

PROBING THE
PROPERTIES OF
QUARK-GLUON PLASMA
USING EXPERIMENTS
AND ADS/CFT CALCULATIONS

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KITP, 8/28/06

WHAT IS QUARK-GLUON PLASMA?

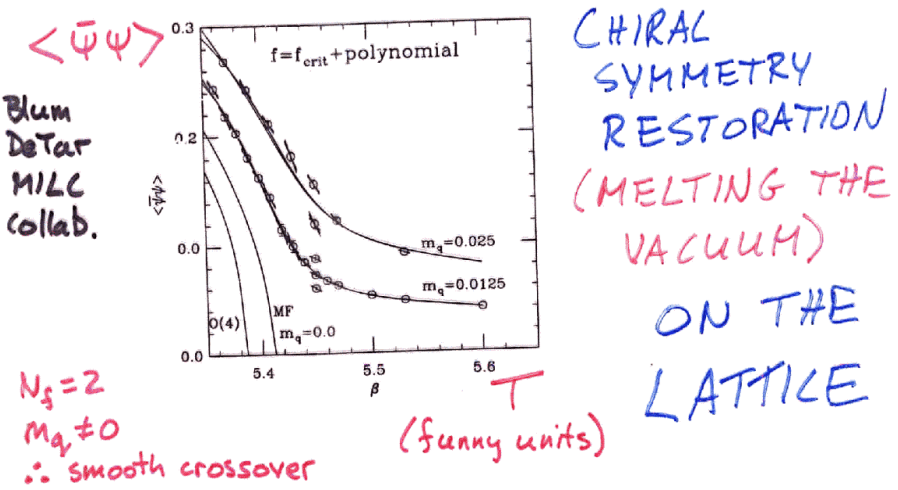
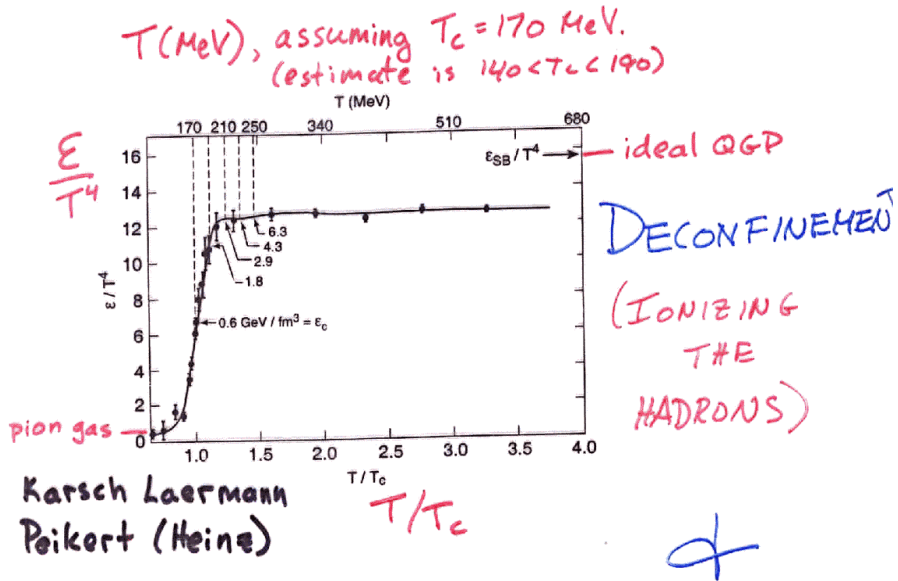
An operational definition:

Matter in (or close to) thermal equilibrium at a temperature above the QCD "transition," which is in fact a crossover occurring in a narrow range of temperatures around 170 MeV.

HOW TO PROBE ITS PROPERTIES?

Static (ie thermodynamic) properties well-suited to lattice QCD analyses. →

But, there is more to matter than thermodynamics ...



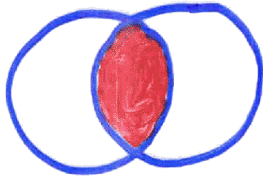
EXPLORING QGP PROPERTIES

"Making QGP" is not a yes/no question:
No sharp boundary between hadrons, QGP.
Goal of RHIC: create matter ^① that is above the crossover ^② and study its properties. ^③

- ①: RHIC data (on V_2) tell us interactions sufficient to yield ~equilibrated matter, expanding collectively as a fluid, by a time $\sim 0.6 - 1 \text{ fm}$. After that hydrodynamics (ideal hydro; zero mean free path; ideal liquid not ideal gas) describes "bulk" of particles ($p_T \lesssim 1 - 2 \text{ GeV}$) well.
- ②: RHIC data (dE_T/dy) tell us $E(1 \text{ fm}) > 5 \text{ GeV}/\text{fm}^3 \Rightarrow$ above crossover
So, on to ③.....
NB

TOWARD MEASURING SHEAR VISCOSITY

Elliptic flow indicates extent of early equilibration.
 look at non-head-on collisions:

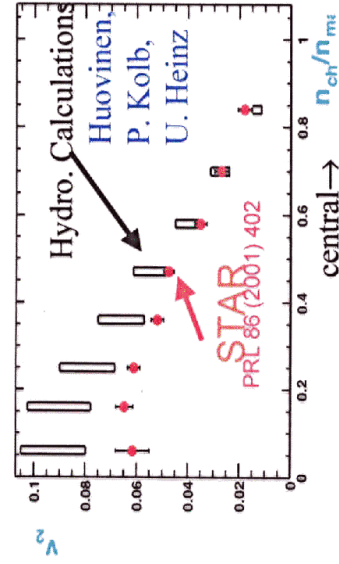


If just lots of p-p collisions followed by free streaming, then final state momenta uniformly distributed in azimuth angle ϕ .

