A HETEROTIC STANDARD MODEL AND THE COSMOLOGICAL CONSTANT

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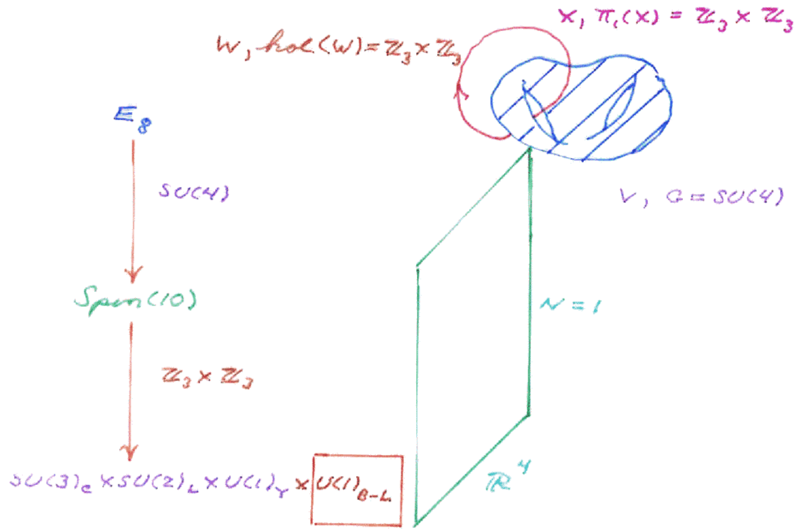
Observable Sector:



Spectrum:

1. $\underline{3}$ families of quark/leptons. Each family is

$Q = (3,2,1,1)$, $u = (3,1,-4,-1)$, $d = (\bar{3},1,2,-1)$

$L = (1,2,-3,-3)$, $e = (1,1,6,-3)$, $\boxed{\nu = (1,1,0,3)}$

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2. 1 pair of Higgs-$\overline{\text{Higgs}}$ fields

$$H = (1, 2, 3, 0), \quad \bar{H} = (1, \bar{2}, -3, 0)$$

NO

EXOTIC MATTER / VECTOR-LIKE FIELDS !

$\Downarrow$

$\boxed{\text{EXACT MSSM SPECTRUM}}$

3. 6 geometric and 13 vector bundle moduli

Physical Properties:

a. Higgs μ-terms

Lazy spectral "quantum numbers" forbid all

$$\langle \phi_i \rangle H \bar{H}$$

interactions.

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However, can have

$$\frac{\langle \phi_i \rangle^p H \bar{H}}{M_c^{\,p-1}}, \quad p \geq 2$$

$\Longrightarrow$ naturally small μ-terms

6. Yukawa couplings

Lazy spectral "quantum numbers" forbid all Yukawa couplings except

$$\lambda_{\binom{u}{d}1,j}\, Q_1 \binom{H}{\bar{H}} \binom{u_j}{d_j} + \lambda_{\binom{u}{d}j,1}\, Q_j \binom{H}{\bar{H}} \binom{u_1}{d_1}$$

$+$

$$\lambda_{\binom{\nu}{e}1,j}\, L_1 \binom{H}{\bar{H}} \binom{\nu_j}{e_j} + \lambda_{\binom{\nu}{e}j,1}\, L_j \binom{H}{\bar{H}} \binom{\nu_1}{e_1}$$

where $j = 2, 3$. $\Longrightarrow$ a "texture" of quark/lepton masses.

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example: up-quark mass matrix. $\langle H\rangle \neq 0 \Rightarrow$

$$\begin{pmatrix} 0 & \lambda_{u\,1,2}\langle H\rangle & \lambda_{u\,1,3}\langle H\rangle \\ \lambda_{u\,2,1}\langle H\rangle & 0 & 0 \\ \lambda_{u\,3,1}\langle H\rangle & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} \boxed{0} & 0 & 0 \\ 0 & \lambda\langle H\rangle & 0 \\ 0 & 0 & \lambda'\langle H\rangle \end{pmatrix}$$

similar result for down-quark and lepton mass matrices. However, have unrestricted terms

$$\lambda \frac{\langle \phi_i\rangle^p}{M_c^{p-1}} Q \begin{pmatrix} H \\ \bar{H} \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix} + \lambda' \frac{\langle \phi_i\rangle^p}{M_c^{p-1}} L \begin{pmatrix} H \\ \bar{H} \end{pmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix}$$

for $p \geq 2$. $\Rightarrow$ naturally small first family masses

c. Proton decay

a) $M_c \sim \mathcal{O}(10^{16}\,GeV) \Rightarrow$

dim 6 decay suppressed

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b) Leray spectral "quantum numbers" + $U(1)_{B-L}$ forbid the $\Delta B = 1$, $\Delta L = 1$ cubic terms

$$\alpha\, Q L d + \beta\, L L e + \gamma\, u d d$$

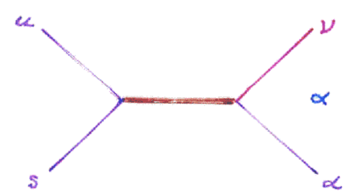
$\Rightarrow$

dim 4 decay forbidden

remains sufficiently small if $U(1)_{B-L}$ is broken at $\mathcal{O}(10^3 - 10^4\,GeV)$.

