Local Grand Unification

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TECHNISCHE UNIVERSITÄT MÜNCHEN

Santa Barbara, String Pheno, August 29, 2006

Based on:
W. Buchmüller, K. Hamaguchi, O. Lebedev & M.R. :

- Nucl. Phys. B 712, 139 &
- Phys. Rev. Lett. 96, 121602 &
- hep-ph/0512149 &
- hep-th/0606187

O. Lebedev, H. P. Nilles, S. Raby, S. Ramos-Sánchez, M. R., P. Vaudrevange & A. Wingerter :

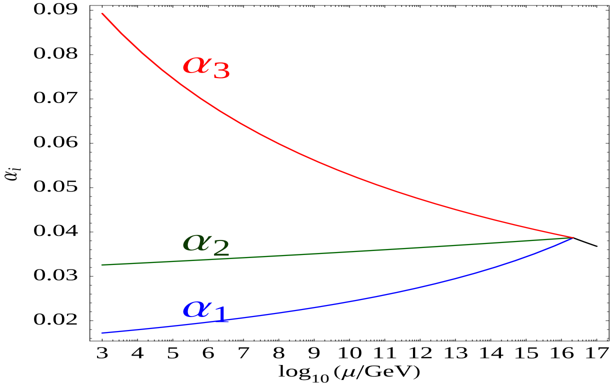
- in preparation

Outline

1. Motivation
2. Local grand unification
   (using heterotic $\mathbb{Z}_3 \times \mathbb{Z}_2$ orbifold)
3. Brief discussion of the MSSM from the heterotic string
4. Summary and outlook

Beautiful and ugly aspects of GUTs

☺ MSSM gauge coupling unification @ $M_{\mathrm{GUT}} \sim 10^{16}\,\mathrm{GeV}$



Beautiful and ugly aspects of GUTs

- ☺ MSSM gauge coupling unification @ $M_{\mathrm{GUT}} \sim 10^{16}\,\mathrm{GeV}$
- ☺ One generation of observed matter fits into $\mathbf{16}$ of $\mathrm{SO}(10)$

$$
\begin{aligned}
\mathrm{SO}(10) \quad &\rightarrow \quad \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_{\mathrm{Y}} = G_{\mathrm{SM}} \\
\mathbf{16} \quad &\rightarrow \quad (\mathbf{3},\mathbf{2})_{1/6} \oplus (\overline{\mathbf{3}},\mathbf{1})_{-2/3} \oplus (\overline{\mathbf{3}},\mathbf{1})_{1/3} \\
&\qquad \oplus (\mathbf{1},\mathbf{1})_{1} \oplus (\mathbf{1},\mathbf{2})_{-1/2} \oplus (\mathbf{1},\mathbf{1})_{0}
\end{aligned}
$$

Beautiful and ugly aspects of GUTs

- ☺ MSSM gauge coupling unification @ $M_{\mathrm{GUT}} \sim 10^{16}\,\mathrm{GeV}$
- ☺ One generation of observed matter fits into $\mathbf{16}$ of $\mathrm{SO}(10)$
- ☹ However: Higgs only as doublet(s)

$$\mathbf{10} \rightarrow \underbrace{(\mathbf{1},\mathbf{2})_{1/2} \oplus (\mathbf{1},\mathbf{2})_{-1/2}}_{\text{doublets: needed}} \oplus \underbrace{(\mathbf{3},\mathbf{1})_{-1/3} \oplus (\overline{\mathbf{3}},\mathbf{1})_{1/3}}_{\text{triplets: excluded}}$$

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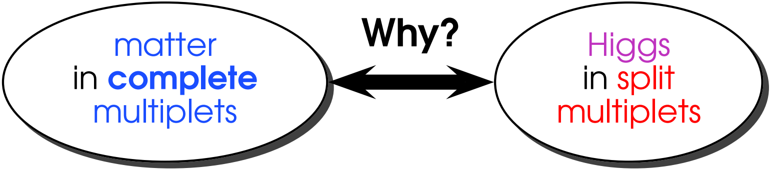
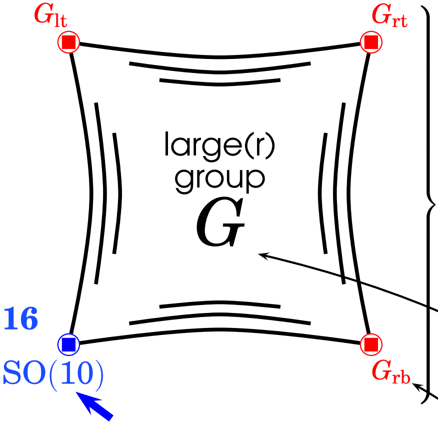


convincing answer:

'localized gauge groups'

Local grand unification (using **small** extra dimensions)

Buchmüller, Hamaguchi, Lebedev, M.R. (2004-2006)
Lebedev, Nilles, Raby, Ramos-Sánchez,
M.R., Vaudrevange, Wingerter (to appear)



$\xrightarrow[\text{effective theory}]{\text{‘low-energy’}}$

standard model as an intersection of $G_{\mathrm{rb}}, G_{\mathrm{rt}}, G_{\mathrm{lt}}$ & $\mathrm{SO}(10)$ in G

SM generation(s):

localized in region with $\mathrm{SO}(10)$ symmetry

Higgs doublets:

live somewhere else

Higher-dimensional GUTs vs. heterotic orbifolds

top-down

→ Orbifold compactifications of the heterotic string

Dixon, Harvey, Vafa, Witten (1985-86)
Ibáñez, Nilles, Quevedo (1987)
Ibáñez, Kim, Nilles, Quevedo (1987)
Font, Ibáñez, Nilles, Quevedo (1988)
Font, Ibáñez, Quevedo, Sierra (1990)
Katsuki, Kawamura, Kobayashi, Ohtsubo, Ono, Tanioka (1990)
...

- has UV completion
- automatically consistent
- explain representations

bottom-up

→ Orbifold GUTs

Kawamura (1999-2001)
Altarelli, Feruglio (2001)
Hall, Nomura (2001)
Hebecker, March-Russell (2001)
Asaka, Buchmüller, Covi (2001)
Hall, Nomura, Okui, Smith (2001)
...

- simple geometrical interpretation
- shares many features with 4D GUTs

combine both approaches

implement field-theoretic GUTs in non-prime orbifold compactifications of the heterotic string

Kobayashi, Raby, Zhang (2004)
Förste, Nilles, Vaudrevange, Wingerter (2004)
Nilles (2004)
Buchmüller, Hamaguchi, Lebedev, M.R. (2004-2006)
Faraggi, Förste, Timirgaziu (2006)
Kim, Kyae (2006)
Lebedev, Nilles, Raby, Ramos-Sánchez, M.R., Vaudrevange, Wingerter (to appear)

Higher-dimensional GUTs vs. heterotic orbifolds

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What's new?

- systematic analysis of non-prime orbifolds with Wilson lines
- geometric picture with various orbifold GUT limits
- anisotropic compactification may mitigate the discrepancy between GUT and string scales

  Witten (1996)
  ...
  Hebecker, Trapletti (2005)
- localized **16**-plets as the origin of complete generations

Local Grand Unification

in

$$\mathbb{Z}_{\mathbf{6}} - \mathbf{II} = \mathbb{Z}_{\mathbf{3}} \times \mathbb{Z}_{\mathbf{2}}$$

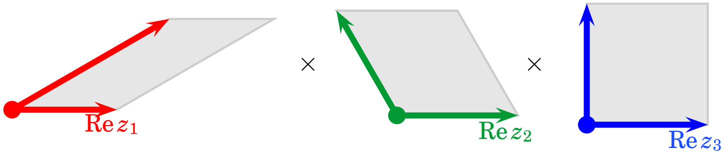
orbifolds

Compactification on $\mathbb{T}^6/\mathbb{Z}_6$ orbifold ($\mathbb{Z}_6$ – II)

$\mathbb{T}^6$ torus is defined by the root lattice

Kobayashi, Raby & Zhang (2004)

$\Lambda_{\mathrm{G}_2\times\mathrm{SU}(3)\times\mathrm{SO}(4)}$:= root lattice of Lie algebra of $\mathrm{G}_2\times\mathrm{SU}(3)\times\mathrm{SO}(4)$



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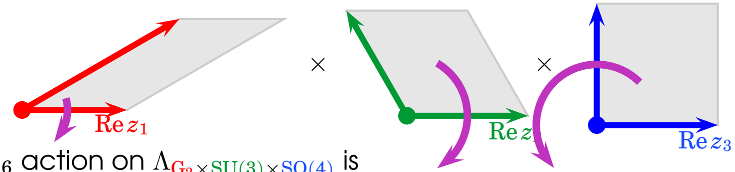

The $\mathbb{Z}_6$ action on $\Lambda_{\mathrm{G}_2\times\mathrm{SU}(3)\times\mathrm{SO}(4)}$ is

$$z_i \to e^{2\pi\mathrm{i}\, v_6{}^i}\, z_i \qquad \text{with} \qquad v_6 = \frac{1}{6}(-1,-2,3)$$

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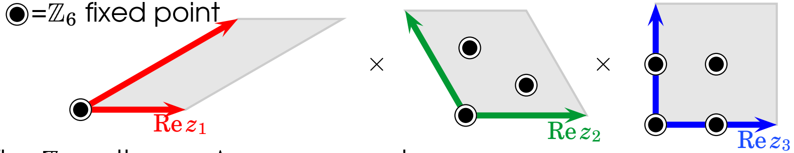

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and has $\mathbb{Z}_k$ ($k = 2,3,6$) fixed points:

$$z^i_{\mathbb{Z}_k\mathrm{f.p.}} - e^{2\pi\mathrm{i}\,\frac{6}{k}v_6{}^i}\,z^i_{\mathbb{Z}_k\mathrm{f.p.}} \in \Lambda_{\mathrm{G}_2\times\mathrm{SU}(3)\times\mathrm{SO}(4)}$$

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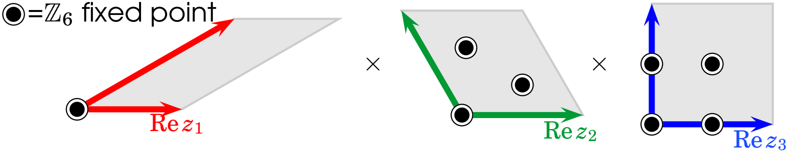


twist action is embedded into the gauge degrees of freedom

left-movers $X^I \to X^I + \pi\, V_6^I$ (where $6V_6 \in \Lambda_{\mathrm{E}_8\times\mathrm{E}_8}$)

torus translations are associated to Wilson lines, e.g.