If interaction \rightarrow equilibration \rightarrow pressure, pressure gradients \rightarrow collective flow.

If this happens early, before circularizes by free streaming, then nonzero $v_2 \sim \langle \cos 2\phi \rangle$.

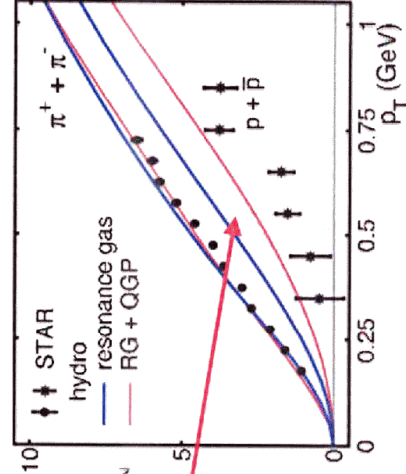
v_2 predicted by hydrodynamics



Hydro can reproduce magnitude of elliptic flow for π, p . BUT **must add QGP to hadronic EOS!**

Similar conclusion reached by CM Ko, et al., Kapusta, et al., Bleicher, et al., among others...

pressure buildup \rightarrow explosion happens fast \rightarrow early equilibration!



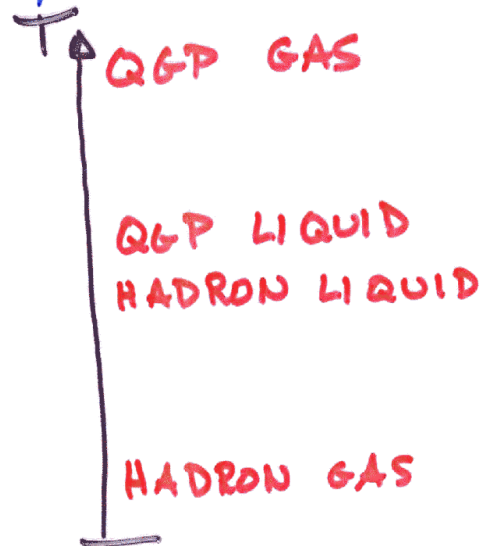
talk by B. Jacak

- Ideal hydrodynamics based on assumption of local eqbm.
- Hydro never agreed with data before RHIC. (At SPS, $v_2^{\text{data}} \sim \frac{v_2^{\text{hydro}}}{2}$)
- At RHIC, hydro does good job of describing v_2 , spectra for $P_T < 1-2 \text{ GeV}$
- MEANS: "hydro works" by $t \sim 0.6-1 \text{ fm}$
Heine Kolb
- Challenge to theory: how can \sim equilibration occur so quickly?
Strong interactions? Strong color fields \rightarrow plasma instabilities?
Mrowczynski, Redhan, Pometschke, Strickland, Arnold, Moore, Yaffe, ...
- MEANS: "small" shear viscosity η .
Teaney: $\eta/s < \mathcal{O}(1/10)$
[cf water: $\eta/s > 10$]
- CHALLENGES: Real extraction of η requires hydro calculations with $\eta \neq 0$.
Muronga; Heine Song Chaudhuri

Should we be surprised if/that the QGP turns out to be liquid-like?

- 1) No. At $T \sim \text{few } T_c$, coupling not small
 - 2) But ... Lattice shows ϵ/T^4 reaches 80% of its value in an ideal-gas-QGP (ie noninteracting) already just above T_c .
Doesn't this imply interactions are "just a 20% correction" ???
 - 3) $N=4$ SUSY QCD can teach us a lesson:
 - $\epsilon/T^4 = 75\%$ of its value in a noninteracting SUSY-QGP
 - interactions very strong.
- Gubser, Kribs, Teoytin
PolICASTRO $\eta/s = \frac{1}{4\pi} \rightarrow$ m.f.p. \sim spacing
Son Starinets
Kovtun - a liquid with lower viscosity per entropy than water
Son Starinets
- ideal hydro!
• Teaney uses v_2 data to suggest η/s of real world QGP \sim as small.