c) Natural doublet-triplet splitting projects out color triplet Higgs $C, \bar{C}$. However, these quantum states appear in the Kaluza-Klein tower

$\Rightarrow$ dim 5 operators such as



$$\propto \frac{\lambda^2}{M_c}\, u d s \nu \Rightarrow p \rightarrow K^+ + \nu_{\mu,\tau}$$

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Vector Bundle Stability :

Slope-stable bundle V $\Rightarrow$ connection solves $\boxed{g^{a\bar{b}}F_{a\bar{b}}=0}$

The Kahler cone of X is found to be

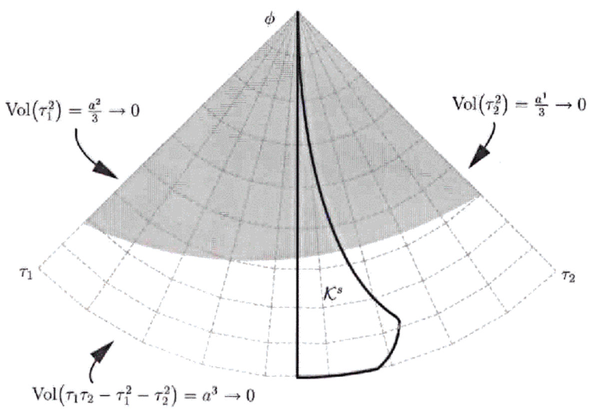

Figure 1: *Kähler Cone. The observable sector vector bundle is slope-stable in the region $\mathcal{K}^s$.*

V is slope-stable with respect to each Kahler modulus in

$$\mathcal{K}^3 \subset H^2(X, \mathbb{R}) \simeq \mathbb{R}^3$$

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What is the hidden sector ?

To motivate this must discuss

Moduli Stability :

A) 5-brane only

moduli :

$$S, \; T^I, \; Z_\alpha, \; \phi, \; Y$$

dilaton, $h^{1,1}$, $h^{1,2}$, vector bundle, 5-brane

simplification - assume 1 vector bundle modulus

Can construct

$$K = K_{S,T} + K_Z + K_5 + K_\phi$$

and

$$W = W_f + W_g + W_{np} + W_5^{(1)} + W_5^{(2)}$$

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For example

$$K_Z = -M_{Pl}^2 \ln\left(-i\int_X \Omega\wedge\bar{\Omega}\right)$$

where Ω is the holomorphic 3-form and

$$W_f = \frac{1}{v_{CY}^{1/2}}\int_X H\wedge\Omega$$

H is the B-flux.

Result:

- Can always solve

$$D_F W = 0$$

for all $F = S, T^I, Z_\alpha, \phi, Y$.

- These equations fix all

$$\langle S\rangle, \langle T^I\rangle, \langle Z_\alpha\rangle, \langle \phi\rangle, \langle Y\rangle$$

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- $\langle F\rangle$ have phenomenologically acceptable values such as

$$Re\langle S\rangle \sim 1, \quad R \sim 1, \quad 0 < Re\, Y < R$$

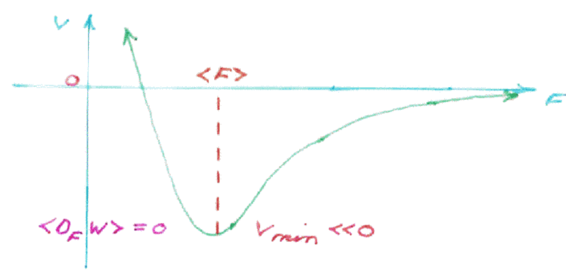
- $\langle D_F W\rangle = 0 \Rightarrow$ SUSY unbroken $\Rightarrow$

$$V_{min} \propto -\frac{3|\langle W\rangle|^2}{M_{Pl}^2}$$

- For these values of $\langle F\rangle$

$$V_{min} \sim -(10^{-4} M_{Pl})^4 \sim -10^{60}\ GeV^4$$

$\Rightarrow$ a deep, negative cosmological constant



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B) 5-brane + anti 5-brane $(+ V' \sim \mathcal{O}_X)$