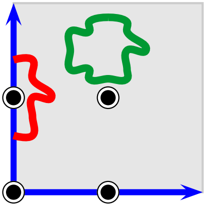
$$z_3 \to z_3 + 1 \quad \leftrightarrow \quad X^I \to X^I + \pi\, W_2 \quad \text{(where } 2W_2 \in \Lambda_{\mathrm{E}_8\times\mathrm{E}_8})$$

Light states

Light states of effective field theory ($k \equiv 0$ for untwisted sector)

$$\Psi_{r,s}(x; z_1, z_2, z_3)$$

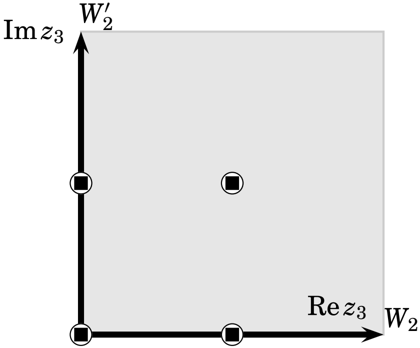
$$r = p + k\,V_6 \qquad s = q + k\,v_6$$



heterotic string	field theory
untwisted sector = strings closed on the torus	extra components of gauge fields
T_k twisted sector = strings which are only closed on the orbifold	'brane fields' (hard to understand in field-theoretical framework)

Local gauge symmetry (breaking)

Analyze invariance conditions **locally** (for illustration just in $\mathrm{SO}(4)$ plane)



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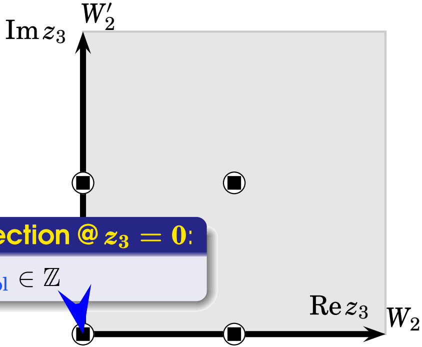

projection @ $z_3 = 0$:

$p \cdot V_{\mathrm{bl}} \in \mathbb{Z}$

local shift

$$e^{2\pi \mathrm{i}\, p \cdot V_{\mathrm{bl}}} A_\mu^p(x; \dots, e^{2\pi \mathrm{i}/2} z_3) = A_\mu^p(x; \dots, z_3)$$

gauge group generator

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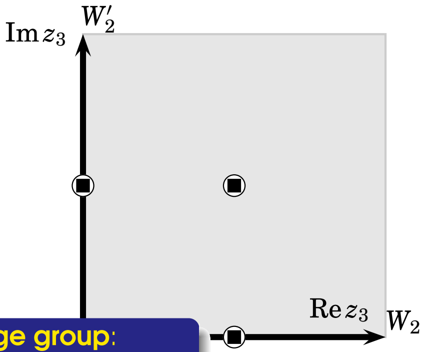

emerging *local* gauge group:

$G_{\mathrm{bl}} \subset \mathrm{E}_8 \times \mathrm{E}_8$

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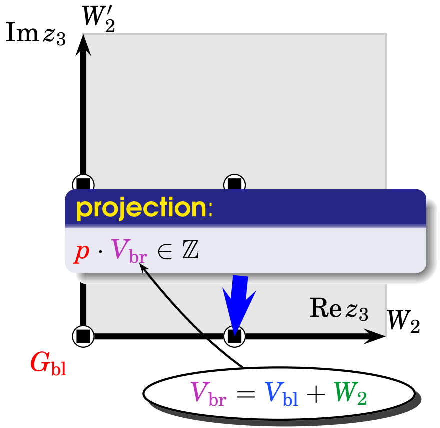

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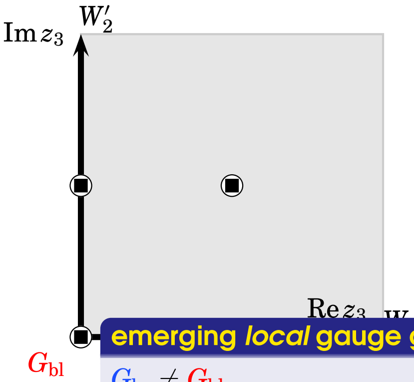

emerging *local* gauge group:

$G_{\mathrm{br}} \neq G_{\mathrm{bl}}$

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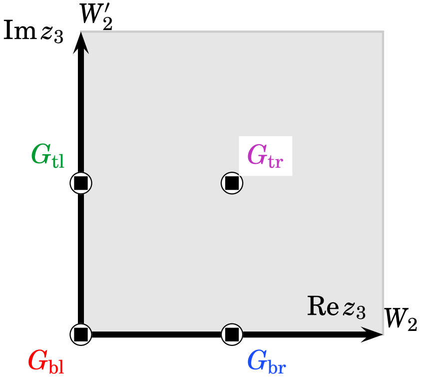

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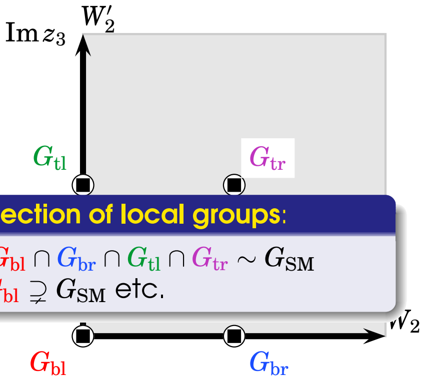

intersection of local groups:

e.g. $G_{\mathrm{bl}} \cap G_{\mathrm{br}} \cap G_{\mathrm{tl}} \cap G_{\mathrm{tr}} \sim G_{\mathrm{SM}}$
but $G_{\mathrm{bl}} \supsetneq G_{\mathrm{SM}}$ etc.

G_{bl} etc. : 'local GUTs'

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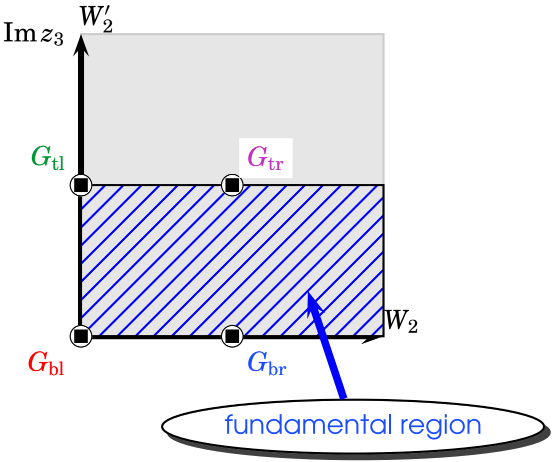

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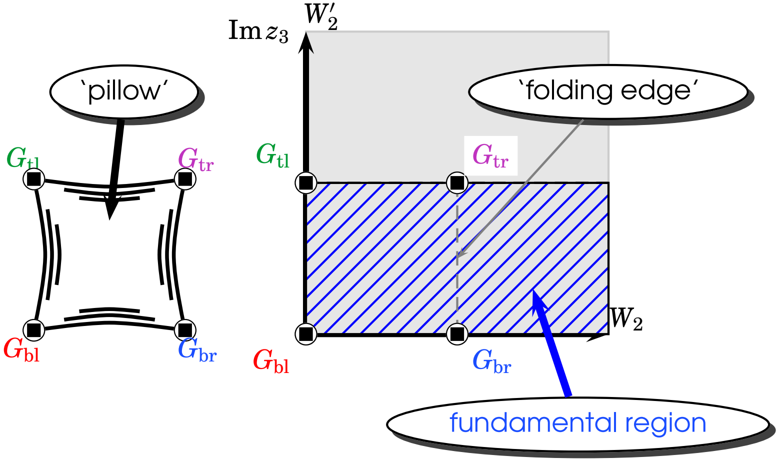
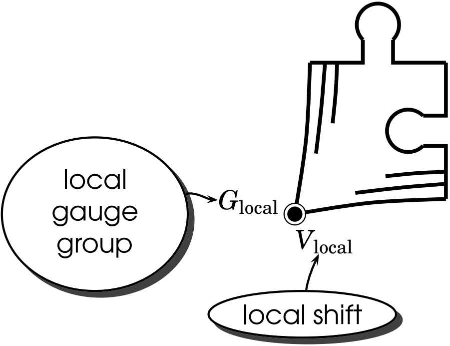