So, a posteriori, (ie after the data) it is not surprising to find a QGP liquid. In fact, given that the transition is a crossover, it probably has to be this way:

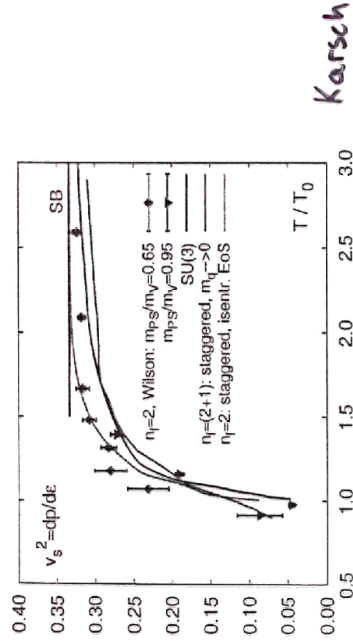


Also a posteriori (in this case, after the string theorists) we realize that $\frac{\epsilon}{T^4} = 80\%$ of noninteracting is closer to 75% (strong coupling) than to 1.

η/s IN $N=4$ SYM vs IN QCD

- $N=4$ is supersymmetric, but ~~SUSY~~ @ $T \neq 0$
- $N=4$ has 16 adjoint d. of f.; QCD has 2.
 - cancels in ratio η/s
- $N=4$ has no fundamental d. of f.
 - Buchel, Liu, Starinets showed $\eta/s = 1/4\pi$ in any gauge theory with a gravity dual.
- η/s calculated for $N_c \rightarrow \infty$
 - unfortunate that $1/N_c$ corrections hard
- $N=4$ SYM is conformal. QCD is not, for $T < T_c$ and $T \rightarrow \infty$.
 - But, for $2T_c < T < 10+T_c$, QGP thermodynamics quite conformal.
- $\eta/s = 1/4\pi$ for $\lambda \equiv g^2 N_c \rightarrow \infty$
 - NB: $N_c = 3, \alpha_s = 1/2 \leftrightarrow \lambda = 6\pi$
 - $\eta/s = \frac{1}{4\pi} (1 + .09 (\frac{6\pi}{\lambda})^{3/2} + \dots)$

Buchel Liu Starinets



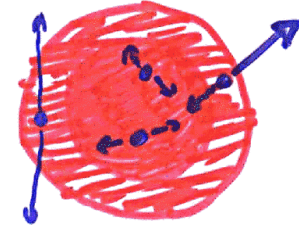
Karsch

FIGURE 6. The velocity of sound in QCD vs. temperature expressed in units of the transition temperature T_0 . Shown are results from calculations with Wilson [22] and staggered fermions [10] as well as for a pure SU(3) gauge theory [21]. Also shown is the resulting v_s^2 deduced from Eq. 3 [19].

TOWARD MEASURING OPACITY
AND PERHAPS v_{sound} AND T^3

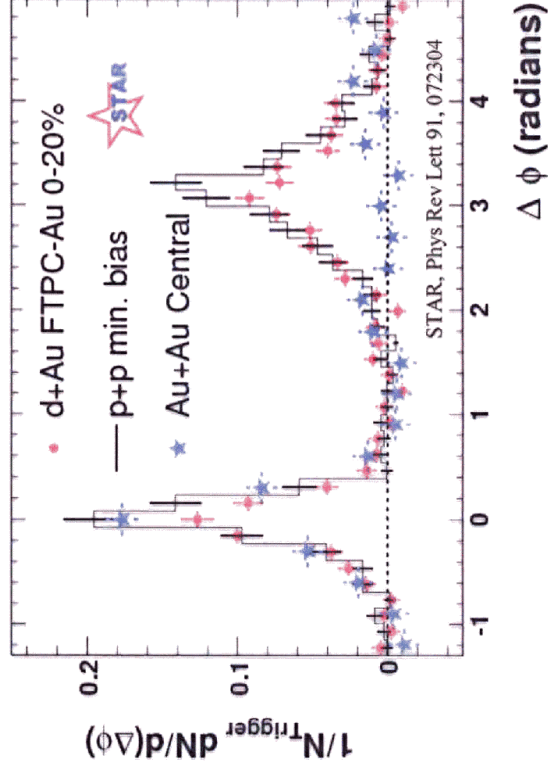
"Jet quenching": RHIC data suggests that the rare high- P_T particles produced in initial hard scatterings are efficiently stopped. "Parton energy loss" to the point that matter is opaque.

Picture Suggested:



Ingoing, and interior, jets quenched. Should see some back to back jets at any P_T , and more at higher P_T .

What is known: recoiling hadrons are suppressed



Compare to d+Au: suppression is final-state effect

M. van Leeuwen, LBNL

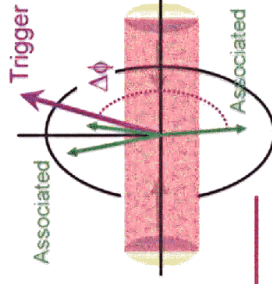
High- p_T at SPS, RHIC and LHC

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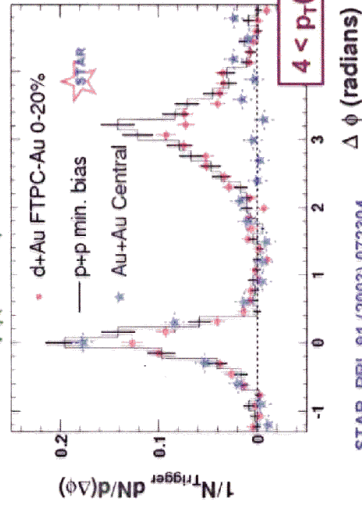
Evolution of $\Delta\phi$ correlations at RHIC