The anti 5-brane adds the term

$$\Delta U_{\bar{5}} = \frac{4 T_5}{(\mathrm{Re} S)^{4/3} R^2} \frac{1}{v_{CY}^{2/3}} \int_X \omega \wedge J$$

to the $N=1$ supersymmetric Lagrangian, where

$$J = c_2(V) - c_2(TX) + \underbrace{[W]}_{5} + \underbrace{[\bar{W}]}_{\bar{5}} ,$$

ω is the Kahler form

$$\omega = a^I \omega_I$$

and T_5 is the 5-brane tension. The anomaly cancellation condition $\Rightarrow$

$$c_2(V) - c_2(TX) + [W] - [\bar{W}] = 0$$

$\Rightarrow$

$$J = 2[\bar{W}]$$

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Therefore

$$\Delta U_{\bar{5}} = + \frac{8 T_5}{(\mathrm{Re} S)^{4/3} R^2} \mathcal{V}_{\bar{5}}$$

where

$$\mathcal{V}_{\bar{5}} = \frac{1}{v_{CY}^{2/3}} \int_X \omega \wedge [\bar{W}] = \int_{z_{\bar{5}}} \omega$$

Add to the Lagrangian & solve the equations of motion $\Rightarrow$

Result:

- Meta-stable minimum with

1. $\langle S \rangle, \langle T^I \rangle, \langle Z_\alpha \rangle, \langle \phi \rangle, \langle Y \rangle$ all fixed
2. $\langle D_F W \rangle \neq 0 \Rightarrow$ SUSY broken
3. $\langle F \rangle$ have phenomenologically acceptable values

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- The cosmological constant can be made small as long as one chooses

$$T_5 \mathcal{V}_{\bar{5}} \sim 10^{-16} M_{Pl}^4$$

or, equivalently

$$\boxed{\frac{\mathcal{V}_{\bar{5}}}{\mathcal{V}_{CY}^{1/3}} \sim 10^{-7}} \quad *$$

- Since $\langle F \rangle$ have acceptable values $\Rightarrow$

$$\boxed{\frac{\mathcal{V}_5}{\mathcal{V}_{CY}^{1/3}} \sim 1} \quad **$$

where

$$\mathcal{V}_5 = \frac{1}{\mathcal{V}_{CY}^{2/3}} \int_X \omega \wedge [W] = \int_{z_5} \omega$$

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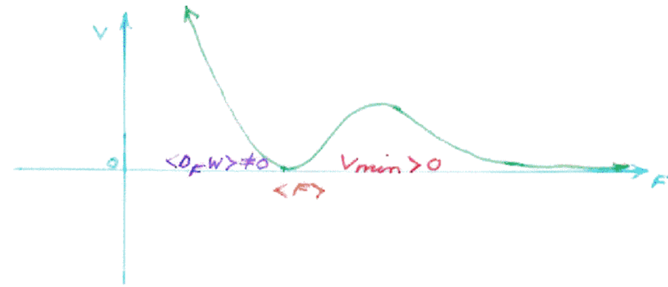

Question: For a realistic vacuum can * and ** be solved such that the observable sector vector bundle is slope-stable with respect to ω ?

Answer: Yes!

Consider the MSSM heterotic standard model. In the hidden sector choose

$$V' \sim \mathcal{O}_X$$

which is trivially slope-stable.

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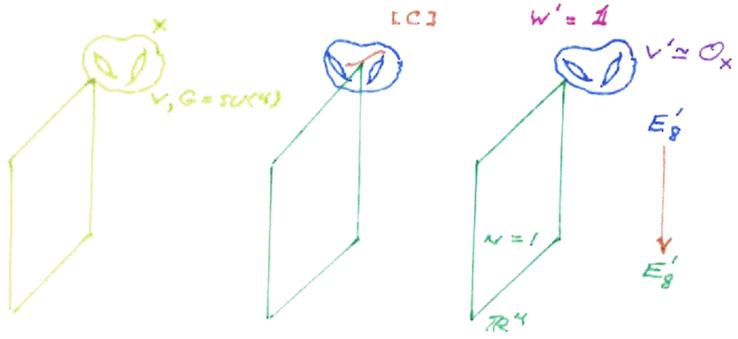
Hidden Sector:



