Review of

some recent attempts to understand

Inflation in String Theory

I Early Universe Inflation

\( \ddot{a} > 0, \dot{a} > 0, 10^{-35}\) sec

** Number of e-foldings \( N \geq 60 \)

*** Exit to deceleration \( \ddot{a} < 0, \dot{a} > 0 \)

II Late time inflation (accelerating universe)

\( \ddot{a} > 0, \dot{a} > 0, 7-14\) billion years

** Number of e-foldings \( N \sim 1 \) so far

*** Exit? Future of the universe?
Outline

1) KKLT model of dS space
   - Landscape of string theory

2) Warm up: inflation in N=1 d=4 supergravity
   - F-term, P-term

3) Issues raised in KKLT paper
   - On inflation in string theory
   - Brane inflation models

4) Brane inflation + P-brane inflation
   - With some exceptions

5) de Sitter space and inflation in string theory

No-go theorems: Maldacena & Nunez 2001
   - Gibbons 1985

Problem: In terms of canonical scalars (moduli) representing dilaton and volume of the compactified space

\[ V = e^{-\sqrt{2} \Phi - \sqrt{6} \varphi} \tilde{V}(\phi, \eta) \]

- Type II B

Exponents are too steep, runaway behavior

\[ \Phi \rightarrow -\infty \]

\[ \varphi \rightarrow +\infty \]

Interpretation: Decompactification

4D \Rightarrow 10D
**Possible solution**

Kachru, Kallosh, A., L., Trivedi

Use IIB compactifications with nontrivial NS and RR fluxes and calculate nonperturbative contributions to the moduli potential in presence of D3 branes

**Result:** Dilaton is fixed

Giddings, Kachru, Polchinski 2002

and volume modulus is fixed as well:

Kachru, Kallosh, A., L., Trivedi

\[ V = e^K \left( |DW|^2 - 3|W|^2 \right) \]

\[ K = -3 \ln (g + \overline{g}) \]

Non-perturbative string corrections to the superpotential:

\[ W_g = W_0 + A e^{-g} \]

instantons, gaugino cond.

The volume is fixed at AdS minimum with unbroken supersymmetry

\[ V_{AdS} = -3e^K |W|^2 < 0 \]
Add to the picture a D3 brane at the tip of the resolved conifold.

New contribution:

\[ \Delta V = \frac{D}{S^3} \]

Explicit breaking of supersymmetry.

de Sitter space is metastable.

KKLT model may describe late inflation with \( \sum = -1 \).

Burgess, A.K., Quevedo work in progress.

An alternative to KKLT with spontaneous breaking of supersymmetry.

Replace D3 by fluxes of gauge fields within the D7 brane.

The uplifting of AdS to dS:

\[ \Delta V = \frac{D}{S^x} \]

can be understood as a field-dependent D-term in dS vacuum D-flatness condition is broken spontaneously!

New features in landscape:

Susskind, Giddings, Douglas

NYT
$N=1$ supergravity in $d=4$

Effective superstring theory

Kähler potential $K(z, \bar{z})$

Superpotential $W(z)$ holomorphic

Kinetic term function $f_{\mu\nu}(z)$ holomorphic

Most general potential

$$V_F = e^K (|DW|^2 - 3|W|^2) \sim |F|^2$$

$$V_D = \frac{1}{4} \text{Re} f_{\mu\nu}(z) D_{\mu} D_{\nu} \geq 0 \sim |D|^2$$

\[-\frac{1}{2} \text{Re} f_{\mu\nu}(z) F_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{i}{4} \text{Im} f_{\mu\nu}(z) F_{\mu\nu} \tilde{F}^{\mu\nu}\]

Chiral superfield auxiliary field $F = P_1 + i P_2$

Vector --- $u$ --- $D = P_3$

Supergravity and Cosmology $\frac{1}{W}$ issue


COSMOLOGICAL MODEL

Hybrid inflation

Linde 1991

- Long slow-roll near de Sitter inflationary stage

- Change of regime near the bifurcation point of the potential

- End of inflation: waterfall stage towards a ground state, tachyon condensation and reheating
F-term hybrid inflation