$\Delta\phi$ correlations

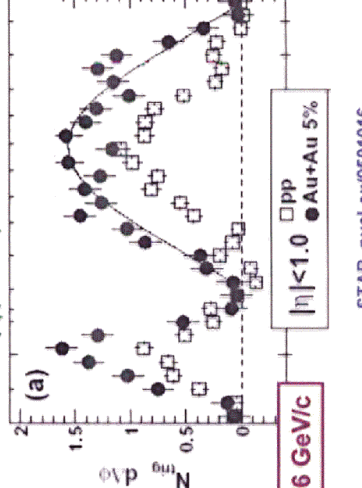
- “Trigger-associated” technique valuable for tagging jets in high-multiplicity environment (vs. jet-cone algorithms)
- Probes the jet’s interaction with the QCD medium
- Provides stringent test of energy-loss models



Higher $p_T \rightarrow$ Away-side suppression
 $p_T(\text{assoc}) > 2 \text{ GeV}/c$



Lower $p_T \rightarrow$ Away-side enhancement
 $p_T(\text{assoc}) > 0.15 \text{ GeV}/c$



QM 2005 Budapest

STAR, PRL 91 (2003) 072304

$\Delta\phi$ (radians)

STAR, nucl-ex/0501016

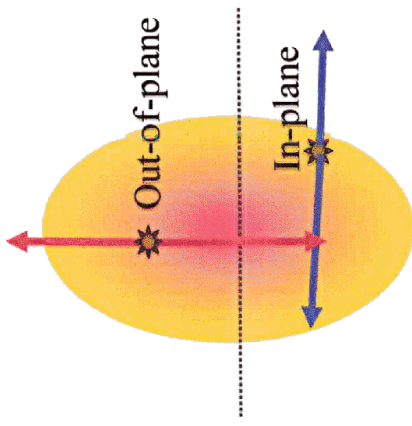
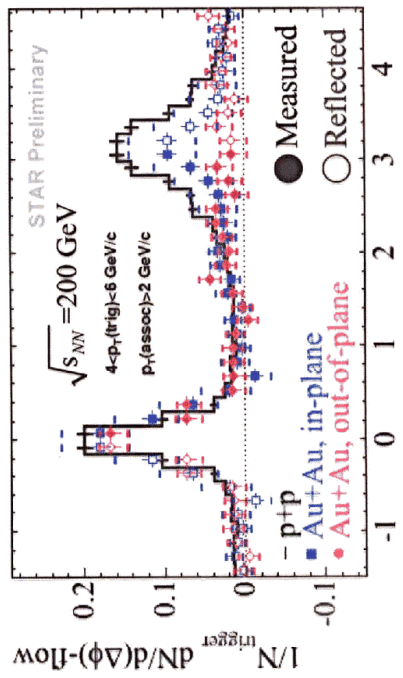
Dan Magestro, STAR

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Path Length Dependence

Background Subtracted
See J. Bielcikova *et al.*,
(nucl-ex/0311007) for
background derivation

di-hadron, 20-60% Central



K. Filimonov DNP03 $\Delta\phi$ (radians)

Suppression larger out-of-plane

$$\Delta E_{GLV} \sim L^n(\phi)$$



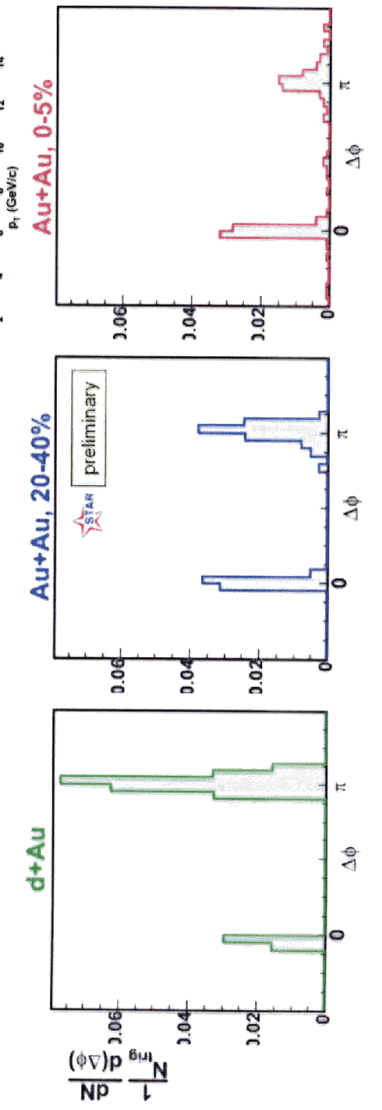
Mike Miller 9

January 2004

Emergence of dijets w/ increasing $p_T(\text{assoc})$

$\Delta\phi$ correlations (not background subtracted)