Anomaly Cancellation

$$[C] = c_2(TX) - c_2(V)$$

Kahler Cone

$$\omega = a^1\tau_1 + a^2\tau_2 + a^3\phi \in \mathcal{K}$$

We find that

$$c_2(V) = \tau_1^2 + 4\tau_2^2 + 4\tau_1\tau_2\ , \quad c_2(TX) = 12(\tau_1^2 + \tau_2^2)$$

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$\Longrightarrow$

$$[C] = [W] - [\bar{W}]$$

where

$$[W] = \overset{5}{\overbrace{7\tau_1^2 + 4\tau_2^2}}\ , \quad [\bar{W}] = \overset{\bar{5}}{\overbrace{4(\tau_1\tau_2 - \tau_1^2 - \tau_2^2)}}$$

Then using the τ_1, τ_2, ϕ intersection numbers $\Longrightarrow$

$$\frac{V_{\bar{5}}}{v_{CY}^{1/3}} = \frac{1}{v_{CY}}\int_X \omega \wedge [\bar{W}] = 4a^3$$

and

$$\frac{V_5}{v_{CY}^{1/3}} = \frac{1}{v_{CY}}\int_X \omega \wedge [W] = \frac{4}{3}a^1 + \frac{7}{3}a^2$$

Consider the region

$$\mathcal{K}_5 \subset \mathcal{K}$$

As one approaches the bottom of $\mathcal{K}_5 \Longrightarrow a^3 \to 0$

$\Longrightarrow$

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one can choose

$$\frac{\nu_{\bar{5}}}{\nu_{CY}^{1/3}} \sim 10^{-7} \quad \checkmark$$

Note that

$$Re\, S = \frac{1}{6}\left((a^1)^2 a^2 + a^1 (a^2)^2 + 6 a^1 a^2 a^3 \right)$$

and

$$Re\, S \sim 1$$

K_S is bounded on the left by the vertical line where

$$a^1 = a^2$$

$\Rightarrow$ on vertical line as $a^3 \to 0$

$$a^1 = a^2 \longrightarrow (3)^{1/3}$$

The moduli

$$a^1 = (3)^{1/3} - \frac{\epsilon}{2}, \quad a^2 = (3)^{1/3} + \frac{\epsilon}{2}$$

$\Rightarrow$ $Re\, S \sim 1$ and are in K_S. For this region

$$\frac{\nu_S}{\nu_{CY}^{1/3}} \sim 1 \quad \checkmark$$

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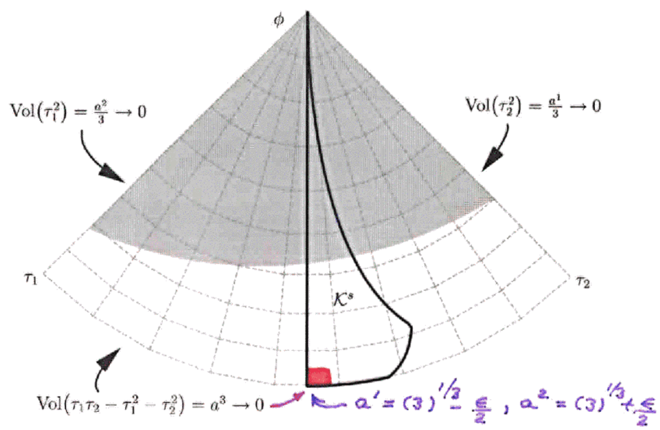

Figure 1: Kähler Cone. The observable sector vector bundle is slope-stable in the region $\mathcal{K}^s$.

In the region

$$\frac{\nu_{\bar{5}}}{\nu_{CY}^{1/3}} \sim 10^{-7}, \qquad \frac{\nu_S}{\nu_{CY}^{1/3}} \sim 1$$

$$\Downarrow \qquad\qquad\qquad \Downarrow$$

$$0 < \Lambda / M_{Pl}^4 \ll 1 \qquad\qquad Re\, S \sim 1$$

and

observable V is slope-stable

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Conclusion:

For the MSSM heterotic standard model

- Take $V' \simeq \mathcal{O}_X$. Anomaly cancellation $\Rightarrow$ both 5-brane and anti 5-brane on $S^1/\mathbb{Z}_2$ interval and fixes their cohomology classes.

- Neglecting the anti 5-brane, all moduli are stabilized, but at $N=1$ preserving minimum with $V_{min} \sim -10^{-16} M_{Pl}^4$.

- Add anti 5-brane lifts the minimum to a meta-stable vacuum with a positive cosmological constant. The moduli are fixed in this vacuum and have phenomenologically acceptable values.

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- There is a region of the Kahler cone for which the cosmological constant has its observed value and for which the observable sector vector bundle is slope-stable.

- One expects the Kahler moduli can be fine-tuned to lie in this region.