Copeland, Liddle, Lyth, Stewart, Wands, Dvali, Shaﬁ, Sch¨afer

\[ K = \frac{|\Sigma|^2 + |\Phi|^2 + |\bar{\Phi}|^2}{m_p^2} \]

\[ W = \frac{1}{12g} S(\Phi\Phi - \frac{7}{2}) \]

\[ V_F = 2g e^{\frac{1}{m_p^2}} \left[ |\Sigma|^2 \left( \frac{dW}{dS} \right)^2 \left( 1 + \frac{|\Sigma|^2}{m_p^2} \right)^2 - 3 \frac{|\Sigma|^2}{m_p^2} \right] \]

\[ V_F = V_0 \left[ 1 + \frac{|\Sigma|^2}{m_p^2} + \frac{2|\Sigma|^2}{m_p^2} - 3 \frac{|\Sigma|^2}{m_p^2} + \frac{|\Sigma|^4}{2 m_p^2} + \ldots \right] \]

S is an inflaton

\[ m_s^2 \sim H^2 \quad \text{kills inflation} \]

Fine-tuning \( K, W \)

\[ m_s^2 \sim 0 \]

D-term inflation

Binetruy, Dvali, Halyo

\[ V_{F+D} = e^K \left[ |DW|^2 + m^2 |W|^2 \right] + \frac{1}{2} \text{Re} f(z) D^2 \]

\[ f(z) = 1 \quad \text{or} \quad f(z) \text{ may depend on dilaton, volume moduli but not on inflaton!} \]

No problem \( m^2 \sim H^2 \)

No need for fine-tuning!

Fayet–Iliopoulos terms

\[ D = (|\Phi|^2 - |\bar{\Phi}|^2) \]

\[ K = \frac{|\Sigma|^2 + |\Phi|^2 + |\bar{\Phi}|^2}{m_p^2} \]

\[ W = \lambda S \Phi + \Phi - \ldots \]

Inflation at \( W \sim DW \sim 0 \)

\[ D^2 \neq 0 \]
P-term Hybrid Inflation

R. Kallosh and A. L. hep-th/0306058

- Unification of F- and D-term models of Brane Inflation with $\lambda = \sqrt{2}g$

- Gravity coupling: new parameter, $0 \leq f \leq 1$

  D-term $\to f = 0$, F-term $\to f = 1$

- Cosmological applications
  
  Controllable running of the spectral index:
  
  $\frac{dn_s}{dk} \sim f$

  Suppression of cosmic strings $n_s = 1$

THE POTENTIAL IN P-TERM INFLATION

Salam-Strathdee-Fayet N=2 susy gauge model

Auxiliary field triplet $\tilde{p}$, FI triplet $\xi$

$$\tilde{p} = -g[(\Phi^\dag \sigma \Phi) - \tilde{\xi}]$$

Potential

$$V^P = 2g^2 \left[ \Phi^\dag \Phi |\xi|^2 + \left( \frac{1}{4} \Phi^\dag \sigma \Phi - \frac{\tilde{\xi}}{g} \right)^2 \right]$$

F and D

$$2g^2 \left( |S \Phi_+|^2 + |S \Phi_-|^2 + |\Phi_+ \Phi_- - \frac{\xi_1 + i \xi_2}{2}|^2 \right) +$$

$$\frac{g^2}{2} \{ |\Phi_+|^2 - |\Phi_-|^2 - \xi_3 \}^2.$$
Add 1-loop gauge corrections and coupling to gravity

\[ V = \frac{g^2 \xi^2}{2} \left( 1 + \frac{g^2}{8\pi^2} \ln \frac{|S^2|}{|S_c^2|} + f \frac{|S|^4}{2M_p^4} + \ldots \right), \]

\[ f = \frac{\xi_1^2 + \xi_2^2}{\xi_2^2}, \quad 0 \leq f \leq 1, \]

D-term inflation, \( f = 0 \), minimal running of \( n_s \)

F-term inflation, \( f = 1 \), maximal running of \( n_s \)  
(A. L. and A. Riotto, 1997)

P-term inflation model interpolates between these two cases

Choice of \( \xi_1, \xi_2, \xi_3 \Rightarrow \) Fluxes on a brane

Towards Inflation in String Theory

Kachru, R.K., Linde, Maldecena, McAllister, Trivedi, hep-th/0308055

An investigation of inflation based on KKLT

Braun inflation (Dvali, Tye 1998, ...)  
Review: Quevedo 2002

Brauns and/or anti-brauns in 6d internal space of finite volume

The issue of volume stabilization was not addressed in previous work \( \Rightarrow \) "naive Brauns inflation models"