The 'orbifold construction kit'



basic structure: one 'corner' with shift V

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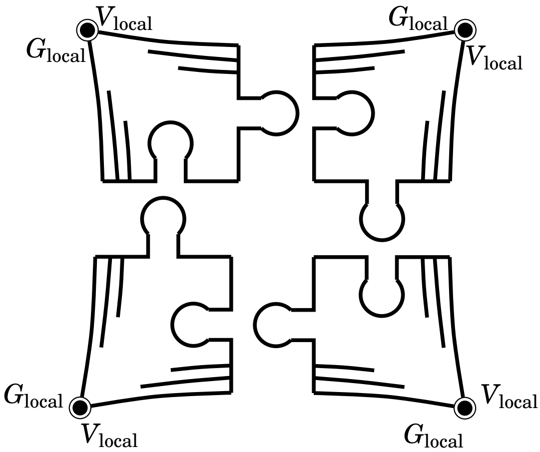

simplest possibility: consider identical corners

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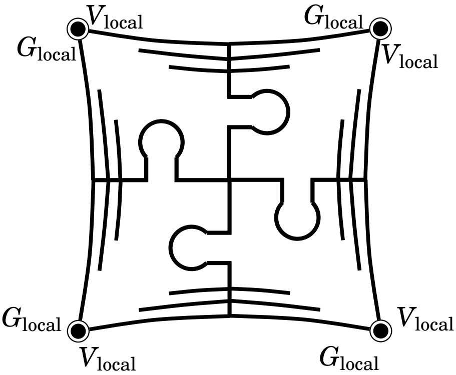

the combination corresponds to an

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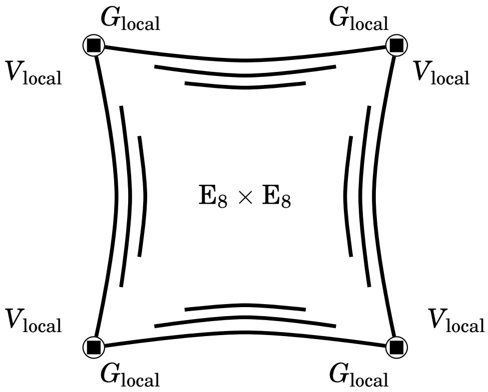

orbifold without Wilson lines

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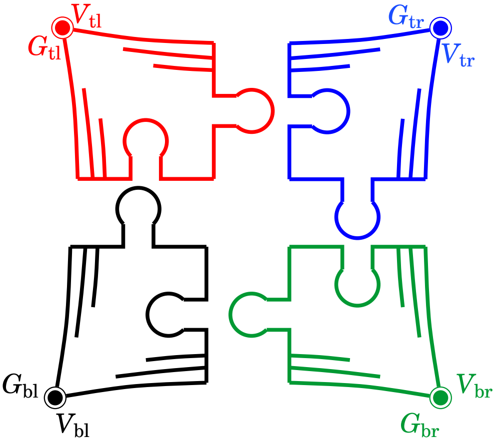
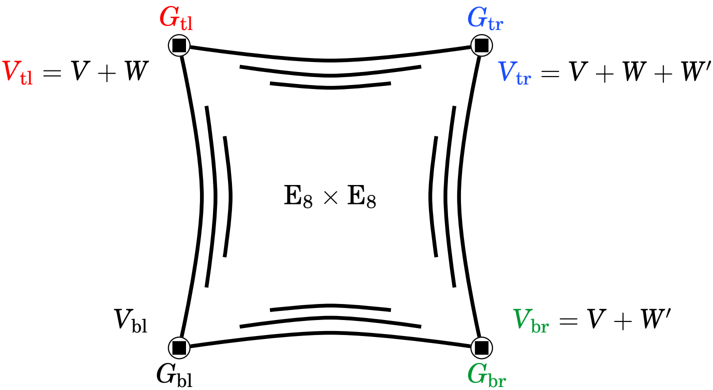


one can combine different 'corners'

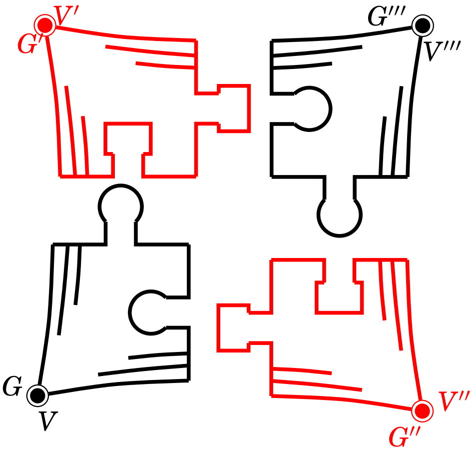
The 'orbifold construction kit'



this leads to an orbifold **with Wilson lines**
where the Wilson lines correspond to the differences of local shifts and

$$G_{\mathrm{low-energy}} = G \cap G' \cap G'' \cap G'''$$

The 'orbifold construction kit'



but there are restrictions from modular invariance
i.e., one may combine the 'corners' not arbitrarily

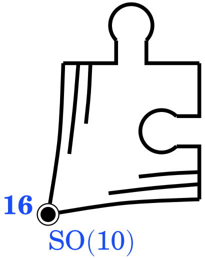
The role of localized $\mathbf{16}$-plets of $\mathrm{SO}(10)$

Buchmüller, Hamaguchi, Lebedev, M.R. (2004)

- **basic observation**: the states of the 1^{st} twisted sector appear as **complete** multiplets of the local gauge group

- **main idea**: use localized $\mathbf{16}$-plets of $\mathrm{SO}(10)$ to explain generations

- $\mathbb{Z}_3 \times \mathbb{Z}_2$: there are two shifts which 'produce' local $\mathrm{SO}(10)$ and $\mathbf{16}$-plet in 1^{st} twisted sector

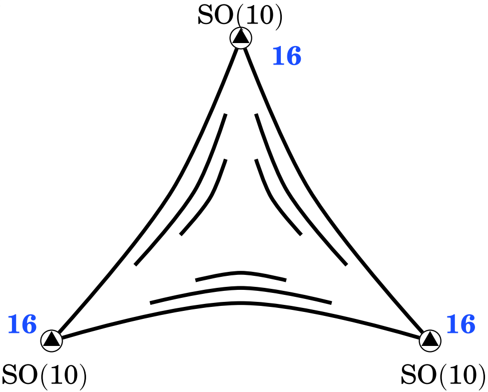


$$V_6 = \frac{1}{6}(3,3,2,0,0,0,0,0)\,(2,0,0,0,0,0,0,0)$$
$$V_6' = \frac{1}{6}(2,2,2,0,0,0,0,0)\,(1,1,0,0,0,0,0,0)$$

cf. Katsuki, Kawamura, Kobayashi, Ohtsubo, Ono, Tanioka (1990)

No-Go for three sequential families

- **simplest implementation**: three 'sequential' **16**-plets
- Only possible in $\mathbb{Z}_3 \times \mathbb{Z}_2$ etc. but not in $\mathbb{Z}_3$, $\mathbb{Z}_4$, $\mathbb{Z}_2 \times \mathbb{Z}_2$ etc.



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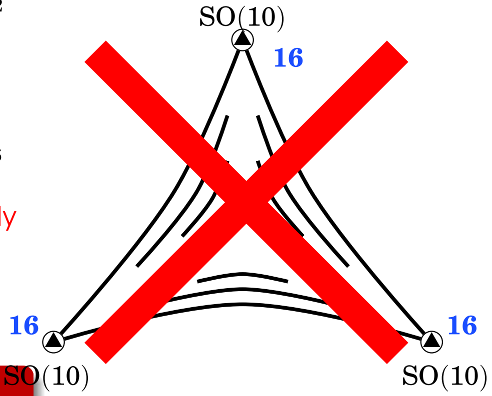


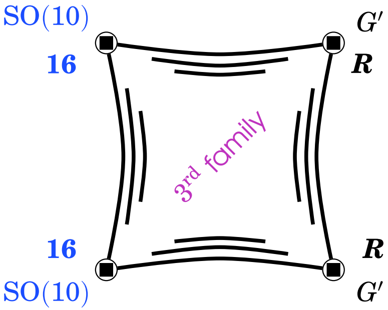
bottom-line

not possible in $\mathbb{Z}_{N\leq 7}$ orbifolds

2+1 family models

Features:

- Two families come from two equivalent fixed points
- 3^{rd} family has to come from 'somewhere else' (untwisted sector, $T_{k>1}$)



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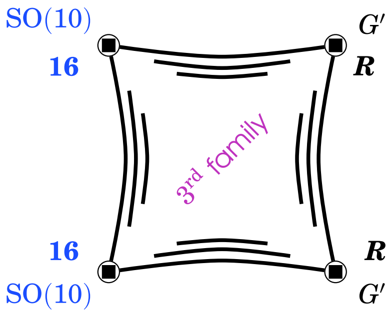

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- ☞ Two families come from two equivalent fixed points
- ☞ 3^{rd} family has to come from 'somewhere else' (untwisted sector, $T_{k>1}$)
- ☺ $\mathcal{O}(100)$ models with:
  - $\mathrm{E}_8 \to G_{\mathrm{SM}} \times \mathrm{U}(1)^4$
  - 3 generations + vector-like exotics $X_i, \overline{X}_j$
  - $X_i, \overline{X}_j$ have couplings

$$X_i\,\overline{X}_j \quad \underbrace{s_{i_1}\ldots s_{i_n}}_{\text{vev}\to\text{mass term}}$$

SM singlet

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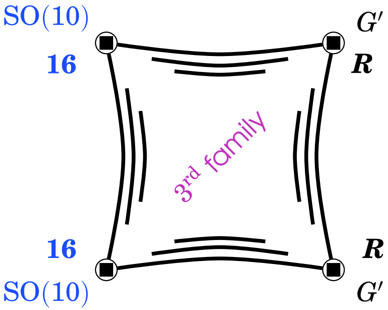


$$X_i \overline{X}_j \quad \underbrace{s_{i_1} \ldots s_{i_n}}_{\text{vev}\to\text{mass term}}$$

SM singlet

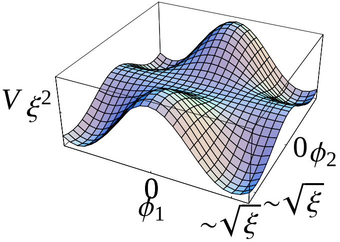
? are the exotics' mass terms consistent with supersymmetry?