$8 < p_T(\text{trig}) < 15 \text{ GeV}/c$
 $p_T(\text{assoc}) > 7 \text{ GeV}/c$



Narrow peak emerges cleanly above vanishing background

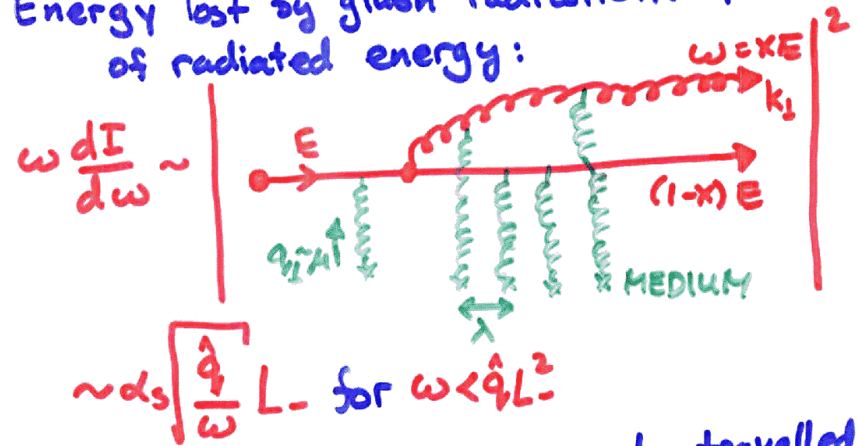
v_{sound} ?

Some reports of a "Mach cone" on the away side, where the supersonic jet was heading before it was quenched. If this persists as the data is further analyzed (via 3-particle correlations) then measure opening angle of the sonic boom $\rightarrow v_{\text{sound}}$.

WHAT DO WE LEARN ABOUT THE MEDIUM FROM HOW A HARD PARTON LOSES ENERGY PLOWING THROUGH IT?

Perturbative formalism for calculating parton energy loss: Baier Dokshitzer Mueller Peigné Schiff Zakharov Wiedemann Gyulassy Wang Wang Levai Vitev Salgado ...

Energy lost by gluon radiation. Spectrum of radiated energy:



where \hat{q} is p_T^2 picked up per L -travelled, so $\sim \mu^2 \lambda$, and L = distance travelled.

ASSUMES: E, k_{\perp} large, so QCD weakly coupled at these scales. (At RHIC, $E \sim 20 \text{ GeV}$. At LHC, $E \sim 100 + 6 \text{ GeV}$)

Parton energy loss sensitive to the medium (ie to strongly interacting physics at scales αT) through one parameter: \hat{q} .

Energy loss: $\Delta E \sim \alpha_s \hat{q} L^2$

Intuition: $\hat{q} \sim \frac{\mu^2}{\lambda} \leftarrow (\text{Debye screening length})^{-2}$
 \leftarrow "mean free path"
 $\sim n_{\text{scatterers}} \cdot (\text{Dimensionless measure of } \sigma)$

Implications of RHIC data: \rightarrow FIG
 Eskola Honkanen Salgado Wiedemann Dainese Loizides Paic

$$\bar{\hat{q}} \sim (5-15) \text{ GeV}^2/\text{fm}$$

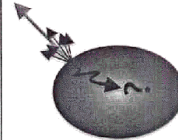
($\bar{\hat{q}}$ is a time-averaged \hat{q} . Relation to $\hat{q}(\tau)$ depends on assumptions, but reasonable to think $\bar{\hat{q}} \sim \hat{q}(1 \text{ fm})$.)

WANTED: strong coupling calculation of \hat{q}

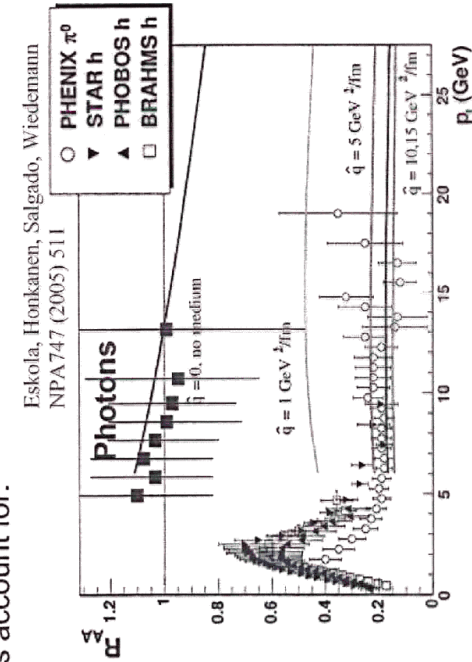
The suppression of leading hadrons

Parton energy loss calculations account for:

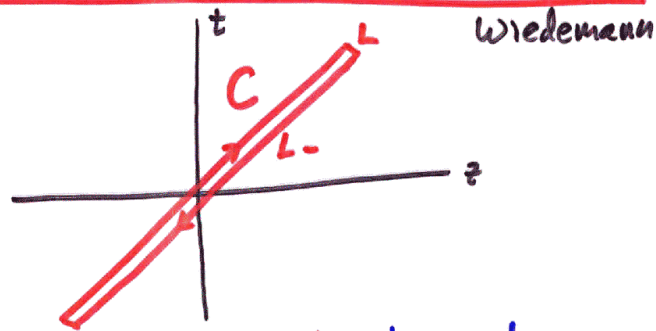
- Nuclear modification factor
- Centrality dependence
- Back-to-back correlations
- $R_{AA} = 0.2$ is a natural limit due to surface emission



indicates very opaque medium.