Inflation \( \Rightarrow \) distance between brauns, \( \ell \) 
Linear size of internal space, \( L \)
Two main points

1) Even in "naive brane-anti-brane inflation models" there was a problem to make small slow roll parameter
   \[ \theta = \frac{r''}{r} \ll 1 \]

   it was necessary to have distance between \( \mathcal{O}_3 - \mathcal{O}_3 \) larger than the scale of CY
   \[ \ell \gg L \]

   which does not make sense

   We studied \( \mathcal{O}_3 - \mathcal{O}_3 \) model in warped geometry (Klebanov-Strassler)

   \[ \theta_{\text{warped}} = \theta \left( 1 - \frac{2\pi \kappa}{3g \mathcal{M}} \right) \ll 1 \quad \text{GKP} \]

   \( \kappa, \mathcal{M} \) are the background fluxes
   easy to make \( \theta \) very small

2) A general argument why ALL "naive brane inflation models" may not work was suggested

   String theory version of \( m^2 \sim H^2 \) problem

   Any source of inflationary energy in GKP setting
   \[ \frac{C}{r^\alpha} \quad \alpha > 0 \]

   \[ r \sim R^\alpha \]

   \[ ds^2_{10} = R^{-6} \tilde{g}_{\mu \nu} dx^\mu dx^\nu + \frac{R^2}{g_{\text{ads}}} dy^a dy^b \]

   \[ 2 \varphi = \rho + \rho - \varphi \quad \text{inflaton assumption} \]

   \[ r \sim R^\gamma \rightarrow \text{volume modulus} \]
KKLT breaking of "no-scale"

\[ V \sim \frac{\mathcal{C} [\rho]}{(\rho + \bar{\rho} - \gamma \Phi)^x} \]

If \( \rho \) is stabilized as in KKLT

\[ V \sim V_0 \left( 1 + |\Phi|^2 + \cdots \right) \]

where \( \Phi \) is a canonically normalized inflation field

\[ m^2 \sim H^2 \]

We need an analog of fine-tuning in F-term supergravity

Specific example

\[ W[\rho, \varphi] \]

\( \rho \) and \( \varphi \) dependent superpotential

Problem and its possible resolution

We may extend the KKLT model to include the dependence on fields \( \varphi, \bar{\varphi} \) describing D-brane position

\[ \kappa = -3 \log (\rho + \bar{\rho} - \gamma \Phi) \]

\[ W = W_0 + A e^{-q \rho} \left( 1 + \delta \varphi^2 \right) \]

The calculation of the supergravity potential \( V_\gamma \) shows that

\[ m^2 = 2 H^2 - \frac{\alpha}{3} \left| V_{\text{AdS}} \right| (\beta - 2 \beta^2) \]

\[ \beta = -\frac{5}{\alpha} \]

Subexponential dependence of \( W \) on \( \varphi \)

may save the day

Stringy model?
Can we avoid fine-tuning altogether?

The source of the problem was $e^K$ contribution to the F-term.

Our model is similar to F-term hybrid inflation, which requires fine-tuning.

It is known, however, that there is no $e^K$ contribution in D-term hybrid inflation. Binetruy, Dvali 96.


Everything might work without any fine-tuning if FI terms are possible in string theory.

Recent SB discussion

Burgess, Koch, R.K., Quevedo, Maldacena

To embed "naive brave inflation models" into full string theory with volume stabilization one has to resolve the following issue.

Take GKP setting with the fixed dilatou as a background for D7: $F_{\mu\nu} F^{\mu\nu} g^{\alpha\beta} g_{\alpha\beta} \rightarrow$

in 4d Einstein frame $\frac{1}{g^2} \sim r \sim R^4$.

However "the volume" / inflaton $r = \frac{1}{4} (\rho + \bar{\rho} - m \bar{\phi})$ is not a real part of the holomorphic superfield!

Contradiction with supersymmetry

$\frac{1}{2} \text{Re} f(z) F_{\mu\nu}^2$

for any Dp brave $P \neq 3$. 

Interpretation of energy from fluxes on D-branes in IIB with fixed dilaton

\[ \frac{F_a \alpha_s}{r^a} \]

with inflaton dependence is inconsistent with GKP + supersymmetry.