Orbifold vacua, decoupling and $\mathrm{U}(1)$ breaking

orbifold point is 'saddle point'

$$V_D = g^2 \left(\sum q_{\mathrm{anom}}^{(i)} |\phi_i|^2 + g\,\xi\right)^2 + \dots$$

$$\xi = \frac{M_\mathrm{P}^2 \sum q_{\mathrm{anom}}^{(i)}}{192\pi^2} \sim 10^{-2\cdots-1} M_\mathrm{P}^2$$



☞ Exotics' masses are $\sim \langle \phi_1 \dots \phi_n \rangle$

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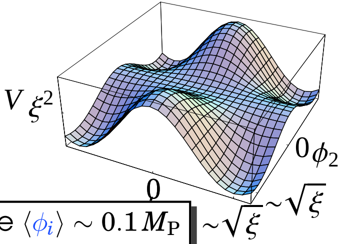

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➡ some $\langle\phi_i\rangle \sim 0.1\,M_\mathrm{P}$

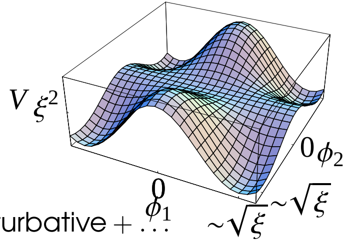
☞ Exotics' masses are $\sim \langle\phi_1 \dots \phi_n\rangle$

↪ Scale of vector-like exotics' masses is $\sim$ few $\times\, M_\mathrm{GUT} \simeq 10^{16}\,\mathrm{GeV}$

Orbifold vacua, decoupling and $\mathrm{U}(1)$ breaking

orbifold point is \`saddle point'

$$V_D = g^2 \left(\sum q_{\mathrm{anom}}^{(i)} |\phi_i|^2 + g\,\xi\right)^2 + \dots$$



$$V_F = \sum_i \left|\frac{\partial W}{\partial \phi_i}\right|^2 + \text{SUGRA} + \text{non-perturbative} + \dots$$

- It is possible to \`rescale' solutions of $\partial W/\partial\phi_i = 0$ to $V_D = 0$ by \`complexified gauge transformations'

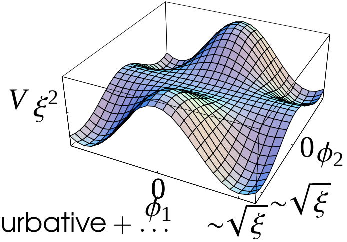
  e.g. Wess & Bagger … Gray, He, Jejjala, Nelson (06)

- These solutions are manifolds or points in field space

Orbifold vacua, decoupling and $\mathrm{U}(1)$ breaking

orbifold point is 'saddle point'

$$V_D = g^2 \left(\sum q_{\mathrm{anom}}^{(i)} |\phi_i|^2 + g\,\xi\right)^2 + \dots$$



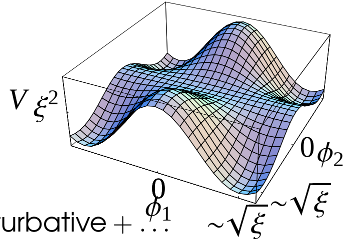
$$V_F = \sum_i \left|\frac{\partial W}{\partial \phi_i}\right|^2 + \text{SUGRA} + \text{non-perturbative} + \dots$$

- ☞ In models with discrete Wilson lines: there are usually no fields charged only under $\mathrm{U}(1)_{\mathrm{anom}}$
- ↪ @ the minimum of V: $n > 1$ gauge factors are broken (rank reduction)

Orbifold vacua, decoupling and $\mathrm{U}(1)$ breaking

orbifold point is 'saddle point'

$$V_D = g^2 \left(\sum q_{\mathrm{anom}}^{(i)} |\phi_i|^2 + g\,\xi\right)^2 + \dots$$



$$V_F = \sum_i \left|\frac{\partial W}{\partial \phi_i}\right|^2 + \text{SUGRA} + \text{non-perturbative} + \dots$$

bottom-line

there are many 'vacua' without exotics

➡

Remainder of this talk:

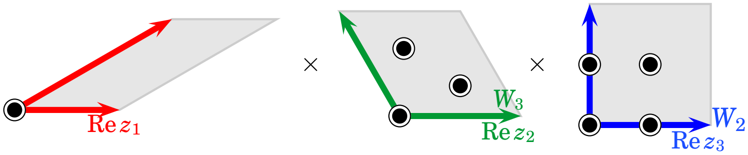
one specific model

The MSSM

from the

heterotic string

Lattice, shift and Wilson lines



$$
\begin{aligned}
V_6 &= \left(\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{3},0,0,0,0,0\right)\left(\tfrac{1}{3},0,0,0,0,0,0,0\right),\\
W_2 &= \left(\tfrac{1}{2},0,\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2},0,0,0\right)\left(-\tfrac{3}{4},\tfrac{1}{4},\tfrac{1}{4},-\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4},\tfrac{1}{4},-\tfrac{1}{4}\right),\\
W_3 &= \left(\tfrac{1}{3},0,0,\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3}\right)\left(1,\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3},0,0,0,0\right),
\end{aligned}
$$

Gauge group after compactification:

$$\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)_Y\times[\mathrm{SU}(4)\times\mathrm{SU}(2)\times\mathrm{U}(1)^8]$$

Spectrum @ orbifold point

The model exhibits 3 generations + vectorlike matter

name	irrep	count		name	irrep	count
q_i	$(\mathbf{3},\mathbf{2};\mathbf{1},\mathbf{1})_{1/6}$	3		$\bar{u}_i$	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{-2/3}$	3
$\bar{d}_i$	$(\overline{\mathbf{3}},\mathbf{1};\mathbf{1},\mathbf{1})_{1/3}$	3+4		d_i	$(\mathbf{3},\mathbf{1};\mathbf{1},\mathbf{1})_{-1/3}$	4
$\bar{\ell}_i$	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{1/2}$	1+4		ℓ_i	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{-1/2}$	3+1+4
m_i	$(\mathbf{1},\mathbf{2};\mathbf{1},\mathbf{1})_{0}$	8		$\bar{e}_i$	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{1}$	3
s_i^-	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{-1/2}$	16		s_i^+	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{1/2}$	16
s_i^0	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1})_{0}$	69		h_i	$(\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{2})_{0}$	14
f_i	$(\mathbf{1},\mathbf{1};\mathbf{4},\mathbf{1})_{0}$	4		$\bar{f}_i$	$(\mathbf{1},\mathbf{1};\overline{\mathbf{4}},\mathbf{1})_{0}$	4
w_i	$(\mathbf{1},\mathbf{1};\mathbf{6},\mathbf{1})_{0}$	5				

remarks:

- extra states vectorlike $\rightarrow \mathrm{U}(1)_Y$ non-anomalous
- none of the oscillators is charged under G_{SM}

…and if all oscillators get vevs, $G = G_{\mathrm{SM}} \times \mathrm{SU}(4) \times \mathrm{U}(1)_{\mathrm{hidden}}$

Vacua with $B-L$ symmetry at high energies

? How to distinguish between lepton and Higgs doublets?

☞ one possibility

1. break at high scale to

$$G_{\mathrm{SM}} \times \mathrm{U}(1)_{B\text{–}L} \times [\mathrm{SU}(4)]$$

observed proton ~~SUSY~~

- three generations
- one pair of $d+\bar{d}$ and $\ell+\bar{\ell}$
- one pair of Higgs doublets
- three extra pairs $d+\bar{d}$ and $\ell+\bar{\ell}$ with $B-L$ charges $\mp 2/3$ and 0

field	$B-L$ charges
q_i	$3 \times \left(+\frac{1}{3}\right)$
$\bar{u}_i$	$3 \times \left(-\frac{1}{3}\right)$
$\bar{d}_i$	$(3+1) \times \left(-\frac{1}{3}\right) + 3 \times \left(+\frac{2}{3}\right)$
d_i	$1 \times \left(+\frac{1}{3}\right) + 3 \times \left(-\frac{2}{3}\right)$
ℓ_i	$(3+1) \times (-1) + (1+3) \times 0$
$\bar{\ell}_i$	$1 \times (+1) + (1+3) \times 0$
$\bar{e}_i$	$3 \times (+1)$

2. break $\mathrm{U}(1)_{B\text{–}L}$ at a hierarchically smaller scale

Decoupling of exotics

☞ mass terms for the exotic states

$$W = x_i\,\bar{x}_j\,\mathcal{M}_x^{ij}(\widetilde{s}) \quad \text{with} \quad \mathcal{M}_x^{ij}(\widetilde{s}) = \sum \widetilde{s}_{i_1}\cdots\widetilde{s}_{i_n}$$

vector-like exotics

$B-L$ neutral singlets

Decoupling of exotics

- mass terms for the exotic states

$$W = x_i\,\bar{x}_j\,\mathcal{M}_x^{ij}(\widetilde{s}) \quad \text{with} \quad \mathcal{M}_x^{ij}(\widetilde{s}) = \sum \widetilde{s}_{i_1}\cdots\widetilde{s}_{i_n}$$

$$\mathcal{M}_d^{ij}(\widetilde{s}) = \begin{pmatrix} 0 & 0 & \widetilde{s}^6 & 0 & 0 & \widetilde{s}^6 & \widetilde{s}^6 \\ 0 & 0 & \widetilde{s}^6 & 0 & 0 & \widetilde{s}^7 & \widetilde{s}^7 \\ 0 & 0 & \widetilde{s}^6 & 0 & 0 & \widetilde{s}^7 & \widetilde{s}^7 \\ \widetilde{s}^8 & 0 & 0 & \widetilde{s}^6 & \widetilde{s}^6 & 0 & 0 \end{pmatrix}$$