\bar{q} ↔ LIGHT-LIKE ADJOINT WILSON LOOP



Contour C: a rectangle, L - long along lightcone, L wide in a transverse direction

$L \gg L; L \sim \frac{1}{k_{\perp}} \ll \frac{1}{T}$

$\langle W^A(C) \rangle_T \propto \exp\left[-\frac{\bar{q}}{4} \frac{L}{\sqrt{2}} L^2\right]$ for $LT \ll 1$

Provides a nonperturbative definition!

Derivation nontrivial, but it is partially correct to think of the two long sides of C as the gluons in the amplitude and (amplitude)* from the diagram



*: this $\sqrt{2}$ missing in paper on arXiv. new version coming soon

\bar{q} IN $N=4$ SYM FROM AdS/CFT

H. Liu KR Wiedemann

Use AdS/CFT to calculate Wilson loop at nonzero T, in large N_c and large $\lambda = g^2 N_c$ limits. [NB: Minkowski space crucial, ∴ inaccessible to lattice QCD]

$\langle W^A(C) \rangle = \langle W^F(C) \rangle^2 + \mathcal{O}\left(\frac{1}{N_c^2}\right)$

Prescription: Maldacena Witten Gubser Klebanov Polyakov Rey Yee ...

$\langle W^F(C) \rangle = \exp[iS]$

where S is action of an extremized world sheet in a 5D AdS + BH metric with boundary C at $r = \infty$.

r : 5th dimension.

R: AdS curvature

r_0 : BH horizon

$T_{\text{gauge theory}} = T_{\text{BH}} = \frac{r_0}{\pi R^2}$

$\sqrt{\lambda}$ gauge theory = R^2/α' , where $\frac{1}{2\pi\alpha'}$ = string tension

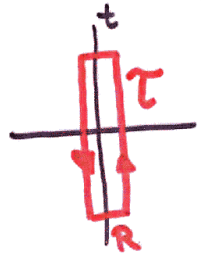
$\lambda/4\pi N_c = g_{\text{string}}$

IMAGINARY S, ie SPACELIKE WORLD SHEET

$$\langle W \rangle \propto \exp\left[-\frac{\hat{q}_L L^-}{4\sqrt{2}} L^2\right] \text{ vs } \langle W \rangle \propto \exp[iS]$$

means we expect (and find) an extremal string world sheet that is spacelike.

In contrast, timelike Wilson loops used to study screening have



$$\langle W \rangle \propto \exp[iS] \text{ with } S \text{ real} \\ = \exp[i\tau E(R)]$$

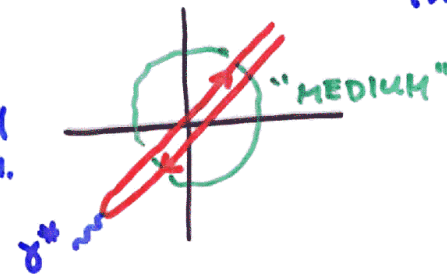
What happens upon boosting the loop?

- First, study screening in a hot wind
 → implications for J/ψ and Υ
 in heavy ion collisions at LHC, RHIC?
 Liu KR Wiedemann

- For quarks of any finite mass, above some wind velocity screening length becomes \ll Compton wavelength. Worldsheet becomes spacelike. Physical interpretation of $\langle W \rangle$ changes.

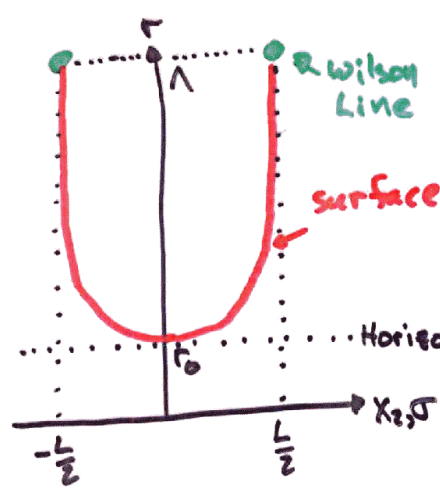
If you boost until Wilson loop is lightlike, you should no longer think of $q\bar{q}$ -meson. Instead, think $q\bar{q}$ -component of a virtual photon, in D.I.S.

Wilson lines describing q & \bar{q} , in Eikonal approximation.