A better understanding of VOLUME MODULUS

in presence of Dp-branes with \( p \neq 3 \) is required to promote brane inflation models into string theory.

What is the correct dependence on inflaton (distance between branes) ?

What is the correct ansatz for the metric ?

How to avoid contradiction with susy ?

Before these problems are resolved, we may study "naive brane inflation models"

D-brane inflation

Exit, reheating

Dp \( \bar{D}p \), \( Dp \bar{D}p \)

at angles

Tachyon condensation

Sen's conjecture \( e^{-T} \)

Min at infinite distance

Kofman, Linde problem with tachyon reheating

Exit and reheating difficult

| Dp \( \bar{D}p+n \)
| \( D_4 \bar{D}_6 \), \( D_3 \bar{D}4 \) |
| Tachyon potential is calculable |
| Min at finite distance |
| No problem with exit and reheating |

D-term inflation or F-term inflation
**D3/D7 Cosmological Model and M Theory**

Dasgupta, Herdeiro, Hirano, R. K.

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D7 worldvolume: \(\mathcal{F}_{67} = \tan \theta\) \(\mathcal{F}_{89} = \tan \theta'\)

\(\mathcal{F}^- \neq 0\): spontaneously broken susy

**D7-D3 Strings Spectrum**

\[ M_{\frac{2}{3}} = \frac{d^2}{(\pi \alpha')^2} \pm \frac{\theta - \theta'}{2\pi \alpha'} \quad M_{\psi}^2 = \frac{d^2}{(\pi \alpha')^2} \]

\[ d^2 = (x^4)^2 + (x^5)^2 \]

\[ d_{\text{critical}} \sim \mathcal{F}^- \]
N=2 susy, 1 vector, 1 hyper with mass splitting $\rightarrow$ UNIQUE POTENTIAL

THE POTENTIAL IN P-TERM INFLATION
Salam-Strathdee-Fayet model

$$V = \frac{g^2}{2} \left[ \Phi^\dagger \Phi |\varphi|^2 + \left( \frac{1}{2} \Phi^\dagger \sigma^r \Phi - \frac{\xi^r}{g} \right)^2 \right]$$

- Unstable Non-supersymmetric Vacuum
  Inflation when coupled to gravity
  De Sitter valley, Coulomb phase

$$P^r = \xi^r \quad |\varphi| >> 0 \quad V = \frac{1}{2} \xi^2$$

- Stable Supersymmetric vacuum, Higgs phase

$$P^r = 0 \quad \varphi = 0 \quad V = 0$$

Known to String Theorists also as D-flatness condition
and ADHM equations in instanton construction

$$P^r = \frac{g}{2} \Phi^\dagger \sigma^r \Phi - \xi^r = 0$$

$D7/D3$ bound state

Hybrid Inflation
A.L. 91
Easier to implement in
supergavity and string cosmology

Cosmological potential with Fayet-Iliopoulos term

De Sitter valley is classically flat; it is lifted by the one-loop correction corresponding to the one-loop potential between $D4$,$D6$. In this figure the valley is along the $|\Phi_3|$ axis; the orthogonal direction is a line passing through the origin of the complex $\Phi_3$ plane and we have put $|\Phi_1| = 0$. The bifurcation point corresponds to $|\Phi_3| = \sqrt{\xi/g}$, $\Phi_3 = 0$. The absolute minimum is at $\Phi_3 = 0$, $\Phi_2 = \sqrt{2\xi/g}$.
HYPERMULTILEPT MASS SPLITTING
controlled by spontaneous susy breaking

\[ \text{STr } M^2 = 0 \]

\[ M_\pm^2 = g^2 |\varphi|^2 \pm g \xi \quad M_\psi = g |\varphi| \]

**Bifurcation point:** hyper becomes massless,
\[ M_\pm^2 = 0 \quad \Rightarrow \quad |\varphi|^2 = \frac{\xi}{g} \]

At \( \varphi \geq \varphi_c \), de Sitter minimum

At \( \varphi \leq \varphi_c \), de Sitter maximum

Beyond the critical point, scalars become tachyonic. The system is unstable and the waterfall stage of the potential, mixed Coulomb-Higgs phase, leads it to a ground state, Higgsed phase.

Instability and \( \kappa \)-symmetry of the worldvolume

With FI parameter D3/D7 system at the distance from each other is unstable.