- $\mathcal{M}_d$ has full rank

➥ all extra d_i-$\bar{d}_j$ decoupled

- **note**: zeros partially dictated by $B-L$

Decoupling of exotics

- mass terms for the exotic states

$$W = x_i\,\bar{x}_j\,\mathcal{M}_x^{ij}(\widetilde{s}) \quad \text{with} \quad \mathcal{M}_x^{ij}(\widetilde{s}) = \sum \widetilde{s}_{i_1}\cdots\widetilde{s}_{i_n}$$

$$\mathcal{M}_\ell^{ij}(\widetilde{s}) = \begin{pmatrix} \widetilde{s}^3 & 0 & 0 & 0 & 0 & \widetilde{s}^8 & 0 & 0 \\ \widetilde{s} & 0 & 0 & 0 & 0 & \widetilde{s}^6 & 0 & 0 \\ \widetilde{s} & 0 & 0 & 0 & 0 & \widetilde{s}^6 & 0 & 0 \\ 0 & \widetilde{s}^8 & \widetilde{s}^8 & 0 & 0 & 0 & \widetilde{s}^6 & \widetilde{s}^6 \\ \widetilde{s} & 0 & 0 & \widetilde{s}^6 & \widetilde{s}^6 & 0 & 0 & 0 \end{pmatrix}$$

- 1 eigenvalue is zero $\curvearrowright$ Higgs
- **note**: we do not need to tune VEVs against each other in order to achieve doublet-triplet splitting
- mass-less $\bar{\ell}$ eigenstate dominated by $\bar{\ell}_1$ (untwisted state)

Gauge-Top unification

- Untwisted sector (=internal components of the gauge bosons)

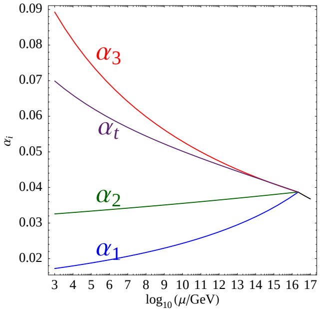
	field-theoretic description	state
U_1	$\sim A_5 + \mathrm{i} A_6$	$\overline{u}_1 + \dots$
U_2	$\sim A_7 + \mathrm{i} A_8$	$q_1 + \dots$
U_3	$\sim A_9 + \mathrm{i} A_{10}$	$\overline{\ell}_1 \simeq H_u + \dots$

Renormalizable coupling

$$y_t \, u_1 \, q_1 \, H_u$$

$y_t \simeq g$ @ M_{comp}

- all other Yukawa couplings are suppressed



Some taste of flavor

$$W_{\mathrm{Yukawa}} = Y_u^{ij}(\widetilde{s})\,\phi_u\,q_i\,\bar{u}_j + Y_d^{ia}(\widetilde{s})\,\phi_d\,q_i\,\bar{d}_a + Y_e^{ib}(\widetilde{s})\,\phi_d\,\bar{e}_i\,\ell_b\ ,$$

Higgs   matter $B-L$ neutral singlets

- matter : right $B-L$ charges
- Higgs : massless $\mathrm{SU}(2)$ doublets with zero $B-L$ charge

Some taste of flavor

$$W_{\text{Yukawa}} = Y_u^{ij}(\widetilde{s})\,\phi_u\,q_i\,\bar{u}_j + Y_d^{ia}(\widetilde{s})\,\phi_d\,q_i\,\bar{d}_a + Y_e^{ib}(\widetilde{s})\,\phi_d\,\bar{e}_i\,\ell_b\;,$$

$$Y_u^{ij}(\widetilde{s}) = \begin{pmatrix} g & \widetilde{s}^6 & \widetilde{s}^4 \\ \widetilde{s}^3 & 0 & \widetilde{s}^7 \\ \widetilde{s}^7 & \widetilde{s}^7 & 0 \end{pmatrix}$$

$$\boxed{y_t \simeq g \;@\; M_{\text{GUT}}}$$

Some taste of flavor

$$W_{\mathrm{Yukawa}} = Y_u^{ij}(\widetilde{s})\,\phi_u\,q_i\,\bar{u}_j + Y_d^{ia}(\widetilde{s})\,\phi_d\,q_i\,\bar{d}_a + Y_e^{ib}(\widetilde{s})\,\phi_d\,\bar{e}_i\,\ell_b\ ,$$

$$Y_d^{ia}(\widetilde{s}) = \begin{pmatrix} 0 & \widetilde{s}^2 & \widetilde{s}^2 & 0 \\ \widetilde{s}^5 & \widetilde{s}^5 & \widetilde{s}^5 & 0 \\ 0 & \widetilde{s} & \widetilde{s} & 0 \end{pmatrix}$$

- Y_d becomes 3×3 matrix after integrating out the heavy d-$\bar{d}$ pair
- Y_d has full rank
- flavor structure à la Froggatt-Nielsen

Some taste of flavor

$$W_{\text{Yukawa}} = Y_u^{ij}(\widetilde{s})\,\phi_u\,q_i\,\bar{u}_j + Y_d^{ia}(\widetilde{s})\,\phi_d\,q_i\,\bar{d}_a + Y_e^{ib}(\widetilde{s})\,\phi_d\,\bar{e}_i\,\ell_b\;,$$

$$Y_e^{ib}(\widetilde{s}) = \begin{pmatrix} 0 & \widetilde{s}^6 & 0 & 0 \\ \widetilde{s}^5 & 0 & 0 & 0 \\ 0 & \widetilde{s}^5 & 0 & 0 \end{pmatrix}$$

- ☞ Y_e becomes 3×3 matrix after integrating out the heavy d-$\bar{d}$ pair
- ☞ Y_e has not full rank $\curvearrowright$ electron massless
- ☞ τ Yukawa coupling seems unrealistic

Summary

and

outlook

Summary

Guided by the idea of local grand unification we have obtained $\mathcal{O}(100)$ models with the following features:

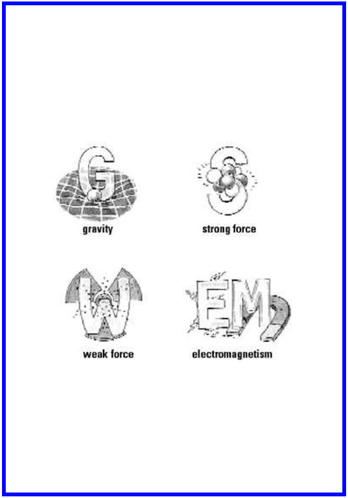
1. $3 \times \mathbf{16} + \text{Higgs} + \text{nothing}$

No exotics

Summary

$\mathcal{O}(100)$ **models with**:

1. $3 \times \mathbf{16} + \text{Higgs} + \text{nothing}$
2. $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y \times G_{\mathrm{hid}}$



Summary

$\mathcal{O}(100)$ **models with**:

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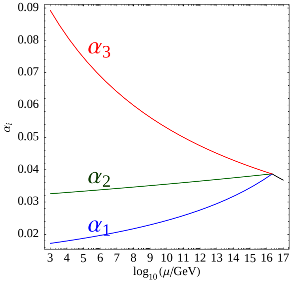
2. $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y \times G_{\text{hid}}$
3. unification



Summary

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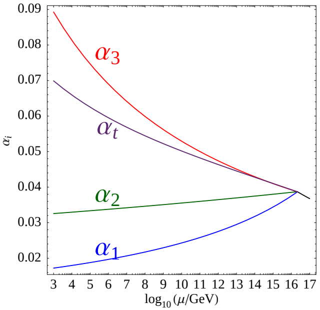
4. $y_t \simeq g$ @ M_{GUT} & realistic flavor structures à la Froggatt-Nielsen



Summary

$\mathcal{O}(100)$ models with:

1. $3 \times \mathbf{16} + \text{Higgs} + \text{nothing}$
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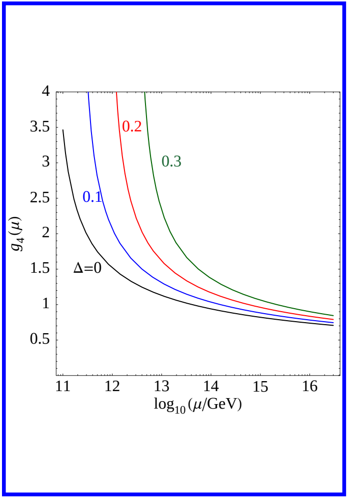
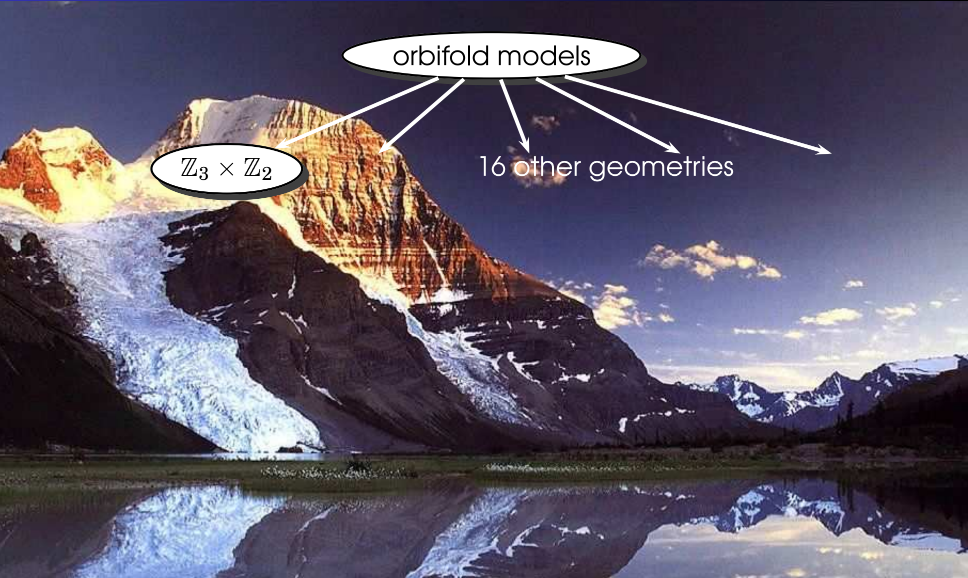
3. unification
4. $y_t \simeq g$ @ M_{GUT} & realistic flavor structures à la Froggatt-Nielsen
5. hidden sector gaugino condensation