Now, if "MEDIUM" is a hadron or cold nucleus, $\langle W \rangle$ is related to σ_{DIS} , the virtual photoabsorption cross-section. The relationship is sensible only if $\langle W \rangle \sim \exp[-\text{real}]$, ie S imag. ie spacelike string world sheet. In this context, $\hat{q}_L L^-$ is called Q_s^2 , saturation scale. SO: \hat{q}_L we calculate describes both energy loss and DIS off QGP.

$$ds^2 = -\left(\frac{r^2}{R^2} + f\right) dx^+ dx^- + \frac{1}{2}\left(\frac{r^2}{R^2} - f\right)(dx_2^2 + dx_3^2) + \frac{r^2}{R^2} d\tau^2$$



where $f = \frac{r^2}{R^2} \left(1 - \frac{r_0^4}{r^4}\right)$

We take $\Lambda \rightarrow \infty$ only after taking $V \rightarrow C$. (Ensures spacelike surface.)

We assume $L \gg L$.

Surface: $X^\mu(\sigma, \tau)$
 $\begin{matrix} \downarrow & & \downarrow \\ X_2 & & X^- \end{matrix}$

$X^+ = \text{const}; X_3 = \text{const}; \tau = \tau(\sigma)$

b.c.: $r(\pm L/2) = \infty$

$$S(c) = -\frac{1}{2\pi\alpha'} \int d\sigma d\tau \sqrt{-\det G_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu}$$

$$= \frac{i\sqrt{2} r_0^2 L^-}{2\pi\alpha' R^2} \int_0^{L/2} d\sigma \sqrt{1 + r'^2 R^2 / f r^2}$$

Extremize. \rightarrow E of M for $r(\sigma)$: $r'^2 = \gamma^2 r^2 f / R^2$
 \uparrow
 const.

$\Rightarrow r' = 0 \Leftrightarrow f = 0 \Leftrightarrow r = r_0$.

\therefore SURFACE KISSES HORIZON, FOR ANY L !

Can now do the integrals:

$$\frac{L}{2} = \int_0^{L/2} d\sigma = \int_{r_0}^{\infty} \frac{dr}{r'} = \frac{R^2}{8} \int_{r_0}^{\infty} \frac{dr}{\sqrt{r^4 - r_0^4}}$$

$$= \frac{a R^2}{8 r_0} \text{ with } a = \frac{\sqrt{\pi} \Gamma(5/4)}{\Gamma(3/4)} \approx 1.311$$

and hence

$$S(c) = \frac{i\pi}{2\sqrt{2}} \sqrt{\lambda} L^- L T^2 \sqrt{1 + 4a^2 / \pi^2 T^2 L^2}$$

Finally, must subtract self-energy S_0 of q and \bar{q} , given by two disconnected surfaces "hanging" from $r = \infty$ to $r = r_0$ at constant X_2 , and also

$\langle W^A \rangle = \langle W_F \rangle^2$ means $S \rightarrow 2S$. So...

$$2(S(c) - S_0) = \frac{i}{4\sqrt{2}} \underbrace{\frac{\pi^2 \sqrt{\lambda} T^3 L^2 L^-}{a}}_{\hat{q}!} + \mathcal{O}(T^5 L^4 L^-)$$

WHAT DO WE LEARN?

$$\hat{q}_{SYM} = \frac{\pi^{3/2} T(3/4)}{T(3/4)} \sqrt{\lambda} T^3 = 26.7 \sqrt{\alpha N_c} T^3$$

- \hat{q} is not proportional to S , or to $n_{scatters}$; as these are $\propto N_c^2$.
- Redo calculation in $(p+1)$ -dim SYM; find \hat{q} 's have different N_c and T dependence for $p \neq 3$.
- \hat{q} is better thought of as measuring T^3 !
- Try some numbers: $N_c = 3, \alpha = 1/2$
 $\hat{q}_{SYM} = 4.5, 10.6 \text{ GeV}^2/\text{fm}$ for $T = 300, 400 \text{ MeV}$
- Right ballpark!
- $\bar{\hat{q}} = \frac{4}{L^2} \int_{\tau_0}^{\tau_0 + L/\sqrt{2}} \tau \hat{q}(\tau) d\tau$; $T(\tau) = T_0 \left(\frac{\tau_0}{\tau}\right)^{1/3}$
 $\Rightarrow \bar{\hat{q}} = 5 \text{ GeV}^2/\text{fm}, L/\sqrt{2} = 2 \text{ fm} \leftrightarrow T(1 \text{ fm}) = 310 \text{ MeV}$
- This is surprisingly close to expectations for $T(1 \text{ fm})$ based on hydrodynamic modelling, but slightly too hot.
R eg Kolb Heinz; Teaney Saurade

ISSUES

We have found $\hat{q}_{N=4 SYM}$ to be close to and perhaps slightly less than \hat{q}_{QCD} extracted by comparison with RHIC data. What are the issues on both sides?

 \hat{q} EXTRACTED FROM RHIC DATA

- Extraction neglects fact that high energy parton feels some transverse "wind". This increases energy loss, meaning extracted \hat{q} is greater than true \hat{q} . Aronstein Salgado Wiedemann
- Extraction neglects energy loss processes other than gluon radiation. Gluon radiation does dominate for high enough energy partons, but do RHIC collisions provide high enough energy partons?
- LHC will help. Also, by allowing one to see jets, and hence jet modification. \rightarrow more discriminating observables than just leading hadrons.