**D7-brane worldvolume action in D3-brane background**

\[-T_7 \left( \int d^8 \sigma e^{-\phi} \sqrt{-\det (g + \mathcal{F})} - \int \sum A_{p+1} \wedge e^\mathcal{F} \right) \]

We turn on the worldvolume gauge field

\[ F_{67} = \xi \quad F_{89} = \xi' \]

**Effective potential**

\[ \text{BI} : \quad T_7 V_3 \int d^4 \sigma \left[ \sqrt{(1 + H^{-1} \xi^2)(1 + H^{-1} \xi'^2)} \right] \]

\[ \text{WZ} : \quad T_7 V_3 \int d^4 \sigma \left[ -H^{-1} \xi \xi' \right] \]

\[ H = 1 + Q/r^4 \quad r^2 = d^2 + (\sigma)^2 \]
When $\xi = \xi'$, the force between the D3 and D7 vanishes.

$$\mathcal{F}^- = 0$$

For arbitrary but small values of the parameters $\xi, \xi'$ and large distances $d$ the finite, non-constant part of the potential becomes

$$V \simeq \pi^2 T_7 V_3 Q (\mathcal{F}^-)^2 \ln d^2$$

$d^2$ is interpreted as an inflaton field $\varphi^2$

$(\mathcal{F}^-)^2$ is interpreted as FI terms $\xi^2$

Same for D7 brane background with $B^- \neq 0$ probed by D3 brane

In Coulomb branch D3/D7 system has broken supersymmetry and slow-roll inflation is possible due to

$$\mathcal{F}^- \neq 0$$

THE BIG QUESTION is: how D3/D7 system can become supersymmetric with $\mathcal{F}^- \neq 0$ in the Higgs branch? The solution is:

**D3/D7 SUPERSYMMETRIC BOUND STATE**

The necessary condition for the bound state to be supersymmetric is that the rotation factor $a$ of $\kappa$-symmetry is $\sigma$-independent.

$$\frac{\partial a(\mathcal{F})}{\partial \sigma} = 0 \quad a = \frac{1}{2} Y_{ij} \Gamma_{ij} \otimes \sigma_3$$

$$Y = Y(\mathcal{F})$$

A 2-form $Y$ is a known non-linear function of $\mathcal{F}$
**κ-Symmetry and Deformation**

Deformed vector field strength \( \hat{F} \)

\[
\hat{F} = Y(F - B) - Y(-B)
\]

D3/D7 bound state is supersymmetric iff

\[
\exp\left(-\frac{1}{2} \sigma_3 \otimes \Gamma^{ij} Y_{ij}^- (B)\right) i\sigma_2 \otimes \Gamma_{01236789} \epsilon = \epsilon
\]

\[
(1 - \Gamma_{6789}) \epsilon = 0
\]

\[\hat{F}^-(\sigma) = 0\]

Non-linear supersymmetric instanton in 6,7,8,9 space of D7

\[
B^- = \frac{1}{2} (B_{67} - B_{89}) \neq 0
\]

Supersymmetry with FI terms present, ADHM equations follow! D3/D7 cosmological model justified: Coulomb branch, tachyon condensation, Higgs branch

It is tempting to speculate

In the context of Instantons on Non-commutative \( R^4 \) Nekrasov-Schwarz, 1998 FI terms are necessary to make the instantons non-singular.

In the context of Dirac-Born-Infeld non-linear instantons Seiberg-Witten, 1999 FI terms are necessary to have a finite non-vanishing instanton number.

This speculation, if taken seriously, would give an explanation of the non-vanishing effective cosmological constant (in the early universe and today): it may be needed to remove certain instanton singularities.
Current work in progress
with Stanford students Hsu, Prokushkin

How to generalize the Dy D6 Brane construction at angles D3 D7 with fluxes
to include more general case with controllable spectral index?

Previous case: only one angle in 6-7 plane
We add 6-8 and 6-9
only F67 - F89
We add F68 - F99
F69 - F78

If observations (Hui, Shtanov) will lead to \( \frac{du}{dk} \rightarrow 0 \) \rightarrow D-term model
if \( \frac{du}{dk} \neq 0 \) \rightarrow mix with F-term
Fluxes on branes \rightarrow Observations

Spectral index \( n \) in F-term, D-term and P-term inflation

Beginning of inflation
F-term
P-term
D-term

\( (lnk \sim N) \)