↪ Spontaneously broken SUSY with TeV scale soft masses



'Orbifold landscape'



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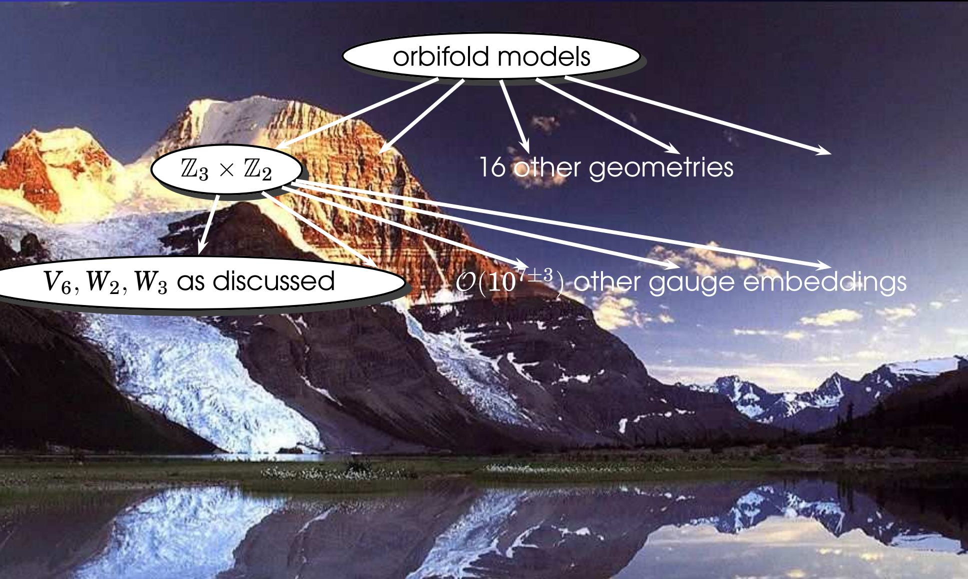
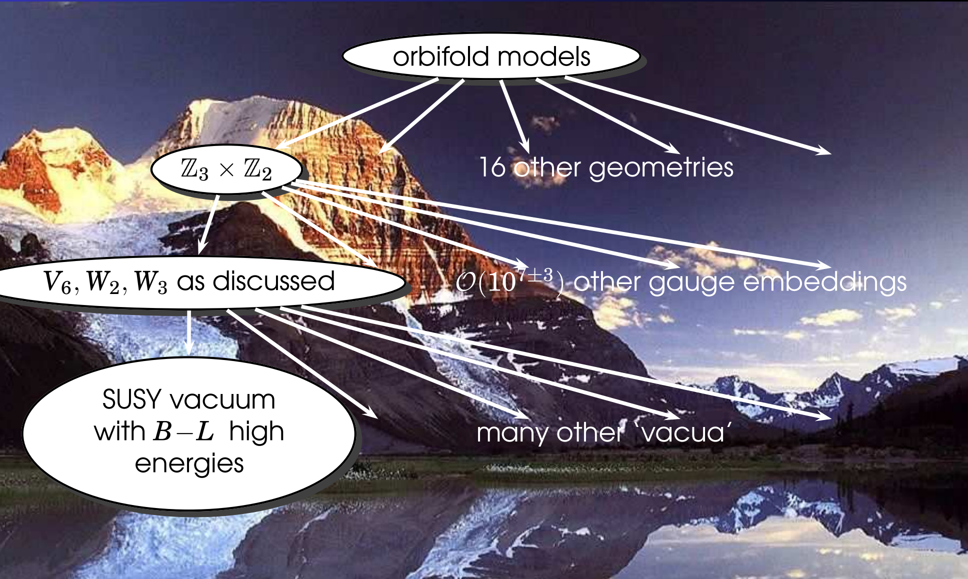


'Orbifold landscape'



Outlook

we're studying the $\mathcal{O}(100)$ MSSM(-like) models...

? Discrete symmetries ↷ talk by S. Raby

? SUSY breakdown ↷ talk by H.P. Nilles

? Neutrino masses... yes, we get see-saw

Further questions:

? Relation to the bundle constructions?

Braun, He, Ovrut, Pantev (2005);Bouchard, Donagi (2005);
Bouchard, Cvetič, Donagi (2006);Blumenhagen, Moster, Weigand (2006)
↷ talks by R. Blumenhagen, M. Cvetič, B. Ovrut

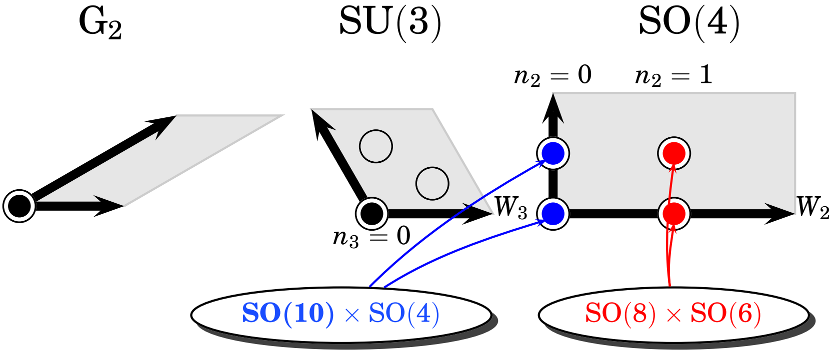
? Relation to free fermionic models?

↷ talk by A. Faraggi

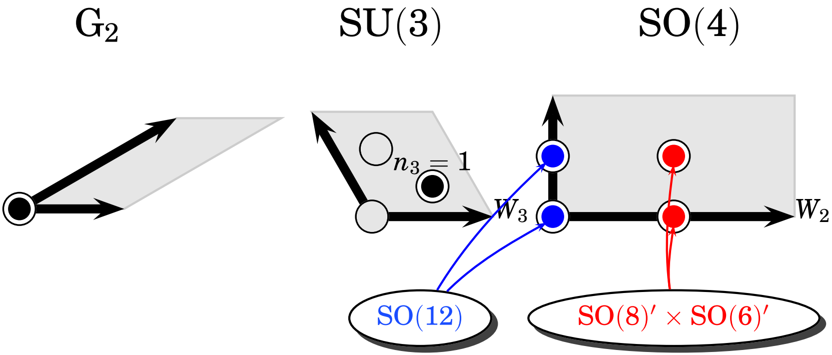
? ...

‘Appendix’

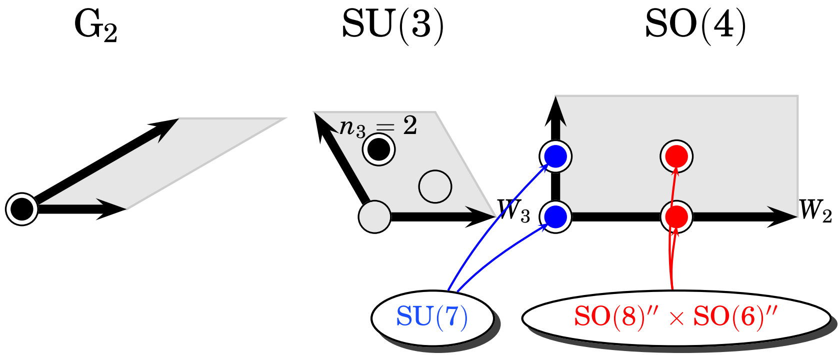
Gauge group topography



Gauge group topography



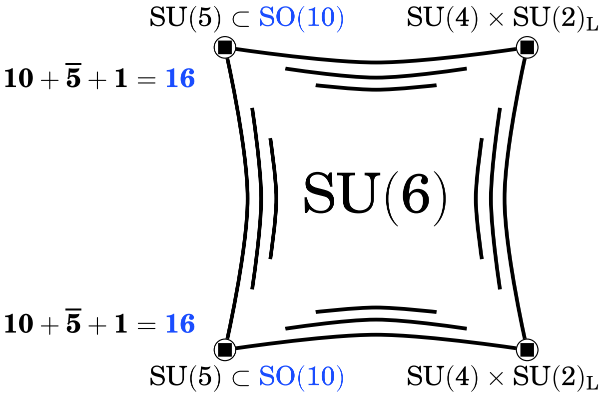
Gauge group topography



Orbifold GUT limit: SO(4) plane 'large'

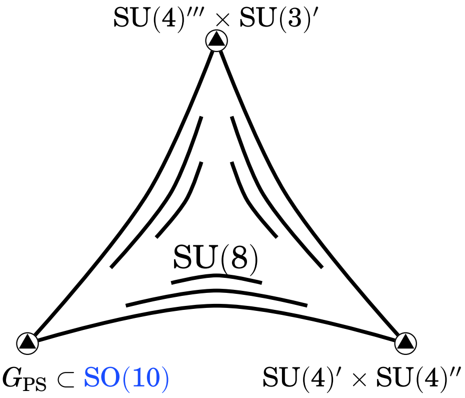
- **Motivation**: anisotropic compactification may allow to understand why $M_{\mathrm{GUT}} < M_{\mathrm{string}}$

Witten (1996)
...
Hebecker, Trapletti (2005)



further orbifold GUT limits

Orbifold GUT limit: $\mathrm{SU}(3)$ plane 'large'



