\[ < E - \frac{\pi R_{BH}^2}{4\pi R_{BH}} \]
\[ = E - \frac{M_{BH}}{2} < 0 \]
Large Backreaction?

E.g., Kraus, Parikh and Wilczek, or Massar and Parentani:

\[ \Gamma_{\text{micro}} \sim e^{\Delta S_{\text{BH}}} < 1 \]

\[ \Gamma_{\text{macro}} \sim e^{(\Delta S_{\text{BH}} + S_{\text{object}})} > e^{(-S_{\text{BH}} + S_{\text{object}})} > 1 \]