FROM \hat{q}_{SYM} TOWARDS \hat{q}_{QCD}

- First correction to $\lambda \rightarrow \infty$ has been calculated: Armedo Edelstein Mas

$$\hat{q}(\lambda) = \hat{q}(\infty) \left(1 - 0.0216 \left(\frac{6\pi}{\lambda} \right)^{3/2} \dots \right)$$
 a very small effect for $\lambda \sim 6\pi$.
- $\hat{q}_{KW} = \sqrt{\frac{27}{32}} \hat{q}_{N=4}$ KW: a different conformal theory
- $\hat{q}_{KS} = \left[1 - 3.123 \left(c_s^2 - \frac{1}{5} \right) \right] \hat{q}_{KW}$ KS: a nonconformal theory
 $\approx 0.85 \hat{q}_{KW}$ for $c_s^2 - \frac{1}{5} \approx 0.05$ as in QCD at $T \sim 1.5 T_c$.
 Maybe corrections due to nonconformality are small? Buchel
- \hat{q} increases with increasing R-charge. Caceres Guijosa; Lin Matsuo; Aramis Stetsos; Armedo Edelstein Mas
- Really want examples where number of adjoint d. of f. is reduced. (It's 2 in QCD, $8 + 8 \cdot \frac{1}{3}$ in N=4 SYM)
 \rightarrow Calculate \hat{q} in $N=2^*$.

- Suppose that after calculating \hat{q} in many, and varied, gauge theories with gravity duals we understood enough to conjecture $\hat{q}_{QCD} = b \sqrt{\lambda} T^3$ for large λ with an estimate of b we trusted at the factor of two level. This would have a big impact.
 $\rightarrow \hat{q}$ would serve as a thermometer for early times with a calibration uncertainty only of order $2^{1/3}$.

CAN WE MEASURE (OR BOUND) ν
AND DEMONSTRATE DECONFINEMENT?

$$\nu \sim \frac{\epsilon}{T^4} \sim \frac{S}{T^3} \sim \frac{S^4}{\epsilon^3}$$

We know: $\epsilon(1\text{fm}) > 5 \text{ GeV} / \text{fm}^3$

We can estimate $S(1\text{fm})$ from final state entropy, assuming equilibration before 1fm :

$$S(1\text{fm}) = 33 \pm 3 \text{ fm}^{-3} \text{ Muller, KR}$$

Can we use jet quenching observable $\underline{\underline{S}}$ to get upper bound on T^3 ?

Challenge to theory: $\hat{q} \leftrightarrow T^3$ in QCD

Challenge to exp + theory:
 reliable upper bound on \hat{q} .
 (Need jet modification, not quenching)

Other routes to T , also hard:
 photons? J/ψ ?

FOR YOUR AMUSEMENT

if $S(1\text{fm}) = 33 \text{ fm}^{-3}$

if $L^- = 2 \text{ fm}$

if $\alpha = 1/2$

if $\hat{q}_{\text{QCD}} = 26.7 \sqrt{3} \alpha^{-1} T^3$

if $\bar{\hat{q}}_{\text{from RHIC}} = 3.2 \text{ GeV}^2 / \text{fm}$

THEN: $T(1\text{fm}) = 270 \text{ MeV}$

AND, using $S = \frac{2\pi^2}{45} \nu T^3$ as the definition of ν ,

$\rightarrow \nu = 30$, as in lattice QCD at $T = 1.5 T_c$.

So: Watch with interest how $\bar{\hat{q}}_{\text{RHIC}}$ evolves, and on the theory side how large the corrections to \hat{q}_{QCD} vs $\hat{q}_{\text{N=4SYM}}$ appear to be.

OTHER "NEARBY" DIRECTIONS BEING INVESTIGATED WITH AdS/CFT

- Calculate drag on quark moving through $N=4$ plasma. Herzog Karch Koutun Kozlov Yaffe; Gubser
I.e. treat whole process of energy loss at strong coupling in $N=4$, rather than just calculation of \hat{q} . This is not a good description for high enough energy partons in QCD (eg at LHC) but is plausibly relevant at RHIC, particularly for heavy quarks. And, allows to address further questions like "where does energy go?" Friess Gubser Pufu Michalobgiorgakis
- Screening in a hot wind. Find $L_{\text{screening}} \propto (1 - v_{\text{wind}})^{1/4}$. New input to understanding J/ψ @ RHIC+LHC.
Liu KR Wiedemann; Peeters Sonnenschein Zamaklar; Cheruicoff Garcia Guijosa; Caceres Natsuume Okamura
- Photon emission rate from $N=4$ plasma.
Caron-Huot Koutun Moore Steriuaets Yaffe

FARTHER AFIELD?

Is there some gauge theory with a gravity dual in which a $T=0$ background (ie no BH) corresponding to either one or many baryons be found? [Eg go from mesons to skyrmion to skyrmion crystal?]
If so, calculate \hat{q} in that background!
I.e. \hat{q} we calculated describes "DIS off QGP"; is it possible instead to set up DIS off a baryon or off cold "nuclear matter"?
If possible, could impact another area under intense experimental investigation. (Theoretical questions related to small- x physics, saturation, color glass condensate, ... abound.)