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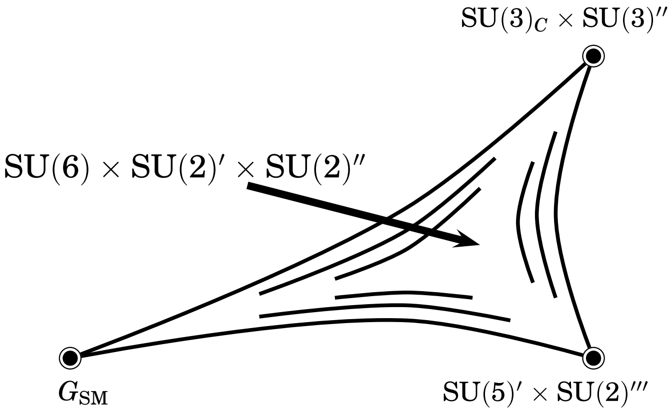
Orbifold GUT limit: G_2 plane 'large'



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Hidden sector gaugino condensation

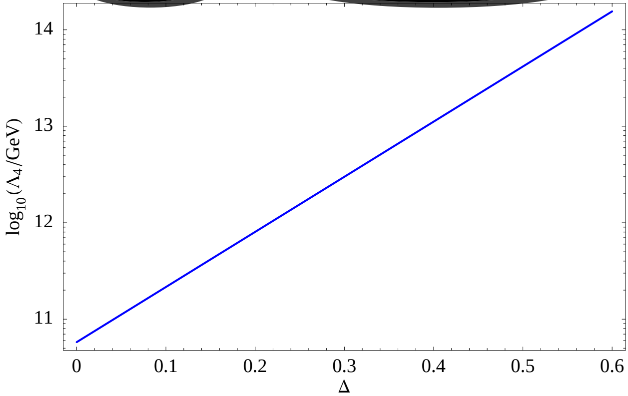
- Hidden sector **stronger** coupled

$$g_{\mathrm{obs/hid}}^{-2} = \operatorname{Re} S \pm \varepsilon \operatorname{Re} T + \cdots =: \operatorname{Re} S \pm \Delta$$

dilaton

Kähler modulus

Ibáñez, Nilles 1986
Dixon, Kaplunovsky, Louis 1991
Mayr, Stieberger 1993
...



Hidden sector gaugino condensation

- Hidden sector **stronger** coupled

$$g_{\mathrm{obs/hid}}^{-2} = \mathrm{Re}\,S \pm \varepsilon\ \mathrm{Re}\,T + \cdots =: \mathrm{Re}\,S \pm \Delta$$

dilaton

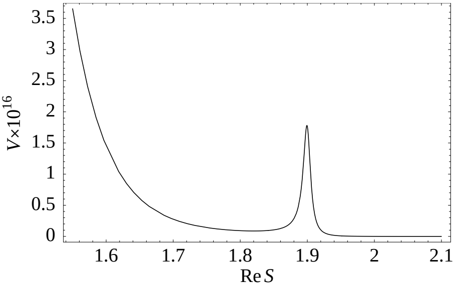
Kähler modulus

Ibáñez, Nilles 1986
Dixon, Kaplunovsky, Louis 1991
Mayr, Stieberger 1993
...

e.g. Kähler stabilization: large coefficients in the relation

$$m_{3/2} \simeq \frac{\Lambda_4^3}{M_\mathrm{P}^2}$$

bottom-line: $m_{3/2}$ is fine

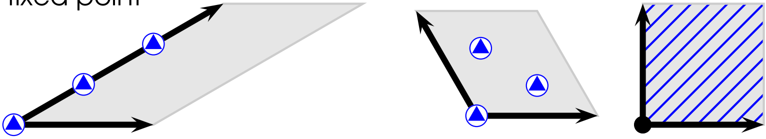


e.g. Barreiro, de Carlos, Copeland 1998

$\mathbb{Z}_3$ and $\mathbb{Z}_2$ subtwists

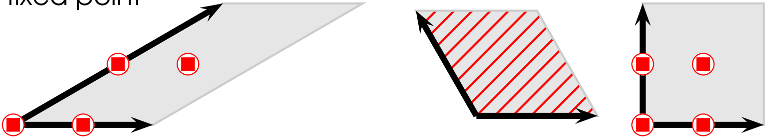
$\mathbb{Z}_3$ subtwist: $v_3 = 2v_6 = \frac{1}{3}(1, 2, -3)$

=$\mathbb{Z}_3$ fixed point



$\mathbb{Z}_2$ subtwist: $v_2 = 3v_6 = \frac{1}{2}(1, 2, -3)$

=$\mathbb{Z}_2$ fixed point



Decoupling of the extra states for 'generic' SM singlet vevs

The $\overline{d}_a d_b$ mass matrix (@ order 8)

- rank of the mass matrix is **maximal**
- ↪ four combinations $\overline{d}_a d_b$ disappear from the low-energy spectrum

	$\overline{d}_1$	$\overline{d}_2$	$\overline{d}_3$	$\overline{d}_4$	$\overline{d}_5$	$\overline{d}_6$	$\overline{d}_7$
d_1	s^5	s^5	s^5	s^5	s^5	s^3	s^3
d_2	s^1	s^1	s^3	s^3	s^3	s^3	s^3
d_3	s^1	s^1	s^3	s^3	s^3	s^3	s^3
d_4	s^6	s^6	s^6	s^3	s^3	s^6	s^6

(An entry s^n means that there is an allowed coupling $\overline{d}_a\, d_b\, s^0_{i_1} \cdots s^0_{i_n}$)

- **note**: high powers of s do not necessarily mean strong suppression

e.g. Cvetič, Everett, Wang (1998)

Decoupling of the extra states for 'generic' SM singlet vevs

The $\overline{\ell}_a \ell_b$ mass matrix (@ order 8)

- rank of the mass matrix is **maximal**
- How to get a rank 4 mass matrix?
  ...see later

	$\overline{\ell}_1$	$\overline{\ell}_2$	$\overline{\ell}_3$	$\overline{\ell}_4$	$\overline{\ell}_5$
ℓ_1	s^3	s	s	s	s
ℓ_2	s^4	s^2	s^2	s^2	s^6
ℓ_3	s^4	s^2	s^2	s^2	s^6
ℓ_4	s	s^5	s^5	s^5	s^3
ℓ_5	s	s^5	s^5	s^5	s^3
ℓ_6	s	s^3	s^3	s^6	s^6
ℓ_7	s	s^3	s^3	s^3	s^3
ℓ_8	s	s^3	s^3	s^3	s^3

Decoupling of the extra states for `generic' SM singlet vevs

The $m_a m_b$ mass matrix (@ order 8)

- rank of the mass matrix is **maximal**
- recall: m_i are $\mathrm{SU}(2)_\mathrm{L}$ doublets with hypercharge 0

	m_1	m_2	m_3	m_4	m_5	m_6	m_7	m_8
m_1	$*$	$*$	—	—	—	$*$	—	$*$
m_2	$*$	$*$	—	—	—	$*$	—	$*$
m_3	—	—	—	—	$*$	—	$*$	—
m_4	—	—	—	—	$*$	—	$*$	—
m_5	—	—	$*$	$*$	$*$	—	$*$	—
m_6	$*$	$*$	—	—	—	—	—	—
m_7	—	—	$*$	$*$	$*$	—	$*$	—
m_8	$*$	$*$	—	—	—	—	—	—

$*$ means `there is a coupling'

Decoupling of the extra states for ‘generic’ SM singlet vevs

The $s_a^+ s_b^-$ mass matrix (@ order 8)

- rank of the mass matrix is **maximal**
- recall: $s_i^\pm$ are $\mathrm{SU}(3) \times \mathrm{SU}(2)_\mathrm{L}$ singlets with hypercharge $\pm 1/2$

	s_1^-	s_2^-	s_3^-	s_4^-	s_5^-	s_6^-	s_7^-	s_8^-	s_9^-	s_{10}^-	s_{11}^-	s_{12}^-	s_{13}^-	s_{14}^-	s_{15}^-	s_{16}^-
s_1^+	*	*	—	—	*	—	*	—	—	—	—	*	—	—	—	
s_2^+	*	*	—	—	*	—	*	—	—	—	—	*	—	—	—	
s_3^+	*	*	—	—	—	—	—	—	—	*	*	*	—	*	*	
s_4^+	*	*	—	—	—	—	—	—	—	*	*	*	—	*	*	
s_5^+	—	—	—	—	—	*	—	*	—	—	—	—	—	—	—	
s_6^+	*	*	—	—	*	—	*	—	—	—	—	*	—	—	—	
s_7^+	—	—	—	—	—	*	—	*	—	—	—	—	—	—	—	
s_8^+	*	*	—	—	*	—	*	—	—	—	—	*	—	—	—	
s_9^+	—	—	—	—	—	—	—	—	*	—	—	—	*	—	—	
s_{10}^+	*	*	*	*	*	—	*	—	—	*	*	*	—	*	*	
s_{11}^+	*	*	*	*	*	—	*	—	—	*	*	*	—	*	*	
s_{12}^+	*	*	*	*	*	—	*	—	—	*	*	*	—	*	*	
s_{13}^+	—	—	—	—	—	—	—	—	*	—	—	—	*	—	—	
s_{14}^+	*	*	*	*	*	—	*	—	—	*	*	*	—	*	*	
s_{15}^+	*	*	*	*	*	—	*	—	—	*	*	*	—	*	*	
s_{16}^+	*	*	*	*	*	—	*	—	—	*	*	*	—	*	*	

Flavor issues (using d-type quarks as an example)

- Higher-order couplings giving rise to d-type Yukawa couplings

$$
\begin{aligned}
W \quad \supset \quad & \bullet\, \ell_4\, \bar{d}_4\, s_5^0\, q_2 + \bullet\, \ell_5\, \bar{d}_4\, s_5^0\, q_2 + \bullet\, \ell_4\, \bar{d}_5\, s_5^0\, q_2 + \bullet\, \ell_5\, \bar{d}_5\, s_5^0\, q_2 \\
& + \bullet\, \ell_4\, \bar{d}_4\, s_{12}^0\, q_3 + \bullet\, \ell_5\, \bar{d}_4\, s_{12}^0\, q_3 + \bullet\, \ell_4\, \bar{d}_5\, s_{12}^0\, q_3 + \bullet\, \ell_5\, \bar{d}_5\, s_{12}^0\, q_3
\end{aligned}
$$

omit coefficients

quark doublets from localized **16**

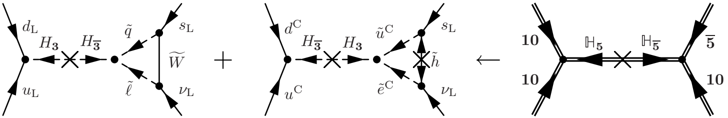
- Promising: $h_d \simeq \ell_1 \curvearrowright$ search for $\ell_1\, \bar{d}_j\, q_k\, (s^0)^n$ appears at order 7, e.g.

$$
W \quad \supset \quad \ell_1\, \bar{d}_1\, q_2\, s_{55}^0\, s_{56}^0\, s_7^0\, s_5^0 + \ldots \text{ (15 terms at order 7)}
$$

Proton decay in orbifold GUTs

- Dimension 5 proton decay operators

see e.g. Hebecker, March-Russell '02



- 4D GUT $m_3\, \mathbb{H}_{\mathbf{3}}\, \mathbb{H}_{\overline{\mathbf{3}}}$

- 5D GUT: $\mathbb{H}_{\mathbf{5}} \oplus \mathbb{H}_{\overline{\mathbf{5}}} \xrightarrow[\text{reduction}]{\text{dimensional}} (\mathbb{H}_{\mathbf{5}}, \mathbb{H}^C_{\mathbf{5}}) \oplus (\mathbb{H}_{\overline{\mathbf{5}}}, \mathbb{H}^C_{\overline{\mathbf{5}}})$ in 4D

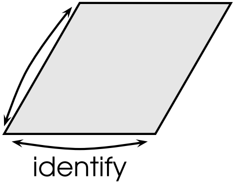
- ↪ 5D orbifold GUT: KK masses $\frac{1}{R}\, \mathbb{H}_{\mathbf{3}}\, \mathbb{H}^C_{\mathbf{3}}$ and $\frac{1}{R}\, \mathbb{H}_{\overline{\mathbf{3}}}\, \mathbb{H}^C_{\overline{\mathbf{3}}}$ in 4D

- ↪ Dimension 5 proton decay operator absent in orbifold GUT

- However: Constraints from dimension 6 operators!

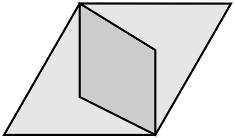
Pillow construction (. . . using a $\mathbb{Z}_3$ orbifold as example)

Starting point is the torus



Pillow construction (…using a $\mathbb{Z}_3$ orbifold as example)

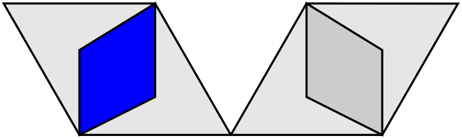
The fundamental region of the orbifold is 1/3 of the fundamental region of the torus



back

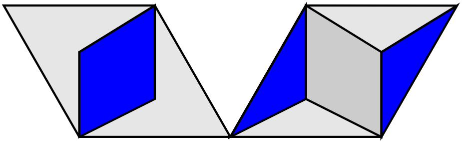
Pillow construction (...using a $\mathbb{Z}_3$ orbifold as example)

Rotating by $2\pi/3$ yields:



Pillow construction (. . . using a $\mathbb{Z}_3$ orbifold as example)

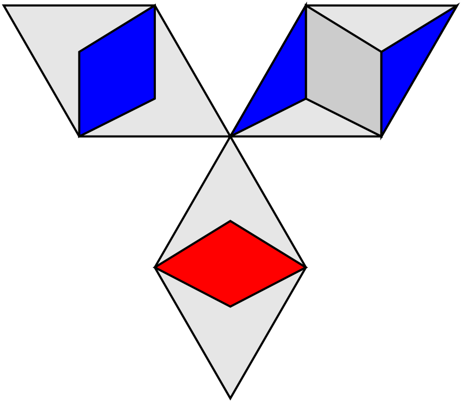
The rotated fundamental region covers a ‘new’ area



back

Pillow construction (. . . using a $\mathbb{Z}_3$ orbifold as example)

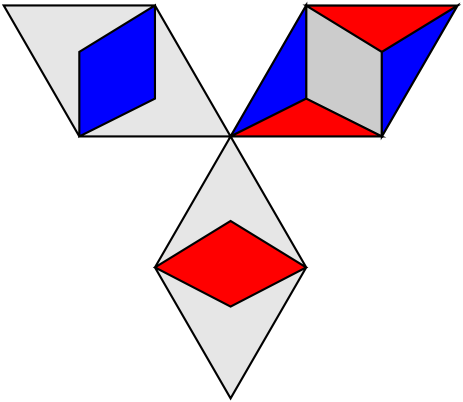
Further rotation yields:



back

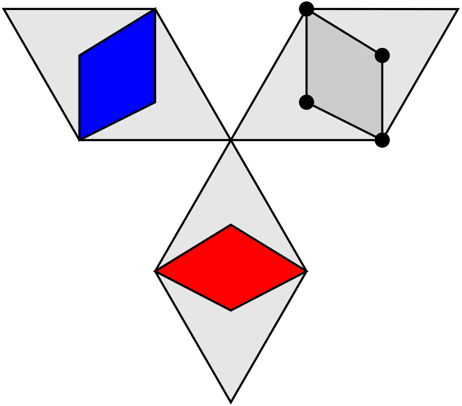
Pillow construction (. . . using a $\mathbb{Z}_3$ orbifold as example)

Now the fundamental region covers the remaining area



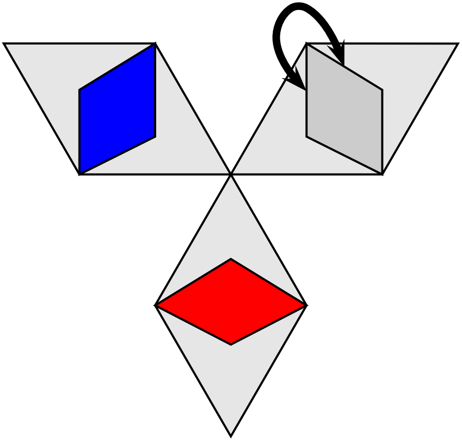
Pillow construction (. . . using a $\mathbb{Z}_3$ orbifold as example)

The corners of the fundamental region are fixed under the $\mathbb{Z}_3$ rotation (on the torus)



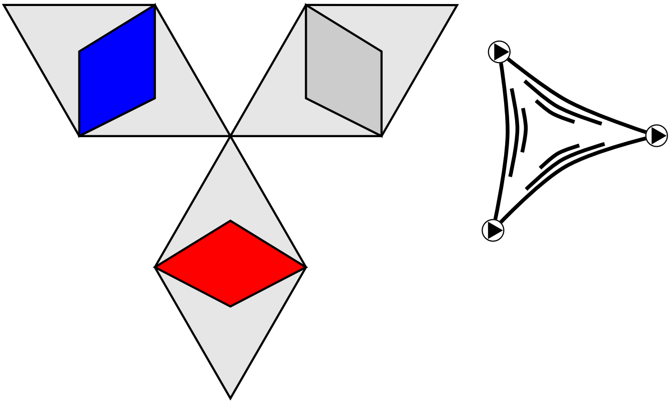
Pillow construction (. . . using a $\mathbb{Z}_3$ orbifold as example)

The edges are pairwise identified

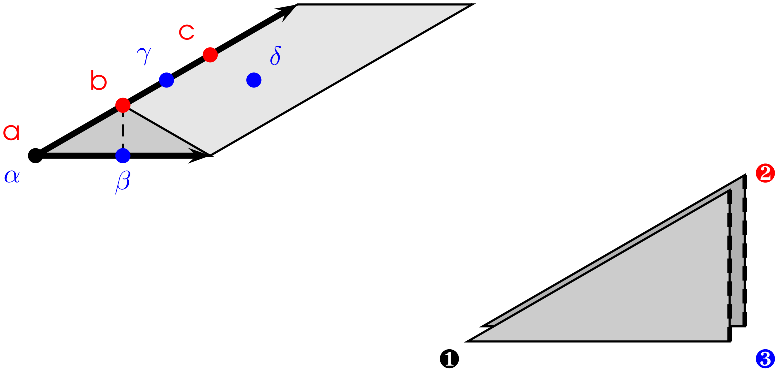


Pillow construction (. . . using a $\mathbb{Z}_3$ orbifold as example)

The geometry is that of a 'pillow'



$\mathbb{Z}_6$ pillow



V_6 vs. V_6'

$$V_6 = \frac{1}{6}(3,3,2,0,0,0,0,0)\,(2,0,0,0,0,0,0,0)$$
$$V_6' = \frac{1}{6}(2,2,2,0,0,0,0,0)\,(1,1,0,0,0,0,0,0)$$

	V_6	V_6'
G	$\mathrm{SO}(10)\times\mathrm{SO}(4)\times\mathrm{U}(1)$	$\mathrm{SO}(10)\times\mathrm{SU}(3)\times\mathrm{U}(1)$
U_1:	$(\mathbf{16},\mathbf{1},\mathbf{2})\oplus\dots$	$(\mathbf{16},\overline{\mathbf{3}})$
U_2:	$(\mathbf{16},\mathbf{2},\mathbf{1})\oplus(\mathbf{10},\mathbf{1},\mathbf{1})$	$(\mathbf{10},\mathbf{3})\oplus\dots$
U_3:	$(\mathbf{10},\mathbf{2},\mathbf{2})$	$(\mathbf{16},\mathbf{1})\oplus(\overline{\mathbf{16}},\mathbf{1})$

cf. Katsuki, Kawamura, Kobayashi, Ohtsubo, Ono, Tanioka (1989)

☞ U_3 states always vector-like

observation

For V_6 one can get Higgs pairs from U_3 (but not for V_6')